Competition in two-sided markets with common network externalities

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Abstract

We study competition in two sided markets with *common network externalities* rather than with the standard inter-group effects. This type of externality occurs when *both groups benefit*, possibly with different intensities, from an *increase* in the size of one group and from a *decrease* in the size of the other. We explain why common externalities are relevant for the health and education sectors. We focus on the symmetric equilibrium and show that when the externality itself satisfies an homogeneity condition then platforms' profits and price structure have some specific properties. Our results reveal how the rents coming from network externalities are redistributed by platforms from one side to other, according to the homogeneity degree. In the specific but realistic case where network externalities are homogeneous of degree 0, platform's profit do not depend on the intensity of the (common) network externalities. This is in sharp contrast to conventional results stating that the presence of network externalities in a two-sided market structure increases the intensity of competition when the externality is positive (and decreases it when the externality is negative). Prices are affected but in such a way that platforms only transfer rents from consumers' group to providers' one.

Jel codes: D42, L11, L12.

1 Introduction

The theory of two-sided markets has been developed in recent years to investigate market structures in which two groups of agents interact via platforms; see for instance Rochet and Tirole (2006). The central theme of this literature is the notion of (network) externality which reflects the property that the benefit from joining a platform for individual of a given group depends on the size of membership (and/or usage) from the other group (Armstrong, 2006). Prominent examples of sectors in which such inter-group externalities occur range from credit cards and software to dating clubs.

We consider a two-sided market with externalities of a different nature. We shall refer to them as a "common network externalities". This type of externality occurs when *both groups benefit*, possibly with different intensities, from an *increase* in the size of one group and from a *decrease* in the size of the other. Such externalities are relevant in a number of two-sided markets. For instance, in the health care sector, hospitals compete for patients on one side and for providers on the other side (see Pezzino and Pignatoro, 2008). It is a conventional assumption that the quality of health care depends on the providers' "workload". This is documented, for instance, by Tarnow-Mordi *et al.* (2000) who use UK data to show that variations in mortality can be explained in part by excess workload in the intensive care unit. Accordingly, health care quality is frequently related to the provider/patient ratio; see Mc Gillis Hall (2004). In other words, it increases when the number of health care professionals increases (for a given number of patients) but decreases when the number of patients increases (for a given number of providers). Both sides benefit from a higher quality albeit for different reasons and possibly with different intensity. This is quite obvious on the patients' side, where one can expect a higher quality to translate into a improvement in patients' health state (or at the very least into a reduction in waiting lines for appointments, *etc...*). Physicians benefit from a higher quality through a reduction in their workload[1] or indirectly, through their altruism.

Similar issue may arise in the education sector. Colleges or universities compete

[1] See for instance Fergusson-Paré (2004) for the nursing workload. Griffin and Swan (2006) also find a strong relationship between nurses' workload and quality of health care.

for students on one side and for professors on the other side. The quality of education depends on the pupil/teacher ratio and one can expect both sides to benefit from a higher quality (Mueller, Chase and Walden, 1988). On one hand, lower pupil/teacher ratios are associated with higher test scores for the children (see for instance Angrist and Lavy, 1999). In addition, it has been observed a smaller class size tend to increase average future earnings (Card and Krueger, 1992). On the other hand, teachers enjoy an improved job satisfaction.[2]

In this paper, we revisit the Armstrong's framework but with *common network externalities* rather than with the standard inter-group effects. Two platforms compete in prices on two distinct Hotelling's lines. The common externality enters the preferences of both groups as a quality parameter. Each group values the common externality with (possibly) different intensities but the underlying notion of quality that matters (the functional form that specifies quality) is the same for both groups. We focus on the symmetric equilibrium and show that when externalities are specified by an homogeneous function, price structure and platforms' profit present some special features. First, network externalities enter in a cumulative way in the price structure. Second, platforms operate a redistribution of this common network externality from one side to other. We identify that the direction of this redistribution depends on the sign of the homogeneity degree of the common network externality. Third, the competition intensity is also affected by this homogeneity degree. Roughly speaking, according to the homogeneity degree, platforms may or may not be able to modify its price structure such that a higher price charged on one side is not outweighed by a lower price on the other side. Finally, our results shed lights on equilibrium properties in education and health sectors where quality is known to mainly depend on consumer/provider ratio *i.e.* homogeneity degree is 0. In this case, platforms' profit do not depend on the intensity of the (common) network externalities. This property is in sharp contrast to the results obtained so far in the two-sided literature. One of the major results findings which has been reiterated in many settings is that the presence of network externalities in a

[2]Buckingham (2003) finds that reduction of class size allows to increase slightly achievment but also increase teachers work conditions by lightening their workload and easing classroom managment.

two-sided market structure increases the intensity of competition when the externality is positive (and decreases it when the externality is negative). We show that in a context of common network externalities of degree 0, this is not the case. Under this assumption, prices are affected by the externality but in such a way that platforms only transfer rents from one group to the other. Roughly speaking, to come back to the sectors previously mentioned, some rents due to common network externalities are extracted from the "consumers' side" and transferred to "providers".

Before proceeding, let us have a closer look at the relationship of our paper to the existing literature. As pointed out by Rochet and Tirole (2003), the two-sided literature is at the intersection between multi-product pricing and network theories. The main focus of this paper lies on the second aspect. Several types of network externalities have been analyzed in the two-sided literature. The standard one is the *inter-group* network externalities which we have mentioned above. It has also been pointed out in the literature that negative *intra-group* network "externalities" can occur in equilibrium. This may be the case when members of a given group compete with each other. An additional member on one side then not only creates a positive inter-group externality but at the same time, it can adversely affect welfare of the other members of the considered group.[3] For instance, in Bardey and Rochet (2009), health plans compete for policy holders on one side and for physicians on the other side. When a health plan enlists more physicians, this directly increases welfare of its policy holders. However, at the same time, it may tend to attract riskier policy holders who place a higher value on the diversity of physicians. The induced adverse selection problem can be seen as a negative intra-group network "externality" that occurs, in equilibrium, on the policy holders' side.

These intra group effects are of course strictly speaking not externalities as they operate through the price system. However, some recent papers have also considered proper negative intra-group network externalities. Belleflamme and Toulemonde (2007) develop a model where agents value positively the presence of agents of the other group but may value negatively agents of their own group. For instance, both advertisers

[3] Most of time, this effect occurs because it increases the number of competitors.

and consumers benefit from a large representation of the other group (positive inter-brand externality) but advertisers are in competition for eyeballs (negative intra-brand externality). Belleflamme and Toulemonde show that entry of a new platform might be impossible as long as intra-group negative externalities are too strong in comparison with inter-group ones. Kurucu (2008) analyses a matching problem in which an agent on one side prefers more agents on the other side but less on its own side. Such a configuration of externalities can occur for matrimonial or job matching agencies.

Our paper is inspired by Belleflamme and Toulemonde (2007) and Kurucu (2008) from whom we borrow the presence of negative intra- and positive inter-groups network externalities. However, we combine the same ingredients in a different way. In our framework, an additional consumer generates a negative intra-group and a positive inter-group network externality. Roughly speaking, the utility of a consumer is increasing in the number of providers and is decreasing in the number of the other consumers affiliated with the same platform. On the providers' side, network externalities work on the opposite direction. In other words, the utility of a provider is increasing in the number of providers affiliated to the same platform (positive intra-group network externality), while it is decreasing in the number of consumers present on the other side (negative inter-group network externalities). The combination of these two characteristics leads to our concept of *common externality*: both groups benefit, possibly with different intensities, from an increase in the size of one group and from a decrease in the size of the other group.

The paper is organized as follows. The model is presented in Section 2. In Section 3, we determine the equilibrium and study its properties. Some illustrations are provided in Section 4.

2 Model

Consider two platforms $j = \{1, 2\}$ located at both endpoints of the Hotelling's segment. They compete for two groups of agents $i = \{A, B\}$ of mass 1 (group A) and m (group B) respectively. Agents of each group are uniformly distributed over an interval of length 1. The utilities of both groups exhibit quadratic transportation costs with parameters

t_A and t_B respectively. For the sake of simplicity, we shall refer to members of group A as "customers" while group B individuals are considered as "providers". We shall return to this interpretation later.

The utility of a group A individual (a customer), located at z, who patronizes platform j (consumes one unit of its product) is given by

$$V = \overline{V} + \gamma q_j - P_j - t_A (z - x_j)^2 ,$$

where P_j denotes platform j's price, while γ measures the preference intensity for a quality q_j. An individual of group B (a provider), located at y, who works (a given number of hours) for platform j has utility[4]

$$U = \overline{U} + \theta q_j + w_j - t_B (y - x_j)^2 ,$$

where w_j denotes the wage paid by platform j, while θ is the preference for quality q_j. Without loss of generality, reservation utilities are equal to zero. Consequently, the constants $\overline{V}$ and $\overline{U}$ denote the gross utility on sides A and B; they are assumed to be sufficiently large to ensure full coverage on both sides of the market. Platforms maximize profits and simultaneously set their price/wage vectors (P_j, w_j), $j = 1, 2$.

Let n_j^i denote the *share* of type $i = A, B$ individuals affiliated with platform $j = 1, 2$, while N_j^i denotes the *number* of affiliates. With our normalizations we have $N_j^A = n_j^A$ and $N_j^B = mn_j^B$. The quality offered by platform j depends on its number of affiliates in both groups and is determined by

$$q_j = f(N_j^A, N_j^B) = f\left(n_j^A, mn_j^B\right).$$

This function specifies what we refer to as a "*common network externality*" and which is defined as follows.

Definition 1 *A common network externality, described by the function $q_j = f(N_j^A, N_j^B)$, occurs when both sides value, possibly with different intensities, the same network externality.*

[4] Note that for purpose of simplicity, we do not enter in the details of the providers' time constraint.

An important feature of this definition is that the functional form f is the same on both sides.[5] In other words, customers and providers agree on the ranking of quality levels. However, the taste for quality (measured by γ and θ) can differ between customers and providers.

Prominent examples of such common externalities can be found in the health care and education markets. In the hospital sector for instance, on can think of n_j^A as representing the number of patients while mn_j^B represents the number of physicians. Alternatively, n_j^A can be interpreted as the number of students while mn_j^B stands for the number of teachers. In both of these cases one would expect quality to increase with mn_j^B and to decrease with n_j^A. A formulation often used in the literature on education and health is given by $q_j = \left(cmn_j^B/n_J^A\right)^{\alpha}$. With this specification the quality offered by a hospital or a university depends upon provider/patient or teacher/student ratio, and the function f is homogenous of degree 0.[6] More generally, one can assume that the function specifying the quality is homogenous of degree k, which may or may not be positive. For instance when quality is specified by

$$q_j = \left(N_j^B\right)^{\beta} / \left(N_J^A\right)^{\alpha}, \tag{1}$$

f is homogenous of degree $\beta - \alpha$. We do *not* impose this assumption when determining the equilibrium in the next section. However, it will turn out that the equilibrium has specific properties when the common externality is homogenous of degree k. We shall focus more particularly on the realistic case $k = 0$ which has some strong implications in terms of competition policy.

Using subscripts to denote the derivatives of f with respect to its first and second arguments (N_j^A and N_j^B respectively) and applying Euler's law yields the following property.

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[5] Concerning the nature of network externalities, the difference between our framework and Kurucu (2008) and Belleflamme and Toulemonde (2007) can be understood as follows. In our setting, the two utility functions on both sides *i.e.* $V^A(N_A, N_B)$ and $U^B(N_A, N_B)$ are both decreasing in N_A and increasing in N_B. In their framework (but with our notations), they consider the case where $V^A(N_A, N_B)$ is increasing in N_B but decreasing in N_A while $U^B(N_A, N_B)$ is increasing in N_A but decreasing in N_B.

[6] Krueger (2003) provides a cost-benefit analysis of class size reduction. He shows that the internal rate of return of a class size reduction from 22 to 15 students is around 6%.

Property 1 *When a network externality is homogenous of degree k then $N_j^A f_A(N_j^A, N_j^B) + N_j^B f_B(N_j^A, N_j^B) = kf(N_j^A, N_j^B)$.*

3 Equilibrium

First, we characterize the demand functions (market shares) on both sides. Then, we determine the price equilibrium and study the properties of the corresponding allocation.

3.1 Demand functions

On group A's side, the marginal consumer indifferent between two platforms is determined by

$$\tilde{z} = \frac{1}{2} + \frac{1}{2t_A}\left[\gamma\left(q_1 - q_2\right) - \left(P_1 - P_2\right)\right],$$

while in group B, the marginal provider is given by

$$\tilde{y} = \frac{1}{2} + \frac{1}{2t_B}\left[\theta\left(q_1 - q_2\right) + \left(w_1 - w_2\right)\right].$$

As both sides are fully covered, demand levels are equivalent to market shares. On side A, we have $n_1^A = \tilde{z}$ and $n_2^A = 1 - \tilde{z}$ while on side B, $n_1^B = \tilde{y}$ and $n_2^B = (1 - \tilde{y})$. Defining the quality differential between platforms as

$$g\left(n_1^A, mn_1^B\right) = f\left(n_1^A, mn_1^B\right) - f\left(1 - n_1^A, m(1 - n_1^B)\right) = q_1 - q_2,$$

the demand functions are determined by the following system of implicit equations

$$n_1^A = \frac{1}{2} + \frac{1}{2t_A}\left[\gamma g\left(n_1^A, mn_1^B\right) - \left(P_1 - P_2\right)\right], \tag{2}$$
$$n_1^B = \frac{1}{2} + \frac{1}{2t_B}\left[\theta g\left(n_1^A, mn_1^B\right) + \left(w_1 - w_2\right)\right]. \tag{3}$$

Let $\phi = (\gamma, \theta, t_A, t_B, m)$ denotes the vector of exogenous parameters. Equations (2)–(3) define the demand levels of platform 1, $n_1^A\left(P_1, P_2, w_1, w_2, \phi\right)$ and $n_1^B\left(P_1, P_2, w_1, w_2, \phi\right)$, as functions of both platforms price/wage vectors and of the exogenous variables[7]. With

[7] Note that we assume throughout the paper that demands are well defined and unique for any price levels. When $q_j = mn_j^B - n_j^A$, it is straightforward that demands are uniquely defined. Appendix A describes that it is also the case when $q_j = \left(mn_j^B / n_J^A\right)$.

full market coverage on both sides, demand levels of platform 2 are then also fully determined and given by $n_2^A = 1 - n_1^A$ and $n_2^B = 1 - n_1^B$.

Totally differentiating (2)–(3), solving and defining $g_A = \partial g / \partial N_1^A$, $g_B = \partial g / \partial (N_1^B)$ with

$$\Phi = \left(1 - \frac{\theta}{2t_B} m g_B\right)\left(1 - \frac{\gamma}{2t_A} g_A\right) - \frac{\gamma \theta g_A m g_B}{4 t_A t_B},$$

yields the following properties of the demand functions:

$$\frac{\partial n_1^A}{\partial \gamma} = \frac{g\left(n_1^A, m n_1^B\right)}{2 t_A \Phi}\left(1 - \frac{\theta}{2t_B} m g_B\right), \tag{4}$$

$$\frac{\partial n_1^B}{\partial \gamma} = \frac{g\left(n_1^A, m n_1^B\right)\theta g_A}{4 t_A t_B \Phi}, \tag{5}$$

$$\frac{\partial n_1^A}{\partial P_1} = -\frac{\left(1 - \frac{\theta}{2t_B} m g_B\right)}{2 t_A \Phi}, \tag{6}$$

$$\frac{\partial n_1^B}{\partial P_1} = -\frac{\theta g_A}{4 t_A t_B \Phi}, \tag{7}$$

$$\frac{\partial n_1^A}{\partial w_1} = \frac{\gamma m g_B}{4 t_A t_B \Phi}, \tag{8}$$

$$\frac{\partial n_1^B}{\partial w_1} = \frac{\left(1 - \frac{\gamma}{2t_A} g_A\right)}{2 t_B \Phi}. \tag{9}$$

3.2 Equilibrium prices and allocation

Platform 1 maximizes its profit with respect to P_1 and w_1 and solves

$$\max_{P_1, w_1} \Pi_1 = P_1 n_1^A\left(P_1, P_2, w_1, w_2, \phi\right) - m w_1 n_1^B\left(P_1, P_2, w_1, w_2, \phi\right).$$

The first-order conditions are given by

$$\frac{\partial \Pi_1}{\partial P_1} = n_1^A + P_1 \frac{\partial n_1^A}{\partial P_1} - m w_1 \frac{\partial n_1^B}{\partial P_1} = 0, \tag{10}$$

$$\frac{\partial \Pi_1}{\partial w_1} = P_1 \frac{\partial n_1^A}{\partial w_1} - m w_1 \frac{\partial n_1^B}{\partial w_1} - m n_1^B = 0. \tag{11}$$

The first two terms of equations (10) and (11) represent the traditional marginal income tradeoff, while the third terms capture the two-sided market feature. Specifically, an increase in the price charged on one side of the market also affects the demand on the other side. Equations (10) and (11) determine platform 1's best-reply functions: $P_1 = \widetilde{P}_1\left(P_2, w_2, \phi\right)$ and $w_1 = \widetilde{w}_1\left(P_2, w_2, \phi\right)$. Platform 2's best-reply functions

$P_2 = \widetilde{P}_2(P_1, w_1, \phi)$ and $w_2 = \widetilde{w}_2(P_1, w_1, \phi)$ can be determined in a similar way by the maximization of Π_2. Solving these best-reply functions yields the Nash equilibrium (P_1^*, w_1^*), (P_2^*, w_2^*).

In the remainder of the paper, we concentrate on symmetric equilibria in which both platforms charge the same prices, pay the same wages and equally split the market on both sides ($n_1^A = n_2^A = 1/2$ and $n_1^B = n_2^B = 1/2$) so that quality levels are also identical ($g = 0$). To determine the symmetric equilibrium we solve (10) and (11). The derivatives of n_1^A and n_1^B that appear in these expressions are given by equations (6)–(9); with $n_1^A = 1/2$ and $n_1^B = 1/2$, they are all well determined and the problem reduces to the solution of a system of linear equations.[8]

Using the Cramer's rule, we obtain

$$P_1 = \frac{\frac{1}{2}\left[\frac{\partial n_1^B}{\partial w_1} + m\frac{\partial n_1^B}{\partial P_1}\right]}{D}, \qquad w_1 = \frac{\frac{1}{2}\left[m\frac{\partial n_1^A}{\partial P_1} + \frac{\partial n_1^A}{\partial w_1}\right]}{mD}, \tag{12}$$

where

$$D = \left[-\frac{\partial n_1^A}{\partial P_1}\frac{\partial n_1^B}{\partial w_1} + \frac{\partial n_1^A}{\partial w_1}\frac{\partial n_1^B}{\partial P_1}\right]. \tag{13}$$

Substituting from (6)–(9) and rearranging yields

$$\frac{1}{2}\left[\frac{\partial n_1^B}{\partial w_1} + m\frac{\partial n_1^B}{\partial P_1}\right] = \frac{1}{2}\left[\frac{2t_A - \gamma g_A - \theta m g_A}{4t_A t_B \Phi}\right], \tag{14}$$

$$\frac{1}{2}\left[m\frac{\partial n_1^A}{\partial P_1} + \frac{\partial n_1^A}{\partial w_1}\right] = \frac{1}{2}\left[\frac{-2t_B + \theta m g_B + \gamma g_B}{4t_A t_B \Phi}\right], \tag{15}$$

$$D = \frac{1}{4t_A t_B \Phi}, \tag{16}$$

where g_A and g_B are evaluated at $(1/2, m/2)$. Substituting (14)–(16) into (12), simplifying and defining $g_A^* = g_A(1/2, m/2)$ and $g_B^* = g_B(1/2, m/2)$ establishes the following proposition:

Proposition 1 *Symmetric equilibrium prices are given by*

$$P_j^* = t_A - \frac{1}{2}(\gamma + m\theta)\, g_A^*, \tag{17}$$

$$w_j^* = -t_B + \frac{1}{2}(\gamma + m\theta)\, g_B^*, \ \forall j = 1, 2 \tag{18}$$

[8] It is worth noticing that the derivates depend on n_1^A and n_1^B (which are by definition set at 1/2) but not directly on P_1 and w_1. The underlying reason for this simplification is that for the determination of demands only *differences* in prices and wages matter; see expressions (2)–(3).

As usual, on both sides, platforms take advantage of transportation costs to increase their markup. Externalities, on the other hand, affect prices in a more interesting way. Because of their specific nature *i.e.* common externalities, their impact is "cumulative", as is reflected by the factor $(\gamma + m\theta)$ in the second term. In other words, network externalities affect prices on the side they occur $(\gamma + m\theta)\, g_i^*$ and they are partially or entirely shifted to the other side $(\gamma + m\theta)\, g_h^*$, with $h \neq i$.

Using (17) and (18) we can now express equilibrium profits as

$$\begin{aligned}
\Pi_j^* &= \frac{1}{2}\left(P_j^* - mw_j^*\right) \\
&= \frac{1}{2}\left(t_A + mt_B\right) - \frac{1}{4}\left(\gamma + m\theta\right)\left(mg_B^* + g_A^*\right), \qquad (19)
\end{aligned}$$

so that

$$\frac{\partial \Pi_j^*}{\partial \theta} = m\frac{\partial \Pi_j^*}{\partial \gamma} = -\frac{1}{4}\left(mg_B^* + g_A^*\right).$$

This last expression establishes the following proposition:

Proposition 2 *The impact of individual valuations of quality γ and θ on (symmetric) equilibrium profits is described by*

$$m\frac{\partial \Pi_j^*}{\partial \gamma} = \frac{\partial \Pi_j^*}{\partial \theta} \lesseqqgtr 0 \qquad \textit{if and only if} \qquad -g_A^* \lesseqqgtr mg_B^*,\ \forall j = 1, 2.$$

Proposition 2 shows that the impact of the externality (or, more precisely of the relevant preference parameters) on profits depend on the relative strength of the externalities created by the membership on the two sides. To interpret this proposition, we shall concentrate on the case where the common externality is homogenous of degree k. According to Property 1 we then have $N_1^B f_A + N_1^A f_A = kf$. Moreover, at a symmetric equilibrium, we have:

$$\begin{aligned}
g_A &= 2f_A, \\
g_B &= 2f_B,
\end{aligned}$$

so that

$$mg_B^* + g_A^* = 4kf\left(\frac{1}{2}, \frac{m}{2}\right) \geq 0.$$

Proposition 3 *When f homogenous of degree k the symmetric equilibrium implies $mg_B^* + g_A^* = 4kf\left(\frac{1}{2}, \frac{m}{2}\right)$, so that*

$$sign\left(\frac{\partial \Pi_j}{\partial \gamma}\right) = sign\left(\frac{\partial \Pi_j}{\partial \theta}\right) = -sign\,(k)\,, \qquad j = 1, 2 \tag{20}$$

Proposition 1 has shown that common network externalities have a cumulative way on prices. Proposition 3 shows how profits and competition intensity are affected. When the degree of homogeneity of the common network externalities is positive, platforms' profit decrease in the externalities parameters. In other words, common network externalities increase the competition intensity between platforms. This outcome occurs because in this case, platforms have to pay a higher relative price w_j^*/P_j^* on providers' side. Returning to the health care (or schooling) example, we have a positive externality created by one side and a negative externality generated by the other side. We can think about the case of $k > 0$ as that where the global impact of the externality is positive. In other words, if we increase membership on both sides in the same proportion, f (and thus quality) increases. From that perspective we can think of our finding as a generalization of the conventional result in the literature (relating profits and intensity of competition to the sign of the externality).

When $k < 0$, on the other hand, we have a negative global externality which brings about extra profits for the platforms. The wage paid to providers continues to increase in the network externality parameters. However, this increased cost is now more than fully shifted to the consumers. This is because quality is more sensitive to the number of consumers–and recall that quality decreases with the number of consumers. Consequently, the network externality tends to reduce the intensity of competition on the consumers' side. The firms are then able to extract more extra rents from the consumers than they have to concede to the providers.

Finally, let us consider the special case in which homogeneity degree is equal to 0 i.e. $k = 0$.

Corollary 1 *For a symmetric equilibrium, $\forall j = 1, 2$ with f homogenous of degree 0 we have $mg_B^* + g_A^* = 0$, so that*

$$\frac{\partial \Pi_j}{\partial \gamma} = \frac{\partial \Pi_j}{\partial \theta} = 0.$$

In that case, the intensity of preferences for quality (and thus the intensity of the externality) has no impact on equilibrium profits. Specifically, profit levels are the same when the externality does not matter at all (in which case $\gamma = \theta = 0$) as when one or both of these parameters are positive. In other words, a common network externality that is homogenous of degree 0, has no impact on the intensity of competition which is in stark contrast with conventional results obtained in the two-sided market literature (for alternative forms of externalities). The expressions for the prices (17) and (18) make it clear why this result emerges. Assuming $g_B^* > 0$ (providers produce a positive externality) the externality in itself (or an increase in its valuation on either side) increase "rents" on the providers' side: wages are increased. However, this increase in wages has no impact on profits because it is entirely shifted to consumers: the price increase exactly matches the increase in wages.

4 Examples and illustrations

To illustrate the results and provide some additional intuition, we shall now provide the full analytical solution for three special cases. First, we consider the case there the externality simply depends on the ratio between membership on both sides (so that f is homogenous of degree zero). Then we consider a setting with different degrees of homogeneity. Finally, we provide an example for the non homogenous case.

When $q_j = \left(mn_j^B/n_j^A\right)^\alpha$, the common network externality is homogeneous of degree 0. Proposition 1 implies that equilibrium prices are given by

$$P_j^* = t_A + 2(\gamma + m\theta)\,\alpha\,(m)^\alpha\,,$$
$$w_j^* = -t_B + 2(\gamma + m\theta)\,\alpha\,(m)^{\alpha-1}\,.$$

With this price structure, it is clear that platforms only transfer rents from "consumers" to "providers" and have equilibrium profits independent of the network externalities. To confirm this, note that with this specification 19 reduces to

$$\Pi_j^* = \frac{1}{2}(t_A + mt_B)\,, \qquad j = 1, 2,$$

which does not depend on γ or θ. It is worth noticing that (as discussed in the introduction) this functional form (with quality depending on the ratio), has interesting applications for education and heath care sectors.

We now turn to the more general case where f is specified by (1) so that the externality is homogenous of degree $k = \beta - \alpha$. The nice feature about this specification is that it shows that negative levels of k do not have to be ruled out. Equilibrium prices and profit levels are given by $(j = 1, 2)$

$$P_j^* = t_A - \left(\frac{1}{2}\right)^{\beta-\alpha-1} \alpha m^{\beta} \left(\gamma + m\theta\right), \tag{21}$$

$$w_j^* = -t_B + \left(\frac{1}{2}\right)^{\beta-\alpha-1} \beta m^{\beta-1} \left(\gamma + m\theta\right), \tag{22}$$

$$\Pi_j^* = \frac{1}{2}\left(t_A + mt_B\right) - \left(\frac{1}{2}\right)^{\beta-\alpha} m^{\beta} \left(\gamma + m\theta\right)\left(\beta - \alpha\right). \tag{23}$$

Not surprisingly, network externalities affect profits according to the sign of $k = \beta - \alpha$. When $\alpha > \beta$, quality offered by the platforms are relatively more sensitive to the number of consumers than to the number of providers. Therefore, platforms can charge a higher relative price on the consumers' side for the quality provided without transferring all the network externalities rents get to the providers.

Finally, let us consider a case where the homogeneity property does not hold at all. For instance, think about a case where quality depends positively on the ratio $\left(mn_j^B/n_j^A\right)$ but also depends positively on the "volume" of consumers treated by the provider[9]. In such a case, we have $q_j = \left(mn_j^B/n_j^A\right) + cn_j^A$ with c small enough to ensure that we continue to have a negative intra-group externality on the consumers' side. Equilibrium prices, wages and profits are now given by

$$\begin{aligned} P_j^* &= t_A + 2\left(\gamma + m\theta\right)\left(m - \frac{c}{2}\right), \\ w_j^* &= -t_B + 2\left(\gamma + m\theta\right)(m), \\ \Pi_j^* &= \frac{1}{2}\left(t_A + mt_B\right) - \left(\gamma + m\theta\right)c. \end{aligned}$$

It is worth noticing that because of the parameter c (which reduces the intensity of

[9] This assumption makes sense for a hospital. The quality of care can depend on the volume of patients treated.

negative intra-group externality on the consumers' side) the common network externality does not satisfy the homogeneity property. Then, platforms' profit are reduced in equilibrium because they charge a lower price on the consumers' side. This lower price is not outweighed by a lower wage paid on the providers' side.

5 Conclusion

Our concept of common network externalities applied to a two-sided market structure allows to point out several issues. One of them concerns the rent redistribution that may occur in a two-sided framework. Our model reveals the rent transfer mechanism, operated by platforms, from consumers to providers according to the homogeneity degree of common network externalities. In particular, we have considered the case, popular in the health and education literature, where the externality is homogeneous of degree 0. Under this assumption, platforms merely transfer rents, generated by network externalities, between the two sides of the market. Roughly speaking, we show that the providers' side gains while the consumers' one pays for the network externalities. If the common externality is not homogenous of degree zero, the platform can extract some rents from the side with the largest marginal impact on the of the common externality.

Our analysis could be extended at least in two ways. First, It would be interesting to investigate how the results change when one (or two) of the market is not fully covered. This would for example illustrate the case where the severity of illness of patients is not high enough for them to buy one unit of medical care in both hospitals. On the providers' side, it would capture the case where some of the physicians would prefer to remain independent workers. Second, another issue concerns the type of hospitals that compete in the market for patients. It would be interesting to study the outcome of competition between not for profit or physician's owned hospitals in a setting of mixed oligopolies. This is on our research agenda.

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Appendix

A Unicity of the demand

In the ratio case we have $q_1 = mn_1^B/n_1^A$. From(3), one obtains

$$g\left(n_1^A, mn_1^B\right) = \frac{2t_B}{\theta}\left[n_1^B - \frac{1}{2} - \frac{\Delta w}{2t_B}\right]$$

where $\Delta w = w_1 - w_2$.

Substituting this expression in (2) yields:

$$n_1^A = \frac{1}{2} + \frac{\gamma}{t_A}\frac{t_B}{\theta}\left[n_1^B - \frac{1}{2} - \frac{\Delta w}{2t_B}\right] - \frac{\Delta P}{2t_A} \tag{24}$$

where $\Delta P = P_1 - P_2$. It is worth noticing that if n_1^B is unique, then n_1^A is also unique.

From (3), one has:

$$n_1^B\left[1 - \frac{\theta m}{2t_B}\left(\frac{1}{n_1^A} + \frac{1}{1-n_1^A}\right)\right] = \frac{1}{2} + \frac{1}{2t_B}\left(-\frac{\theta m}{1-n_A^1} + \Delta w\right)$$

Multiplying both sides of this equality by $n_1^A\left(1-n_1^A\right)$ yields:

$$n_1^B\left[n_1^A\left(1-n_1^A\right) - \frac{\theta m}{2t_B}\right] = \frac{1}{2}\left[n_1^A\left(1-n_1^A\right)\left[1 + \frac{\Delta w}{t_B}\right] - \frac{\theta}{t_B}n_1^A\right].$$

Substituting (24) gives n_1^B as a solution to a three degree polynomial. Fastidious computations show that this polynomial admit three solutions but only one belongs to the space of real numbers.[10]

B Second order local conditions

We analyze here the second order local conditions for the class of functions f being homogemous of degree k. For the symmetric equilibrium to be a local maximum, one needs that the matrix H defined by

$$H = \begin{bmatrix} \frac{\partial^2 \Pi_1}{\partial P_1^2} & \frac{\partial^2 \Pi_1}{\partial P_1 \partial w_1} \\ \frac{\partial^2 \Pi_1}{\partial P_1 \partial w_1} & \frac{\partial^2 \Pi_1}{\partial w_1^2} \end{bmatrix}$$

be semi definite negative *i.e* $\partial^2 \Pi_1 / \partial P_1^2 < 0, \partial^2 \Pi_1 / \partial w_1^2 < 0$ and $\det H > 0$.

We thus assume that g is homogenous of degree k *i.e*

$$N_j^A g_A(N_j^A, N_j^B) + N_j^B g_B(N_j^A, N_j^B) = kg(N_j^A, N_j^B).$$

We also know, that g_A and g_B are homogenous functions of degree $k-1$ so that for all β :

$$g_A(\beta N_j^A, \beta N_j^B) = \beta^{k-1} g_A,$$
$$g_B(\beta N_j^A, \beta N_j^B) = \beta^{k-1} g_A.$$

Differentiating with respect to β and letting $\beta = 1$, leads respectively to:

$$g_A\,(k-1) = N_1^A g_{AA} + N_1^B g_{AB}$$
$$g_B\,(k-1) = N_1^A g_{AB} + N_1^B g_{BB}$$

Thus at the symmetric equilibrium where $g_A^* = -m g_B^*$, one has:

$$N_1^A g_{AA} + N_1^B g_{AB} + m N_1^A g_{AB} + m N_1^B g_{BB} = 0$$

[10] Note that the market share belongs to $[0,1]$. If the solution of the polynomial is negative or superior to 1, it means that we are in presence of a corner solution at the demands level.

so that

$$\frac{1}{2}g_{AA}^{*}+\frac{m^{2}}{2}g_{BB}^{*}+mg_{AB}^{*}=0, \tag{25}$$

$$2\left(k-1\right)g_{A}^{*}=g_{AA}^{*}+mg_{AB}^{*}, \tag{26}$$

$$2\left(k-1\right)g_{B}^{*}=g_{AB}^{*}+mg_{BB}^{*}. \tag{27}$$

The second order conditions are:

$$\begin{aligned}
\frac{\partial^2\Pi_1}{\partial P_1^2} &= 2\frac{\partial n_1^A}{\partial P_1}+P_1\frac{\partial^2 n_1^A}{\partial P_1^2}-w_1 m\frac{\partial^2 n_1^B}{\partial P_1^2},\\
\frac{\partial^2\Pi_1}{\partial w_1^2} &= -2m\frac{\partial n_1^B}{\partial w_1}+P_1\frac{\partial^2 n_1^A}{\partial w_1^2}-w_1 m\frac{\partial^2 n_1^B}{\partial w_1^2},\\
\frac{\partial^2\Pi_1}{\partial P_1\partial w_1} &= \frac{\partial n_1^A}{\partial w_1}-m\frac{\partial n_1^B}{\partial P_1}+P_1\frac{\partial^2 n_1^A}{\partial P_1\partial w_1}-w_1 m\frac{\partial^2 n_1^B}{\partial P_1\partial w_1}.
\end{aligned}$$

Differentiating (6) with respect to P_1 yields:

$$\begin{aligned}
\frac{\partial^2 n_1^A}{\partial P_1^2} &= \frac{1}{2t_A\Phi^2}\left[m\frac{\theta}{2t_B}\left(g_{BA}\frac{\partial n_1^A}{\partial P_1}+mg_{BB}\frac{\partial n_1^B}{\partial P_1}\right)\left(1-\frac{\theta}{2t_B}mg_B-\frac{\gamma}{2t_A}g_A\right)\right]\\
&\quad+\frac{1}{2t_A\Phi^2}\left(1-\frac{\theta}{2t_B}mg_B\right)\left[-\frac{\theta}{2t_B}m\left(g_{BA}\frac{\partial n_1^A}{\partial P_1}+mg_{BB}\frac{\partial n_1^B}{\partial P_1}\right)\right]\\
&\quad+\frac{1}{2t_A\Phi^2}\left(1-\frac{\theta}{2t_B}mg_B\right)\left[-\frac{\gamma}{2t_A}\left(g_{AA}\frac{\partial n_1^A}{\partial P_1}+mg_{AB}\frac{\partial n_1^B}{\partial P_1}\right)\right],
\end{aligned}$$

or

$$\frac{\partial^2 n_1^A}{\partial P_1^2}=\frac{1}{2t_A\Phi^2}\left[\begin{array}{c}-\frac{\theta\gamma}{4t_At_B}g_A\left(\frac{\partial n_1^A}{\partial P_1}\left(mg_{BA}+g_{AA}\right)+m\frac{\partial n_1^B}{\partial P_1}\left(mg_{BB}+g_{AB}\right)\right)\\ -\frac{\gamma}{2t_A}\left(g_{AA}\frac{\partial n_1^A}{\partial P_1}+mg_{AB}\frac{\partial n_1^B}{\partial P_1}\right)\end{array}\right]$$

According to (26) and (27), we obtain

$$\frac{\partial^2 n_1^A}{\partial P_1^2}=\frac{1}{2t_A\Phi^2}\left[\begin{array}{c}-\frac{\theta\gamma}{4t_At_B}2g_A^2(k-1)\left(\frac{\partial n_1^A}{\partial P_1}-\frac{\partial n_1^B}{\partial P_1}\right)\\ +\frac{\gamma}{2t_A}\left(\frac{g_{AA}\left(1+\frac{\theta}{2t_B}g_A\right)+mg_{AB}\frac{\theta}{2t_B}g_A}{2t_A\Phi}\right)\end{array}\right]$$

Using the fact that

$$\frac{\partial n_1^A}{\partial P_1}-\frac{\partial n_1^B}{\partial P_1}=-\frac{1}{2t_A\Phi}$$

it yields

$$\frac{\partial^2 n_1^A}{\partial P_1^2}=\frac{1}{4t_A^2\Phi^3}\left[\frac{4\theta\gamma}{4t_At_B}g_A^2(k-1)+\frac{\gamma}{2t_A}g_{AA}\right]$$

Using the same method, one finds

$$\frac{\partial^2 n_1^B}{\partial P_1^2} = \frac{4\left(\frac{\theta}{2t_B}\right)^2 g_A^2(k-1) + \frac{\theta}{2t_B} g_{AA}}{4t_A^2\Phi^3} \tag{28}$$

$$\frac{\partial^2 n_1^A}{\partial w_1^2} = \frac{4\left(\frac{\gamma}{2t_A}\right)^2 g_A^2(k-1) + \frac{\gamma}{2t_A} m^2 g_{BB}}{4t_B^2\Phi^3} \tag{29}$$

$$\frac{\partial^2 n_1^B}{\partial w_1^2} = \frac{4\left(\frac{\gamma\theta}{4t_A t_B}\right) g_A^2(k-1) + \frac{\theta}{2t_B} m^2 g_{BB}}{4t_B^2\Phi^3} \tag{30}$$

$$\frac{\partial^2 n_1^A}{\partial P_1 \partial w_1} = \frac{2g_A^2(k-1)\frac{\gamma}{2t_A}\left(\frac{\gamma}{2t_A} + \frac{\theta}{2t_B}\right) - \frac{\gamma}{2t_A} m g_{AB}}{4t_B t_A\Phi^3} \tag{31}$$

$$\frac{\partial^2 n_1^B}{\partial P_1 \partial w_1} = \frac{2g_A^2(k-1)\frac{\theta}{2t_B}\left(\frac{\gamma}{2t_A} + \frac{\theta}{2t_B}\right) - \frac{\theta}{2t_B} m g_{AB}}{4t_B t_A\Phi^3} \tag{32}$$

As a result, using (17) and (18) together with (**??**) and (28), one ends up with:

$$P_1\frac{\partial^2 n_1^A}{\partial P_1^2} - w_1 m\frac{\partial^2 n_1^B}{\partial P_1^2} = \frac{(\gamma+\theta m)\left[\frac{\theta}{t_B} g_A^2(k-1) + \frac{g_{AA}}{2}\right]}{4t_A^2\Phi^2}$$

$$P_1\frac{\partial^2 n_1^A}{\partial w_1^2} - w_1 m\frac{\partial^2 n_1^B}{\partial w_1^2} = \frac{(\gamma+\theta m)\left[\frac{\gamma}{t_A} g_A^2(k-1) + \frac{m^2 g_{BB}}{2}\right]}{4t_B^2\Phi^2}$$

$$P_1\frac{\partial^2 n_1^A}{\partial P_1\partial w_1} - w_1 m\frac{\partial^2 n_1^B}{\partial P_1\partial w_1} = \frac{(\gamma+\theta m)\left[\left(\frac{\gamma}{t_A} + \frac{\theta}{t_B}\right) g_A^2(k-1) - \frac{m g_{AB}}{2}\right]}{4t_B t_A\Phi^2}$$

So

$$\begin{aligned}
\det H = &\left(\frac{-2\left(1+\frac{\theta}{2t_B}g_A\right)}{2t_A\Phi} + \frac{(\gamma+\theta m)\left[\frac{\theta}{t_B} g_A^2(k-1) + \frac{g_{AA}}{2}\right]}{4t_A^2\Phi^2}\right) \\
&\left(\frac{-2m\left(1-\frac{\gamma}{2t_A}g_A\right)}{2t_B\Phi} + \frac{(\gamma+\theta m)\left[\frac{\gamma}{t_A} g_A^2(k-1) + \frac{m^2 g_{BB}}{2}\right]}{4t_B^2\Phi^2}\right) \\
&- \left(\frac{g_A\left(\theta m - \gamma\right)}{4t_A t_B\Phi} + \frac{(\gamma+\theta m)\left[\left(\frac{\gamma}{t_A} + \frac{\theta}{t_B}\right) g_A^2(k-1) - \frac{m g_{AB}}{2}\right]}{4t_B t_A\Phi^2}\right)^2
\end{aligned}$$