Recipes for acid-base titration* General Chemistry

Dan Dill Chemistry Department, Boston University Boston MA 02215

January 28, 1997

Contents

Introduction

Here is everything you need to do simple calculations of pH in acid-base titrations. For each kind of titration, there are four circumstances that need to be handled differently, according to whether you are at

- the start of the titration,
- before the equivalence point,
- at the equivalence point, or
- after the equivalence point.

It is crucial that you first calculate the reaction of the acid and base as a **limiting reagent problem**, using **moles** of acid and base, to determine which of these four circumstances you have. Then, you need to set up and solve the **equilibrium equations** for that circumstance, using **concentrations**.

As you study these notes, try to master the method rather than to memorize formulas. It is easy to write down the formulas you need once you understand the method.

^{*}Copyright © 1993, 1997 Dan Dill. You can print a formatted version of this document on acs.bu.edu with the command 'lpr ~dan/titration-recipes.ps'. You can display this document on acs.bu.edu with the command 'acroread ~dan/titration-recipes.pdf'.

1 Titration of strong acid with strong base

The first thing to do – **always** – is to calculate the reaction of the acid and base,

$$H_3O^+(aq) + OH^-(aq) \rightarrow H_2O(l) + H_2O(l),$$

as a **limiting reagent problem** (ignore the anion of the acid and the cation of the base). Do this by computing the number n_a of moles of acid and the number n_b of moles of base. Then,

• If $n_{\rm b} = 0$, we have **pure strong acid**. The pH is

$$pH = -\log c_a$$
.

If n_b ≠ 0 but n_b < n_a, we have a unreacted acid. The reaction of the base leaves n_a - n_b moles of unreacted acid, so that

$$c_{\rm a} = \frac{n_{\rm a} - n_{\rm b}}{V_{\rm a} + V_{\rm b}}$$

and hence

$$pH = -\log c_a$$
.

• If $n_b = n_a$, we have a **pure water**. The reaction of the base produces $n_a - n_b = 0$ moles of unreacted acid and $n_b - n_a = 0$ moles of unreacted base, so that

$$pH = 7$$

If n_b > n_a, we have a excess base. The reaction of the base consumes all of the acid and leaves n_b - n_a moles of unreacted base, so that

$$c_{\rm b} = \frac{n_{\rm b} - n_{\rm a}}{V_{\rm a} + V_{\rm b}}$$

and hence

$$pH = 14 + \log c_b.$$

2 Titration of weak acid with strong base

The first thing to do – **always** – is to calculate the reaction of the acid and base,

$$HA(aq) + OH^{-}(aq) \longrightarrow H_2O(l) + A^{-}(aq),$$

as a **limiting reagent problem** (ignore the cation of the strong base). Do this by computing the number n_a of moles of acid and the number n_b of moles of base. Then,

• If $n_{\rm b} = 0$, we have **pure weak acid**. The equilibrium relation is

$$HA(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + A^-(aq)$$
$$K_a = \frac{x^2}{c_a - x} \approx \frac{x^2}{c_a} \Longrightarrow x \approx \sqrt{K_a c_a}$$
1

and so

$$\mathrm{pH} = -\log x = -\frac{1}{2}\log\left(K_{\mathrm{a}}c_{\mathrm{a}}\right).$$

If n_b ≠ 0 but n_b < n_a, we have a **buffer**. The reaction of the strong base produces n_b moles of the conjugate base, A⁻, of the weak acid and leaves n_a - n_b moles of unreacted weak acid, so that

$$c_{\rm a} = \frac{n_{\rm a} - n_{\rm b}}{V_{\rm a} + V_{\rm b}}$$
 and $c_{\rm cb} = \frac{n_{\rm b}}{V_{\rm a} + V_{\rm b}}$.

The equilibrium relation is

$$HA(aq) + H_2O(l) \rightleftharpoons H_3O^+(aq) + A^-(aq).$$
$$K_a = \frac{x(c_{cb} + x)}{c_a - x} \approx \frac{xc_{cb}}{c_a} \Longrightarrow x \approx \frac{K_ac_a}{c_{cb}}$$

and so

$$\mathrm{pH} = -\log x = -\log\left(\frac{K_{\mathrm{a}}c_{\mathrm{a}}}{c_{\mathrm{cb}}}\right).$$

• If $n_b = n_a$, we have **hydrolysis**. The reaction of the strong base produces $n_b = n_a$ moles of the conjugate base, A⁻, of the weak acid and leaves $n_a - n_b = 0$ moles of unreacted weak acid, so that

$$c_{\rm cb} = \frac{n_{\rm b}}{V_{\rm a} + V_{\rm b}}$$

The equilibrium relation is

$$\mathrm{H}_{2}\mathrm{O}(l) + \mathrm{A}^{-}(aq) \rightleftharpoons \mathrm{HA}(aq) + \mathrm{OH}^{-}(aq).$$

$$K_{\rm cb} = rac{K_{\rm w}}{K_{\rm a}} = rac{x^2}{c_{\rm cb} - x} pprox rac{x^2}{c_{\rm cb}} \Longrightarrow x pprox \sqrt{rac{K_{\rm w} c_{\rm cb}}{K_{\rm a}}}$$

and so

$$\mathrm{pH} = 14 + \log x = 14 + \frac{1}{2} \log \left(\frac{K_{\mathrm{w}} c_{\mathrm{cb}}}{K_{\mathrm{a}}}\right)$$

• If $n_b > n_a$, we have a **excess strong base**. The reaction of the strong base produces n_a moles of the conjugate base, A⁻, of the weak acid, leaves 0 moles of unreacted acid (since $n_a - n_b < 0$) and leaves $n_b - n_a$ moles of unreacted strong base, so that

$$c_{\rm b}=\frac{n_{\rm b}-n_{\rm a}}{V_{\rm a}+V_{\rm b}}.$$

The excess strong base overwhelms the OH⁻ from the hydrolysis of the conjugate base, A⁻, of the weak acid, so we can **ignore hydrolysis** and compute

$$\mathrm{pH} = 14 + \log c_\mathrm{b}.$$

3 Titration of weak base with strong acid

The first thing to do – **always** – is to calculate the reaction of the acid and base,

$$H_3O^+(aq) + B(aq) \longrightarrow HB^+(aq) + H_2O(l),$$

as a **limiting reagent problem** (ignore the anion of the strong acid). Do this by computing the number n_a of moles of acid and the number n_b of moles of base. Then,

• If $n_a = 0$, we have **pure weak base**. The equilibrium relation is

$$H_2O(l) + B(aq) \rightleftharpoons HB^+(aq) + OH^-(aq)$$
$$K_b = \frac{x^2}{c_b - x} \approx \frac{x^2}{c_b} \Longrightarrow x \approx \sqrt{K_b c_b}$$

and so

$$pH = 14 + \log x = 14 + \frac{1}{2} \log (K_b c_b).$$

If n_a ≠ 0 but n_a < n_b, we have a **buffer**. The reaction of the strong acid produces n_a moles of the conjugate acid, HB⁺, of the weak base and leaves n_b - n_a moles of unreacted weak base, so that

$$c_{\rm b} = \frac{n_{\rm b} - n_{\rm a}}{V_{\rm a} + V_{\rm b}}$$
 and $c_{\rm ca} = \frac{n_{\rm a}}{V_{\rm a} + V_{\rm b}}$.

The equilibrium relation is

$$H_{2}O(l) + B(aq) \rightleftharpoons HB^{+}(aq) + OH^{-}(aq).$$

$$K_{b} = \frac{x(c_{ca} + x)}{c_{b} - x} \approx \frac{xc_{ca}}{c_{b}} \Longrightarrow x \approx \frac{K_{b}c_{b}}{c_{ca}}$$

$$FH_{ca} = \frac{14}{c_{b}} + \log x = \frac{14}{c_{b}} + \log \left(\frac{K_{b}c_{b}}{c_{b}}\right)$$

and so

$$pH = 14 + \log x = 14 + \log\left(\frac{K_{\rm b}c_{\rm b}}{c_{\rm ca}}\right).$$

• If $n_a = n_b$, we have **hydrolysis**. The reaction of the strong acid produces $n_a = n_b$ moles of the conjugate acid, HB⁺, of the weak base and leaves $n_b - n_a = 0$ moles of unreacted weak base base, so that

$$c_{\rm ca}=\frac{n_{\rm a}}{V_{\rm a}+V_{\rm b}}.$$

The equilibrium relation is

$$HB^{+} + H_{2}O(l) \rightleftharpoons H_{3}O^{+}(aq) + B(aq).$$
$$K_{ca} = \frac{K_{w}}{K_{b}} = \frac{x^{2}}{c_{ca} - x} \approx \frac{x^{2}}{c_{ca}} \Longrightarrow x \approx \sqrt{\frac{K_{w}c_{ca}}{K_{b}}}$$

and so

$$pH = -\log x = -\frac{1}{2}\log\left(\frac{K_{\rm w}c_{\rm cb}}{K_{\rm b}}\right)$$

• If $n_a > n_b$, we have a **excess strong acid**. The reaction of the strong acid produces n_b moles of the conjugate acid, HB⁺, of the weak base leaves 0 moles of unreacted weak base (since $n_b - n_a < 0$), and leaves $n_a - n_b$ moles of unreacted strong acid, so that

$$c_{\rm a}=\frac{n_{\rm a}-n_{\rm b}}{V_{\rm a}+V_{\rm b}}.$$

The excess strong acid overwhelms the H_3O^+ from the hydrolysis of the conjugate acid of the weak base, so HB^+ , we can **ignore hydrolysis** and compute

$$pH = -\log c_a$$
.