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Wang and Wittgenstein[1]

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> *I would like to begin and end with a classification of what philosophy has to attend to. The guiding principle is, I believe, to do justice to what we know, what we believe, and how we feel.*
>
> Hao Wang, *Beyond Analytic Philosophy*

1. Introduction

No account of Hao Wang's philosophy can be complete without discussion of his serious engagement with Wittgenstein. This began in the period 1953-1958, immediately following the publication of *Philosophical Investigations* [1953] and *Remarks on the Foundations of Mathematics* [1956], and culminated in a second phase of engagement from 1981 to 1995. It closed in the final chapter, "Alternative philosophies as complementary," of Wang's posthumously published *A Logical Journey: From Gödel to Philosophy* ([1996a], cited hereafter as LJ). Wang's work on Wittgenstein thus preceded the discussions he had with Gödel (starting from 1967), and at least partly shaped his articulations of Gödel's philosophy. Unlike Gödel, Kreisel, and Bernays—with whom he worked on logic and philosophy and whose readings and criticisms of Wittgenstein Wang took seriously—Wang was inclined increasingly over time to claim that Wittgenstein, despite certain limitations of his approach, had made fundamental and constructive contributions to philosophy.

[1]I owe numerous debts to the editors for their great patience and help in bettering this essay and in producing this volume of essays. Thanks are also due for comments on a late draft by my colleague Tian Cao.

The purpose of this essay is to characterize what Wang thought those contributions were, to say something about why he held Wittgenstein in such esteem, and to evaluate Wang's contribution to our understanding of Wittgenstein. This will preclude discussing Wang's most important contributions to logic, mathematics, philosophy, and their history. Fortunately others have broached discussion of these, especially Wang's discussions of and with Gödel.[2] I hope to be able to provide a snapshot of his engagement with a particular philosopher that will convey something of Wang's own philosophical ambitions and temperament. Though his influence on philosophy proper is not widely recognized today—partly because of the unorthodoxy of his rejection of much of the analytic philosophy of his day, and partly because of difficulties internal to his own thought—Wang's role in shaping more than one generation's understanding of the fundamental problems in logic and their history through the 1950s was significant, and not as widely acknowledged as it should be.[3] Perhaps more important, philosophy was for Wang himself the most central and significant subject, so that if we wish to measure his own sense of his accomplishments, this part of his work cannot be ignored.

Before I begin, certain qualifications are in order. Wang, to his credit, was never a "Wittgensteinian" in the sense of being a single-minded *devoté*. He always denied that he was an expert scholar of Wittgenstein, and was even proud of the fact that in his essay [1961b], an assembly and analysis of passages from *Remarks on the Foundations of Mathematics*, he never once mentions the philosopher's name.[4] He admired Wittgenstein mainly for the challenges

[2]See Parsons's essay "Hao Wang" in this volume, as well as his [1996] and [1998] and Shieh [2000].

[3]Parsons has described his brief studies with Wang in the mid 1950s. (See his [1998], note 10, quoted and amplified in note 2 to the Preface in this volume.) In conversation Burton Dreben stressed with me more than once the important role Wang played for him and others interested in logic at Harvard beginning in 1946. Wang educated students in basic proof theory, an are in which Quine was neither especially focused nor especially adept. Because Wang taught at Harvard and Oxford, the cumulative impact of his teaching on the dissemination of logic was significant. Perhaps as important, Wang supported study of the subject as internal to philosophy proper. Hide Ishiguro has stressed to me how supportive Wang was of Michael Dummett during his early years teaching at Oxford, when logic was not a very popular subject among philosophers there.

[4]Wang died five years before the release of the electronic version of Wittgen-

he posed and the suggestions that he made, but only against a wider backdrop of Wang's own philosophical projects and his interests in the history of philosophy, mathematics, and science. There were parts of Wittgenstein's philosophy that Wang positively rejected, as well as parts he viewed as too one-sided, even if valuable. Certainly he felt that Wittgenstein's philosophy, while of fundamental importance, was unclearly formulated and deserving of better articulation in light of its alternatives. All of his treatments of Wittgenstein take place from within a broader philosophical project, and most of his discussions of Wittgenstein are, implicitly or explicitly, comparative. This illustrates his approach in philosophy more generally. Readers of Wang's accounts of Gödel's philosophical thoughts should, I think, bear this in mind. Wang was not reading either Wittgenstein or Gödel neutrally, but charitably and critically, in terms of his own philosophical ideas. He was doing philosophy, not merely describing it.

In what follows I shall focus on a number of themes pertaining to Wittgenstein that extend through both phases of Wang's evolution, tying these to Wang's own writings. I shall be illustrative, rather than fully explanatory or directed at details, as it is impossible to attempt anything like an exhaustive characterization of Wang's enormous corpus of philosophical writing, even limiting myself to the role of Wittgenstein. The suitability of this approach may be questioned, of course. But my sense is that directly critical assessments of Wang's particular arguments, while valuable, may miss the wide forest of his views for the trees. For it was not argumentation, but reflection, discernment, and synthesis of knowledge, that were his *fortes*. I shall focus on reconstructing what I take to be some of the most central insights and challenges for Wang's readers.

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stein's *Nachlass* (2000), and in most of his writings had to work with memoirs of Wittgenstein's students, rather than the biographies by McGuinness [1988] and Monk [1990]. But his knowledge of the corpus in the early 1990s was remarkably well-informed, and I doubt that viewing the e-version would have altered in any significant ways his understanding of Wittgenstein any more than did the biographies. The remarks from the *Nachlass* that would have surely most interested him are those on Gödel's theorem that were unknown until after 2000, and those on Turing's use of the diagonal method. Floyd [1995] was directly inspired by conversations with Wang. I discuss the latter in Floyd [2001]. Floyd [forthcoming a]

These include Wang's notions of "perspicuousness," "factualism," "conceptualism," and "intuition," as well as his idea of the "dialectic" of the formal and the intuitive. These were the notions used by Wang to present his own view of the nature of knowledge, including philosophical knowledge. ~~These were the notions used by Wang to present his own view of the nature of knowledge, including philosophical knowledge.~~

2. Wang's reading of Wittgenstein in context

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Wang and Other Readings of Wittgenstein

2.1. Wang belonged to a gifted generation of philosophers who constructively yet critically engaged with the first and most intensive wave of Wittgenstein's reception in the mid-1950s: Anscombe, Cavell, Dummett, Feyerabend, and, though less explicitly in his published writings, Rawls.[5] Like them, Wang offers a fresh and critical approach to Wittgenstein, different both from those of Oxford ordinary language analysis and from Kantian readings of Wittgenstein common in the 1960s and 1970s. The philosophical standards against which Wang's reactions to Wittgenstein are to be measured are thus of a high caliber, and his analyses, especially of Wittgenstein's remarks on mathematics, are to be counted as lasting, even if neither definitive nor ultimately correct.

Like these contemporaries, Wang took Wittgenstein's anti-empiricism and anti-reductive conceptual pluralism, as well as his concern with probing the character of objectivity and agreement as embodied in actual practice, as central philosophical concerns. Like them, Wang did not see ordinary, everyday language as offering an ultimate subject of theorizing or a constraint on speculation, but

[5] At Harvard the initial reception of Wittgenstein was significantly shaped by Rogers Albritton and Burton Dreben, at least in the classroom, and not by Wang. Until the early 1970s, however (by which time Wang had left Harvard), Dreben mainly taught logic, with an occasional seminar on *Remarks on the Foundations of Mathematics* (Wang had begun teaching Dreben logic while he was still an undergraduate, and provided him with an introduction to Paul Bernays before 1950). Albritton (and Cavell) were the major influences on the study of the later Wittgenstein's significance for epistemology. Hilary Putnam took a logic course with Wang at Harvard and later discussed logic with him while at Oxford, but Wang's philosophy seems to have had little impact on him.

rather, at best, a touchstone or challenge for reflection.[6] He did not take the concept of truth to be exhaustively characterized by any one theory, or even a central topic or approach.[7] And he was not concerned to dismiss ontology.[8] He was always loath to dismiss a philosophical perspective with the term "nonsense."

Though he wrote on Hume's problem of induction, Wang did not have much interest in drawing philosophical lessons from a confrontation with general forms of skepticism ([1950c], [1974a] (cited hereafter as FMP); cf. LJ, p. 371). His readings of Wittgenstein reflect this, differing not only from those of Cavell—who makes skepticism of central importance to Wittgenstein—but also from the later readings of Kripke and Wright, who take one of Wittgenstein's central contributions to have been a new form of skepticism about following a rule.[9] The contrasts with the latter, widely known reading, are instructive, both for what they reveal about Wang's general philosophy and for how he read Wittgenstein.

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Wang did not take Wittgenstein's form of constructivism to be based upon a general concept of rule-following, logical instantiation or necessitation, or conceptual grasp. There are of course more than a few ways to criticize a reductive idea that "meaning is use," and

[6]In fact, Wang was more passionately against "linguistic philosophy" than any of these philosophers, even favorably quoting Gellner's [1959] irreverent anthropological comparison between ordinary language philosophy and a secular, established religion for gentlemanliness (FMP, p. 393). He explicitly praised Rawls's *A Theory of Justice* [1971] for its willingness to go beyond the "special kind of piecemeal linguistic or conceptual analysis" Wang took to have wrongly dominated Anglo-American philosophy in the 1960s (LJ, p. 326).

[7]Wang emphasized here the importance of Wittgenstein's remarks on truth in his [1980], p. 75: "Philosophy is not a choice between different 'theories'. It is wrong to say that there is any one theory of truth. For truth is not a concept." However, Wang certainly did think that truth is a concept, and that Gödel, for example, had shown us at least some of its essence.

[8]Nor did he take Wittgenstein to have been preoccupied with such a dismissal. He welcomed, for example, von Wright's pointing out to him that the term "metaphysical" was an erroneous transcription of the word "metaphorical" in early editions of *Culture and Value*: Wittgenstein remarked that "there is no religious denomination in which the misuse of metaphorical [JF: not "metaphysical"] expressions has been responsible for so much sin as it has in mathematics." Cf. LJ, p. 181 and Wittgenstein [2003] MS 106, p. 58 (1929), correctly transcribed and translated in Wittgenstein [1998], p. 3e.

[9]During the conversations I had with Wang between 1990 and 1995, I often asked him to comment on these readings, but he resisted discussion of them.

rule-following skepticism is one of them. But another way is *via* a sophisticated form of conceptualism in which grasp of concepts is understood in terms of something other than the purely deductive model. This latter approach was Wang's. He always took Wittgenstein to have been a conventionalist about at least part of the content of mathematical knowledge, but this did not exhaust his understanding of what Wittgenstein had to say.

By now there is much in Wang's writings on Wittgenstein that seems dated, but in other respects his remarks on Wittgenstein are still relevant. Wang would certainly have shunned fictionalist accounts of mathematics, including those inspired by the idea—arguably Wittgensteinian—that mathematics belongs to our artifactual capacity to invent and represent, rather than attaching directly to actual truth and knowing. Rather than ascribing to Wittgenstein a response-dependent, anti-realist, or assertion-conditional account of meaning, Wang took him to be investigating phenomena of certainty that figure in logical and mathematical objectivity at the basis. Wang would have welcomed recent work in philosophy of mathematics in which visual elements of diagramming are made central to the theory of reasoning, computer modeling is regarded as of fundamental philosophical interest, and in which, more generally, the philosophy of mathematical practice is given center stage. 2.2 History and Philosophical Method

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Fact and fiction, concept and object, actuality and possibility, were for Wang notions best placed within the frame of logic and the theory of knowledge. But though he took them to be structured by the historical fact of idealization in science and the ubiquitousness of its methods of mathematization, including formalization of proof, Wang resisted the reduction of philosophical methods to logico-formal methods throughout his life. This was an important element of the affinity he found with Wittgenstein and with Gödel and explains the disaffection he had with the main figures of the analytic tradition in twentieth century philosophy, including Quine, who was one of his most important teachers.

Unlike Wittgenstein, Wang used historical perspective as a weapon in mounting his criticisms.[10] This expressed part of his

[10]In conversation Wang explicitly rejected a remark Wittgenstein reportedly made in the early 1930s in reaction to Broad ([1980], p. 74-75):

> If philosophy were a matter of choice between rival theories, then

factualist ideal. Perhaps for this reason his writings are distinctive ~~and unusual~~ in containing a wealth of unusually measured, insightful, informative, and objective—as opposed to dramatically narrated—history. Here is a contrast with Feyerabend, as also with Kuhn. Like them, under the influence of Wittgenstein, Wang resisted the positivist's ideals of empiricism and the use of formalized languages to solve fundamental philosophical problems. Yet Wang urged resistance from a variety of different points of view, and with full knowledge of the contributions of mathematical logic to philosophy and to mathematics, as well as a serious understanding of the history of mathematics proper. Typically his arguments are laced with specific examples drawn from the history of mathematics, logic, and philosophy, and express no general account of science or scientific method.

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Early on Wang adduced limitative results on decision procedures: "the quest for an ideal language is probably futile. The problem of formalization is rather to construct suitable artificial languages to meet individual problems" ([1955e], p. 236). Historically, he pointed out, the distinction between "artificial" and "natural" languages is a matter of degree, familiarity, and culture.[11] He also highlighted the role of mathematical notation (not merely formal logical notation) as something crucial to, even if not exhaustive of, our grasp of thought, especially in allowing us "perspicuous" grasp of massive details ([1961b]). Later he emphasized that "the philosophically more central and more difficult task is to grasp the right ideas intuitively; how far they can be or how well they are

> it would be sound to teach it historically. But if it is not, then it is a fault to teach it historically, because it is quite unnecessary; we can tackle the subject direct, without any need to consider history.

[11]Noting the long development of the spoken Chinese language over time, he stressed that although each alteration of the language seemed at the time of its introduction "natural," if one had tried to introduce the changes all at once, one would have been attempting to make a kind of revolution, and this would probably have failed. But "on the other hand, when an artificial language meets existing urgent problems, it will soon get generally accepted and be no longer considered artificial," so that "it may be more to the point if we compare artificial languages with Utopian projects" ([1955e], pp. 236-37). Wang's picture here is that ideal aspirations may realize themselves over time, at least to some extent, but never fully and in every detail.

formalized is an auxiliary secondary consideration, which is admittedly very helpful sometimes" ([1985a], p. 122).[12] In the history of philosophy, Wang believed in presenting not only results and principles, but biographical facts as well, thus using individuals, not archetypes, as his narrative frame. This expressed a conviction that philosophy should be relevant to individual human life and human experience, not merely to the facts, but also a belief that biographical and/or cultural facts about a philosopher may be relevant to an assessment of his or her philosophy.

In his later writings on Wittgenstein and Gödel, Wang found congenial what he took to be their firm respect for facts, including facts of everyday life and experience in their own lives, however idiosyncratic ([1991?], p. 23). This makes reading his accounts of their philosophies difficult. Ordinary biographies of these figures are easier to take in for those interested in the sweep of their intellectual lives, and critical conceptual analyses are by contrast the norm for philosophers. In Wang's writings there is a mixing of the genres, with his own sense of history and philosophy overlain on top. This does not mean that these writings fail to contain a wealth of detail from which many different readers can learn. But it does imply that one should bear in mind Wang's own philosophical ideas in assessing their value. It also explains why Wang's writings are not, and may well never be, tremendously influential. He had great ambitions and a discerning eye for fundamental work, but he found in the end that a convincing synthesis and articulation of his own perspective eluded him.

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2.3 Objectivity Before Objecthood

The philosophical frame surrounding Wang's philosophy is difficult to characterize, for it is multifaceted and schematic. In the end, it offers no more—and no less—than a "flexible" yet comprehensive framework for thinking about philosophy, both in its scientific and its literary articulations ([1991?], p. vi). Certain general things may, however, be said. Unlike Dummett, whom he knew well and admired, Wang did not take the theory of meaning to be a fundamental branch of philosophy. He stressed the importance of alternative characterizations of the relation between language and thought to twentieth century philosophy, focusing on the multiplicity and vicissitudes of languages, not their unity. Facts as we know

[12]This work will be cited hereafter as BAP.

them must, on his view, form a direct object of reflection quite apart from their linguistic expression, placing non-trivial constraints on philosophical theorizing.

The primacy of facts over objects was something Wang admired in Wittgenstein's *Tractatus*, and tended to encapsulate in sayings reminiscent of one long associated with Kreisel: "objectivity before objecthood" ([1991b], pp. 260ff.; [1991?], p. 71). The idea was to take objectivity to require only a bifurcation of propositions into true and false (by the law of excluded middle), thereby leaving open how best to articulate the nature of objects. Wang's "substantial factualism," as he called it in FMP, was intended to develop this idea in a systematic way, providing an alternative to the forms of linguistic philosophy he resisted. He proposed calling it "anthropocentric magnifactualism," thereby tying it to his earlier reading of Wittgenstein as an advocate of "anthropologism" (FMP, p. 1). But Wang knew that Wittgenstein did not regard the notion of "fact" as of much use in elucidating the nature of mathematics.

Wang's "conceptualism," indebted to Dedekind and Gödel, is hardly to be classified as Wittgensteinian, except in the amorphousness of its edges and the absence of a general account of concepthood. It turns away from the methods by means of which Wittgenstein drew distinctions between factual and conceptual investigations. Yet Wang believed that the very idea of conceptual knowledge implies a contrast and a connection with technical or combinatorial skill, and he proposed late in life to characterize his own perspective with the term "connectivism," rather than "conceptualism," a Wittgensteinian sounding revision ([1991?], pp. 262-3). He was never sympathetic to Wittgenstein's critical remarks about Dedekind, regarding as too one-sided their persistent griping about the dominance of the extensional point of view.[13] Yet Wang understood that what Wittgenstein opposed was what Wang himself

[13]It is interesting to note that Bernays [1959] is relatively charitable to these particular remarks of Wittgenstein, though he regards them as ultimately unsatisfactory, as it seems one must. Bernays takes Wittgenstein to be resisting the mixing up of intensional and extensional approaches, an "applicable" criticism, according to Bernays, in certain versions of the Dedekind theory of numbers "that create a stronger character of the procedure than is actually achieved." Bernays thinks Wittgenstein's considerations are of potential value in combating the kind of dogmatism that sometimes accompanies reductions—an idea Wang would pick up on and develop in his own writing on Wittgenstein.

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regarded as a kind of dogmatism, exemplified by philosophers like Quine. Wang certainly regarded reasoning by means of infinitary objects as a primary given that forces us to accept certain principles in the philosophy of mathematical science, as Wittgenstein did not. And he felt, quite understandably, that Gödel was right to suggest that Wittgenstein sinned against his own philosophical stance in coming close to denying the existence of facts about sets ([1991?], p. 35). Wang certainly felt that Wittgenstein too often disregarded a principle he should have embraced fully: that "the requirement of leaving things alone demands also a fuller appreciation of the alternative views to one's own favorite" (ibid.).[14] And he believed that Wittgenstein saw "true philosophy as insulated from mathematics," as he could not (LJ, p. 19). Wang endorsed, rather than merely questioned, Dedekindean views that model mathematical objectivity on law-preserving extensions of concepts, and he seemed prepared to take such views more or less at face value, as Wittgenstein did not.

On the question of philosophical approaches to the concept of objectivity, however, a more nuanced difference may be discerned. Wang appreciated the positive role of formalization in carrying forward the logicist project, but appreciated Wittgenstein's criticisms of a reductive attitude toward formal proofs. He understood the limitations of a vague appeal to "law governedness" in explaining the objectivity of structures, and he was not inclined to take second-order logic to have provided us with a philosophically sufficient basis for the theory of concepts. Insofar as the Dedekindean ideal of rigor is taken to be primarily that of the axiomatic method, Wang appreciated Wittgenstein's willingness to question the application of that ideal across the board in philosophy, as he believed Gödel had not. Gödel, Wang believed, was overly optimistic about the use of the axiomatic method in philosophy—an assumption Wang felt the Vienna Circle had shared (BAP, p. 104). But "the essential immunity of mathematics to the contingent vicissitudes of language cannot be shared by philosophy," according to Wang (LJ, pp. 210-11).

[14] It is also true, however, that in discussion with me in the early 1990s Wang was particularly puzzled about the status of Wittgenstein's writings on mathematics in the early 1940s. He sensed that Wittgenstein had invested a great deal of effort here, and felt these writings had not yet been well enough understood. That judgment still stands today, I believe.

Thus Wittgenstein served in Wang's mind as a useful corrective to overly optimistic, uncritical over-extensions or oversimplifications of method. In general, proof and axiomatization are in Wang's view important tools for distilling explicit acknowledgment of principles which can then be assessed and discussed, but they are never means for unearthing self-evident, unrevisable facts. Their application is generally limited to mathematics and to purely logical questions. But the foundations of logic: these lie in broader features of language and thought not reducible to mathematical science and unlikely to be resolved in a definitive way.

2.2. *Logic and Foundations.* What made Wang's engagement with Wittgenstein most distinctive, then, is that he reached his interpretive conclusions by grappling directly with Wittgenstein's remarks on mathematics and logic, and then attempted to draw general lessons for philosophy as a whole. Wittgenstein's philosophy could potentially be used, he believed, to make significant contributions to the foundations of logic at a basic level in a way that could contribute to a philosophical program of potentially wide methodological significance.[15] He seems to have felt that Wittgenstein could help him bridge the gap between his technically specialized knowledge of mathematics and logic and overarching, general philosophy, and he believed that this was where the value of Wittgenstein's writings on mathematics were to be found. His focus on Wittgenstein was thus narrower than that of any of these readers of Wittgenstein I have mentioned, though his ambitions were as broad.

Throughout his philosophy, and certainly in his readings of Wittgenstein, Wang's inspiration was the *Grundlagenstreit* in the foundations of mathematics. He was deeply inspired by Paul Bernays's call in [1935], in the face of the bitter controversies between Hilbert and Brouwer, for an informed and broad-minded ap-

[15] von Wright and Hintikka spring to mind as similar readers in this respect. But the idea of solving philosophical problems directly via logic was anathema to Wang. He explicitly differed with von Wright's views about the history of logic, finding them too narrow and disengaged from the history of mathematics, as well as overly pessimistic about the potential relationship between logic and philosophy in the twenty-first century. This is clear from comments Wang made on the manuscript of von Wright [1994] in a letter of 10 September 1991 that I discussed with him.

proach to foundational issues in mathematics and philosophy, one that would avoid exaggerated claims about "crises" in foundations, exercise minimal partisanship and maximal reasonableness of approach, and portray the history of logic and mathematics as capable of proceeding, ultimately, in harmony and security despite a plurality of approaches.[16] He followed Bernays's idea that broad talk should be replaced with a careful and informed mathematical exploration of the consequences and possibilities for a variety of logical and philosophical alternatives.

Nevertheless, Wang was not inclined to regard mathematics or logic as capable of settling philosophical disagreements on their own: logic in particular had philosophical foundations, and just here was where he located Wittgenstein's specific contributions. He admired Wittgenstein's ways of exploring the elements of fundamental philosophical choices and moves and emphasizing the complexity and variety of ways in which thought may be seen to be expressed in language. He believed that a "neutral viewpoint" that adopted a "detached position" from certain philosophical controversies would be productive (LJ, p. 214). When explanations give out, the philosopher should describe, and not explain or defend. The best that may be done is to present a range of alternatives and arrange them, if possible, in a synoptic, step-by-step manner. This, Wang felt, would allow for the defense of what he called "open-minded, stepwise Platonism" ([1991?], p. 54), and avoid overreactions to apparent paradoxes. But such arrangement itself belongs to what may broadly be conceived of as logic.

2.3. *Logic as Metaphilosophy.* Increasingly over time Wang became explicit that he had devoted himself to articulating an approach, nowadays deemed (perhaps tendentiously) "quietist," in which the totality of possible approaches would be carefully canvassed, distilling what might be deemed best in each. He did not mean by this that all philosophical problems could be reduced to matters of language use. In fact, he considered Quine's proposal that existence be analyzed by way of the uses of pronouns in particular languages

[16] Bernay says at the beginning of [1935] that the mathematical sciences are growing in harmony and security, though he does not claim this about logic or foundations. Wang, however, conceived of logic as at least ideally playing an adjudicative, mediating, and harmonizing foundational role in philosophy.

to be "harmful and dangerous," on the ground that, given how we now speak and think, we are in "no good position to anticipate the form of all confusions ahead of time" ([1991?], pp. 142-3).

Yet while recognizably drawing upon and reacting to Wittgenstein, Wang's version of "substantial factualism" is complex and hardly reducible to any simple-minded descriptivist view (for more see Parsons [1998]). He did not believe that as the sense of crisis in foundations passed, philosophical disputes would fall away as unnecessary residue or merely pragmatic dross, to be replaced with strictly mathematical, cooperative work. Instead, he believed that the nature of philosophical dispute should itself be normatively reassessed in light of the history and philosophy of logic and mathematics, as well as the broader history of philosophy.[17]

Like Bernays, he stressed that Wittgenstein too often rejected speculation. And he was steadfastly critical of the trend in analytic philosophy, shaped by Carnap, that erected or rejected philosophical distinctions primarily by formal means, grounding choice of framework on vaguely enunciated, pragmatic appeals to the scientific enterprise as a whole or to a general holism about evidential support. Already in his [1958a] Wang complained about the trend toward "piecemeal exercises" in philosophy, having in mind not only the exercises of ordinary language philosophers, but also those of technically minded analytic philosophers who were, in his mind, too reductive and sanguine about the usefulness of methods of formalization in philosophy and in mathematics ([1958a], p. 468).

Wang regarded resistance to dogmatism and partisanship in philosophy, whether it was theoretical or practical, not only as essential to the health of the subject, but essential for overcoming the political and humanitarian challenges bequeathed by philosophies of the twentieth century. He had in mind, of course, not only disasters perpetuated in the name of philosophy in the West, but also in China. Top-down, totalizing philosophies that fail to respond to individual perspectives and feelings, philosophies that ignore the practical need to reach agreement on collective action through dialogue, philosophies that claim infallible insight, denying the possibility of

[17]In conversation Wang told me that one reason he had found his professorship at Harvard unsatisfying is that he was appointed in the Department of Applied Mathematics, and not in Philosophy, where he felt the bulk of his efforts and his contributions had been made.

a reasonable range of disagreement: these were all anathema to Wang. Much of his philosophical motivation came from a wish to devise a philosophy responsive to these concerns.

These attitudes partly explain, I believe, why Wang was attracted to Wittgenstein's later philosophical writing, designed as it was to investigate and to question the philosophical "must."[18] Wittgenstein's later methods and style of writing were attractive to Wang in their way of asking for philosophical responses from readers one by one, from the bottom up, without dogmatism. As he saw it, they invite the idea of trying to reflect upon a whole range of possible individual responses to philosophy, and, in fact, to the experience of human life as such. Wang at one point defined philosophy as "an attempt to attain a perspicuous view of (all of) human experience" ([1991?], p. 21), which he equated with "the quest for (comprehensive) perspicuous objectivity" ([1991?], p. v), or a worldview.[19] Wittgenstein's remarks thus fulfilled a certain methodological ideal to which Wang was attracted: state all reasons for and against the available positions and then let the reader make up her own view on a subject ([1991?], p. 144).

Wang himself did not, however, shun comprehensive and systematic theory in philosophy, and he faulted Wittgenstein for doing so. In the end, however, Wang himself did not achieve a formulation of a philosophy that satisfied him, or could be easily taken in. He was perhaps for this reason attracted at the end of his life to the later writings of Rawls. These Wang explicitly adduced in the epilogue to his final book as an especially valuable exemplar of philosophizing with the right aims and methods (LJ, chapter 10).

Like Rawls, Wang regarded the idea of a social contract understood in terms of coordination and pure convention, a mere *modus vivendi*, as inadequate. A deeper measure of agreement based on a normative theory was needed, but one that would be based on the here and now, that is, on how we as individuals respond to

[18] Wang goes so far as to say that a major task in the philosophy of mathematics is to either justify or to explain away the common feeling that elementary truths about small integers are intrinsically obvious and convincing ([1991?], p. 155).

[19] At [1991?], p. 44, Wang wrote, "It is, I believe, perfectly reasonable to view Gödel's conceptual realism and Wittgenstein's earlier and later philosophies as alternative attempts to give a perspicuous view of the human experience (and therewith of the world)."

the idea of the legitimacy of an historically given structure or system. In his early work, Wang tended to treat this as something like "the sociological fact" that a result is accepted ([1961b], p. 329; [1987b], pp. 88ff.), but as he progressed, objective agreement became the quest of philosophy proper, and in this context something more elusive, more normative, more open-ended, and difficult to articulate.

What the later Wang emphasized in the work of the later Rawls is, in fact, more Wittgensteinian than Kantian. For Wang was steadfastly against the idea of utopian thinking where it is unwilling to see itself shaped by the friction of everyday life ([1955e], pp. 235-6; [1991?], pp. 154-155). Wang rightly took Rawls's distinction between treating a theory of justice as a comprehensive or metaphysical doctrine and treating it as a political conception forwarded from within the ongoing, constructed framework of "public reason" to be of fundamental methodological importance, a signal contribution to the history of philosophy as a whole, and not merely to political theory. At the same time, he agreed with Rawls that clarity will not leave philosophy, or the world, as it is, but potentially alter it ([1991?], p. 32).

In his own writing, however, Wang did not proceed systematically, as did Rawls, but synoptically and intuitively, as he took Wittgenstein to have done (cf. Parsons [1998], p. 20). He took one of Wittgenstein's observations more literally to heart than his contemporaries: in the face of certain fundamental questions and distinctions, where explanations end, supplement pure philosophy with descriptions and observations. Wang's efforts in this direction lack the literary sophistication of Cavell's and Feyerabend's writing and not infrequently tend toward being overly descriptive or biographizing, especially in his later writings on Wittgenstein and Gödel. Yet Wang's assemblies of reminders are not lacking in stylistic sophistication, and they are never trivial. For they are everywhere informed by his extensive knowledge of history and forwarded in service of his wider philosophical point of view.

2.4. *Wang's evolution: Gödel and Wittgenstein.* Wang's work on Wittgenstein may be organized into two phases, forming a pair of parentheses around his engagement with Gödel. In the first (typ- 2.6

ified by his [1955e], [1958a] and especially [1961b][20]), indebted to Bernays, his central concern was to make sense of Wittgenstein's remarks on mathematics by analyzing them in relation to Bernays's [1935] idea of a hierarchical, five-fold way of regarding the distinction between constructivist and non-constructivist positions: anthropologism (Wittgenstein's alleged position, "strict finitism" in the language of Kreisel [1958]), finitism, intuitionism, predicativism, and Platonism. He used Wittgenstein to structure his understanding of the notion of "effectiveness" and his understanding of what he took, throughout his life, to be a fundamental "dialectic" between the "formal" and the "intuitive."

The second phase (after 1981) took place after his discussions with Gödel and his formulation of factualism. Here Wang aimed to draw from Wittgenstein's later writings on the nature of logic and philosophy methodological lessons for philosophy as a whole.[21] At this juncture the comparison between Wittgenstein's philosophy and Gödel's was of central interest, and a characterization of their contributions to twentieth century philosophy the overarching aim. Gödel and Wittgenstein seemed to Wang to provide examples of true depth in philosophy, complementary alternatives to the then prevailing empiricism, emphasis on formal methods, conventionalism, and holism that he associated with Carnap and Quine, and vehemently disliked.

Wang tended to think of the contrast between Wittgenstein and Gödel as exemplifying the wider and important thematic dialectic between the pull of the "intuitive" and the pull of the "formal" or "idealized" or "conceptualized," a dialectic he applied to account for the history of science, East and West, and the history of phi-

[20] Wang mentions Wittgenstein rather dismissively in his earliest essay ([1945]/[2005], p. 139), a review of Russell's *Inquiry into Meaning and Truth*. He grouped him with Bergson as a philosopher holding, self-contradictorily, that "there is some knowledge that language cannot be used to express." Wang later came to soften his view of Wittgenstein by devising a distinction between the intuitive and the formal, but it is not clear how the paradox is overcome by this means. In the 1990s Wang was especially interested in discussing with me the then "new" interpretations of the *Tractatus* that aim to resist the kind of flatly contradictory reading he had once proposed.

[21] Chapter two of BAP works out ideas about Wittgenstein's early philosophy as a kind of "digression"; Wang announces a plan to write more substantially on Wittgenstein in a later place (p. 75).

losophy.[22] The contrast also showed the importance of a general conception of logic to philosophy. In his final writings, Wang drew on Wittgenstein's *On Certainty*, less as a response to skepticism than as a starting point for presenting his own "factualist" ideal in terms of the concept of perspicuousness and his own conception of "logic as metaphilosophy" (LJ, chapter 10), a comparative and adjudicative method Wang adopted as his own.

In the end Wang rejected two aspects of Wittgenstein's philosophy that were crucial to it: 1. Wittgenstein's attempt to work without a fixed structure of overarching epistemological distinctions, a general theory of the mind, or of logic; and 2. Wittgenstein's insistence on pursuing philosophy and/or logic through an investigation of grammar, or logical distinction-drawing, without relying directly on traditional, historically given systems of thought. Wang was always skeptical of the idea that a focus on how we use language to express thoughts might yield philosophical illumination. So, in contrast, he developed an overarching set of epistemological distinctions, highly indebted to the past, and attempted to treat the philosophical data in terms of the framework they provided. Even if he regarded his frame as "flexible" and merely schematic ([1991?], p. vii), even if he attempted to revise and critically revisit traditional philosophical categories, he took these as a given, and worked with them.[23]

Wang himself located his most fundamental philosophical differences with Wittgenstein at the same level as he located his most fundamental differences with Gödel: in the greater generality and richness of view he thought his own philosophy had achieved:

> My own perspective differs from theirs in what I see as a different conception that favors a sharper delineation of the several distinct kinds of human experience. For instance, within the frame of experiential objectivism I aim to give the distinguishing traits of our mathematical experience their

[22] Wang for example thought of Western medicine as more "formal" and conceptualized, and Chinese medicine as more "intuitive" ([1984a], p. 528).

[23] Hence the justness, I believe, of Dreben's opinion that Wang, although "constantly deeply attracted to Wittgenstein," never accepted the "full force" of Wittgenstein (Parsons [1998], p. 21n). Wang's insistence on retaining a notion of "intuition" at the basis of his epistemology is a striking instance of this. See section 5 below, and note 10.

> due, and shun away from seeing it as representative of all experience or as a 'degenerate' though pervasive kind of experience that forms a part of 'grammar.' In particular, I take it to be a philosophically relevant and significant fact that mathematics (and physics) are 'intellectual sciences,' in which the danger of confusion due to (the use of) language and the irresolvability of conflicting interpretations are not so serious as elsewhere (say in the humanities and in art). ([1991?], p. viii)

3. Anthropologism

Wang appreciated early on that Wittgenstein was not an intuitionist, but in fact a critic of intuitionism, rejecting as an incorrect idealization of mathematical proof the constructivist constraints on reasoning offered by Brouwer and his followers. Wang was indebted on this point to Kreisel's and Bernays's critical reviews of *Remarks on the Foundations of Mathematics.*[24] They took Wittgenstein to have advocated an even more restrictive form of "strict finitism," in which the use of an induction scheme allowing for generalizations over all numbers is rejected in favor of what is "feasible." Wang agreed and chose for this position a term that is a "bit more colorful," namely "anthropologism" ([1958a], p. 474; [1961b]). In Wang's [1958a] the notion of "anthropologism" as an approach to the foundations of mathematics is first framed. This is a definite step beyond Bernays [1935], who mentions that one might question unfeasible recursions but doesn't suggest this as a possible foundational stance.

One sees in his choice of terminology Wang's search for a broader philosophical perspective from which the commitments of such a position might be appreciated (elsewhere he speaks of "ethnological" perspectives, alluding to Wittgenstein). Bernays had used the phrase "anthropomorphic" to describe, more critically than Wang

[24]See Kreisel [1958] and Bernays [1959]. As late as [1987b], pp. 85, 89-90, Wang took himself to be "in essential agreement" with Bernays's review. Later on Wang would maintain that Wittgenstein was closer to the terms of variable-free finitism, associated with Skolem, and deny that Wittgenstein was a strict finitist. An example is ([1991?], 87ff.), which was however removed from its successor passages published in (LJ, pp. 212ff.).

would have, Wittgenstein's reliance on the character of grammar and the fact of actual agreement in mathematics in connection with logic ([1959], p. 30). Wang applauded the conceptual pluralism implied by Wittgenstein's idea, and its focus on practical agreement, as Bernays did not. For Bernays, there was a threat of irrationalism here. This Wang did not believe. For him Wittgenstein's insistence on "intuitiveness" was in a sense arational, insofar as it grappled with that which escapes conceptualization by one means or another, and in another sense perfectly rational, since such residue's presence was evident.

Wang never accepted "anthropologism" as a correct point of view. He saw it only an illuminating perspective along the way, an unearthing of steps of idealization. But one of the fundamental contributions Wang saw early on in Wittgenstein's finitistic-sounding remarks on mathematics was their illuminating the character of our need for an abstractive "big jump" (in Gödel's terms) to the totality of all (finite) numbers. Even Hilbert's finitism and Brouwer's intuitionism had taken this big jump for granted.

"Anthropologism" is an interesting and distinctive view, both of mathematics and of Wittgenstein, and Wang explored its presuppositions and consequences at just the time he did his most important work in what would now be called computer science. Though he deemed anthropologism a position clearly too restrictive to be satisfactory as a comprehensive view, he felt it worthwhile to consider as an antidote to the one-sidedness of reductionist aims in logical analysis he associated with Carnap. The notions of "perspicuousness" and "feasibility" brought out the importance of practice and concreteness of understanding, as opposed to theory.

Thus, unlike Kreisel, Wang did not see Wittgenstein's remarks on mathematics as "the surprisingly insignificant product of a sparkling mind" (Kreisel [1958], p. 158). Moreover, he did not consider the later Wittgenstein to have tended toward irrationalism or anti-scientific dogmatism, as did Bernays and Gödel. Instead, Wang drew out the constructive philosophical significance of Wittgenstein's notion of perspicuousness, both for the philosophy of mathematics and logic and for philosophy as a whole, interpreting it in terms of a notion of humanly "feasible" actions or computations, i.e., those that can "actually be carried out and kept in mind" ([1958a], p. 474). Wang's contributions to our understanding of

Wittgenstein's thought are in this respect more substantial and wide-ranging than Bernays's or Kreisel's were. He aimed to explore the practical and philosophical consequences of Wittgenstein's remarks by situating them within a wider philosophical frame, where their attractiveness and commitments could be clearly understood.

Wang admired Wittgenstein's deflation of forms of logicism that construe it as offering a complete semantic analysis and/or grounding of arithmetical knowledge. In fact he agreed that no systematic theory of mathematical reasoning can ground, in any interestingly fundamental sense, our everyday knowledge of elementary arithmetical facts. The key notion he used in forwarding this criticism is that of "perspicuousness."[25] As early as his [1958a, pp. 469ff.] he wrote:

> When a reduction gives the impression of being of profound philosophical interest, there is reason to suspect...some trickery. The talk of logical foundations is misleading at least on two accounts: it gives the impression that number theory and set theory do not provide their own foundations but we must look for foundations elsewhere, viz., in logic; it implies that the grand structure of mathematics would collapse unless we quickly replace the sand underneath by a solid foundation. Neither thought corresponds to the actual situation. Indeed, if we adopt the linear mode of thinking to proceed from the logical foundation to the mathematical superstructure, there is surely something glaringly circular in the mathematical treatment of mathematics itself which makes up mathematical logic...
>
> The basic circularity suggests that formalization rather than reduction is the appropriate method, since we are, in foundational studies, primarily interested in irreducible concepts.

[25]German terms in Wittgenstein associated with this notion are *Übersichtlichkeit*, *übersehbar*, *durchsichtig*, *überblickbar*, *anschaulich*, and *einprägsam*. These terms are used to modify presentations, proofs, and models, though differently in different contexts of discussion, especially in mathematics and philosophy. Wang and I discussed the usual translations into English in the early 1990s ("perspicuous," "synoptic," and "surveyable"), and he continued to use these terms, especially "perspicuous," as central to his understanding, both of Wittgenstein and of philosophy itself. The connections with understanding ("taking in") and mastery in a practical sense were of central importance to Wang here, but also the epistemic "intuitive" element in the sense of commanding a "firm" grasp of a concept ([1991?], p. 124). See note 28.

Of interest here is the contrast Wang draws between "formalization" and "reduction," and his focus on "irreducible" foundational concepts. Wang tended to stress a series of contrasts between "conceptual" ("scientific," "theoretical") and "technical" ("practical," "intuitive") knowledge, treating the contrasts themselves as forming either a continuum or a dialectical pairing. Wittgenstein was always placed on the "practical," "intuitive," "concrete" side of the pairing, implying that his philosophy is fundamentally flawed in failing to do justice to the need for speculation, systematicity, truth, and theory. This Wang certainly believed.[26]

It is striking that in this early period Wang grouped under the broad term "formalization" a very wide array of phenomena not usually classified with the term: mathematization, conceptualization, but also all verbalization of thought and idealization. "To put thoughts in words or to describe a particular experience involves formalization of intuition," he held; and "it is impossible to formalize without residue the complete intuition at the moment" ([1955e], p. 231). This breadth of usage, historically indebted to Brouwer and the intuitionists, turns also on the idea that formalization offers a "translation" rather than a "reduction" (cf. [1958a], p. 470). Here are crucial indices of Wang's self-conception as a philosopher. They mark points at which he departed from Wittgenstein, whose broad conception of language included, e.g., the particular samples and paradigms used ostensively in the teaching and use of language. For Wang, Wittgenstein's later investigations of the variety of ways in which we might conceive the expression of thought in language were investigations of "formalizations," but not critical explorations of the notions of "language," "residue," "intuition" or meaning as such, as most readers would hold. Wang took the "dialectic" of the formal and the intuitive, the theoretical and the practical, the idealized and the intuitive, to be crucial for Wittgenstein, and then he adopted these *Leitmotifs* his own (LJ, chapter 7.1).

And yet in his early, most explicitly Wittgenstein-influenced essay [1961b], perhaps his overall finest as a single piece, a complex treatment of perspicuousness emerges through analysis of actual logical and mathematical practice, including the type of applica-

[26] Wang [1987b] states that the "anthropic" element in Kant's and Wittgenstein's philosophies of mathematics does not allow for the concept of arithmetical truth (p. 90).

tions of logic in computer proofs which Wang had himself made.[27] The question whether and in what sense *Principia Mathematica* might be fully formalized became for Wang an actual question, and the program he devised to derive some of its theorems a vindication of the usefulness of Wittgenstein's perspective on mathematics, which Wang proposed dubbing "praximism." The essay [1961b] is a subtle, prescient and informative piece of philosophical writing, laced with examples and suggestions for further thought. Wang is often quoting from or directly alluding to remarks of Wittgenstein (never with citation) while critically reflecting and commenting on their merits and limitations, often by confronting them with examples. Especially impressive is Wang's deft distinction-drawing in thinking through the interplay between logic and mathematics. Set theory is classified as mathematics, logic as a matter of breaking proofs down into small steps; "surveyability" and "perspicuousness" attach to the concept of proof, but to proof in practice, and not as necessary or sufficient conditions. Wang is careful not to reduce the issue merely to a matter of length of proof, the reproducibility of signs, formal systems, or epistemic certainty, though each of these issues figures in his choice of examples.[28] He sees that the notion of "perspicuousness" should be tied to reflection on sense-making, value, significance, interest, and meaning, and that its use lies in resisting reductions, not forwarding a one-sided point of view. An idea that he first formulated here would remain an important part of Wang's philosophical conception for the rest of his life: in the techniques of mathematics, but not necessarily those of logic, the

[27]Wang's [1960b] reports that all the theorems of propositional and predicate logic in *Principia Mathematica* were proved in under nine minutes by the program he wrote. In this sense the *Principia* was given a precise formalization. In the early 1990s Wang said to me in conversation that this indicated to him the essential lack of conceptual richness involved in the *Principia*'s formulation of the deductive elements of mathematics.

[28]Marion [2009, forthcoming] and Mühlhölzer [2006, 2010] discuss a variety of understandings of Wittgenstein's notions. It would be interesting to compare Wang's uses with theirs, for it is less precise in emphasizing neither the visual side of the idea [as in Marion] nor the copying, reproducing pictorial idea (as in Mühlhölzer). Floyd [2000], an essay partly indebted to discussions with Wang, offers a broader use of the notion, focusing on the analogy between the perspicuousness of proof in mathematics and the perspicuousness of a presentation in philosophy.

practical and the theoretical merge.[29] But accounting for this merging requires of the philosopher careful attention to purposes, aims, and the richness of ordinary mathematical experience. Without factoring these in, misleading and dogmatic statements will and do abound. When Wang focuses on the distinction between calculation and experiment, he is taking calculation to be crucial to the logical aspect of proof, but not holding that all mathematics is calculation, or even that the heart of mathematics is calculation. The logician's idea that definitions are mere abbreviations, that decidability should be construed in terms of purely formal, explicit logical operations; that the possibility of correct interpretations of arithmetic requires the impossibility of incorrect interpretations of logic or of set theory; that the emergence of contradictions and/or lack of sharp boundaries with a concept immediately indicate a risk of conceptual incoherence it is obvious how to interpret—these ideas are given their due by Wang, but criticized. The point of emphasizing the "perspicuousness" of proofs in mathematics, then, was to avoid one-sidedness, not to promote a single point of view. It was to show how dogmatic and/or overextended the interpretation of a logician's perspective can be, but not to dismiss that perspective altogether.

Among the most important of the ideas Wang always rejected in Wittgenstein's later writings on mathematics are i) those voiced in Wittgenstein's harsh and dismissive remarks on Dedekind and set theory, with their suggestion that these parts of mathematics should be dismissed or restricted in favor of a more limited fragment based on a fully intensionalist view; ii) the idea that philosophical clarification "leaves everything as it is," dealing only with linguistic puzzles and rejecting speculation; iii) the idea that the notion of "convention" could be applied to account for at least a part of the fundamental content of mathematics; and iv) Wittgenstein's unwillingness to work with any notion of "intuition." Yet it is significant that even these rejected ideas were taken by Wang to incorporate elements of truth, and so aspects of Wittgenstein's thought that were

[29] In the concept of "effectiveness" or "computability" as analyzed by Turing, this merging was most clear, according to Wang (cf. LJ, pp. 372-73, and [1961b], p. 339). Moreover, "if a machine is to do mathematics, it is necessary that methods of logic be explicitly included" ([1961b], p. 332). This was an excellent forecast when it was written.

worthy of serious consideration or transformation into something better. In later writings Wang went on to situate Wittgenstein's remarks against the backdrop of a wider, schematic distinction between practice versus theory ("doing and being") in the philosophy of mathematics. He was thus able to see Wittgenstein early on as a critic, rather than an advocate, of 'linguistic' philosophy, one who took seriously the realm of the familiar, but nevertheless made useful reflections on meaning and language, some of which could be turned in a constructive, practical direction in, e.g., computer science. This, Wang held, shed critical light on the limitations of reductionist ways of approaching "foundations" of mathematics. Of course, taken as a comprehensive view, Wittgenstein's would be unnecessarily restrictive. But just this illustrated, for Wang, the importance of aiming for a kind of comprehensiveness that would be useful, but partial.

double quotes

Wang's "paradox," called by Wang himself "the paradox of small numbers" and an instance of the sorites paradox, was made famous by Dummett [1975]. The conceptual difficulty is alluded to, though not explicitly stated, in Bernays [1935], p. 60, and in Wang [1958a], p. 473 (which cites Bernays): there is no precise limit to be drawn between "feasible" and "non-feasible" (or "accessible" and "non-accessible") numbers, even though intuitionists proceed without regard to this limitation of idealization (or application of the induction schema). It may be stated as a paradox thus. Some numbers are, intuitively, small, and others large. If a natural number n is small, then so is $n+1$. But then all natural numbers n are small, by induction.

The use of the induction schema is not viewed as essential to the paradox by Dummett [1975], who also shows how to formulate it with observational predicates such as "red" and "apodictic"—arguably concepts we do not apply *via* inference. Dummett argues that the paradox shows that strict finitists are committed to an inconsistent pair of beliefs, and so, more generally, are any users of a language whose vague concepts display a similar absence of a clear boundary in application. When generalized to treat observational terms, or the notion of "perspicuous," as in Wang ([1991?], p. 86), it seems that there is a systematic problem given the demand for consistency: concepts that throw up sorites-type phenomena are incoherent (cf. Wright [1980], p. 137).

Strict finitists, for example, hold that the meanings of all terms in mathematics must be given in relation to constructions that we are capable of actually effecting, and depend upon our capacity to recognize such constructions as providing proofs of those statements. But they must then also hold that there are non-empty sets of natural numbers (of, e.g., those that are "small" or "apodictic") which are closed under the successor operation, but also are bounded above.

The paradox did not pertain for Wang directly to the theory of meaning, as it does for Dummett, but instead to the theory of knowledge. This, however, as we have seen, is in Wang's hands a complicated matter. His position is not to be equated with contemporary epistemicist interpretations of the paradox, which hold that there is a point where the successor step breaks down, but we do not (yet) know where it is. For Wang the paradox shows that our responses to the use of a vague predicate are to be tested against two fundamental parameters, the "formal" and the "intuitive," and reveal the one-sidedness of a demand for global consistency proofs. Our experience and our concepts cannot be fully reduced to the formal.[30] And yet this does not mean that a formal rigorization of the notion of a "feasible" number might not be of interest, as in the first-order theory of bounded arithmetic (cf. Parikh [1971], Buss [1999]).

The paradox was perhaps of more importance to Wang than Dummett suggested at the end of his [1975], where he acknowledged Wang's framing of it. For Wang the paradox shows the inherent vagueness surrounding our uses of the notions of "perspicuousness," "smallness of number," and "understanding" (cf. [1991?],

[30]In this respect Wang's attitude bears comparison to the later account of Wright [2007]. But Wang did not reject the possibility of articulating a rule that might be followed in applying vague or other predicates. Nor did he propose a special logic of negation (intuitionistic, for example) to handle the difficulty (cf. Putnam [1983], Dummett [2007]). He chose instead to reject the demand for a univocal or reductive application of a formal theory of deduction to a series of perceptual (or "intuitive" mathematical) experiences. As an historical point, Wright's prefatory remarks (p. 415) about philosophers in the 1950s generally viewing vagueness as "a marginal, slightly irritating phenomenon," attended to only by those who pursued "the amateur linguistics enjoyed by philosophers in Oxford in the 1950s" and idealized away by those favoring the use of formalized languages, do not apply to Wang.

p. 86). But it thereby shows, not the need for a theory of the meaning of vague predicates, but instead the force of a kind of Platonism about the totality of numbers and the difficulty of attempting to adhere to what is "really" intuitive given the vagaries of language (cf. LJ, pp. 212-213). It also reinforced his idea of the purpose- and interest-relativity of formalization. Thus, in general, it illuminates the importance of the concrete and the complexities of its relation to theorizing. It allows the notion of meaning to be approached with subtlety, and the distinction between confusion and contradiction to be sharpened in its applications.

Mathematical induction is a schema, a "formal" method used in proofs. It may produce "paradoxes" when applied directly to natural language, but the contradictions that arise here should not engender skepticism. They should rather lead us to reflect on the limits of idealization and the need for it. The paradox suggests that there is "a gray area between the analytic and the synthetic" (BAP, p. 128), according to Wang. The idea that we could single out the operation of adding 1 as a clear basis for getting "larger" and "larger" numbers is itself "an act of abstraction, to give form to a range of nebulous relations of order" (LJ, p. 213). The use of the paradox is to illuminate that this is a specific step, and one that is not self-evident. It is also to unmask the misplaced nature of demands for proofs of absolute perspicuousness and/or consistency in the general foundations of knowledge or meaning. FMP attributes to Gödel an appreciation of this point (p. 44).

4. Factualism and Conceptualism

By "facts" Wang intended something "gross," not something metaphysical (FMP, p. 3). Wang's "factualism" contains more than a touch of Moore, at least in the sense that Wang believed that philosophy should not be allowed to override all that is familiarly taken to encompass "what we know," and that reminders of what we know serve a crucial philosophical function. Yet unlike Moore, Wang stressed how difficult it is to do justice to what we do know, and he embraced a general fallibilism, even while wanting to hold that we possess at least some knowledge that may be regarded as unconditional. He also insisted that facts may be meaningfully understood

and discussed only from within wider structures of their interconnections, which he was willing to call, in a very broad sense, "logical." His philosophy contains nothing of the inveterately plain man or the particularist; Wang was a "conceptualist." Like Dedekind, he believed that our knowledge of number, as autonomous, cannot possibly be seen to stem from any particular sensory experience or capacity for empirical knowledge, but springs from and concerns concepts or intensions alone, and these insofar as they figure in law-governed structures of pure thought, rather than knowledge of the physical world or the structure of space as we perceive it.[31] Of course, Wang appreciated the point that "law-governedness" is a contentious notion in philosophy, where it receives a variety of interpretations. His interest in Wittgenstein partly turned on his efforts to grapple with this contentiousness, though it is not clear exactly how he resolved it in the end.

What is clear is that Wang took this broad form of Dedekindean anti-empiricism and revision of Kant's philosophy of mathematics as a starting point. His reasons for resisting empiricism thus lay in his knowledge of modern mathematics, and the seriousness with which he took the challenge of incorporating into proof reasoning about infinitary objects. Like Dedekind, Frege, and Gödel, Wang took the laws of number to arise, if not autonomously, then independently of any particular sensory experience or form of possible human sensation.

Wittgenstein would have agreed with this conclusion, but he argued for it very differently, and the contrasts with Wang's methods of argumentation are instructive. First, Wittgenstein questioned the idea that Dedekind's construction of number forces the philosopher into any particular position on the nature of mathematical objects generally. Second, he argued by elementary example, devising numerous pictures and images which fail to secure on their own a significant mathematical application or proof (cf. [1978], Part I). His general point is to indicate that no mental account of representations, perceptions, or impressions is able to account for our knowledge here—not, as Wang held, that appeals to inner mental life are, ideally, to be banished from philosophy altogether (cf. [1991?], pp. 170ff.; LJ, p. 356). Wittgenstein's point involves neither re-

[31] Wang [1957a] is still a model of pedagogical clarity about Dedekind's contributions to the theory of number in their historical context.

ductive behaviorism nor reductive appeal to sociological facts, but instead a philosophical generalization of Frege's context principle: a picture or image or impression only has meaning within an articulated proof or systematic structure of articulated procedures.

Yet Wang, suspicious of appeals to language, was inclined to find this generalization of a Fregean approach inadequate.[32] This was partly because he aimed to bring logic and philosophy into accord with psychology. In his latest work, Wang felt that "what is at stake" in characterizing the nature of logic "may be construed as a determination of the universal receptive scheme of the human mind, which is to capture the underlying intersection of the diverse schemes actually employed by human beings, which are presumed to be potentially convergent" ([1996a], p. 366). The propositions of logic, he wanted to agree with Wittgenstein (in a remark he quoted from *On Certainty* (§401), "form the foundation of all operating with thoughts" (LJ, p. 367). But Wang meant this literally, as part of the foundations of psychology. Wittgenstein was by contrast more likely to regard assemblages of remarks about mental terms as part of an exploration of the effects of grammar and everyday speech on theory.

insert ")"

A focus on concepts from within a logical analysis of their role(s) was, however, attractive to Wang. And from within that focus, he was content to speak of "intuition," especially where he focused on the "perspicuous." Though he appreciated Wittgenstein's efforts to find "perspicuous," easily grasped renditions of fundamental philosophical problems, he always read him in a behavioristic vein, just as had Bernays [1959]. This created a kind of looming tension, if not inconsistency in his reading of Wittgenstein. Wang knew that Wittgenstein considered talk about "intuition" as misleading and likely to be harmful. But he felt that Wittgenstein relies everywhere on "introspection" and the appeal to (his own) "intuitions," and so cannot deny their presence ([1991?], p. 171).

Where in the end there was some apparent meeting of minds was in Wittgenstein's willingness to treat philosophy as an investi-

[32]For this reason Wang rejected Tait's deflationary idea that Wittgenstein's attack on the Augustinian picture of reference justifies a conception of the Platonism of mathematics as a "truism" (cf. Tait [2005], chapter 3; Wang [1991?], p. 2). His Platonistic reading of Frege, indebted to conversations with Palle Yourgrau, shaped his responses to the issues in [1991?].

gation of the very notion of "concept" itself, as well as apparently fundamental philosophical concepts. Wittgenstein's later uses of language games allowed for a flexible and fallibilistic approach to the fundamental notions that attracted Wang.

Wang was explicit that, in following Gödel's idea of "concept," he did not want to prejudge the issue whether there are concepts, or rather in what sense there are concepts ([1991?], p. 161). Attention to what we in fact do helped him make sense of the idea of such an approach. And perhaps it allowed him to proceed with conceptualism although the foundations of this position were not fully clarified. Wang never worked out what he regarded as a satisfactory account, either of the notion of "concept" or of "intension": he was well aware of the plurality of views expressed in this regard and shied away from any definite account. Here too he resembles Wittgenstein, who was an inveterate critic of extensionalism insofar as it is regarded as a sufficient or fully adequate point of view, while at the same time lacking any firm account of the notions of concept or meaning.

Wang certainly did not think there was any such thing as an "intuitive" grasp of the notion of concept itself, even if certain particular concepts, such as that of computability, could receive "more or less definitive" analysis by means of "intuitive" representations (cf. [1991?], p. 139).[33] He seemed to think that any play in the notions of intension and concept could be narrowed down by beginning with at least the broadest features of what is taken to be known. And he read Wittgenstein, with his notion of language game, as having agreed here. An interesting feature of his emphasis upon looking at mathematical practice is that it tempered his understanding of Gödel's view of concepts. On Wang's reading, Gödel's "Platonism" and conceptualism are not brute, but more holistic and fallibilist (cf. Shieh [2000]).

Wang's fallibilism and commitment to a broad, pluralistic conception of logic gave him at least the beginnings of a reply to the

[33]To justify this claim Wang quotes from Turing [1937], where Turing says (§9) that he makes a "direct appeal to intuition" (Wang [1991?], pp. 139 ff.; cf. FMP, pp. 81-5, 90-95). I do not think, however, that Turing had in mind any notion resembling Wang's, but a much more down-to-earth idea of "common sense," and in fact one indebted to Wittgenstein. On this see my [forthcoming a and forthcoming b].

add space

obvious objection, How are we (and who are we) to get at or determine what the facts *are*? As Wang saw it, each individual inevitably operates with his or her own range of immediate reactions, always wearing subjectively tinged glasses, and in one way this is the starting point of philosophy ([1996a], p. 355). Rejecting Wittgenstein's distrust of the notion, Wang called these "intuitions." He considered one of the most important aims of philosophy to be the development of better immediate reactions in and for all of us. And for this philosophical task to have sufficient material to begin, Wang held that there must be a certain degree of universal commonality of structure to these intuitions. Modelling his account on the *Grundlagenstreit*, Wang held that familiar facts of elementary arithmetic ("$2+2=4$") form an important example here. Here the influence of Bernays is felt: Bernays had also maintained that knowledge of elementary arithmetic is of intuitive, or special evidential origin. But once again the philosophical frame Wang would ultimately bring to bear on the idea was generalized. "Immediate apprehension," or "intuition," could be "sensation, knowledge, or even mystical rapport," according to Wang (LJ, p. 372).

5. The notion of "intuition"

Wang's notion of "intuition" is perhaps the greatest stumbling block for readers trained in twentieth century Anglo-American philosophy, and it certainly marks the place where I myself find his philosophy most difficult to understand. He was well aware of Wittgenstein's and his likely readers' antipathy to the notion. Sometimes he would insist that the notion of "intuition" could be eliminated without loss from Gödel's or Wittgenstein's philosophies. Sometimes he used the notion in an everyday way, as a mathematician does when looking for an "intuitive" way of seeing a proof (or parts of a proof). Nevertheless, Wang's particular uses of the notion are quasi-systematic and central to his own thought and to his accounts, not only of Wittgenstein, but other philosophers, including Gödel.

The traditional European division of philosophy by way of a distinction between "theoretical" and "practical" branches of the subject was foreign to the explicit organization of faculty structures and specializations in Anglo-American philosophy of Wang's

day, but remained widespread in many other parts of the world and in American curricula influenced by the German model. This distinction was natural for Wang, although as he used it, it was being investigated, sharpened and extended, encompassing at times Eastern as well as Western philosophy. It did not correspond, for example, to a distinction between ethics and political philosophy versus metaphysics or philosophy of science. Instead, Wang often articulated its significance in terms of a "dialectic" or continuum or pair of forces, rather than an overarching dichotomy. He saw this dialectic as a driving force in the history of science given the importance of mathematization and idealization. But the distinction itself expressed a conviction that conceptualization always leaves something out. Wang's notion of "intuition" is not, however, a theory of non-conceptual content: the degree and extent to which "intuition" can become conceptualized is left open, as is the degree and extent to which concepts can be concretely presented in "intuition." Wang lacked a theory of perception, basing his philosophy on particular examples of our grasp of mathematical truth and on what Wang took to be everyday experiences of meaning, knowledge, and life.

Wang read Wittgenstein as holding, in his early philosophy as well as later on, that something intuitive always remains after conceptualization. This is Wang's understanding of the distinction between showing and saying, and it shaped his understanding of Wittgenstein, earlier and later, from his very first writings. In his review [1945] of Russell's *Inquiry into Meaning and Truth* he was already suggesting that philosophers should seek "possible sources of knowledge besides language" ([1945], p. 147 of translation). And in his latest writing he takes the novelty of Wittgenstein's approach to have been that he "begins and ends with the perceptual immediacy of our intuition of the actual use of words in a given situation," an approach Wang saw as a "way of pursuing the traditional quest for certainty in philosophy" (LJ, p. 329).

The later Wang takes there to be a "familiar gap" between "seeing and saying," between the understanding of a thought and the clarification of its meaning, its grasp vs. its communication. Because of the "subjective and fluid character" of seeing, philosophers can be driven to assure communication by "a direct appeal to the connection between words and deeds, to bypass the interference

from passing through the mental" (LJ, p. 356). This bypassing, he believed, is illustrated in Wittgenstein's later thought. As to philosophical method, Wang emphasized that "saying...is only one way of communicating":

> Literature, for instance, tries to show the universal by saying the particular; similes and metaphors show one thing by saying something else; action, tone, and gesture can be shown in a drama or film but they can only be said or told in a novel... (LJ, p. 356)

This mode of communication was, he rightly felt, of central importance to Wittgenstein—although it is questionable whether Wittgenstein required a theory of non-conceptualized content to forward it. In his latest writings on Wittgenstein and Gödel, the connection between aesthetic experiences, intuitive presentations, and philosophical method are stressed by Wang. In fact, the appeal of conceptual thinking as such is its promise of finding for us "a perspicuous view of a larger whole" ([1991?], p. 172). But Wang seemed to feel by the end that philosophy required literary formulations to achieve its full potential to communicate concepts.

Thus unlike Bernays, whose use of the notion of "intuition" had a more specifically Kantian cast, Wang's notion is tied, at least when connected to Wittgenstein, to something like "respect for the particularity and immediacy of everyday experience." In epistemic terms, Wang defined it as a "summary 'perspicuous' grasp of massive details" ([1991?], p. 124). In this way, partly inspired by Wittgenstein, he followed Bernays's and Gödel's idea that Kant had been wrong to draw a sharp distinction between concept and intuition, but he held on to the ambition, common to Husserl, Gödel, and Wittgenstein, to use philosophy to clarify experience of everyday life. Wang put forward here a view intended to contrast with forms of conventionalism about analyticity and meaning, such as Carnap's, that he despised.

In his later writings Wang emphasized that Wittgenstein's *On Certainty* had taken up the subtle challenge of beginning with the actually (not the possibly) familiar, bringing it into the orbit of our understanding of logic itself. Wittgenstein, he believed, had made substantial progress beyond Moore in sketching how general philosophy might articulate a very broad conception of the logical as both

adjudicative and able to unearth the structure of what is known unconditionally. This went well beyond a doctrine of "common sense" that involves the risk of foot-stamping. Wang took Wittgenstein to have successfully refined, especially in *On Certainty*, our understanding the problem of where philosophy should begin, and where it should end. The answer, as Wang understood it, was in logic, conceived as a broadly adjudicative inquiry. But Wittgenstein's examples ("Here is one hand", "This is green") displayed a practical, concrete, empirical or "intuitive" element in knowledge, hence in logic itself. Factualism could be seen to incorporate this method within itself, without becoming dogmatic. Our capacity for clarity and knowledge of everyday facts could be a starting point, but be treated, on any given occasion, as fallible.[34]

Wang's use of the notion of "intuition" also gave him a way to use Wittgenstein to resist and delimit Husserl's approach. There are a variety of notions of rigor in philosophy, and not all of them are to be equated. Wang regarded his own notion of "intuition" as a useful counterweight to Husserl's phenomenology, whose impact on Gödel he did not see as uniformly salutary. Husserl, for Wang, hoped to build up objectivity from an immediately given, subjective, inner experience, but this Wang deemed wrongheaded and in fact "disagreeable," both because it failed to begin with what we know, intersubjectively, at the start, and because it encouraged, he believed, conceptions of infallible intuitive insight and experience (BAP, p. 37). Wang did not believe that philosophy could aspire to become a "rigorous science" in the sense of a system (FMP, p. x). Here he differed with readers of Husserl, such as Føllesdal, who commended Husserl for his focus on problems of perception and his fallibilist conception of intentionality (LJ, pp. 350-352). For Wang, empiricism was a non-starter in light of the fundamental importance of the philosophy of mathematics and there was no general theory of intentionality apart from an account of the nature of logic. Percep-

[34]The view might be seen to have affinities with those enunciated in McDowell [2011], in terms of identifying a place in the space of possible positions for the manifestation of perception as a capacity for knowledge. Neither McDowell, nor Sellars, whom McDowell is representing, relied on a wider frame of intuitive vs. conceptual knowledge, although Sellars' idea of "the manifest image" bears an interesting comparison to Wang's later ideas. Yet neither Sellars nor McDowell are, like Wang, attempting to integrate an account of our knowledge of infinitary objects within the orbit of their theories.

tion was an interesting phenomenon for psychologists, but not for philosophers. Here too we see Wang's affinities with Wittgenstein.

"Intuition" is an attempt to express whatever the formal, theoretical, conceptualized misses or fails to represent. Wang admired Wittgenstein's criticisms of Russell on the notion of "self-evidence" in logic, and took to his resistance to logicism regarded as a program to eliminate the use of intuition in arithmetic. For Wang, intuition is not determinative, and it carries no form of justification with it. There is no gap in its application to the world or the facts. But it is not always to be regarded as less reliable than proof, much less as merely psychological.

Still, "intuition" is intended to fit within a general account of human mental receptivity as such. And here there does seem to lurk a genuine difficulty with Wang's views. Sometimes a formal principle such as mathematical induction is exactly what gives us an "intuitive" perspective on a property, and it is the particular cases that seem, collectively, obscure. Sometimes what seems transparently grasped through purely formal means is not—as when we see that the principle of mathematical induction is, perhaps surprisingly, actually logically equivalent to the pigeonhole principle. It is hardly the case that the formal principle never lies on the side of the practical, the intuitive, the concrete, or the particular. The boundaries of Wang's dichotomies are permeable, and in fact liable to shift: they are useful, but occasion sensitive. Thus the "dialectic" of the formal and the intuitive is not explanatory but parasitic on particular cases. And this has methodological importance for the philosopher. Wang held that "felt psychological certainty should be appealed to in a non-parochial way" ([1991?], pp. 79-80). But one might better hold, with Wittgenstein, that our handle on psychological concepts and semantical theories, even on the notion of proof in mathematics, requires the parochial (cf. Travis [2006]).

Yet the notion of "intuition" has a broad meaning in Wang's hands. It is not to be identified with anything more, or anything less, than the immediate, considered *appearance* of truth (to us), or certainty, rather than truth itself. It thus serves as an entering wedge for the beginnings of epistemology. As in Wang's readings of Gödel, "intuition" is, though immediate, fallible, liable to revision in light of reflection, and attaches to judgments, rather than to objects as such. In a sense, Wang's uses of the notion of "intuition"

stick closely to the Kantian framework, within which the notion is tied, both to immediacy of representation and to the instantiation or exemplification of a concrete particular.

The purpose of philosophy is, however, tied directly to the notion for Wang: philosophy is to explore and to structure what is shared despite differences in our "intuitions," to make immediate, "perspicuous" sense of the rich but complex experiences of life. To achieve this one must also use and devise "intuitions," perhaps even aesthetic presentations, to communicate concepts. Mathematics, especially the familiar areas of elementary arithmetic on which Wittgenstein focused, offer a starting point from which the "factualist" may begin by reflecting on the nature and character of at least nearly universal agreement, and to titrate and measure differences as they appear. Positions in the philosophy of mathematics do not of course exhaust mathematical knowledge. But philosophy, even philosophy of mathematics, is not complete without providing something "intuitive," concrete, and clear that is extramathematical.

Wang thus had no use for direct forms of Platonism which picture us intuiting objects of knowledge directly with the mind's eye. Here he was certainly sympathetic to Wittgenstein's doubts. His use of the notion of "intuition" was not accompanied by any theory of perception. Yet there are tensions in his thought in this regard. Wang tended to think of Wittgenstein's notion of aspect perception as a matter for the psychology of vision rather than a theoretically basic phenomenon that could illuminate the notions of "intuition," "understanding," "concept," or "perspicuousness." Thus his reading of Gödel reflects a broadly logicizing view of "intuition"—but only so long as one understands that Wang's notion of the "logical" covers more than formal or symbolic logic, encompassing a variety of methods of analysis and elucidation (cf. Shieh [2000]).

In Wang's philosophy there is a constant adverting to the uncomfortable idea of a mystical or unutterable content that eludes conceptualization altogether. While theorists of perception who advocate the idea of non-conceptual content tend to rely on psychological data, avoiding commitment to such a paradoxical treatment of experience, it is not clear that Wang could do so, insofar as he was offering an account of philosophical experience itself. To him this risk of mysticism was all to the good, and connected his phi-

losophizing with certain strands of Buddhism, as well as Gödel's thought. But it seems clear that at least Wittgenstein, by contrast, had far greater faith in the effort to use our means of representation to accurately reflect on, and philosophize about, the structure of our thoughts.

It is interesting that since Wang's death there has been a revival of interest in the structure of appeals to "intuition" in analytic philosophy, as conceptualism and the development of general metaphysics have been revitalized. Non-conceptual content is discussed as part of philosophical method much more frequently than during Wang's lifetime. Within epistemology, the relation of "knowing how" to "knowing that" receives much more attention than heretofore. Perhaps on these scores Wang was ahead of his time. It would be interesting to analyze in another place how his appeals to intuition compare with those of recent theorists, and to see how much farther they have gotten than he. This would be the real way to assess the worth of Wang's philosophy.

6. Philosophy as a Quest for Perspicuous Objectivity

Throughout Wang's philosophy, as we have seen, the most central philosophical problem is what might be termed "the problem of disagreement," the (partly practical) problem of how philosophy might go about distilling or "decomposing" a range of possibly universally held, agreed upon judgments from the fact of multiple disagreements between individuals with different immediate reactions, feelings, histories, experiences, and "intuitive" perceptions. He was less interested in what might be termed "the problem of agreement," which also bothered Wittgenstein, namely, what if the agreements we express cover up profound misunderstandings?[35]

Capitalize "W"

The epilogue to Wang's LJ (Chapter 10) assesses what progress he thinks was made in twentieth century philosophy on this problem, and assesses prospects for the future. It is unsurprising that

[35]If one sees this as a problem for Wittgenstein, then Wang's picture of him as a conventionalist or ordinarily language contractualist about the notion of "agreement" is unattractive. See Wittgenstein [1980], §1107, for an explicit place where it is asked, of a perceptual case, "What if this full agreement ("It's like that for me too!") were based on a misunderstanding?" See also [1953], §241, also pp. 226-27 and 230.

Wittgenstein, whose notion of "agreement in judgment" is notoriously central but problematic, would play a central role in Wang's thoughts here. For Wang, the later Wittgenstein saw the "natural" inclination to generalize as the main source of confusion in philosophy, and Wittgenstein's recommendation, a novel one, was to insist that philosophy begin and end "with the perceptual immediacy of our intuition of the actual use of words in a given situation" (LJ, p. 329). This methodological way of understanding Wittgenstein's notion of the "perspicuous" came more and more to the fore in Wang's later writings, as it was taken by him to show the importance of concreteness and practice for knowledge.

In fact in the unpublished manuscript [1991?] Wang defines philosophy as "the quest for (comprehensive) perspicuous objectivity" (p. v). By focusing on a "dissection" of conceptual objectivism in mathematics, he hoped to point toward a "flexible frame" for philosophy. And his initial idea was that the "less audacious parts" of the philosophies of Gödel and Wittgenstein might be excerpted, seen to be compatible, and used to erect that frame. In LJ there is much overlap with this manuscript but an important difference, in that in this later, more mature work, the comparison between Gödel and Wittgenstein is absorbed into a wider book framework, reduced to an explicit chapter and then folded into more methodological remarks in a final epilogue.

Wang believed that philosophy progresses most fruitfully by clarifying, presenting, and resolving differences intuitively and perspicuously, though not by restricting itself to the actual uses of words. The aim should be to alter the interplay between practice and theory while not promising forms of certainty that cannot be reasonably achieved. Philosophy should then ideally begin with what is familiar and agreed upon, and move from there, cataloging, analyzing, and decomposing disagreements as they emerge. And it should be held, at least eventually, to the aim of altering everyday life, as well as theory.

7. Wang's style and his final motto

In addition to the subtlety of overlaying Wang's aims upon Wittgenstein's, there is the challenge of penetrating Wang's own style,

which, overlaid on Wittgenstein's—itself hardly a usual or clear one—is not easy to characterize.[36] Wang's philosophical goal was to draw a portrait of a philosopher or view and then to identify which of its features he found more, and which less, attractive, thereby revealing his self, freely admitting from the start that his own immediate "intuitions" and philosophizing shaped his reactions. Wittgenstein's later method was to react to his own (and others') remarks in a multilogue, polyphonic interlocutory fashion, carefully orchestrated for literary effect, in order to represent the nature and character of philosophical thought. When arguments are found in Wang, they are deductions from his observations, or relative comparisons of the interest and correctness of different responses, concepts, or principles. The tone of the whole is always tentative, pluralistic, and, as Parsons [1998] has emphasized, synoptic rather than systematic in aim.

Yet Wang's literary ambitions became stronger over time, and importantly shape his final writings. These bear an important relation to his ideas about "intuition" and his interest in Wittgenstein, whom he came to regard as "art centered" rather than "science centered" in his conception of philosophy (BAP, p. 75). Wang felt the literary effects of Wittgenstein's writing were not irrelevant to the content of his philosophy. I think we can assume that Wang felt the same way about his own books.

In the manner of Walter Benjamin,[37] or perhaps better, of his Chinese forebears, Wang often proceeded by arranging quotations, without interpretation, in an effort to draw out the reader's reflection and response, thereby showing, but not himself directly stating. Wang's attachment to this way of writing philosophy indicates one reason he may have found Wittgenstein's later interlocutory style less off-putting than many philosophers do. He liked its intuitiveness, and he modeled his own writing on its way of inviting reflection without necessitating dogmatic adherence. It showed rather than stated, was intuitive rather than theoretical. Wang himself sought to probe philosophical systems dialectically and intuitively, without an overarching theory. He was explicit that he considered his own approach to be "more effective" than Dummett's in the quest for clarification of differences and distinctions of meaning ([1991?],

[36]Here I agree with Parsons [1998] and Shieh [2000].

[37]This is recounted by Arendt in her introduction to Benjamin [1969], p. 4.

p. 131).

The device of quotation, used by Wittgenstein at the opening of *Philosophical Investigations*, was one Wang admired and often used, both when discussing Wittgenstein and when attempting to characterize his own views. His mottos are often useful clues to his thoughts.

As early as [1958a], when introducing anthropologism, Wang quotes from Butler the saying Wittgenstein once considered taking as a motto to the *Philosophical Investigations*: "Everything is what it is, and not another thing," thus endorsing the importance of attending to particulars and their differences from one another.[38]

He closes chapter 2 of BAP with an interpretation of the end of the *Tractatus* and Wittgenstein's form of "mysticism" by drawing a comparison between the early Wittgenstein and the Buddhist Wu Yunzeng, both of whom are, for Wang, "more concrete" than traditional philosophers like Plato, Aristotle, conceptualizers of Tao, and Spinoza when it comes to thinking about traditional ideals of an unattainable limit (BAP, p. 100). He does this by means of a quote from Wu Yunzeng.

The structure of quotations and mottos in LJ is perhaps most illuminating of his style. In Wang's opening motto to this work, he pairs a quote from Hegel about objective logic embracing the "wealth of the particular" with a quote from Lu Jiuyuang stating that the minds of all sages from the East and the West "have the same kind of intuition." The introduction that follows this allusion to universalistic particularism opens with another contrasting and interlocking pair of quotations, this time from Gödel and Wittgenstein, concerning generalization in philosophy. Gödel's begins: "Philosophers should have the audacity to generalize things without any inhibition: go on along the direction on the lower level, and generalize along different directions in a uniquely determined manner." Wittgenstein's runs: "Hegel seems to me to be always wanting to say that things which look different are really the same. Whereas my interest is in showing that things which look the same are really different."

Wang's epilogue, or final chapter, opens with four quotations

[38] As Wang knew, Wittgenstein had considered for the motto of the *Investigations* the saying from Lear, "I'll teach you differences!" (Monk [1990], pp. 536-37).

concerning knowledge and understanding, designed, it seems, to show the points where his reflection was directed at the end of his life. One is from Confucius's *Analects*, one from Bernays's essay on rationality [1974], one from Wang's own *Beyond Analytic Philosophy* ([1985a], quoted above as my motto for this essay), and the last is from Rawls's *Political Liberalism* [1993], a remark about the method of "reflective equilibrium."

Wang's final motto for his own corpus, taken as my own motto in this essay, is obviously intended as a critical comment on Kant's characterization of the questions that had unified his own critical project, namely "What can I know? What ought I do? For what may I hope?"[39] To orient us with respect to Wang's overarching philosophical aims at the end of his life, let us compare and contrast the structures of these two quotations.

The first and perhaps most important thing to note is that Wang offers a "classification of what philosophy has to attend to." The suggestion is that philosophy will have to go outside of itself in order truly to know itself: the subject is not autonomous, or self-authenticating.[40] Wang takes there to be extra-philosophical matters that no philosopher can or should ignore or repress. So the central focus and concern of philosophy is not itself purely philosophical, as, at least arguably, was Kant's. Wang does not offer a list of questions bound to generate ultimately unsolveable conundrums and a focus on a transcendent, "higher" world of hopes and dreams. He also does not claim that his classification is exhaustive of any interests of reason or philosophy, as does Kant: he frames a "guiding principle" or rule of thumb, rather than an *a priori* characterization of superordinate challenges or questions. Finally, Wang's use of the idea of "doing justice" is intentionally amorphous and flexible, yet central to his own conception of philosophy, which in his own mind bore, even if indirectly, on ethical, moral, and political questions. His philosophy aimed to cope, practically and intellectu-

[39]"All the interests of my reason, speculative as well as practical, combine in the three following questions: 1. What can I know? 2. What ought I to do? 3. For what may I hope?" (*Critique of Pure Reason* A804-05/B 832-33.)

[40]As Wang elsewhere wrote, to know mathematics we must go outside mathematics to the world, by way of our experience. Thus philosophy of mathematics is not self-sufficient as a philosophy. Ending the manuscript in which he hoped to compare Wittgenstein and Gödel, he quoted Gödel: "in order to know what mathematics is, one has to know what the world is" ([1991?], p. 175).

ally, with the fact that there are incompatible perspectives, as well as the facts of reality as science knows them.

Like Kant's characterization of his philosophy, Wang's is tripartite. The first element of his guiding principle concerns knowledge. His "substantial factualism" and "conceptualism" aimed less at a limiting of knowledge, or discussion of possible knowledge, than at the faithful and clear exposition of what we *do* know. This shifts the modality of Kant's question in a way clearly appropriate to the contemporary world in which the quantity of information and knowledge increases faster than the ability to survey it and lay down *a priori*, universal principles for its justification. It also states a constraint on philosophizing that Wang felt Wittgenstein had come too close to violating: never restrict what we know in light of a philosophical theory. Of course, this constraint also resonates with a part of Wittgenstein Wang came to emphasize, especially in his later writings: the importance and difficulty of beginning with what is familiar, shared, and actually given to us.

What of the second element of Wang's "guiding principle"? Here Wang suggests that philosophers replace Kant's call for specification of ethical action with an exploration of "what we believe." The problem of what in fact we do believe, the task of openly specifying where and how we disagree and agree, was central to Wang's conception of philosophy, in practice and in theory. He was inclined to think that twentieth century history, including both politics and the history of science, had shown the importance of open exchange and discussion as prior to action, as well as the limits of philosophy treated as a purely foundational enterprise. Finally and thirdly, there is Wang's interest in "how we feel." This echoes Kant's idea in his (third) *Critique of Judgment* that the bridge between theory and practice in the critical philosophy may built through attention to aesthetics, emotion, and art. While Wang's writings about psychology and aesthetics are neither extensive nor especially penetrating, he did take the subjects seriously, and was interested in the "queer resemblance" Wittgenstein once claimed to have perceived between an investigation in mathematics and one in aesthetics.[41]

[41] In fact, Wang ends his [1991?] with a digression on the "queer resemblance" Wittgenstein noted between an investigation in philosophy ("perhaps especially in mathematics") and in aesthetics (Wittgenstein [1998], p. 29e; MS 116,56 in [2003]), and a comparison with Gödel's views. In the early 1990s I frequently

Wang read persistently in psychology, viewing it as an open and developing science to which philosophers should attend; he would surely have had much to say about the recent rise of "experimental philosophy," and probably not have wanted it dismissed out of hand. In general Wang had no use for an eliminative view of psychology in philosophy, including philosophy of mathematics. This is evidenced by his occasional remarks about Freud's importance and his knowledge of, and praise for, books that concern the psychology of invention and discovery in mathematics. His interpretation of Wittgenstein, insofar as it took to be fundamental the idea of aspect perception as part of the notion of *perspicuousness*, turned on a notion that in the eyes of a Fregean, or for that matter a contemporary reader of Wittgenstein on rule-following, is suspiciously phenomenological, a potentially subjective or quasi-psychologistic admixture out of place in discussions of logic and mathematics. For Wang, it was admirable that Wittgenstein had toyed with the idea of taking his later writings to offer an investigation of the foundations of psychology, and important that he had been knowledgeable of at least some psychological results and experiments.

In general Wang took appearances, feelings, lived experiences, emotions, strikings, and so on as fundamental data in philosophy to which we must try to do justice. Against Wittgenstein, he called these "intuitions." Wang had a strong tendency to biographize, especially in his later work, but it must be said that the idea of setting a particular, exemplary philosophical life before the reader, familiar now in certain branches of virtue ethics, was part of what he felt a responsible historian of philosophy should do. Unlike most analytic philosophers, for Wang one test of knowledge is its usefulness for life.

8. Final thoughts

An historical note is in place here. I was a bystander to Wang's interest in Wittgenstein during the period 1990-1994, when I had the great pleasure of spending once a week, sometimes more, in

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spent time in the enjoyable company of Wang and his wife Hanne Tierney, an accomplished artist (`http://www.hannetierney.com/artist-bio.html`). Wang was inclined to consider artists with as much respect as he did scientists.

philosophical conversation with him at Rockefeller University. He approached me with the idea of discussions just after my arrival as a young assistant professor at C.U.N.Y., asking for comments on section 6 of his [1991b], and knowing that I was writing on Wittgenstein's remarks on the foundations of mathematics. During this period he was planning a book, variously titled at various times, in which the comparison of Gödel and Wittgenstein would take center stage.[42] He was interested in learning more about the later Wittgenstein, and in discussing the present state of philosophy.

What was remarkable, as I reflect back on our conversations, was the tenacity, energy, supportiveness, and openness of Wang as a teacher. He was a good teacher by being a good listener. He had specific scholarly questions and his own body of writing he was working on, and he was interested in hearing what someone much younger than himself would say about his ideas and their own.

I am personally and intellectually greatly indebted to Wang's example, as a teacher, a colleague, and a philosopher. My own work on Wittgenstein has been deeply shaped by his, and his encouragement for that work, undertaken while I was a junior faculty member, was unstinting and generous. No young philosopher could have had a finer mentor. Hao Wang provides an authentic example to me of what the Chinese call *xiansheng*, and the Japanese *sensei:* someone who *knows* something.

[42]The titles he discussed with me included *For Perspicuous Objectivity: Discussions with Gödel and Wittgenstein* (a manuscript with this title, cited here as [1991?], was given to me by Charles Parsons), and also Gödel, Wittgenstein and Purity of Mind: Logic as the Heart of Philosophy. Wang had great difficulty producing this manuscript, and in the end set the comparative project aside, cutting out some of the parts on Gödel and Wittgenstein later published in LJ. My sense is that he would have wanted to return to the comparative manuscript at a later stage, but he did not manage to do so.

Italicize title

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