Price Discovery in Dynamic Power Markets with Low-Voltage Distribution-Network Participants

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***Abstract*— We focus on extending Locational Marginal Price (LMP) principles to (i) incorporate marginal costs of real and reactive power, line losses, voltage control, and distribution asset life degradation, and (ii) to enable distribution network connected loads and generators to participate in marginal cost based real and reactive power transactions. To this end, we define a distribution market that can discover spatiotemporal real and reactive power prices dynamically to optimize cost and utility of distributed generation assets, consumers, transformer and other asset life degradation, line losses, and voltage control, while observing full AC load flow constraints. We solve the socially optimal day-ahead market (DAM) clearing problem on a distribution feeder containing industrial, commercial and residential sub-feeders serving conventional non-elastic and flexible reschedulable loads and featuring devices capable of using excess capacity relative to their primary use to provide VAr compensation and voltage support. Numerical results on several DAM clearing scenarios elaborate the value of a fully functional distribution market in providing efficient operation incentives, intertemporal scheduling of demand and optimal location signals to distributed renewable generation and last but not least distribution network rent.**

Keywords-distribution network locational marginal prices; power flow; reactive power compensation; voltage control; distributed generation; dual use of power electronics; transformer loss of life; distribution network rent

I. Introduction

Following a long discussion in the literature originating in Vickrey's work on dynamic pricing of utility services [1] and its detailed application to Electric Power [2, 3] dynamic Locational-Marginal-Price (LMP) based Wholesale Power Markets were introduced in England in 1990 and in the US in 1997 [4] resulting in significant productivity dividends: Competitive Power pools were able to accept individual participant bids and offers to clear markets and discover dynamic LMPs that promoted more efficient and reliable service with fewer capacity reserves, located new generation to relieve transmission congestion and to lower supply cost to consumers, and more.

Nevertheless, mature, yet unexploited opportunities can be sought in the potential of extensive load-side market participation and the use of Distribution network Locational Marginal Prices (DLMP). In particular, we note that distribution network costs, accounting for as much as 35% of low voltage power costs, are priced today at their average cost. Present distribution network average pricing practice deprives millions of consumers from the opportunity to match their preferences to distribution system marginal costs and wastes the opportunity to capture significant cost reducing efficiencies and to assist the cost efficient integration of clean distributed generation.

Critical developments that have occurred since wholesale power markets made their debut, such as affordable communication, computation, sensing, actuation, and the advent of flexible loads and ubiquitous power electronics advocate a major power market reform. The potential of smart appliance demand response has been recognized and studied extensively under DOE funding [5]. However, most research till now has focused on direct control or centrally broadcasted Utility signals directed to various smart appliance types.

This paper is complementary in scope to PNNL research. It focuses on the extension of LMP principles to (*i*) incorporate marginal costs of real and reactive power, line losses, voltage control, and distribution asset life degradation, and (*ii*) enable medium and low voltage consumers and distributed generation to provide reactive power compensation, voltage control and line loss reduction. With this paper we aspire to contribute to the debate on whether detailed DLMP-based markets are worth considering. To this end, we propose an explicit market structure for a realistic distribution network feeder adopted from Southern California Edison data [6] that can discover space and time sensitive prices, reflecting marginal costs of (*i*) ancillary services, such as distribution network reactive power compensation and voltage control, (*ii*) marginal distribution network line losses and (*iii*) distribution asset congestion and life degradation. We solve for day-ahead market clearing prices and quantities for real and reactive power consumed or produced at each load or generation point in the network so as to optimize distribution utility cost minus distributed participant utility subject to full AC load flow relations and voltage magnitude constraints.

We wish to acknowledge similar work by Steven Low and collaborators [6] who have formulated and solved a distribution network line losses minimization problem subject to detailed distribution feeder AC power flow constraints. Distribution network marginal cost based decisions and pricing inroads have also been reported amongst others in [15-23]. In this paper we generalize proposing a complete market framework that (*i*) includes distribution utility and distributed participant costs and benefits and (*ii*) models degrees of freedom ranging from the ability to delay and reschedule consumption to putting excess power electronics resources to dual use for VAr compensation.

Numerical results for typical load trajectory and other input scenarios are reported to elaborate on the quantitative

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unbundling of DLMPs to their specific marginal cost components. This enables unbundling of market participant revenues and distribution network components.

The rest of this paper is organized as follows. Section II formulates the market clearing problem. Section III describes a specific distribution feeder that our numerical results are based on and section IV presents and interprets these numerical results. Section V concludes and discusses interesting future work that may enhance the implementability of the DLMP market.

II. The Distribution Day-Ahead Market Clearing Problem

A. *Notation Conventions*

<u>General</u>

ε: Arbitrarily small positive quantity.

$1_{\square}$: Indicator function. When its subscript holds true, the value of the indicator function is 1, else it is 0.

k: constant

<u>Network related Indices, Subscripts and Sets</u>

h: argument indicating a specific hour in the day ahead market, h=1,2,3,…,24.

$\infty, b, (b,b')$: Subscripts denoting respectively the substation bus, a typical bus, and a line connecting bus *b* to bus b'.

d_i, g_i, f, e_i : Subscripts denoting respectively a specific distributed load, distributed generation, shunt capacitor or distributed power electronics. For example, $d_i(b)$ means that load d_i is located at bus *b*.

$\{\ell\},\{tr\}$:Sets indicating lines and transformers in the distribution feeder topology.

<u>Network Parameters</u>

$c_{g_i(b)}, u_{d_i(b)}$: Marginal Cost and marginal Utility, respectively, associated with generation type/load type g_i, d_i during hour *h*.

c_{∞}^{V}: cost of substation voltage rise.

c_{∞}^{G}: Substation generator fuel costs.

$C_{\infty}, C_{g_i(b)}, C_{f(b)}, C_{e_i(b)}$: Capacity of reactive power compensating generator at the substation bus, distributed generator g_i, capacitor f and power electronics e_i located at bus *b*.

$\mathbf{G}, \mathbf{B}$: Matrices whose elements $G_{b,b'}, B_{b,b'}$ denote respectively the real and imaginary components of line or transformer admittance. $G_{b,b'}, B_{b,b'}$ are defined for all (b,b') pairs, but are only non-zero when the buses b and b' are connected directly. As such zero and non-zero elements define the topology of the distribution network.

<u>Network variables</u>

P,Q,S: denote real, reactive and apparent power, respectively. For example $P_{d_i(b)}$ denotes real power withdrawn from bus b by distributed load type d_i.

$A_b(h), V_b(h)$: voltage angle and magnitude at bus b during hour h.

$\phi_{d_i(b)}$: fixed (+ or -) Current/Voltage phase shift introduced by (capacitive or inductive) load d_i connected to bus *b*.

<u>Electric Vehicles</u>

$i(\tau)$: Superscript denoting a reschedulable load with a deadline τ and utility depending on an associated dynamic state variable evolving with real power consumption (e.g., state of discharge of EV batteries associated with a high cost if it is positive during the hour h=τ, the designated departure of the EV, hence the deadline).

$x_b^{i(\tau)}(h)$: State of discharge (i.e., battery capacity not yet charged) of EV batteries connected to bus b with desired departure at time τ.

$\Delta x_b^{i(\tau)}(h)$: The arrival of new EV battery charging demand during hour h that is associated with desired departure at τ.

$u_b^{i(\tau)}(x_b^{i(\tau)}(h))$: Cost of EV wishing to depart during hour τ when its state of discharge is $x_b^{i(\tau)}(h)$ during hour h.

<u>Transformers</u>

$c_{b,b'}^{tr}$: Cost of transformer *tr* per hour of its economic life.

$\theta_{b,b'}^{H}(h), \theta^{A}(h)$: Hottest spot and ambient temperature respectively of transformer represented by line $(b,b') \in \{tr\}$.

$S_{b,b'}^{N}$: Nominal apparent flow rating of transformer represented by line $(b,b') \in \{tr\}$.

$\Gamma_{b,b'}(\theta_{b,b'}^{H}(S_{b,b'}(h)))$: Loss of life of Transformer represented by line $(b,b') \in \{tr\}$, measured in hours of economic life per hour of clock time, when the apparent power flow through the transformer is $S_{b,b'}(h)$ inducing a hottest spot temperature $\theta_{b,b'}^{H}$. Specific relations are [9,10]

$$\Gamma_{b,b'} = \exp\left(\frac{15000}{383} - \frac{15000}{273 + \theta_{b,b'}^{H}(h)}\right), \forall (b,b') \in \{tr\}$$

with $\theta_{b,b'}^{H}(h) = \theta^{A}(h) + k_{1,b,b'} + k_{2,b,b'}\left(\frac{S_{b,b'}(h)}{S_{b,b'}^{N}}\right)^2$ where $k_{1,b,b'}, k_{2,b,b'}$ are coefficients relating $\theta_{b,b'}^{H}$ to $S_{b,b'}(h)$.

Transmission Level Parameters

$\pi_{\infty}^{P}(h)$: Transmission level LMP of real power during hour h at the bus that the substation is connected to. For clarity in exposition, we decouple the Transmission and Distribution systems and treat $\pi_{\infty}^{P}(h)$ values as exogenous. Coupling of T&D is possible as shown in [15].

$\pi_{\infty}^{R}(h)$: Transmission level clearing price of reserves during hour h at the bus that the substation is connected to. They are also treated as exogenous parameters.

B. Market Clearing Problem

We formulate the Day-Ahead Distribution Market Clearing problem as the minimization over distribution network location-specific real and reactive power injections of:
(*i*) the cost of real power procured at the substation, plus
(*ii*) the cost of required voltage increase at the substation as needed for voltage control throughout the network, minus
(*iii*) real power consumer utility, plus
(*iv*) the distribution operator opportunity cost associated with the production of reactive power at the substation as needed [11, 12], plus
(*v*) reactive power production fuel costs
(*vi*) the cost of transformer loss of life, plus
(*vii*) EV uncharged battery costs, and finally
(*viii*) distributed generation costs.

If the substation generator has a higher fuel cost than the substation real energy price, then it will be providing reserves, and thus the opportunity cost for reactive power production will be associated with the clearing price for reserves at the substation. Otherwise, if the substation generator has smaller fuel cost than the substation real energy price, then it will be providing real power and as such the opportunity cost will be associated with the loss of real power sales minus the decreased fuel costs for lesser real power production. The decreased fuel costs are going to be the fuel cost times the real power provided by the substation times a constant representing the effect of losses.

The objective function described above is minimized subject to constraints (1)-(13) that can be described in words as follows:
(*i*) AC load flow relationships, (1), (2), (8), (9) and (12)
(*ii*) real and reactive power injections by loads and generators (3), (4) and (5)
(*iii*) power conditioning assets accompanying loads such as asynchronous motor HVAC systems, elevator banks, PV installations, and EVs (6)
(*iv*) reactive power output of shunt capacitors related to their bus voltage (7), (7'). If the capacitors are only switchable on and off, then their reactive power output will be equal to zero when switched off and equal to the minimum of their max capacity and their rated capacity times the square of their connection bus voltage, when they are switched on. If, on the other hand, we assume that the output of the capacitor can be controlled in increments small enough such that we can model it as a continuous variable, then those quantities become the upper and lower limits to the reactive power output of the capacitor.
(*v*) voltage magnitude constraints (10),(11), and
(*vi*) intertemporally coupled states of flexible schedulable loads such as in EV Battery charging where the state of charge during hour h depends on the electricity consumed during hour (h-1) (13).

More specifically, the market clearing problem is the solution to the following constrained optimization problem:

$$\min_{P_{g_i(b)}(h),P_{d_i(b)}(h),P_{d^{i^*(\tau)}(b)}(h),Q_{g_i(b)}(h),Q_{e_i(b)}(h),Q_{f(b)}(h),V_\infty(h)} \sum_h \{ \overbrace{P_\infty(h)\pi_\infty^P(h)}^{(i)} +$$

$$+\underbrace{c_\infty^V\left(V_\infty(h)-1\right)^2}_{(ii)} - \underbrace{\sum_{b,i\neq i*} u_{d_i(b)}(h)P_{d_i(b)}(h)}_{(iii)}$$

$$+\underbrace{\left[1_{\pi_\infty^P(h)>c_\infty^G}(\pi_\infty^P(h)-kc_\infty^G)+1_{\pi_\infty^P(h)<c_\infty^G}\pi_\infty^R(h)\right]\cdot\left(C_\infty-\sqrt{C_\infty^2-Q_\infty^2(h)}\right)}_{(iv)}$$

$$+\underbrace{\varepsilon(Q_\infty(h))Q_\infty(h)}_{(v)}+\underbrace{\sum_{b,b'\in\{\mathrm{tr}\}} c_{b,b'}^{\mathrm{tr}}\Gamma_{b,b'}(h)}_{(vi)}$$

$$+\underbrace{\sum_{b,i(\tau)} u_b^{i^*(\tau)}(x_b^{i^*(\tau)}(h))}_{(vii)}+\underbrace{\sum_{b,i} c_{g_i(b)}(h)P_{g_i(b)}(h)}_{(viii)}\}$$

Subject to

$$P_{b,b'}(h)=\left(V_b(h)\right)^2 G_{b,b'}-V_b(h)V_{b'}(h)G_{b,b'}\cos(A_b(h)-A_{b'}(h)) \\ -V_b(h)V_{b'}(h)B_{b,b'}\sin(A_b(h)-A_{b'}(h)) \quad (1)$$

$$Q_{b,b'}(h)=-\left(V_b(h)\right)^2 B_{b,b'}+V_b(h)V_{b'}(h)B_{b,b'}\cos(A_b(h)-A_{b'}(h)) \\ -V_b(h)V_{b'}(h)G_{b,b'}\sin(A_b(h)-A_{b'}(h)) \quad (2)$$

$$0\le\left(P_{g_i(b)}(h)\right)^2+\left(Q_{g_i(b)}(h)\right)^2\le\left(C_{g_i(b)}\right)^2 \quad (3)$$

$$\underline{P}_{d_i(b)}\le P_{d_i(b)}\le\overline{P}_{d_i(b)} \quad (4)$$

$$Q_{d_i(b)}=P_{d_i(b)}\tan(\varphi_{d_i(b)}) \quad (5)$$

$$0\le\left(P_{e_i(b)}(h)\right)^2+\left(Q_{e_i(b)}(h)\right)^2\le\left(C_{e_i(b)}\right)^2 \quad (6)$$

$$Q_{f(b)}(h)\in[0,\min\{C_{f(b)},C_{f(b)}V_b^2(h)\}] \quad (7)$$

$$0\le Q_{f(b)}(h)\le\min\{C_{f(b)},C_{f(b)}V_b^2(h)\} \quad (7')$$

$$\sum_i P_{g_i(b)}+\sum_i P_{e_i(b)}-\sum_i P_{d_i(b)}-P_b^{d^{i(\tau)}}(h)= \\ =P_b(h)=\sum_{b'\in\{b':(b,b')\in\{\ell\}\}} P_{b,b'}(h),\forall b\neq\infty \quad (8)$$

$$\sum_i Q_{g_i(b)} + \sum_i Q_{e_i(b)} + Q_{f(b)} - \sum_i Q_{d_i(b)} = = Q_b(h) = \sum_{b'\in\{b':(b,b')\in\{\ell\}\}} Q_{b,b'}(h), \forall b \neq \infty \quad (9)$$

$$\underline{V}_b \leq V_b(h) \quad (10)$$

$$V_b(h) \leq \overline{V}_b \quad (11)$$

$$A_\infty = 0 \quad (12)$$

$$x_b^{i^*(\tau)}(h+1) = x_b^{i^*(\tau)}(h) - P_{d^{i^*(\tau)}(b)}(h) + \Delta x_b^{i^*(\tau)}(h) \quad (13)$$

For the remainder of this work, we assume that the additional fuel costs for the production of reactive power due to higher winding losses are of lower order of magnitude compared to the fuel costs for real power production, i.e. $\varepsilon(Q_\infty) << c_\infty^G$, and neglect them by removing the objective function component $\varepsilon(Q_\infty(h))Q_\infty(h)$ from all analytical and numerical solution discussions.

C. Unbundling of DLMP

The problem formulated above is solved using the AIMMS modeling framework. As long as there are no multiple solutions, a condition that holds in a radial network with no loops [7, 8, 19] as is the case here, the optimal solution is obtained together with the dual variables associated with constraints (1) through (13). Real and reactive power DLMPs for each hour are thus given by the Lagrange multipliers of (8) and (9).

At each bus, we consider a fictitious generator $\tilde{g}^P$, of small capacity ε and zero cost, providing real power only. Also, at each bus, we consider a fictitious generator $\tilde{g}^Q$, of small capacity ε and zero cost, providing reactive power only. By appending constraints to form the Lagrangean, and using the first order optimality conditions, we can conclude that the real and reactive DLMPs can be seen to equal (see [25] for derivation):

$$\pi_\beta^P = \overbrace{\pi_\infty^P \frac{\partial P_\infty}{\partial P_{\tilde{g}^P(\beta)}}}^{(i)} + \overbrace{\frac{\pi_\infty^P C_\infty}{\sqrt{C_\infty^2 - Q_\infty^2}} \frac{\partial Q_\infty}{\partial P_{\tilde{g}^P(\beta)}}}^{(ii)} + \overbrace{\sum_{tr} c_{tr} \frac{\partial \Gamma_{tr}}{\partial P_{\tilde{g}^P(\beta)}}}^{(iii)} \underbrace{+ 2c_\infty^V (V_\infty - 1)\frac{\partial V_\infty}{\partial P_{\tilde{g}^P(\beta)}} + \sum_b \mu_b \frac{\partial V_b}{\partial P_{\tilde{g}^P(\beta)}}}_{(iv)} \quad (14)$$

$$\pi_\beta^Q = \overbrace{\pi_\infty^P \frac{\partial P_\infty}{\partial Q_{\tilde{g}^Q(\beta)}}}^{(i)} + \overbrace{\frac{\pi_\infty^P C_\infty}{\sqrt{C_\infty^2 - Q_\infty^2}} \frac{\partial Q_\infty}{\partial Q_{\tilde{g}^Q(\beta)}}}^{(ii)} + \overbrace{\sum_{tr} c_{tr} \frac{\partial \Gamma_{tr}}{\partial Q_{\tilde{g}^Q(\beta)}}}^{(iii)} \underbrace{+ 2c_\infty^V (V_\infty - 1)\frac{\partial V_\infty}{\partial Q_{\tilde{g}^Q(\beta)}} + \sum_b \mu_b \frac{\partial V_b}{\partial Q_{\tilde{g}^Q(\beta)}}}_{(iv)} \quad (15)$$

Inspection of (14) and (15) implies that DLMPs can be unbundled to the following components:

(*i*) marginal Cost of Real Power (i.e. of marginal real losses),

(*ii*) marginal Cost of Reactive Power (i.e. cost of compensating for marginal reactive power at the substation),

(*iii*) marginal cost of Transformer loss of life, and

(*iv*) Marginal cost of voltage control through both elevating voltage at the substation and maintaining the voltage constraints at all busses where these are binding, respectively.

III. Numerical results

A. Test Distribution Network

In order to check the applicability of our DLMP model, we applied it to a realistic distribution feeder. The 253 bus test network, whose single line approximation is shown in Figure 10, consists of an industrial and a residential feeder.

The topology of the industrial feeder, so far as the location the loads and the location of the photovoltaics and the capacitors, is obtained from Southern California Edison data as published in [6]. The peak load data can be found in Table V.

The residential feeder, is a duplicate of the industrial feeder. Given the average household consumption and the fact that residential lines typically cover up to 10 houses, the relevant loads were substituted by a medium to low voltage transformer (100kVA, X=5.75%pu) and a sub feeder line segment serving several residential loads located at additional low voltage busses. A high to medium voltage transformer (47MVA, X=18.5%pu) was added to connect the feeders to a high voltage substation.

Figure 8 shows the expanded feeder including transformer lines, PV, and capacitor locations. Low Voltage sub feeder line segments are shown as a range of bus numbers. All medium voltage lines are identical and have a resistance of $R = 0.2\Omega$ and a reactance of $X = 0.3\Omega$. All low voltage line segments -not shown explicitly for lack of space- are identical and have a resistance of $R = 0.002\Omega$ and a reactance of $X = 0.003\Omega$.

On Figure 10 we also point out the busses that we will examine in more detail later in this work. Bus 17 is a bus in the industrial feeder, close to the substation. Bus 101 is its equivalent bus in the residential network, i.e. bus in the residential feeder close to the substation. Bus 43 is a bus in the industrial feeder and far from the substation and bus 101 is its equivalent bus in the residential network.

In addition to the distributed resources described in [6], we also use electric vehicles that could provide reactive power provision while charging. Table I below shows the characteristics of the vehicles.

Connection Bus		95-101	208-220	23	40
Arrival Time	1st class	6pm	6pm	9am	9am
	2nd class	-	-	5pm	5pm
Departure Time	1st class	8am	8am	5pm	5pm
	2nd class	-	-	1am	1am
Number of EVs per bus		1	1	20	20
Battery Capacity (kWh)		12	12	12	12
Charger Capacity (kVA)		3.3	3.3	3.3	3.3

TABLE I. Electric Vehicles characteristics

For our input data, we use a typical summer day 24 hour trajectory for the prices of real power. We model the evolution of the demand and of the real power output of the photovoltaics as a percentage of their respective maximum.

B. Numerical Results

We initially solve the 24 hour Day-Ahead Market clearing problem on the test Distribution Network without any distributed resources and with load fixed to the values of the first column of Table V (i.e. loads are parameters). We will heretofore refer to this base case as Scenario 0. Table II below shows the total costs incurred, as well as the real and reactive DLMPs for buses 17, 43, 101, 233 for Scenario (0).

Total Costs ($)	21370.62			
	Bus 17	Bus 101	Bus 43	Bus 233
Real DLMPs ($/kWh)	0.031199	0.034946	0.033174	0.035924
Reactive DLMPs ($/kVarh)	0.002339	0.003768	0.003441	0.005171

TABLE II. Total costs, Real and Reactive DLMPs for Scenario 0 (fixed loads).

With a projected annual increase of 1.5%, within 20 years, the peak load values of the first column of Table V will rise to the peak load values of the second column of Table V. In this case, the base case scenario will be infeasible for fixed loads, meaning that the increased load cannot be met. In order to quantify the increasing benefit of using distributed enery resources, we consider the real demand to be an upper bounded decision variable to the 24 hour Day-Ahead market clearing problem and then solve it on the test Distribution Network for the following six scenarios:

Scenario	Photovoltaics	Capacitors	Dual Use of Power Electronics
(i)	✗	✗	✗
(ii)	✗	✓	✗
(iii)	✓	✗	✗
(iv)	✓	✓	✗
(v)	✓	✗	✓
(vi)	✓	✓	✓

TABLE III. Distributed resources allowed for each scenario.

Figure 1 below reports the cost incurred by the System Operator broken down to its individual components.

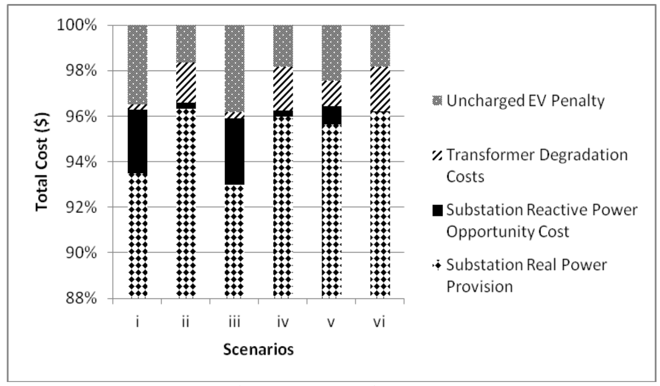

Figure 1. Total cost as a sum of all the objective function cost terms.

We notice that several cost components not explicitly priced in today's markets, like reactive power and equipment loss of life, can make up for a big amount of the total costs. Also, as we allow more distributed recourses, the total cost decreases, but the individual cost components can increase. For example, transformer loss of life costs might increase because of the real and reactive injections of the distributed resources that have to flow through the transformer and further burden them.

Figure 2 below shows the evolution of the transmission system costs and the load met for each of the six scenarios.

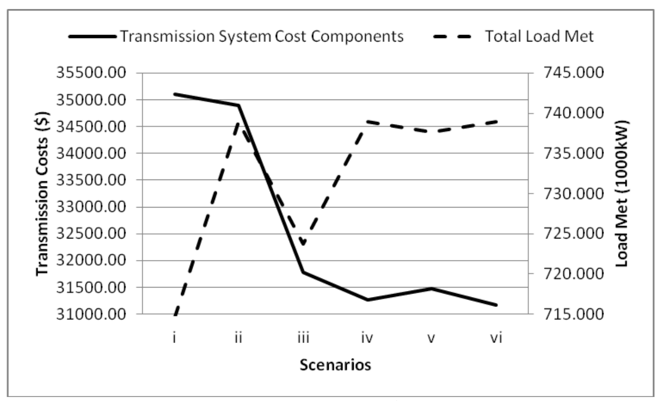

Figure 2. Transmission system costs and Load Met for each scenario.

Our numerical results show an increase in the load met the more distributed resources we allow. Serving more load at lesser cost means that the use of distributed resources increases system resilience to load increases over time.

Figure 3 below reports on the net payments (payments minus receipts) of the Distribution Level participants. Figure 4 further elaborates on the income the demand side receives for participating in real and reactive power provision per device type.

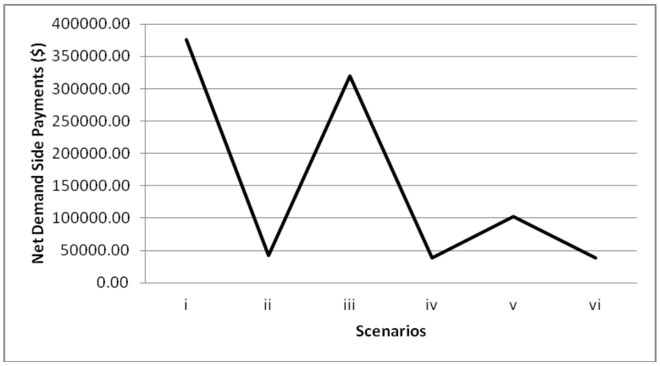

Figure 3. Net Demand Side Payments (demand side payments-receipts).

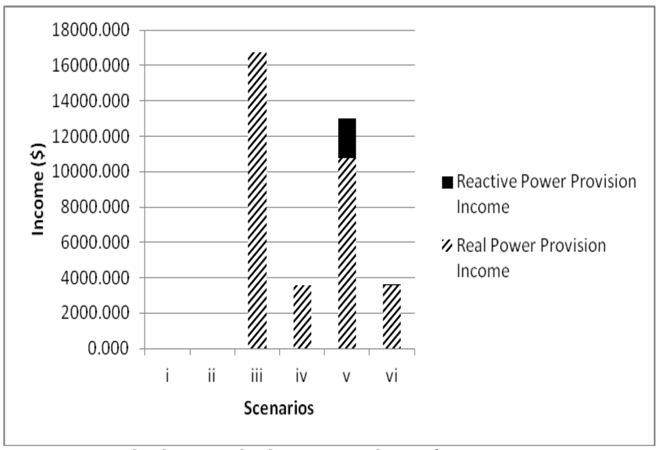

Figure 4. Demand Side Income by device type and type of provision.

Allowing the use of distributed devices providing VAr compensation causes a significant decrease in the demand side payments, showing the importance of distributed VAr compensation. As we allow more distributed resources, we notice that the demand side payments are decreasing together with the demand side income, for a decrease in the net demand side payments as we move from scenario (i) to (vi). The demand side income results quantify the value of the putting power electronics to dual use. The income can be interpreted by the demand side as a market signal to help them make investment decisions in resource type/provision, size and location [6, 24].

Figure 5 shows the evolution of the gross average real energy cost per kWh

$$x_1 = \frac{\sum_b \left\{ \pi_b^P \sum_i P_{d_i(b)} + \pi_b^Q \sum_i Q_{d_i(b)} \right\}}{\sum_{b,i} P_{d_i(b)}}$$

and the net average real energy cost (payments minus receipts) per KWh

$$x_2 = \frac{\sum_b \left\{ \pi_b^P \left(\sum_i P_{d_i(b)} - \sum_i P_{g_i(b)}\right) + \pi_b^Q \left(\sum_i Q_{d_i(b)} - \sum_i Q_{g_i(b)}\right) \right\}}{\sum_{b,i} P_{d_i(b)}}$$

for each of the six scenarios. These results may also be used alternatively as market signals for the demand side.

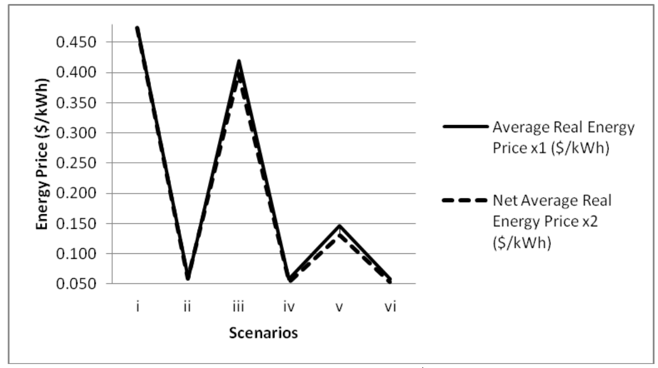

Figure 5. Gross and Net Average Real Energy Price in $/kWh.

Figures 6 and 7 below show the peak hour real and reactive DLMPs of a bus in the industrial feeder close to the substation and its equivalent bus in the residential feeder, specifically buses 17 and 101, and compares them to DLMPs at busses away from the substation, specifically buses 43 and 233. The location of these busses we examine has been pointed out in Figure 10.

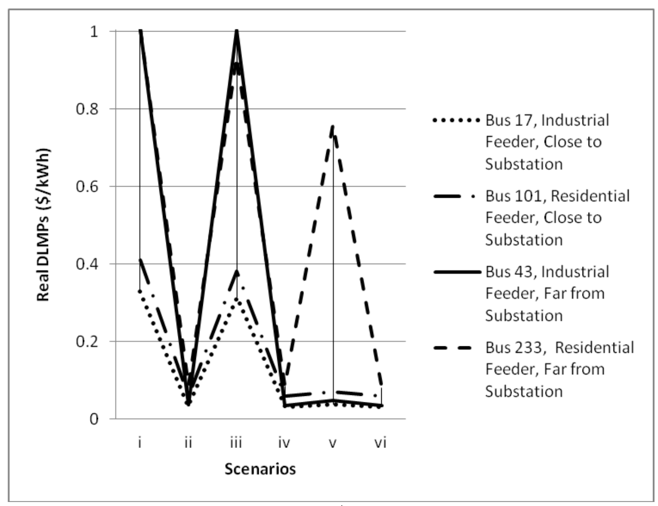

Figure 6. Peak Hour Real Power DLMPs in $/kWh per scenario and bus.

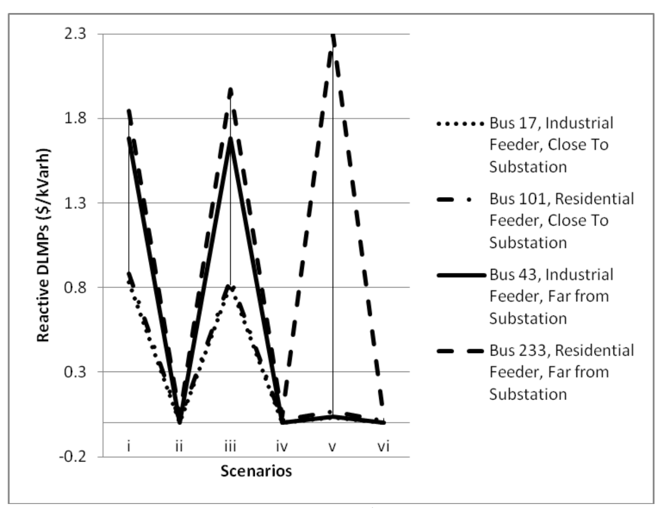

Figure 7. Peak Hour Reactive Power DLMPs in $/kVarh per scenario and bus.

When the voltage constraints and the absence of sufficient distributed generation result in demand rationing, the DLMPs are very high, as is the case in scenario (i). As such, the DLMPs of Scenario (i) are much higher than the DLMPs of the base case Scenario (0), that both have no distributed resources. Comparing scenarios that allow/disallow the use of Capacitors, we see that changes in reactive DLMPs is much greater than in real DLPMs. This results from the combined effect of VAr compensation and voltage control. Comparing the real and reactive DLMPs of busses at the same relative position to the substation but in different feeders, we conclude that the real and reactive DLMPs at the residential feeder is always going to be higher because of the involvement of the medium to low voltage transformers. However, the relationship between the real and reactive DLMPs of busses in the same feeder and different distances from the substation might not always be intuitive, because of the real and reactive injections of the distributed energy resources.
The utility is assumed to be constant at $u_{d_i(b)}(h) = 2 \, {}^{\$}\!/_{kWh}$ for all loads. A load is shed when the DLMP at its connection bus is higher than the utility of the load. In our case, since all the load are considered to be curtailable loads of constant power factor, the relevant comparison need be between the utility $u_{d_i(b)}(h)$ and the composite price of $\pi_b^E(h) + \tan(\varphi_{d_i(b)})\pi_b^Q(h)$. The following table reports on this composite price for busses 17, 43, 101 and 233 for each of the six scenarios. When load is shed, the composite price at that bus is equal to the utility.

Composite price to be compared to the load's utility in $/kWh				
Scenario \ Bus	**Bus 17**	**Bus 101**	**Bus 43**	**Bus 233**
i	0.777	0.886	1.915	2.000
ii	0.034	0.067	0.037	0.106
iii	0.755	0.840	1.912	2.000
iv	0.033	0.067	0.036	0.104
v	0.057	0.107	0.068	2.000
vi	0.031	0.065	0.034	0.095

TABLE IV. Price to be compared to the load's utility.

Figure 8 shows the voltage magnitude results for the line connecting buses 1-13.

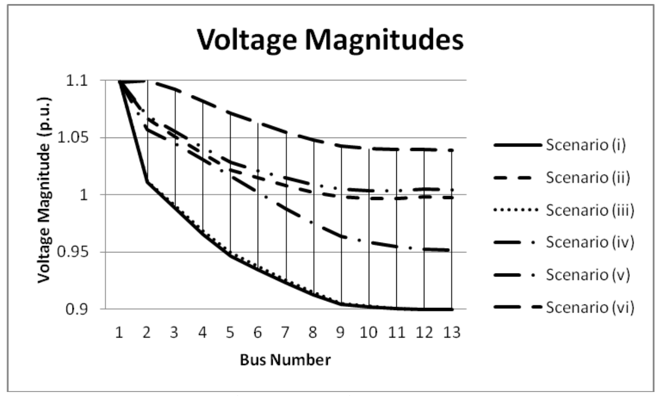

Figure 8. Voltage magnitudes for buses 1-13 for all scenarios.

We notice that enabling the use of distributed resources allows for a flatter voltage magnitude profile, i.e. less voltage drops, throughout the line. Using reactive power providing resources results in higher voltage magnitudes because of the combined effect of voltage control and VAr compensation.
In order to reveal the market size for reactive power, we examine the volatility of reactive power prices with respect to the addition of real and reactive power providing distributed resources. To this end, Figure 9 below shows the peak hourly reactive DLMP for several scenarios.

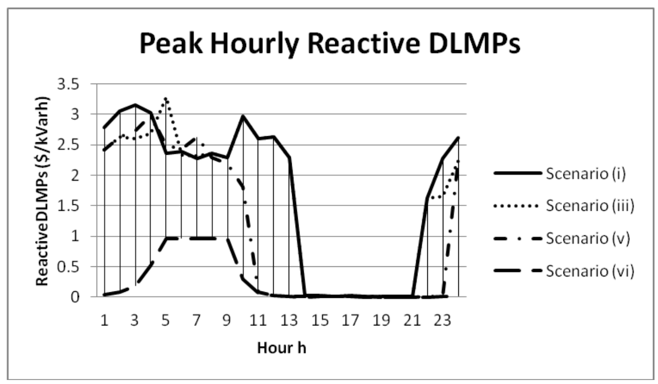

Figure 9. Peak Hour Reactive DLMPs for several scenarios.

As mentioned before, the introduction of reactive power providing distributed resources results in significantly lower reactive DLMPs. During off-peak hours and when reactive power providing resources are allowed, the highest reactive DMLP is still less than $1 \, {}^{cent}\!/_{kVarh}$.
We focus next on unbundling real and reactive DLMPs as shown in section II.C. Figure 7 reports DLMP components for the peak hour (hour 7, 6pm-7pm) at busses 17, 101, 43 and 233 in scenario (ii). Comparing an industrial and residential bus in the same relative position, we notice a higher transformer cost component in the real and reactive DLMP for the residential busses. Since additional medium to low voltage transformers are involved this is expected [23]. The further a load is from the substation, the larger the losses and the transformer's loss of life cost with respect to incremental load.
Next, we focus on the computational burden imposed by the non-convexity of the constraint set. Figure 8 shows the

increase in the number of variable with the corresponding increase in the number of iterations and the computational time required to reach optimality. We note here that the CONOPT solver used for our problem can only guarantee local optimality at termination, which we then know to be global optimality given the network's radial structure.

IV. Conclusions and Future work

We have proposed a detailed DLMP based market and applied it to a 253-bus distribution feeder. Numerical results and observed computational burden support the tractability of the proposed market framework and elaborate on the ability of the DLMP market to provide consistent rewards to innovations in demand response, optimally located and sized PV installations and hardware improvements. Future work should focus on further elaboration of distribution network variable costs, representation of flexible loads and distributed resources, and explicit coupling with the transmission system power markets. Advances shown in [7,16,17,18,19] make us optimistic about the prospects of DLMP adoption.

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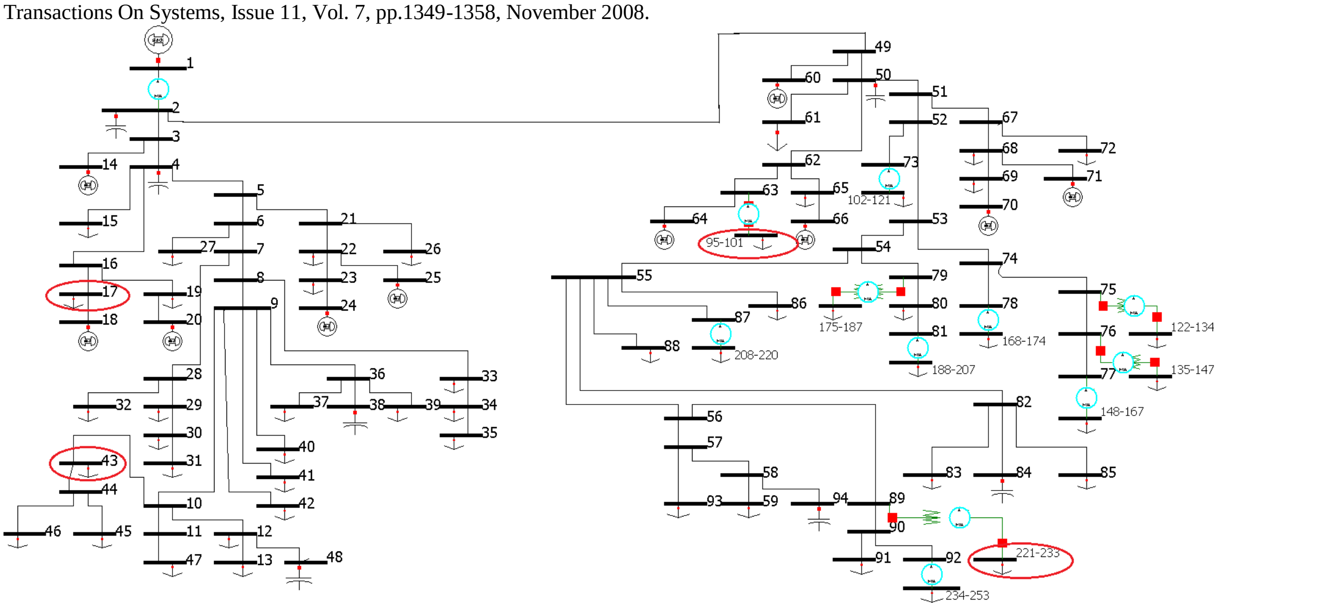

Figure 8. Network Topology