MA 581 Qualifying Exam Problems

1.

Suppose the r.v. X has pdf f(x)= cx^4, for 0<x<3, and f(x)=0 otherwise, where c is a constant. Compute:

a. The value of c.

b. E(X)

c. E(1/X)

d. Stdev(X)

e. Var(-5X)

f. The median of X (defined as the value of t satisfying F(t)=1/2, where F is the cdf of X.)

2.

Let Y be a Poisson random variable with parameter $\lambda > 0$, that is, $\mathrm{P}[Y = k] = \dfrac{e^{-\lambda}\lambda^k}{k!}$ for $k = 0, 1, 2, \ldots$.

1) Derive its moment generating function (MGF)

2) Use the MGF to obtain Var Y.

3) Let X_i, $i = 1, 2, \ldots$ be i.i.d. with uniform distribution on the interval $[0, 1]$ and independent of Y. Compute

$$\mathbb{E}\sum_{i=1}^{Y} X_i.$$

4) Let Y and X_i be as above. Compute

$$\mathrm{Var}(2X_1 + 3Y - 4).$$