Lesson Plan

Title: Spring-Mass Oscillators Resonant Frequency
Primary Subject Area: Physics
Grade Level: 9, 12
Overview: By solving the second-order differential equation, students are able to determine the frequency (in radians/second) at which a mass of mass "m" which is connected to a spring of spring constant "k": $\omega = \sqrt{(k/m)}$. By measuring the dimensions of the mass on the MEMS device and using the known density of polysilicon, students will be able to determine mass "m". By measuring the dimensions of the spring and using the known elastic modulus of polysilicon, students will be able to determine the spring constant "k" (http://silicon.mhopeng.ml1.net/Silicon/). Students will then measure the resonant frequency of the oscillator using an arbitrary waveform generator and an oscilloscope and compare the value to the predicted. For ninth grade students, the measurement of the resonant frequency (without first predicting it) will allow them to determine the effect of increasing mass and increasing spring constant on the resonant frequency.
Approximate Duration: 2 hours (One long block or two regular blocks)

MA Frameworks:

Scientific Inquiry Skills Standards

High School

SIS1. Make observations, raise questions, and formulate hypotheses.

- Pose questions and form hypotheses based on personal observations, scientific articles, experiments, and knowledge.

SIS2. Design and conduct scientific investigations.

- Articulate and explain the major concepts being investigated and the purpose of an investigation.
- Select required materials, equipment, and conditions for conducting an experiment.

SIS3. Analyze and interpret results of scientific investigations.

- Present relationships between and among variables in appropriate forms.
- Use results of an experiment to develop a conclusion to an investigation that addresses the initial questions and supports or refutes the stated hypothesis.

SIS4. Communicate and apply the results of scientific investigations.

- Develop descriptions of and explanations for scientific concepts that were the focus of one or more investigations.

Review information, explain statistical analysis, and summarize data collected and analyzed as the result of an investigation.

Physics Concept Standards:

1. Motion and Forces

Central Concept: Newton's laws of motion and gravitation describe and predict the motion of most objects.

1.1 Compare and contrast vector quantities (e.g., displacement, velocity, acceleration force, linear momentum) and scalar quantities (e.g., distance, speed, energy, mass, work).
1.2 Distinguish between displacement, distance, velocity, speed, and acceleration. Solve problems involving displacement, distance, velocity, speed, and constant acceleration.
1.3 Create and interpret graphs of 1-dimensional motion, such as position vs. time, distance vs. time, speed vs. time, velocity vs. time, and acceleration vs. time where acceleration is constant.
1.4 Interpret and apply Newton's three laws of motion.

2. Conservation of Energy and Momentum

Central Concept: The laws of conservation of energy and momentum provide alternate approaches to predict and describe the movement of objects.

2.1 Interpret and provide examples that illustrate the law of conservation of energy.
2.2 Interpret and provide examples of how energy can be converted from gravitational potential energy to kinetic energy and vice versa.

Objectives for the AP AP Course ® Physics Courses B C

F. Oscillations and Gravitation

1. Simple harmonic motion (dynamics and energy relationships)

Students should understand simple harmonic motion, so they can:

a) Sketch or identify a graph of displacement as a function of time, and determine from such a graph the amplitude, period, and frequency of the motion

b) Write down an appropriate expression for displacement of the form $A \sin \omega t$ or $A \cos \omega t$ to describe the motion.

c) Find an expression for velocity as a function of time.

d) State the relations between acceleration, velocity, and displacement, and identify points in the motion where these quantities are zero or achieve their greatest positive and negative values.

e) State and apply the relation between frequency and period.
f) Recognize that a system that obeys a differential equation of the form

$$m\frac{d^2x}{dt^2} = -kx$$

must execute simple harmonic motion, and determine the frequency and period of such motion.
g) State how the total energy of an oscillating system depends on the amplitude of the motion, sketch or identify a graph of kinetic or potential energy as a function of time, and identify points in the motion where this energy is all potential or all kinetic.
h) Calculate the kinetic and potential energies of an oscillating system as functions of time, sketch or identify graphs of these functions, and prove that the sum of kinetic and potential energy is constant.
i) Calculate the maximum displacement or velocity of a particle that moves in simple harmonic motion with specified initial position and velocity.
j) Develop a qualitative understanding of resonance so they can identify situations in which a system will resonate in response to a sinusoidal external force.

2. Mass on a spring
Students should be able to apply their knowledge of simple harmonic motion to the case of a mass on a spring, so they can:
a) Derive the expression for the period of oscillation of a mass on a spring.
b) Apply the expression for the period of oscillation of a mass on a spring.
c) Analyze problems in which a mass hangs from a spring and oscillates vertically.
d) Analyze problems in which a mass attached to a spring oscillates horizontally.
e) Determine the period of oscillation for systems involving series or parallel combinations of identical springs, or springs of differing lengths.

3. Pendulum and other oscillations
Students should be able to apply their knowledge of simple harmonic motion to the case of a pendulum, so they can:
a) Derive the expression for the period of a simple pendulum.
b) Apply the expression for the period of a simple pendulum.
c) State what approximation must be made in deriving the period.
d) Analyze the motion of a torsional pendulum or physical pendulum in order to determine the period of small oscillations.

Interdisciplinary Connections:
Students will use the calculus skills they have gained in mathematics and apply them to real-world problems. Students will take real-world data to their calculus classes.

Lesson Objectives:
To be able to calculate the frequency of oscillation for a spring-mass oscillator. To become convinced that the equations actually apply to real-world systems.

Lesson Materials and Resources:
Oscilloscope, MEMS device, Arbitrary Waveform Generator

Technology Tools and Materials:
Skype, High-powered USB microscope, see above

Background Information

Useful Vocabulary

New Vocabulary Word	Meaning
Resonance	A phenomenon where energy added to the system results in large-amplitude oscillations
Resonant Frequency	The frequency at which resonance occurs
Second-order differential equation	An equation that has both a function and its second derivative in it.
Elastic Modulus	The effective spring constant per unit of length of a material
Spring constant	The amount of restoring force given by a spring for each meter of stretch or compression.
Volumetric Mass Density	The mass of one cubic meter of a material
MEMS	Micro Electro-Mechanical System
Hertz	A unit for measuring the number of cycles completed in one second.
Equation of Motion	The equation that connects the position of an object to the time.

Essential Questions to be answered; Grand Challenges:

Understanding that the equations derived from solving the differential equation apply to ALL spring-mass oscillators.
What does the frequency of an oscillator depend on?
How can I calculate the mass of a shape?
How can I calculate the spring constant of a shape?
How do the laws of Physics change for very small objects?
What is a Hertz?
What is the equation of motion for a simple harmonic oscillator?

Misconceptions:

Frequency varies linearly with changing spring constant
Frequency is always measured in cycles/second
The laws of physics change when the scale becomes very small

Lesson Procedures:

1. Looking under a microscope at their MEMS device, the student will make note of the length and width of the square mass
2. The student will look up the density of polysilicon
3. The student will look up the thickness of the POLY2 layer of the MEMS device
4. The student will calculate the mass of the square mass

1. Looking under a microscope at their MEMS device, the student will make note of the length and width of the spring connecting the mass to the base
2. The student will look up the elastic modulus of polysilicon
3. The student will look up the thickness of the POLY2 layer of the MEMS device
4. The student will calculate the spring constant of the square mass
5. The student will solve the differential equation to find the equation of motion for the square mass
6. The student will find the resonant frequency of the spring-mass oscillator

1. The student will connect the ground of the wave-form generator to the base of the MEMS device
2. The student will connect the ground of the oscilloscope to the base of the MEMS device
3. The student will connect the output of the wave-form generator to the excitation pin of the MEMS device
4. The student will connect the oscilloscope input to the test point on the MEMS device.
5. Tuning the wave-form generator to near the previously calculated expected value of the oscillator's frequency, the student will sweep the frequency up and down while observing the output wave on the oscilloscope and the motion of the plate under the microscope
6. The student will make note of the frequency at which the amplitude of the motion of the mass and the displayed wave on the oscilloscope is at a maximum.

1. Repeat for the four other oscillators on the die.

Assessment Procedures:

Students will prepare a formal lab report of their findings, following the grading rubric

Accommodations/Modifications

For younger students, simply finding the frequency of oscillation for each of the oscillators, and noting how the frequency changes with A) increasing spring constant and B) increasing mass without the calculations required of the advanced students.

Reproducible Materials

http://joelbradford.org/BU_RET_2012.html
(click on Equation_of_Motion.ppt link)

Explorations and Extensions

Students may design oscillators or other structures to be built in silicon.

Lesson Development Resources

Reflections

Contact Information

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