Relating Work, Change in Internal Energy, and Heat Radiated for Dispersion Force Situations

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This article describes how Casimir-like forces can be calculated for quasistatic situations of macroscopic bodies composed of different materials. The framework of stochastic electrodynamics (SED) is used for much of this discussion in an attempt to provide a very clear physical picture when considering quantities like forces, work done, changes in internal energy, and heat flow. By relating these quantities, one can readily understand why the different methods of calculating dispersion forces agree, such as when obtaining forces via changes in electromagnetic zero-point energy versus computing the average of the Maxwell stress tensor. In addition, a number of physical subtleties involving dispersion forces are discussed, that were certainly not recognized in early work on blackbody radiation, and that still may not be fully appreciated.

INTRODUCTION

The intent of the present article is to provide a physical perspective and explanation behind the often very complicated calculations encountered when evaluating van der Waals and Casimir forces. In the present section, motivation for this examination is provided. Briefly stated, for complex electromechanical situations such as in the atomic force microscope or sonoluminescence, it is easy to lose track of the relationships between work done, changes in stored field energy, and radiated energy; this article is aimed at helping to clarify these relationships.

The physical significance of electromagnetic zero-point (ZP) radiation has certainly been recognized for quite some time, not just in terms of its occurrence in quantum field theory, but in terms of verifiable experiments. During the late 1940s, the Lamb shift (energy shift in hydrogen between the $2s_{1/2}$ and $2p_{1/2}$ levels) and the Casimir force between neutral conducting plates, became recognized as key examples where electromagnetic ZP radiation has a definite physically measurable effect. Prior to that, most physicists seemed to feel that ZP energy had no real observable consequences and could simply be discarded via taking a different lowest energy level; specifically, the infinite energy term of $\sum_{\mathbf{k},\lambda}\frac{1}{2}\hbar\omega_{\mathbf{k},\lambda}$ for electromagnetic ZP radiation could simply be dropped. Actually, perhaps a better description of the prevailing attitude was that there was no obvious reason for retaining what was felt to be a rather ugly infinity occurring in the existing field theory, as physicists were not yet aware of physical effects caused by this term. Moreover, if the term was retained, there would be difficulty in reconciling it with appropriate sources for gravitation.

We now know that changes in position of macroscopic bodies and even atoms and molecules, can change the effective state of equilibrium that exists between matter and radiation. For cavities without matter inside, or with a few molecules introduced, as now studied extensively in cavity quantum electrodynamics (QED), the following procedure is often followed: namely, the change in normal modes of the standing waves of the radiation in the cavity is found and compared with the situation where the cavity is very large, and/or when no particles have been introduced. The apparent singularities attributed to vacuum energy fluctuations is now well recognized to be an important contributor to mass and charge renormalization in QED. Why these infinities cancel during renormalization is still not really understood at a fundamental level. However, following this renormalization procedure is well known to yield reliable and accurate predictions of experimental situations.

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There is little question that the long-range Casimir and van der Waals forces are critically important in our everyday lives. Indeed, in a review article, Elizalde and Romeo wrote (Elizalde, 1991), "van der Waals forces play a very important role in biology and medical sciences. They are in general particularly significant in surface phenomena, such as adhesion, colloidal stability, and foam formation. One could dare say that they are the most fundamental physical forces controlling living beings and life processes." Indeed, these intermolecular forces are extremely important for gases and liquids. Properties such as surface tension, capillary action, solubility, viscosity, heat of evaporation, and the deviation from the ideal gas law, are largely explained by these forces.

There are three classes of van der Waals forces, namely: (1) an orientation force, involving molecules with permanent dipole moments, (2) an induction force, involving a dipole or quadrupole moment in one molecule, that induces a moment in a second molecule, and (3) a dispersion force, involving molecules/atoms without permanent moments, that induce moments in each other due to correlation effects in their fluctuating electromagnetic interactions. This last category is our main interest here. Generally, this force is quite weak, at least in comparison with the other interatomic interactions (*e.g.*, ionic and covalent binding). However, despite this weakness, in a sense, this interaction is fairly universal, in that two atoms will always induce correlated interactions with each other, at essentially all distances where the two systems can be considered separated. Why this universality is typically not recognized or considered, is that the interaction is usually completely masked by ionic and covalent interactions, so that van der Waals interactions often become quite secondary. Nevertheless, for neutral atomic and molecular systems, where ionic forces will not play a role, and where electrons in each system are all paired so that covalent bonds will not form, then the relatively weak van der Waals interaction can become the dominant interaction.

When combining systems of molecules and atoms into, for example, uncharged dielectric walls, the forces do not add vectorally, but combine in a much more complicated manner. Nevertheless, even though the forces are not vectorally additive, in general the more atomic systems, the larger will be the net force that the structure will exert on another structure separated in space from the first one. Forces between such macroscopic, geometry dependent structures are often referred to as Casimir forces, due to Casimir's prediction in 1948 of the attraction of two uncharged conducting parallel plates (Casimir, 1948). Thus, the forces can become reasonably large enough to be measurable. However, the measurements are far from trivial, as nicely described by Derjaguin (Derjaguin, 1960). Making surfaces sufficiently smooth, keeping them free from electrostatic charge and dust particles, and accurately monitoring distances in spite of vibrations, all make complications for experimentalists. For these reasons, and others, fairly definitive experimental confirmation between theoretical predictions and experiment for conducting plates didn't become available until Lamoreaux's work in 1997 (Lamoreaux, 1997). Sparnaay's work in 1958 certainly showed agreement, but was not sufficiently accurate to really pin down the specific power law dependence of the force upon the plate separation (Sparnaay, 1958).

Despite these complications, and despite the weakness of the force, there is still considerable interest in Casimir and van der Waals forces, perhaps for several reasons. Some of these reasons will be the motivation for this article. First, as mentioned, van der Waals forces are very important for biological systems, liquids, and gases, and some solids. Second, ingenious measurement techniques have been developed that rely heavily on van der Waals and Casimir forces, namely, the atomic force microscope, and its related family of measurement techniques. Also, new effects are being investigated in the area of cavity quantum electrodynamics (QED) that involve the interaction of particles with walls and radiation (Sukenik, 1993). Third, the fact that at temperature $T=0$, van der Waals and Casimir forces are naturally described in terms of interactions between atoms and molecules, mediated via the electromagnetic ZP radiation, continues to interest physicists. Careful attention needs to be paid to the singularities that arise in the calculations, to work out the small, but finite forces that result. How and why these infinities cancel out has interested physicists for some time, as has the related singularities that must be dealt with in the program of renormalization for mass and charge in QED. Fourth, many physicists have done extensive investigations into whether the relatively weak Casimir effect in atomic and molecular systems may become critically more important in fundamental particle physics. All quantum fields have vacuum fluctuations and zero-point energies. For quark and gluon fields, this Casimir-like effect has been thought to become much more significant, with the idea that it may play an important role in subatomic particle structure, such as with regard to the bag model of hadrons (Milton, 1983). Finally, scientists are interested in Casimir forces, again, because of the tight relation to ZP radiation, but also because ZP fields are becoming the subject of other areas of physics. In particular, recent observations in astronomy have strongly indicated that our universe is undergoing an accelerated rate of expansion, to the surprise of most scientists (Perlmutter, 1998a and 1998b). The vacuum may have a role in this phenomena (Rueda, 1999), although much work remains to be done to fully analyze this possibility. Other astrophysical phenomena may also owe their origin, at least in part, to effects from electromagnetic ZP radiation, such as cosmic rays and cosmic voids (Rueda, 1990a, 1990b, 1995; Cole, 1995).

Understanding the connection between Casimir forces and ZP radiation more deeply for atomic and molecular systems, has served, and will probably continue to serve, as a testing ground for the effects of ZP radiation in other physical areas. For that reason, many physicists have spent considerable effort reexamining the theoretical description of Casimir forces from many different angles, as nicely described in Milonni's text on the vacuum (Milonni, 1994). In particular, Milonni's book analyses the relation between vacuum and source fields, explaining how phenomena such as the Lamb shift, the Casimir and van der Waals forces, and even the fundamental linewidth of a laser, can be explained from both of these perspectives.

A rather amazing observation regarding Casimir forces in QED is that these forces are generally quite weak, but they originate from changes in infinite, or extremely large quantities. Indeed, in Ref. (Cole, 1999b), a discussion was given on whether there were restrictions on extracting energy from the ZP radiation. Two such constraining conditions were identified that largely applied to reversible thermodynamic operations (Cole, 1990b). For irreversible thermodynamic operations, restrictions for energy extraction are far less apparent. As described elsewhere in Refs. (Cole, 1993b, 1999a, and 1999b), energy contained within electromagnetic zero-point radiation has become of increasing interest to engineers, wondering whether this direction could lead to greater energy sources. Indeed, many of the normal irreversible thermodynamic energy extraction mechanisms that we are familiar with, such as chemical reactions involving batteries, lighting a match, chemical explosions, etc., may be attributed at least in part to electromagnetic ZP radiation. Undoubtedly this viewpoint seems surprising to most, as our normal description of such phenomena is not usually expressed in this manner. However, the vacuum field can be shown to be formally necessary for stability of atoms, as briefly discussed in (Cole, 1999a) and in much more detail in (Milonni, 1994). Without taking the fluctuating vacuum into account, radiation reaction will cause canonical commutators like $[x, p_x]$ to decay to zero. In this sense, the vacuum acts as, roughly stated, a "stabilizer" for physical systems. Changes in physical constraints, such as changes in mechanical, thermal, or chemical equilibrium conditions, temporarily changes stability conditions and enables irreversible processes to occur. The radiation of a metal surface with light (*i.e.*, the photoelectric effect) or the chemical reactions that result from striking a match or closing a circuit switch containing a chemical battery, are examples of such changes in equilibrium conditions. Operations like these result in heat flow and work being done by or on the system.

Two key aspects that are important for making further progress in creating, manipulating, and experimenting with interesting and useful devices that utilize aspects of dispersion forces, are (1) the physical understanding of the phenomena, and (2) improved and versatile means of carrying out detailed calculations. Interestingly enough, both aspects are presently affecting each other considerably. For example, until the conducting sphere problem was fully carried out (Boyer, 1968), physicists did not know whether the net force from electromagnetic ZP radiation would be attractive or repulsive; most everyone undoubtedly assumed it would be attractive, as in the case of two parallel conducting plates. As the shapes of conducting shells are changed, such as from a rectangular parallel-piped, to a sphere, the signature and magnitude of the force can change completely. Our intuition, or what we feel is our understanding of physical situations, is often built up by doing repeated examples, such as for a sphere, an ellipsoid, a cylinder, a box, etc., and examining what happens as these shapes are manipulated. However, our intuition cannot be built up easily if the calculations and experiments are difficult to carry out, as they are for Casimir forces. Still, researchers continue to investigate further, since modification of shapes, materials, and placement, including dynamic variations (*e.g.*, as in the fluctuating lever of the atomic force microscope), promise to yield very interesting insights and applications.

CASIMIR FORCE CALCULATIONS

An illuminating place to begin with regard to Casimir force calculations is simply the conventional uncharged, conducting parallel plate situation (Casimir, 1948). The calculations have been carried out in several different ways. Figure 1 illustrates the situation often treated [see, for example, p. 480 in (Boyer, 1970) or p. 167 in (de la Peña, 1996)]. The force existing between the two plates separated by a distance d is what is desired; the distance R is taken to be quite large, as is the area $L \times L$ of each plate. Let $E(L_x, L_y, L_z)$ be the average electromagnetic ZP energy in a box of size $L_x \times L_y \times L_z$, surrounded by perfectly conducting walls. The standard calculation often performed is then

$$U = E(R-d, L, L) + E(d, L, L) - E(R, L, L) \ , \tag{1}$$

so that the difference is found between the sum of the two energies in both regions, minus the energy in a single region where the center plate is not present. What is interesting here, is that each of the three terms above is infinite, but the combination of the three yield a finite, measurable result. Each term in Eq. (1), in SED, is equal to the ensemble average of the electromagnetic ZP energy, $\left\langle \int_V d^3x \left(\mathbf{E}^2 + \mathbf{B}^2\right) \right\rangle$, taken over the respective volume. This

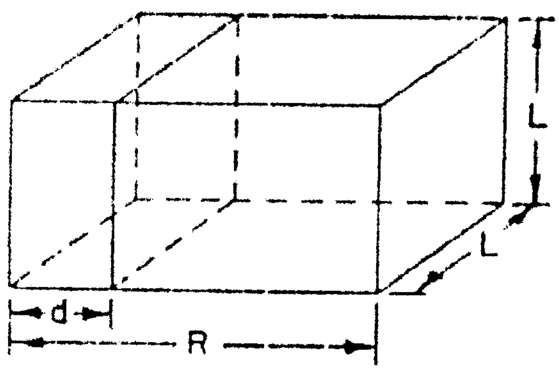

FIGURE 1. Three plates, separated by distances *d* and (*R-d*).

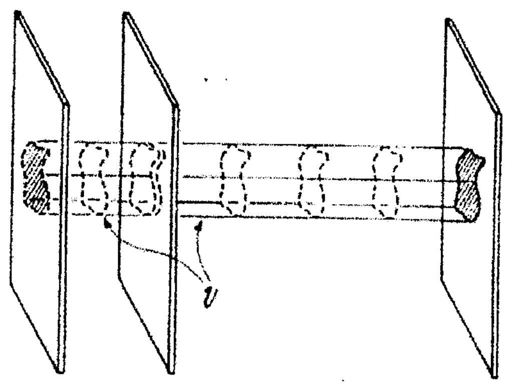

FIGURE 2. A volume *V* cutting through the three plates in Fig. 1.

quantity turns out to equal the sum over all electromagnetic modes, of the average energy existing per mode: $\sum_{\mathbf{k},\lambda} \frac{1}{2} \hbar \omega_{\mathbf{k},\lambda}$. Alternatively to Eq. (1), and perhaps more expediently, one can simply calculate:

$$\frac{\Delta U}{\Delta d} = \frac{\Delta}{\Delta d}\left[E(R-d, L, L) + E(d, L, L)\right] \ . \tag{2}$$

This quantity is taken to be equal to the negative of the average force that would be measured as existing between the plates. Each of the two terms on the right side of Eq. (2) is infinite, but, again, the singularities cancel, leaving a finite result.

Other related ways for finding the force are via calculating the radiation pressure on the plates (Milonni, 1988) and via the closely related way of making use of the Maxwell stress tensor (Lifshitz, 1956; Brown, 1969). Each of these methods encounter infinities, but the singularities cancel out during the calculations.

Here we will relate these methods via making use of the classical electrodynamic theory often called stochastic electrodynamics (SED) (de la Peña, 1996). Analyzing Casimir force situations via the use of SED has the advantage of providing a more intuitive and physical description for many engineers and scientists. SED describes nature via Maxwell's equation, with the relativistic version of Newton's second law of motion for the trajectory of particles. The key difference of this theory versus conventional classical electrodynamics is that the restriction is not made that radiation must vanish at a temperature T=0. The relaxation of this subtle restriction enables the deduction to be made that classical electromagnetic ZP radiation must be present to satisfy a number of thermodynamic conditions (Cole, 1990a and 1990b). In turn, the inclusion of this property enables a more accurate description of our physical world, while still retaining a classical view.

However, a cautionary comment should of course be made here: research *to date* has concluded that agreement between QED, which scientists clearly trust, and SED, only holds for the situation involving linear equations of motion for the modeled atomic systems and linear equations governing the electromagnetic fields for the modeled macroscopic systems (Boyer, 1975; Cole, 1993a; de la Peña, 1996). Thus, if one treats the macroscopic materials as being composed of a set of electric dipole oscillators, or as macroscopic bodies with linear dielectric properties, then we can have confidence in the agreement. Of course, this situation is quite limiting in many critically important ways, but what is interesting is that even in QED, handling the general situation of nonhomogeneous, dispersive, and absorbing material has, to my knowledge, not been successfully tackled yet. The problem is far from trivial, both from the physical description point of view (*i.e.*, appropriate quantization method for this complicated many-particle system, and relation to measurable physical quantities), as well as the mathematical complication of solving the resulting governing equations. Much of the research in this area still relies on solving the problem in QED by precisely the model where QED and SED agree, namely, where atomic systems are treated as being composed of simple harmonic electric dipole oscillators. Shortly we will discuss why this situation exists, but for now, with this reassurance that the agreement between the two theories of SED and QED lies precisely in the area that to date has been quite important, provides incentive for much of the following discussion.

Now, to be precise, we really need to follow a procedure more along the lines of (Cole, 1993a), where renormalization in the classical equations of motion in the charged particles are considered. The expressions, as presented there, become quite lengthy and fairly involved. For the present purposes, where we are trying to reveal more of the physical aspects of a procedure such as Eq. (2), let us proceed much more roughly, without worrying about mass renormalization of charges.

In the subsequent discussion, Maxwell's microscopic equations will be assumed, so that **E** and **B** represent the microscopic electric and magnetic fields, respectively, while $\rho(\mathbf{x},t)$ and $\mathbf{J}(\mathbf{x},t)$ are the microscopic charge and current densities. Following treatments such as in (Jackson, 1998), one obtains:

$$\int_V d^3x\left[\rho(\mathbf{x},t)\mathbf{E}(\mathbf{x},t)+\frac{1}{c}\mathbf{J}(\mathbf{x},t)\times\mathbf{B}(\mathbf{x},t)\right]_i = \oint_S d^2x\sum_{j=1}^{3} n_j T_{ij}(\mathbf{x},t) - \frac{1}{c^2}\frac{d}{dt}\int_V d^3x S_i(\mathbf{x},t) \ , \tag{3}$$

where

$$T_{ij} = \frac{1}{4\pi}\left[E_iE_j + B_iB_j - \delta_{ij}\frac{1}{2}(\mathbf{E}\cdot\mathbf{E}+\mathbf{B}\cdot\mathbf{B})\right] \tag{4}$$

is the Maxwell stress tensor, and

$$\mathbf{S}(\mathbf{x},t) = \frac{c}{4\pi}(\mathbf{E}\times\mathbf{B}) \tag{5}$$

is the Poynting vector. Now, in SED, each of the quantities in these equations are very rapidly varying in time. The average of them, either a time average, or an ensemble average, correspond to quantities most accessible to experimental measurement. For perfectly conducting plates, there would be rapidly fluctuating currents on the surface of the plates, that would vary in just such a way as to make the fields equal to zero within the plates. If V in Eq. (3) is taken to be the region illustrated in Fig. 2, then the ensemble average of Eq. (3) yields, for the left side, the average Lorentz force acting on the region of the center plate within the volume V.

As for the right side, if the plates are being held fixed, then

$$\frac{d}{dt}\int_V d^3x\langle S_i(\mathbf{x},t)\rangle = 0 \ , \tag{6}$$

since $\langle S_i(\mathbf{x},t)\rangle$ must be independent of time. Thus, we obtain a direct relationship between the Lorentz force and the average of the surface integral over the Maxwell stress tensor in Eq (3).

To relate this result to Eq. (2), we can proceed in the following way. Again from Maxwell's equations, one can derive the microscopic formulation of Poynting's theorem:

$$\int_V d^3x\mathbf{J}(\mathbf{x},t)\cdot\mathbf{E}(\mathbf{x},t) = -\frac{\partial}{\partial t}U(\mathbf{x},t) - \int_S d^2x\hat{\mathbf{n}}\cdot\mathbf{S}(\mathbf{x},t) \ , \tag{7}$$

where

$$U(\mathbf{x},t) = \frac{1}{8\pi}\int_V d^3x(\mathbf{E}\cdot\mathbf{E}+\mathbf{B}\cdot\mathbf{B}) \ , \tag{8}$$

and S is the surface of V, and $\hat{\mathbf{n}}$ is the outward surface normal. The ensemble average of the quantity in Eq. (8) is tightly connected with the quantities in Eqs. (1) and (2). In particular, if we quasistatically displace the center plate in Fig. 2, so that at each instant of time, the system can be treated as being in thermodynamic equilibrium, and if we integrate the ensemble average of Eq. (7) in time over the period of displacement, then:

$$\langle U(\mathbf{x},t_{\text{II}})\rangle - \langle U(\mathbf{x},t_{\text{I}})\rangle = -\int_{t_{\text{I}}}^{t_{\text{II}}} dt \int_V d^3x \langle \mathbf{J}(\mathbf{x},t)\cdot\mathbf{E}(\mathbf{x},t)\rangle - \int_{t_{\text{I}}}^{t_{\text{II}}} dt \int_S d^2x \hat{\mathbf{n}} \cdot \langle \mathbf{S}(\mathbf{x},t)\rangle \quad . \tag{9}$$

In this way, the left side becomes equal to the quantity in brackets on the right side of Eq. (2). The first term on the right becomes equal to the work done by the electromagnetic fields of the system, acting on the charges in the center plate, as the plate is displaced. This quantity is approximately equal to the negative of the work done by external forces while quasistatically displacing the plate. The last term of $\int_{t_{\text{I}}}^{t_{\text{II}}} dt \int_S d^2x \hat{\mathbf{n}} \cdot \langle \mathbf{S}(\mathbf{x},t)\rangle$ equals the ensemble average of the electromagnetic energy that flows *out* of the volume V.

Thus, Eq (9) *roughly* becomes equal to the first law of thermodynamics,

$$\Delta U_{\text{internal}} = W + Q \quad , \tag{10}$$

where $\Delta U_{\text{internal}}$ is the change in the internal energy of this electromagnetic system enclosed within the volume V, W is the work done *on* the system while moving the plates, and Q is the heat flow *into* the volume V. The reason for the qualification of "roughly," is that Eq. (9) has not taken into account all of the internal energy of the system, nor accounted for mass renormalization in the case of point charges. These specific steps are taken into account in (Cole, 1993a), but, as mentioned, certainly complicate the description. Nevertheless, it is important to at least point out that a full description does require these aspects, and that if one is to compare these terms to experimentally measured quantities, then the correct understanding of these terms needs to be addressed. In particular, the internal energy of a system also needs to include the internal kinetic energy due to charge motion, as would take place in the plate to ensure the good conductivity of the plate. More specifically, our rough description above ignored the specific heat of the section of the conducting plate contained in Fig. 2. In the same vein, some of the work done on the plate could end up as work being done on internal vibratory motion, and would be contained in the first terms on the right of Eqs. (9) and (10). These aspects are certainly included in Refs. (Cole, 1993a, 1990a, and 1990b). Brown (1969) contains a related discussion on the first law of thermodynamics, as applied to the present problem.

Ignoring these complications, the left side of Eqs. (9) and (10) becomes equal to ΔU in Eq. (2), for a displacement Δd. The first term on the right of these equations becomes roughly equal to the average force acting on the plate, as obtained in the discussion involving Eq. (3) and the Maxwell stress tensor, times the displacement of the plates.

As will be noted, in order for this change in internal energy to equal the work done on the system, requires that the last term on the right side of Eqs. (9) and (10), namely, Q, must equal zero. If the temperature of the system is held fixed during this quasistatic operation, so that T neither increases nor decreases, then the condition that Q=0 for any displacement is only satisfied at a temperature of T=0. This was the condition that was imposed in Refs. (Cole, 1990a, 1992a, and 1992b) to provide derivations for the classical electromagnetic ZP radiation spectrum.

Now, what is also useful about Eq. (10), is that it actually applies for any geometry, rather than being applicable only for the parallel plate situation. This rough description enables us to understand different means for calculating forces existing on, for example, a spherical conducting shell, as discussed in (Boyer, 1970), or rectangular parallel-piped shapes, as discussed in (Ambjørn, 1983). One could carry this calculation out at T=0 using either a change in internal energy calculation, where the sphere or cube is made to grow or shrink slightly [left side of Eq. (9)], or via the average of the Maxwell stress tensor integrated over a volume enclosing the structure. Moreover, the equation can be applied to an unusual geometrical structure, such as a conducting shell with irregular and unsymmetrical shaped walls, as opposed to the highly symmetrical case of the parallel plates, or the spherical shell. Also, an odd shaped cavity in a block of material might be considered. A slight displacement of any small area of the wall, such as might be either imagined (thought experiment), or possibly experimentally carried out by applying external stress forces in that area [perhaps originating from a piezioelectrical apparatus, as might possibly be constructed using micro-electromechanical structures (MEMS)], enables Eq. (10) to be applied to determine the force required to make the quasistatic, infinitesimal displacement. The change in internal energy due to an infinitesimal quasistatic displacement of a section of a wall, must equal the work done during the displacement at T=0, via the definition of the T=0 state (Cole, 1990a).. In this way, the component of the force acting in the direction of the displacement can be obtained. This method should agree with the equally valid method using the evaluation of the surface integral of the average of the Maxwell stress tensor over the region of the wall displaced in the previous hypothetical situation.

Regarding difficulties of calculations, however, much of what was said above certainly drastically oversimplifies the required steps. The symmetrical cases of two parallel plates, a spherical conducting shell, or conducting rectangular parallel-pipeds, are all treatable by nearly fully analytical methods. However, even these are far from trivial, as evidenced by the time delay between 1953, when Casimir published the suggestion of computing the spherical conducting shell situation (Casimir, 1953), and 1968, when Boyer solved the proposed problem (Boyer, 1968). Treatment of unusual geometries would be even more difficult. However, certainly the same basic methods should apply. In particular, it is interesting to note that the ensemble average of Eq. (8) will reduce to a sum, $\sum_i \frac{1}{2}\hbar\omega_i$, for the electromagnetic energy in an unusual shaped conducting cavity, where the sum is over all the electromagnetic modes, which are in turn dependent on the basis functions used to describe the fields. Section 3.1.2 in Ref. (de la Peña, 1996) provides background here. This observation does not necessarily simplify the problem, as one still needs to find all the modes, but at least it places the problem into a more familiar framework. Indeed, once the modes are found, then perturbation methods can be applied to calculate changes in energy and applied forces. For example, in Ref. (Casimir, 1951), a perturbation technique is described for finding the changes in the modes associated with a resonant cavity containing electromagnetic energy, when one of the walls of the cavity is dented or a particle is introduced into the cavity.

This general discussion now brings us to the consideration of the more general problem of treating nonhomogeneous, dispersive, and absorptive dielectric materials. Lifshitz first treated the case of two semi-infinite dielectric walls, separated by a small distance, using a macroscopic electromagnetic treatment (Lifshitz, 1956). The results there have been generally found to be valid, as compared with more fundamental treatments. However, the extension of these methods to more general situations in QED of arbitrary geometry and nonhomogeneous conditions, has been quite problematic. Kupiszewska summarizes much of the issues here in Ref. (Kupiszewska, 1992). The issues largely revolve around the appropriate conditions to apply for quantization rules, when the medium may be nonlinear, nonhomogeneous, dispersive, and dissipative. Consequently, a number of articles have appeared in the QED literature that model the medium by treating it as being composed of electric dipole harmonic oscillators. These oscillators model the actual atoms existing in real materials. By placing them in varying density arrangements, and of course coupling all their interactions appropriately, one can treat the case of a nonhomogeneous material, as well as one that is dispersive and dissipative. Renne made the connection (Renne, 1971b) between this more microscopic approach of treating a medium to the more macroscopic electrodynamic treatment by Lifshitz .

Fortunately, Renne's basic approach (Renne, 1971a) involving full retarded van der Waals forces between oscillators, was shown to yield identical results to those of SED by Boyer (Boyer, 1973) for the case of two oscillators, and later for an arbitrary number of oscillators (Cole, 1986). Indeed, Kupiszewska's treatment of this problem in QED can be cast entirely within the description of SED, as described in Sec. 6.1 of Ref. (de la Peña, 1996). Finally, it should be mentioned that provided one composes a macroscopic body by an arbitrary number of electric dipole oscillators, then quasistatic displacements of any of them will yield the general ideas described here, namely, no net radiation will flow out of a large volume containing these oscillators during the quasistatic displacement operation, provided T=0 (Cole, 1990a).

CONCLUDING REMARKS

This article traced some of the physical issues involving Casimir forces between macroscopic materials. The physical description of SED was used in much of this discussion, because the language is familiar to most engineers and scientists. Moreover, the model that is presently used to treat nonhomogeneous, dispersive, dissipative materials is precisely the area that QED and SED have agreement, thereby providing additional incentive for this treatment. The discussion in this article was kept rather general, in order to emphasize some of the physical points; however, references were also provided that contain more detailed description.

A number of physical issues were not discussed here that will now be briefly mentioned. Reference (Cole, 1992a) analyzed thermodynamic issues associated with operations performed on blackbody radiation, which includes the case of radiation at a temperature *T*, within a cavity, with moveable walls. There, a generalized Wien's displacement law was deduced. An early analysis by Planck in 1913, as republished in (Planck, 1988), contains a very deep physical analysis of the thermodynamic aspects of radiation, that is still quite illuminating today. Indeed, with the emphasis now being on cavity QED, some of these arguments are well worth reexamining. For example, one generalization necessary in Planck's work, is that the electromagnetic energy density varies as a function of position within a cavity, and is highly dependent on the shape and size of the cavity. This is well appreciated now in Casimir related problems. A second key point in Planck's analysis was the use of hypothetical "stopcocks," "carbon particles," and rough wall surfaces to scatter radiation; the latter two were used in Planck's analysis to "thermalize" the radiation after thermodynamic operations. In cavity QED, where highly nonthermal states of radiation can be maintained for long periods of time, such as in cavities with very smooth, reflective walls, then deeper analyses needs to be considered for these nonequilibrium situations. Third, it is very interesting that one of the arguments made by Planck and others was that it should not matter what the piston or cylindrical walls were composed of in the Wien's displacement operation; they assumed the force acting on the piston should be the same [see Sec. 66 in Ref. (Planck, 1988)]. We now know that is not the case, as emphasized by Casimir forces being different for parallel plates composed of different materials. Of course, the force on one plate is equal and opposite to the force on the other plate, in order to keep them separated, but if one of the plates is replaced by a plate of another material, the force magnitude on each plate will change. This recognition was missed in the early blackbody radiation work. Of course, it is easy to understand why, since the cylinders being considered were taken to be quite large, so that this material dependent force contribution would then be negligible. For cavity QED situations, however, such factors can be essential.

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