

Appendix 1, Significant figures in numerical calculations

In chemistry numerical calculations almost always involved numerical values determined by measurement in the laboratory. Such a measured numerical value is known only to a specific number of decimal digits, according to how well the measurement has been made. This number of digits is known as the *number of significant figures* in the numerical value.

For example, if we measure a quantity and determine that its value is between 1.87 and 1.85, then we would report as 1.86, that is, with three significant figures, with the understanding that the rightmost (third) digit is uncertain by one unit.

Here are some examples of how to interpret numerical values in terms of the number of significant figures they represent.

| numerical value | significant figures |
|-----------------------|---------------------|
| 1.2 | two |
| 0.0012 | two |
| 0.001200 | four |
| 10. | two |
| 10 | one |
| 1.31×10^{21} | three |
| 1000 | one |
| 12300 | three |
| 123×10^2 | three |
| 123.0×10^2 | four |
| 12300. | five |
| 12300.0 | six |

In doing calculations with such approximately known numerical values, we need to allow for the effect of our inexact knowledge of the numerical values on the final results. The detailed analysis of such cumulative effects can be subtle, but we will use rules of thumb that do a good job taking into account these effects. The rules apply separately to multiplication and division, addition and subtraction, and to logarithms.

■ Significant figures involving multiplication and division

When multiplying or dividing two approximately known numbers, the number of significant figures in the result is the smaller of the number of significant figures in the factors. Here are some examples.

$$1.23 \times 0.9 = 1 \text{ (one significant figure)}$$

$$1.23 \times 0.9612 = 1.18 \text{ (three significant figures)}$$

$$1.23 \times 0.9612 / 0.3 = 4 \text{ (one significant figure)}$$

The last example illustrates that when we know one of the factors of a product or division to just one significant figure, then it is typically very easy to estimate the result without doing a detailed calculations, since we can write

$$1.23 \times 0.9612 / 0.3 \approx 4 \times 1 = 4.$$

This is a very useful technique to master, since quite often a factor in a calculation will be known only to one or two significant figures. At the very least, it allows us to estimate numerical results "on the back of an envelope." In this course, we will take advantage of this to carry all calculations on exams without use the mechanical or electronic aids.

■ Significant figures involving addition and subtraction

When adding or subtracting two approximately known numbers, the number of significant figures in the result is determined by writing it with the same number of digits to the right of the decimal points as the factor with the *smallest* number of digits to the right of the decimal point. Here are some examples.

$$1.23 + 0.9 = 2.1 \text{ (two significant figures)}$$

$$1.2 + 0.9811 = 2.1 \text{ (two significant figures)}$$

$$12 + 0.9811 = 13 \text{ (two significant figures)}$$

The last example is determined by rounding 12.9811 to zero digits to the right of the decimal point.

$$10 + 0.9811 = 10 \text{ (two significant figures)}$$

In this example, 10 is known to one significant figure, and so after rounding 10.9811 becomes 10 in this case.

■ Rounding

In expressing the result of a numerical calculation to the appropriate number of significant figures, we round the result as necessary as follows. If a digit to be dropped is 0, 1, 2, 3, or 4, leave the next remaining digit unchanged, and if a digit to be dropped is 5, 6, 7, 8, or 9, increase the next remaining digit by one. Here are some examples.

$$0.9811 = 0.981 \text{ (three significant figures)}$$

$$1.745 = 1.75 \text{ (three significant figures)}$$

$$14.29 = 14.3 \text{ (three significant figures)}$$

$$21.74 = 21.7 \text{ (three significant figures)}$$

■ Significant figures involving logarithms

In doing calculations with logarithms, we need to know how to determine the number of significant figures. Significant figures involving logarithms are a little subtle, because a logarithm tells us both the power of the base (usually 10 or e) and the number multiplying this.

The starting point is to write the number whose logarithm we wish to evaluate as a number between 1 and 10 times a power of ten. For example, we would write the number 123.4×10^3 as

$$x = 1.234 \times 10^5.$$

Then, we take the logarithm base 10,

$$\log(x) = \log(1.234) + \log(10^5) = 0.0913 + 5.$$

There are two things to notice about this result. First, in the first term we *keep as many digits to the right of the decimal point as there are significant figures in the original number*, here four. Second, there are essentially an infinite number of significant figures in the second term, since the exponent (5) is exactly known. This means that we can write

$$\log(x) = 0.0913 + 5 = 5.0913$$

In taking antilogs, we need to reverse these rules: *We keep as many digits as there are digits to the right of the decimal point of the original logarithm*. For example, the logarithm 28.0913 has four digits to the right of the decimal. We get therefore

$$x = 10^{28.0913} = 1.234 \times 10^{28}.$$

This material is discussed in an article by D. E. Jones in the *Journal of Chemical Education*, 1971, volume 49, page 753. I mention this to underscore that many people (including me) have been confused by this.

