Achievable Data Rate Analysis of Clipped Flip-OFDM in Optical Wireless Communication

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Dec. 3 2012

Acknowledgement: This work was supported in part by the Texas Instrument DSP Leadership University Program.
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1 System model

2 Achievable Data Rate Analysis

3 Numerical Results

4 Conclusion
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1 System model

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3 Numerical Results

4 Conclusion
Intensity modulation (IM) and direct detection (DD) schemes require the electric signal to be real-valued and unipolar (positive-valued)

- Hermitian symmetric
- Bipolar-unipolar conversion

Real-valued unipolar OFDM signal for OWC

- DC biased optical OFDM (DCO-OFDM) [Hranilovic, 2005]
- Asymmetrically clipped optical OFDM (ACO-OFDM) [Armstrong and Lowery, 2006]
- Flip-OFDM [Yong, 2007] [Fernando et al., 2011]
- Unipolar OFDM (U-OFDM) [Tsonev et al., 2012]

Disadvantage of OFDM: high peak-to-average-power ratio (PAPR)

- The OFDM signal often has to be double-sided clipped in order to fit the linear range of LED [Mesleh et al., 2011][Dimitrov et al., 2011] [Yu et al., 2012][Dimitrov et al., 2012]
- Introduces nonlinear distortions
Assume that the digital pre-distortion (DPD) has perfectly linearized the LED between the interval \([P_L, P_H]\) [Elgala et al., 2009].
Flip-OFDM [Fernando et al., 2011]

The unipolar signal $y[n]$ is composed of a positive part and a negative part from $x[n]$

$$x[n] = x^+[n] + x^-[n],$$

where the positive part $x^+[n]$ and the negative part $x^-[n]$ are obtained as

$$x^+[n] = \begin{cases} x[n], & x[n] > 0 \\ 0, & x[n] < 0 \end{cases}$$

$$x^-[n] = \begin{cases} x[n], & x[n] < 0 \\ 0, & x[n] > 0 \end{cases}$$
At the receiver, assume the channel $h[n]$ is constant over two frames, the two received components can be expressed as

$$r^+[n] = x^+[n] \otimes h[n] + w^+[n],$$

$$r^−[n] = -x^−[n] \otimes h[n] + w^−[n]$$

Then the bipolar signal is reconstructed as

$$r[n] = r^+[n] - r^−[n]$$

$$= (x^+[n] + x^−[n]) \otimes h[n] + w^+[n] + w^−[n]$$

$$= x[n] \otimes h[n] + w^*[n],$$

$w^*[n] = w^+[n] + w^−[n]$: has power $2\sigma_w^2$; can be further reduced with noise filtering scheme [Fernando et al., 2012].
In order to fit the signal within the optical power constraints of the transmitter

\[ \bar{y}[n] = \begin{cases} 
  c_u - c_l, & y[n] > c_u \\
  y[n] - c_l, & c_l \leq x[n] \leq c_u \\
  0, & x[n] < c_l 
\end{cases} \]

\[
\begin{align*}
\bar{y}[n] &= \begin{cases} 
  c_u - c_l, & y[n] > c_u \\
  y[n] - c_l, & c_l \leq x[n] \leq c_u \\
  0, & x[n] < c_l 
\end{cases} 
\]
Definitions

- Clipping ratio $\gamma$ and Biasing ratio $\varsigma$
  \[
  \gamma \triangleq \frac{(c_u - c_l)}{2\sigma}, \quad \varsigma \triangleq \frac{-c_l}{c_u - c_l}
  \]
  where $\sigma$ denotes the standard deviation of $x[n]$

- Average optical power
  \[
  O_{\bar{y}} \triangleq \mathcal{E}\{\bar{y}[n]\}
  \]

- Optical signal to noise power ratio (OSNR)
  \[
  \text{OSNR} \triangleq \frac{O_{\bar{y}}}{\sigma_w}
  \]

- Dynamic optical power
  \[
  G_{\bar{y}} \triangleq \max (\bar{y}[n]) - \min (\bar{y}[n])
  \]

- Dynamic signal to noise power ratio (DSNR)
  \[
  \text{DSNR} \triangleq \frac{G_{\bar{y}}}{\sigma_w}
  \]
After the clipping and biasing, the two components of Flip-OFDM can be expressed as:

$$\bar{x}^+[n] = \begin{cases} 
  c_u - c_l, & x^+[n] > c_u \\
  x^+[n] - c_l, & c_l \leq x^+[n] \leq c_u \\
  0, & x^+[n] < c_l 
\end{cases}$$

$$-\bar{x}^-[n] = \begin{cases} 
  c_u - c_l, & -x^-[n] > c_u \\
  -x^-[n] - c_l, & c_l \leq -x^-[n] \leq c_u \\
  0, & -x^-[n] < c_l 
\end{cases}$$

Average optical power of $\bar{y}[n]$ is reduced to:

$$O_{\bar{y}} = \sigma \left( \phi(2\gamma\varsigma) - \phi(2\gamma(1 - \varsigma)) - 2\gamma\varsigma\Phi(-2\gamma\varsigma) + 2\gamma(1 - \varsigma)\Phi(-2\gamma(1 - \varsigma)) + 2\gamma\varsigma \right)$$
Dynamic optical power of $\bar{y}[n]$ is reduced to

$$G_{\bar{y}} = \max (\bar{y}[n]) - \min (\bar{y}[n]) = c_u - c_l = 2\sigma \gamma$$

At the receiver, we can obtain the reconstructed signal

$$\bar{r}[n] = \bar{x}[n] \otimes h[n] + w^{\star}[n],$$

where

$$\bar{x}[n] = \bar{x}^+[n] + \bar{x}^-[n]$$

$$= \begin{cases} 
  c_u - c_l, & x[n] > c_u \\
  x[n] - c_l, & c_l \leq x[n] \leq c_u \\
  0, & -c_l < x[n] < c_l \\
  x[n] + c_l, & -c_u \leq x[n] \leq -c_l \\
  c_l - c_u, & x[n] \leq -c_u 
\end{cases}$$
System Constraints

- Average optical power constraint $P_A$ (limit on power consumption, eye safety regulation, dim illumination requirement, etc.)

$$O_{\bar{y}} \leq P_A$$

- OSNR constraint $\eta_{OSNR} \triangleq P_A/\sigma_w$

- Dynamic optical power constraint $P_H - P_L$ (dynamic range of the LED)

$$G_{\bar{y}} \leq P_H - P_L$$

- DSNR constraint $\eta_{DSNR} \triangleq (P_H - P_L)/\sigma_w$

- The maximum $\sigma/\sigma_w$ value can be obtained as

$$\frac{\sigma}{\sigma_w} = \min \left( \frac{\eta_{OSNR}}{O_{\bar{y}}/\sigma}, \frac{\eta_{DSNR}}{G_{\bar{y}}/\sigma} \right)$$
Signal to Distortion Ratio

- **Bussgang’s Theorem** [Bussgang, 1952]
  \[ \bar{x}[n] = \alpha \cdot x[n] + d[n], \quad n = 0, \ldots, N - 1 \]

- The output auto-correlation function \( R_{\bar{x}\bar{x}}[m] \) is related to the input auto-correlation function \( R_{xx}[m] \) via [Davenport and Root, 1987]
  \[ R_{\bar{x}\bar{x}}[m] = \sum_{\ell=0}^{\infty} \frac{b^2_\ell}{\ell!} \left[ \frac{R_{xx}[m]}{\sigma^2} \right]^\ell \]

- The SDR at the \( k \)th subcarrier
  \[ \text{SDR}_k = \frac{\mathbb{E}[|\alpha \cdot X_k|^2]}{\mathbb{E}[|D_k|^2]} = \frac{\alpha^2 P_{X,k}}{P_{D,k}} = \frac{\alpha^2 P_{X,k}}{P_{\bar{X},k} - \alpha^2 P_{X,k}} \]
The signal-to-noise-and-distortion ratio (SNDR) for the \( k \)th subcarrier is given by:

\[
\text{SNDR}_k = \left( \text{SDR}_k \right)^{-1} + \frac{2\beta(N-2)}{N\alpha^2|H_k|^2} \cdot \max \left( \frac{O^2_y/\sigma^2}{\eta_{OSNR}^2}, \frac{G^2_y/\sigma^2}{\eta_{DSNR}^2} \right)
\]

The achievable data rate is:

\[
\mathcal{R}(\gamma, \varsigma, \eta_{OSNR}, \eta_{DSNR}, H) = \frac{1}{2N} \sum_{k=1}^{N/2-1} \log_2 \left(1 + \text{SNDR}_k\right) \text{bits/subcarrier}
\]

For given \( \eta_{OSNR}, \eta_{DSNR} \) values and channel response \( H \), we can obtain a pair of optimum clipping ratio \( \gamma^\dagger \) and optimum biasing ratio \( \varsigma^\dagger \) that maximize the achievable data rate by:

\[
(\gamma^\dagger, \varsigma^\dagger) = \arg \max_{(\gamma, \varsigma)} \mathcal{R}|_{\eta_{OSNR}, \eta_{DSNR}, H},
\]

and the corresponding achievable data rate.
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Optimal clipping ratio and biasing ratio

- $N = 512$. AWGN channel.
- $\eta_{\text{DSNR}} / \eta_{\text{OSNR}} = (P_H - P_L) / P_A = 18$ dB
Achievable data rate $\left( \eta_{DSNR}/\eta_{OSNR} = 6 \text{ dB} \right)$

Figure: Achievable data rate with optimal clipping ratio and optimal biasing ratio for $\eta_{OSNR} = 0, 1, \ldots, 25 \text{ dB}$ (in step size of 1 dB), and $\eta_{DSNR}/\eta_{OSNR} = 6 \text{ dB}$. 
Achievable data rate \((\eta_{DSNR}/\eta_{OSNR} = 18 \text{ dB})\)

Figure: Achievable data rate with optimal clipping ratio and biasing ratio for \(\eta_{OSNR} = 0, 1, \ldots, 25 \text{ dB}\) (in step size of 1 dB), and \(\eta_{DSNR}/\eta_{OSNR} = 18 \text{ dB}\).
Achievable data rate (no $\eta_{DSNR}$ constraint)

Figure: Achievable data rate with optimal clipping ratio and biasing ratio for $\eta_{OSNR} = 0, 1, \ldots, 25$ dB (in step size of 1 dB), and no $\eta_{DSNR}$ constraint.
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Summary

- Derived the achievable data rate of clipped Flip-OFDM
- Investigated the trade-off between the optical power constraint and distortion
- Analyzed the optimum clipping ratio and biasing ratio and compared the performance of Flip-OFDM and DCO-OFDM techniques
- Numerical results showed that DCO-OFDM outperforms the Flip-OFDM for most of the optical power constraint scenarios
Power efficient optical OFDM.

Bussgang, J. (1952).
Crosscorrelation functions of amplitude-distorted gaussian signals.
*NeuroReport*, 17(2).

*A introduction to the theory of random signals and noise.*
IEEE Press.

A comparison of OFDM-based modulation schemes for OWC with clipping distortion.

Signal shaping and modulation for optical wireless communication.

Non-linearity effects and predistortion in optical OFDM wireless transmission using LEDs.


