Decision Support for Offering Load-Side Regulation Service Reserves in Competitive Power Markets

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Abstract—**This paper, *(i)* quantifies the ability of Smart Buildings to provide electric power Regulation Service (RS) given the characteristics of the building's flexible loads and the expected costs from occupant utility loss and imperfect tracking of System Operator RS requests, and *(ii)* develops an optimization-based decision support model to assist Smart Building Operators (SBO) willing to offer RS reserves in the Hour Ahead Power Market. The model incorporates minimal Regulation Service tracking performance constraints required by current market rule contractual requirements. To quantify these probabilistic constraints, we develop a Normal Distribution Approximation to the dynamic mean of the discrete time $M/M/\infty$ queue of active appliance loads conditional upon the SBO's price directed control aimed at activating appliance loads. Finally, analytic estimates of performance statistics and properties of the dynamic optimal control used in tracking real time System Operator RS requests are developed and used in the decision support model. Numerical results are provided for elaboration purposes.**

Index Terms—**Regulation Service Provision, Hour Ahead Market Regulation Offer, Electricity Markets, Real Time Stochastic Control**

I. Introduction

The integration of renewable energy into the power grid is progressing at an increasing rate. For example, wind generation in NY was 48 MW in 2005, 1,414 MW in 2012, and is expected to reach 4,000 MW by 2018 [1], with growth rates in Texas and the West coast being far more impressive. The intermittency and volatility of renewable generation, however, results in commensurate increase in the reserves that Independent System Operators (ISOs) must secure. Amongst several types of Capacity Reserves auctioned in the Day and Hour Ahead Power Markets, bidirectional and energy neutral Regulation Service (RS) capacity reserves are secured in advance and then managed in real time through ISO requests updated on a 4 to 8 second basis ([2]). Market participants who offered them in the forward markets must respond to these requests with the same time scale dynamics. NYISO[1] secures 116 MW of RS reserves on average (225 MW during the summer season) and expects the requirements to double by 2018 ([3]).

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[1]NYISO is the Independent System Operator of the New York region and serves nearly 7M electric utility customers [1].

RS reserves have been so far provided by flexible generators capable of modulating their output, with RS clearing prices comparable to energy clearing prices ([4]). As a result, projected renewable generation growth will be impeded unless the demand side (e.g. residential or commercial Smart Buildings) is enabled to contribute to the supply of RS reserves. This is the major motivation for the work presented here.

Several approaches have been developed for implementing real-time control of Smart Building demand ([5], [6], [7], [8]). This paper adopts optimal real time demand management by Smart Building Operators (SBO) proposed in [5] to reflect upon a related but longer time scale decision, namely the optimal bid of RS capacity by SBOs in the Hour Ahead Markets where Energy and RS capacity offers are co-optimized. More specifically it develops an optimization-based decision support model to assist Smart Building Operators (SBO) willing to offer RS reserves in the Hour Ahead Power Market. The model incorporates *(i)* minimal Regulation Service tracking probabilistic performance constraints required by current market rule contractual requirements, and *(ii)* estimates of performance cost statistics in tracking real time Independent System Operator (ISO) RS requests. We quantify the probabilistic constraints by developing a Normal Distribution Approximation to the dynamic mean of the discrete time $M/M/\infty$ queue of active appliance loads conditional upon the SBO's price directed control aimed at activating appliance loads. Furthermore, we investigate properties of the dynamic optimal control that the SBO uses in the real time tracking of ISO RS requests and use them to develop analytic estimates of the tracking performance cost statistics used in the decision support model.

Probabilistic constraints and performance cost statistics are coupled and depend crucially on the smart building's capability to modulate its consumption to conform with a larger or smaller RS reserve offer. Hence, the level of the RS offer in the Hour Ahead Market must be commensurate to these costs and capabilities. For example, if the SBO's offer in the Hour Ahead Market exceeds its capabilities, it will end up not being able to track the RS signal with sufficient fidelity, incur high tracking error costs and, at the same time, impose high utility loss to its occupants who will be subjected to exceedingly wide energy service oscillations. If on the other hand the Hour Ahead Market promise is too small, the

SBO will waste the opportunity to offset part of its energy cost by selling RS reserves that are within its capabilities. To this end, we propose an objective function which captures the trade-offs mentioned above.

The RS reserve market rules are described next. RS reserve market participants (whether generators or SBOs) must be certified by passing a qualification test. The test requires each provider to demonstrate that it is capable to track a standard RS signal trajectory at a pre-specified accuracy level[2]. The RS signal specifies the portion of the RS amount that the resource needs to provide until the next signal update. For example, if a SBO bids for an average consumption rate of A kW and offers R kW of Regulation Service Reserve capacity in the Hour Ahead Market, it needs to anticipate that it will be able to modulate its consumption in real time to achieve time varying consumption rates $A + Ry(t)$ kW tracking the dynamic signal $y(t) \in [-1, 1]$. The range of the RS signal indicates that the resource must be able to provide the regulation in both the up and down directions. The RS signal dynamics are constrained by a maximal ramp rate. For example, for a four second update frequency, i.e., $\Delta t = 4$ sec, $|y(t + \Delta t) - y(t)| \leq \xi$, where ξ is the level that would allow the signal to traverse its $[-1, 1]$ range in 2.5 or 5 minutes depending on whether a more valuable fast or less valuable slow RS is offered. A test taker is assessed a score proportional to its tracking error. Exact scoring rules vary across ISOs and may be updated every few months. For example, PJM includes a delay score, a correlation score and a precision score in measuring the performance and requires three consecutive successful tests with a minimum score of 75 % ([9]). Certified RS providers may offer RS levels in the Regulation Market up to the maximal amount they have been certified for. A Certified provider must submit certain information to the ISO in the Hour Ahead Regulation Market, such as, the amount of RS R it may be willing to offer by type, the regulation midpoint or average consumption A, its regulation price offer ($ /MWh).

This paper focuses on the optimal selection of the R offer for a given A. We assume that the SBO can solve for the optimal dynamic control policy presented in [5] for tentative RS offers and obtain requisite cost performance statistics estimates mentioned above and described in detail below. We do not address the specific selection of RS price offer since there are restrictive guidelines specified by ISOs bounding an RS provider's price offers ([10]) and we can safely assume that the SBO can reasonably foresee the RS clearing price. In the interest of information, we note that a RS price offer consists of two parts that compensate *(i)* the additional cost incurred by the provider by selecting to offer a RS reserve R and having to make related adjustments in its average energy consumption, and *(ii)* the cost associated with the anticipation of having to modulate its consumption in real time over the interval $(A+R, A-R)$ quantified by the expected trajectory length $R\sum_{i=0}^{I}|y((i+1)\Delta t) - y(i\Delta t)|$). Leaving this aspect for future work, this paper neglects variations in the trajectory length assuming for simplicity that it can be reasonably estimated by its average value and incorporating it in a single Market Clearing price denoted by π.

[2]PJM, the largest US ISO operating in 13 states has different RS signal tests corresponding to qualification to offer different RS types, requiring faster or slower response, etc. ([9]).

The rest of the paper is organized as follows: In Section II, we review and summarize for the reader's convenience the SBO's dynamic RS signal policy problem investigated in [5]. In Section III, we develop a normal distribution approximation to predict the number of electricity consuming appliances at some future time and under a constant control policy. We also analyze the rate of convergence in $M/M/\infty$ queues. This describes the SBO's certification requirements to offer RS, or, equivalently, the performance of the optimal control policy under RS levels associated with reasonably fast building appliance response capabilities. Section IV formulates probabilistic RS signal tracking constraints included in the Hour Ahead optimization problem. The Hour Ahead optimization problem is formulated explicitly in Section V, numerical experience elaborating our propositions and optimal hour ahead offer model are presented in Section VI and we conclude in Section VII.

II. Problem of RS Provision

For the reader's convenience we summarize, without proofs and detailed propositions, the dynamic SBO policy proposed in [5].

A. Consumption Dynamics and Utility

The Smart Building of interest includes N appliances, $n(t)$ of which are active at time t consuming power at a rate r kW each. $N - n(t)$ appliances are not active, i.e., are not consuming. An inactive appliance realizes utility $\phi(t)$ if it becomes active at time t and stays active over an exponentially distributed time period with mean $1/\mu$. $\phi(t)$ represents the value of consuming an average of r/μ kWh of energy starting at t.

Assumption 1. $\phi(t)$ is assumed to be a uniformly distributed random variable over $[0, U^M]$, with U^M positive.

To activate appliances, the SBO broadcasts a price $u(t) \in [0, U^M]$ updated after each update of the RS signal $y(t)$. Each inactive appliance monitors the price $u(t)$ at discrete time intervals with length distributed exponentially with rate λ^a. At monitoring instances, an inactive appliance becomes active if its utility is greater than the price, i.e., $\phi(t) \geq u(t)$. It then stays active for an exponentially distributed period with mean $1/\mu$ before it returns to inactivity. If an appliance user's utility is smaller than the monitored price, the appliance remains inactive. Appliance activations are therefore Poisson distributed with rate $\lambda^a\left(N - n(t)\right)\left(1 - u(t)/U^M\right)$.

Assumption 2. $n(t)$ satisfies $n^m \leq n(t) \leq n^M$, $t \geq 0$, where $n^M - n^m$ is small relative to N.

This assumption enables us to use the following approximation: The monitoring rate is time invariant and equal to a practically constant rate $\lambda^M = \left(N - (n^m + n^M)/2\right)\lambda^a$. This results in appliance activation rate $\lambda^M\left(1 - u(t)/U^M\right)$ when the SBO broadcasted price is $u(t)$. An important property that we will be referring to frequently is that $n(t)$ exhibits the characteristics of an $M/M/\infty$ queue as we have Poisson appliance activation rates, and deactivations are equivalent to delay processes. Moreover, if a constant price is broadcasted over a long enough period, say $u(t) = u$, then the corresponding $M/M/\infty$ queue will reach steady state and after that $n(t)$ will follow a Poisson distribution with rate $\lambda^M\left(1 - u/U^M\right)/\mu$.

The utility that is realized by an appliance becoming active at time t is seen as a contribution to the building's social welfare. The objective is to maximize the average utility while minimizing tracking error costs that is explained in the next section. Given Assumption 1, the rate of utility realization at time t is given by

$$\lambda^M\left(1 - u(t)/U^M\right)\left(u(t) + U^M\right)/2. \qquad (1)$$

B. RS Signal Dynamics and Tracking Cost

The ISO broadcasts RS signal $y(t)$ as described in the introduction, followed in our model by an SBO price update. The SBO modulates the price it broadcasts to reach consumption level $A + Ry(t)$ kW by time $t + \Delta t$. If it fails, it is assessed a tracking cost. We first describe the dynamics of the RS signal and then define the tracking cost.

The RS signal is in practice a continuous variable taking values in $[-1, 1]$. In our model, the range space of $y(t)$ is discretized to a $2m+1$ set $\{-1, -1+\Delta\bar{y}, \ldots, 0, \ldots, 1-\Delta\bar{y}, 1\}$, where $\Delta\bar{y} = 1/m \geq 0$. When the RS signal is updated, it takes the value $y(t + \Delta t) = y(t) + \Delta y$, where Δy is a random variable taking values in $\{-\Delta\bar{y}, 0, \Delta\bar{y}\}$. The probability distribution of Δy is a function of $y(t)$ and $d(t)$, where $d(t) \in \{-1, +1\}$ is the direction of the RS signal with the following interpretation: When $d(t) = +1$, $y(t)$ is more likely to increase at the next RS signal update, with the opposite holding when $d(t) = -1$. More formally, if $d(t) = +1$, then $Pr\{\Delta y \geq 0 | d(t) = +1, y(t)\} \geq Pr\{\Delta y = -\Delta\bar{y} | d(t) = +1, y(t)\}$, and if $d(t) = -1$, then $Pr\{\Delta y \leq 0 | d(t) = -1, y(t)\} \geq Pr\{\Delta y = \Delta\bar{y} | d(t) = -1, y(t)\}$. Moreover, the direction changes according to the following dynamics

$$d(t+\Delta t) = \begin{cases} +1, & \text{if } y(t+\Delta t) = y(t) \mid d(t) = +1 \\ & \text{or } y(t+\Delta t) > y(t) \\ -1, & \text{if } y(t+\Delta t) = y(t) \mid d(t) = -1 \\ & \text{or } y(t+\Delta t) < y(t). \end{cases} \qquad (2)$$

Assumption 3. The RS signal satisfy Energy Neutrality over an hour, i.e. $\mathbb{E}[y] = 0$.

This assumption implies that the SBO is asked to consume on average at A kW.

The SBO incurs the following tracking cost for over the Δt seconds if its consumption rate at $t + \Delta t$ deviates from its obligation of $A + Ry(t)$:

$$\kappa\left[(n(t+\Delta t))\, r - (A + Ry(t))\right]^2 \Delta t \qquad (3)$$

where $\kappa \geq 0$ is the cost coefficient. The cost coefficient is generally large so that tracking the RS signal is more important than maximizing consumer utility outright. This is because the SBO may lose its certification as RS provider if its performance is persistently below a certain threshold ([9]).

C. Optimal Dynamic Price Policy Optimality Conditions

The average cost infinite horizon Dynamic Programming (DP) characterizing the optimal price policy developed in [5] is summarized next. The system state is the number of the active appliances, the RS signal value and the RS signal direction, which are respectively denoted by $n(t), y(t)$ and $d(t)$. Control decisions, namely the price $u(t)$ updates, are made at discrete time intervals of Δt seconds. Based on the current state and the control, the system evolves into a new state according to realizations of the random variables Δy and Δn. Here, Δn is defined as the change $n(t + \Delta t) - n(t)$ representing the difference of appliances that become active minus the active appliances that disconnect during the time interval $[t, t + \Delta t]$. The Bellman Equation representing optimality conditions is

$$\begin{aligned} & h(n,y,d) + \bar{J} = \\ & \min_{u\in[0,U^M]} \Bigg\{ \mathop{\mathbb{E}}_{\Delta n, \Delta y | u,n,y,d} \Bigg[\Delta t \kappa\left((n+\Delta n)r - (A+Ry)\right)^2 \\ & - \Delta t \lambda^M \left(1 - u/U^M\right)\left(u + U^M\right)/2 \\ & + h(n+\Delta n, y+\Delta y, d') \Bigg] \Bigg\} \end{aligned} \qquad (4)$$

where d' is the new direction, whose dynamics are given in Equation 2. In the Bellman equation, $\bar{J}$ represents the cost per Δt time interval, and $h(n, y, d)$ is the differential cost function ([16]).

A fully discretized problem obtained by discretizing the control space as $\{0, \ldots, U^M\}$ admits a Linear Programming solution of the Bellman equation ([16]) which is discussed in [5].

D. Properties of the Dynamic Pricing Policy

Note that $u(k) \in [0, U^M]$ represents some dynamic pricing policy, where the price is updated at discrete times $k\Delta t$, $k = 0, 1, \ldots, K$ and stays constant in between, i.e. $u(t) = u(k)$, $t \in [k\Delta t, (k+1)\Delta t]$. In addition, we define $\pi \in [0, U^M]$ as a static price policy with the price being constant over the problem horizon, i.e. $u(t) = \pi$, $t \in [0, T]$.

Proposition 1. The expected average consumption rate over the period $[0,T]$ in the Smart Building is the same under a dynamic pricing policy $u(k)$ and a related static price policy $\pi = \frac{1}{T}\sum_{k=0}^{K} u(k)\Delta t$, $T=(K+1)\Delta t$.

An SBO certified for market participation should be able to consume at any point in the range $[A-R, A+R]$. Therefore, if λ^M is too small to allow reaching a consumption rate of $A+R$ kW in reasonable time, then the SBO will not be able to track RS signals appropriately. Hence, we can write the feasibility condition on λ^M as

$$\begin{aligned} \lambda^M r/\mu &\geq A+R \\ \lambda^M &\geq \mu(A+R)/r \end{aligned} \tag{5}$$

which follows from the fact that the corresponding $M/M/\infty$ queue has Poisson distributed queue length whose mean is bounded from above by λ^M/μ.

Proposition 2. The expected utility realized over the period $[0,T]$ is smaller under a dynamic pricing policy $u(k)$ than it would be under the related static price policy $\pi = \frac{1}{T}\sum_{k=0}^{K} u(k)\Delta t$, $T=(K+1)\Delta t$. Moreover, the loss in utility is proportional to the variance of the dynamic price $u(k)$ and is given by

$$\text{Utility Loss} = \frac{\lambda^M \sigma_u^2}{2U^M}. \tag{6}$$

Proofs of the above propositions are given in [5].

III. Look Ahead Probability Distribution of Active Appliances, $n(t)$

In this section, we analyze the probability distribution of $n(t)$ as it is best known at time τ over the period $[\tau, \tau+\Delta\tau]$. During that period a constant price $u \in [0, U^M]$ is broadcasted resulting in a constant appliance activation rate $\lambda = \lambda^M\left(1-u/U^M\right)$. The propositions shown in this section are used in Section IV to formulate the constraints of the optimization problem that determines the desired level of RS to be offered to the Hour Ahead Market.

A. Normal Distribution Approximation of p.d.f. of $n(\tau+\Delta\tau)$

Proposition 3. Let the random variable η represent the number of active appliances in the system at time $\tau+\Delta\tau$, i.e., $\eta = n(\tau+\Delta\tau)$, given $n(\tau)=i$ and the appliance activation rate over $[\tau,\tau+\Delta\tau]$ is λ. The probability distribution of η can be approximated by a normal distribution with mean M_η and variance σ_η^2 where

$$M_\eta = \frac{\lambda}{\mu}\left(1-e^{-\mu\Delta\tau}\right) + ie^{-\mu\Delta\tau} \tag{7}$$

and

$$\begin{aligned} \sigma_\eta^2 = \; & ie^{-\mu\Delta\tau} - (ie^{-\mu\Delta\tau} - \lambda/\mu(e^{-\mu\Delta\tau}-1))^2 \\ & + i(i-1)e^{-2\mu\Delta\tau} + \lambda^2(e^{-\mu\Delta\tau}-1)^2/\mu^2 \\ & - \lambda(e^{-\mu\Delta\tau}-1)/\mu - 2\lambda i(e^{-\mu\Delta\tau}-1)e^{-\mu\Delta\tau}/\mu. \end{aligned} \tag{8}$$

Proof: Let the random variable X represent the number of active appliances at time $\tau+\Delta\tau$ that have actually connected during $[\tau,\tau+\Delta\tau]$, and the random variable Y represent the number of active appliances at time $\tau+\Delta\tau$ that actually connected before time τ, hence $\eta = X+Y$. Starting at time τ, X corresponds to a birth-death process where the system is initially empty. Therefore, X is Poisson distributed with mean $\lambda\left(1-e^{-\mu\Delta\tau}\right)/\mu$ ([13]). Y corresponds to a pure death process as there are initially i appliances at time τ. Noting that any of the i appliances will be still active after $\Delta\tau$ time with probability $e^{-\mu\Delta\tau}$, we conclude that the probability distribution of Y is binomial with parameters i and $e^{-\mu\Delta\tau}$. By virtue of the fact that both the Poisson and Binomial distributions can be approximated well by a normal distribution, the same is true for both X and Y. Hence, η can also be approximated by a normal distribution since $\eta = X+Y$, X and Y are independent and the normal distribution reproduces with addition. Therefore, the mean of η can be approximated by $M_\eta = \mathbb{E}[X]+\mathbb{E}[Y] = \frac{\lambda}{\mu}\left(1-e^{-\mu\Delta\tau}\right)+ie^{-\mu\Delta\tau}$. An analytic derivation of this mean can be found in [13]. In order to find σ_η^2, rather than using $\sigma_X^2+\sigma_Y^2$, we will derive the exact expression for the variance as follows: [13] provides the generating function of an $M/M/\infty$ system where the initial queue size is i as

$$P(s,t) = e^{-\lambda(1-s)\left(1-e^{-\mu t}\right)/\mu}\left(1-(1-s)\,e^{-\mu t}\right)^i. \tag{9}$$

The expression for the variance is then easily obtainable as

$$\sigma_\eta^2 = P''(1,\Delta\tau) + P'(1,\Delta\tau) - \left(P'(1,\Delta\tau)\right)^2 \tag{10}$$

where P' and P'' represents the first and second order derivatives with respect to s, evaluated at $s=1$. ■

Section VI presents numerical results that verify this proposition.

B. Speed of Convergence to Stationarity

Proposition 4. Given that $n(\tau)=i$, $\eta = n(\tau+\Delta\tau)$ and the appliance activation rate over $[\tau,\tau+\Delta\tau]$ is constant and equal to λ, $\mathbb{E}[\eta]$ approaches λ/μ at a rate that is proportional to $\alpha = \mu^2/(\lambda - i\mu)$. Negative values of α indicate that $\mathbb{E}[\eta]$ approaches λ/μ from above while positive values indicate the opposite.

Proof: This result is an extension of the discussion in [14] and [15] on the convergence rate of a birth death process for the case where the system is initially nonempty. In [14], the relaxation time (which is the reciprocal of the speed of convergence) for $M/M/c$ queues is given as $\int_0^\infty \left[\mathbb{E}[n^\infty] - \mathbb{E}[n(t)]\right] dt$ where c is the number of servers and $n^\infty := \lim_{t\to\infty} n(t)$. The result follows by substituting $n(t)$ by the expression given in Equation (7). ■

Corollary 1. As $\mu\to\infty$, η becomes a Poisson distributed random variable with parameter λ/μ.

Proof: Since the speed of convergence to stationarity is dominated by the parameter μ, the system reaches steady state instantaneously as $\mu\to\infty$. ■

This result is quite intuitive: the larger the value of μ the faster the term $e^{-\mu\Delta\tau}$ diminishes in Equation (7) and Equation (8). We thus obtain $\mathbb{E}[\eta] \approx \sigma_\eta^2 \approx \lambda/\mu$, which verifies Corollary 1. In Section III-C we discuss how the speed of convergence affects the optimal dynamic price policy.

C. Properties of Optimal Price Policy Under Fast Convergence Conditions

As Proposition 4 suggests, the time until convergence under a static price can be shortened in two ways: (i) by initializing the system close to the steady state level, since $\alpha \to \infty$ as $i \to \lambda/\mu$, and (ii) by increasing μ, which is what Corollary 1 shows. The intuition of the former is trivial. However, it formally explains the numerical results on a property of the optimal price policies that we demonstrated in [5]. The numerical experience in [5] showed that when $n(t)r - (A + Ry(t)) \approx 0$, i.e., when the SBO is already tracking the RS signal well, the optimal prices are close to the value that will lead the system to the consumption rate of $(A + Ry(t))$ as $t \to \infty$. This is because $(A + Ry(t))/r$ corresponds to the desired steady state value of n. Thus, when the tracking error is close to zero, the convergence rate, α, is large.

Rather than aiming for instantaneous convergence, it is more interesting to find the value of μ that pushes the number of active appliances $n(t + \Delta t$ to within an ϵ neighborhood of the obligation after, regardless of the value of $n(t)$ at t. In view of Assumption 2, we define the maximum possible difference between the obligation and n, as $D = max\{n^M - (A-R)/r, (A+R)/r - n^m\}$.

Proposition 5. Suppose that κ and λ^M are sufficiently large and Assumption 2 holds. Then, if μ satisfies

$$\mu \geq -\frac{1}{\Delta t}\log\frac{\epsilon}{D} \tag{11}$$

and if the price u is constant over $[t, t + \Delta t]$ and it satisfies

$$\frac{\lambda^M}{\mu}\left(1 - \frac{u}{U^M}\right) r = A + Ry(t) \tag{12}$$

then

$$\mathbb{E}[n(t + \Delta t)] \in [n^* - \epsilon, n^* + \epsilon] \tag{13}$$

where

$$n^* := (A + Ry(t))/r. \tag{14}$$

Proof: For large enough κ we ensure that minimizing the tracking error dominates utility minimization in (4). Moreover, λ^M satisfies Inequality (5). Rewriting the expression in Equation (7) as $\lambda/\mu - e^{-\mu\Delta t}(\lambda/\mu - i)$, where $\lambda = \lambda^M(1 - u/U^M)$ and $i = n(t)$, we can conclude that if μ satisfies (11), the exponent term in (7) becomes less than ϵ for any value of $i \in [n^m, n^M]$. Hence, we have $\mathbb{E}[n(t + \Delta t)] \in [n^* - \epsilon, n^* + \epsilon]$, where n^* represent the number of active appliances that minimizes the tracking cost in Equation (3). ■

This proposition implies that under conditions of high rate of convergence, the optimal price policy can be approximated by (12). Although μ is unlikely to satisfy the lower bound in (11) in practice, Proposition 5 helps to understand the relationship between the utility loss and RS provision in Section V-A.

IV. SBO'S CAPABILITY OF PROVIDING RS RESERVE

In this section, we derive the necessary conditions that ensures the SBO's capability of providing R kW Regulation Service reserve. These conditions will be used as constraints of the Hour Ahead optimization problem that we define in Section V.

A. Condition on Maximum SBO Consumption Capacity

As stated in (5), the SBO should be able to consume at a rate of $A + R$ kW in average when the ISO requires full utilization of the reserve in the positive direction by broadcasting RS signal $y = 1$. However, this constraint will become redundant in the presence of the constraint in (15) that will be given in Section IV-B.

B. SBO Capability of Tracking Maximal RS Ramps

As mentioned in the Introduction, the SBO should be capable of modulating its consumption rate by $2R$ kW in τ minutes so that it traverses the RS provision range, namely from $A - R$ kW to $A + R$ kW and from $A + R$ kW to $A - R$ kW, where τ depends on the reserve type that is being provided. In order to ensure the former, we constraint the RS offer optimization problem by the following inequality.

$$\frac{\lambda^M}{\mu}\left(1 - e^{-\mu\tau}\right) + \frac{A-R}{r}e^{-\mu\tau} \geq \frac{A+R}{r} + \varsigma\sigma\left(\lambda^M, \frac{A-R}{r}, \tau\right) \tag{15}$$

where ς is a positive parameter, and $\sigma(\lambda^M, (A-R)/r, \tau)$ represents the standard deviation that can be obtained by setting $\lambda = \lambda^M$, $i = (A-R)/r$, $\Delta\tau = \tau$ in Equation (8) and calculating the square-root. Inequality (15) implies the following: When the SBO is consuming at a rate of $A - R$ kW that corresponds to $i = (A - R)/r$ number of active appliances, if the price is set to 0 so that the appliance activation rate is maximized to λ^M, the consumption rate after τ minutes is expected to be at least $A + R$ kW with certain confidence. In order to construct the confidence interval, we use the normal distribution approximation that is introduced in Proposition 3 and aim to reach at least $(A+R)/(r)+\varsigma\sigma(\lambda^M, (A-R)/r, \tau)$ kW of consumption after τ minutes. $\varsigma = 1$, for example, implies that the consumption rate will exceed $A+R$ kW after τ minutes with a confidence level greater than 68 %.

We use a similar approach to write the constraint to ensure that SBO is capable of reducing the consumption rate from $A+R$ kW to $A-R$ kW in τ minutes. The RS provision offer in the Hour Ahead Market must satisfy the following inequality.

$$\frac{A+R}{r}e^{-\mu\tau} \leq \frac{A-R}{r} - \varsigma\sigma\left(0, \frac{A+R}{r}, \tau\right) \tag{16}$$

where we assume that there are initially $i = (A+R)/r$ active appliances in the system and the price is set to U^M resulting in $\lambda = 0$.

V. Optimization of the RS Offer in the Hour Ahead Market

In preparation for the formulation of the optimization problem for offering RS reserve in the Hour Ahead Market, we provide analytic estimates for the tracking cost and utility loss as a function of R. These estimates are then used in the objective function of the optimization problem.

A. Analytic Estimate of Utility Lost During Dynamic RS Reserve Management

Proposition 2 states that modulating the price causes a utility loss that is proportional to the price variance. This implies that RS provision comes with a utility loss to the SBO as the price needs to be modulated to track RS signals. Moreover, in [5], it is observed that the utility loss increases as the amount of RS provision, R, increases. Now, we propose a model that explains this relationship.

For the cases where the system converges to steady state very fast, i.e., Inequality (11) is satisfied for small ϵ and the optimal prices satisfy Equation 12, the optimal price policy takes the form

$$u(t) = U^M\left(1 - \frac{\mu}{\lambda^M r}\left(A + Ry(t)\right)\right) \tag{17}$$

which leads to

$$\sigma_u^2 = \left(\frac{\mu U^M R}{\lambda^M r}\right)^2 \sigma_y^2. \tag{18}$$

If we replace this expression for σ_u^2 in the utility loss equation (6), we obtain

$$\begin{array}{l}\text{Utility Loss Under}\\ \text{Fast Convergence}\end{array} = \frac{\mu^2\sigma_y^2 U^M}{2\lambda^M r^2}R^2 \tag{19}$$

which suggests that the utility loss increases quadratically as R increases. In our numerical experience, we observe that utility loss increases with R^2 even if we do not assume fast convergence, and the utility loss is given by

$$\text{Utility Loss} = \chi + \gamma R^2 \tag{20}$$

where χ is a constant and γ is some coefficient. These parameters are complicated functions of the problem parameters such as $\lambda^M, \mu, \sigma_y^2, K$ etc. However, once χ and γ are estimated, they can be used in the Hour Ahead Market to decide optimal value of R as far as the consumption characteristics of the Smart Building stay the same.

Equation 20 also implies another important result. There is a fixed amount of utility loss that occurs at the moment that the SBO decides to provide RS, regardless of the amount of the provision. Because, even if R is very small and $A + Ry(t) \approx A, t \geq 0$, the SBO has to modulate the price in order to keep consuming constantly at a rate of A kW, whereas there is not such an obligation when the SBO is not providing RS reserve. Section VI-B verifies this conclusion numerically.

B. Analytic Estimate of Tracking Cost Incurred During Dynamic RS Reserve Management

SBOs receives a reward for providing RS that is proportional to R. However, this reward is decreased when the tracking is not carried out perfectly. Numerical experiments show that the cost of imperfect tracking also increases with R. The SBO should calculate its net income by considering this tracking cost while deciding the amount of RS to offer in the Hour Ahead Market. However, the exact value of the tracking cost can only be obtained by solving the Bellman equation in (4), a task that is computationally expansive. We provide instead an approximation of the tracking cost as a function of R to use it in the objective function of the Hour Ahead optimization problem.

When we solve the Bellman equation in (4) for the optimal price policy, the distribution of the tracking error exhibits a bell shape centered around zero. In other words, the optimal price policy helps to track RS signals with a minimum tracking error most of the time. However due to the stochasticity in the system, positive or negative values for the tracking error are also possible, but less likely. Let us denote the tracking error by e_T. Then, we approximate the distribution of the tracking error by a normal distribution, i.e., $e_T \sim N(M_e, \sigma_e^2)$ where $M_e \approx 0$. Then the approximate average tracking cost can be find as follows.

$$\begin{aligned}\Omega &\approx \kappa\mathbb{E}[(e_T)^2] \\ &= \kappa(\sigma_e^2 + M_e^2)\end{aligned} \tag{21}$$

In addition, we observe that the standard deviation of the normal distribution increases linearly with RS provision R and has the following form

$$\sigma_e = \varpi + \nu R \tag{22}$$

where $\varpi, \nu \geq 0$ are some constants. The SBO can estimate M_e, ϖ and ν by solving the Bellman equation in (4) for some sample cases. Then, these constants can be used to estimate the tracking cost in the Hour Ahead optimization problem as far as the consumption characteristics of the building stay the same. As a last note about this approximation, we have observed that the constant ν is inversely proportional to λ^M. This means that an increase in the consumption capacity of the Smart Building, while keeping the average consumption A fixed, will make the tracking cost less sensitive to RS provision R.

C. Optimization of the RS Offer in the Hour Ahead Market

Using analysis and propositions introduced in Section III, we derived the limits of a Smart Building on the amount of RS reserve that it can provide, given the consumption characteristics of the building and the rules of the reserve market in Section IV. Then, in Section V-A and V-B, we estimated the loss of utility and the tracking cost of the SBO as a function of the RS provision, so that the SBO does not need to solve repeatedly the Bellman equation in (4) to decide the optimal R to offer in the Hour Ahead Market. The optimization problem that determines the level of Hour Ahead Regulation Service offer is formulated below using the aforementioned analytic cost estimates.

$$\underset{R,\Re}{Max} \quad \pi R - (\chi + \kappa M_e^2 + \kappa \varpi^2)\Re \\ -(\gamma + \kappa \nu^2)R^2 - 2\kappa \varpi R \tag{23}$$

subject to (15), (16), $R \leq A$, $R \leq M\Re$, $R \geq 0$ and $\Re \in \{0, 1\}$.

In this formulation, we assume that the SBO relies on an estimate of the expected reserve clearing price, π, based on historical information. At the moment that the SBO decides to provide RS, there two type of costs that must be minimized: *(i)* The tracking cost the ISO charges due to imperfect tracking of the RS signals, and *(ii)* the loss in consumer utility due to frequent modulation of the consumption in the building. Since the exact values of these losses can only be observed after the SBO actually provides the RS, we use the estimates of their expected value in Equations (20) and (21). These costs are calculated using parameters $\chi, M_e^2, \varpi, \gamma, \nu$, whose value the SBO can obtain once and use for as long as the consumption characteristics of the building remain the same. In order to handle the fixed part in the costs, we use the binary variable $\Re$ and a very large number M.

VI. Numerical Experience

The following input was used in this numerical example: $A = 50$ kW, $\Delta \bar{y} = 1/30$, $\kappa =$ \$ 100 /(kW)^2 min, $\lambda^M = 150$ /min, $\mu = 1$ /min, $U^M =$ \$ 50 , $r = 1$ kW and $\tau = 5$ min.

A. Validation of Normal Distribution Approximation of the p.d.f. of $n(t)$

We provide here numerical results that illustrate the accuracy of the normal distribution approximation to the p.d.f. of $n(t)$ for a system with $n(0) = i$ as described in Proposition 3. In Figure 1, the horizontal axis represents the possible number of active appliances in the system after t minutes and the vertical axis shows the corresponding probabilities. The blue continuous curve represents the exact values of the probabilities that are calculated according to

$$P(t) = e^{Qt} I(0) \tag{24}$$

where the vector $I(0)$ is the initial probability distribution of the system whose i^{th} entry is equal to 1 and the rest of the entries are equal to zero, the matrix Q is the generator of the corresponding Markov process and P is the vector for the probability distribution of the number of appliances after t minutes. The red dashed curve is the normal distribution approximation with the mean and variance as given in Equations 7 and 8. In this example $u = 1/3$ and hence $\lambda = 100$. $n(0) = i = 20$. The graph verifies the accuracy of the normal distribution approximation that is given in Proposition 3.

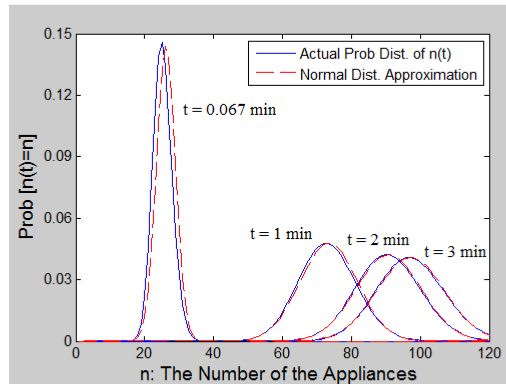

Fig. 1: Normal Distribution Approximation to $n(t)$ for given $n(0)$

B. Average Utility Loss versus RS provision, R

The model suggested in Section V-A, which explains the relationship between the Utility Loss and RS provision R, is numerically verified in Figure 2. At the moment when $R = \epsilon > 0$, there is a χ amount of loss in utility compared to the case where the price $\bar{u}$ is applied constantly, i.e., $\bar{u} = \frac{1}{T}\int_0^T u(t)dt$. The Utility Loss is quadratic in R with γ that is a function of the problem parameters. The accuracy of the average utility loss estimate as a function of R allows the Hour Ahead optimization problem to solve efficiently for the optimal amount of RS provision.

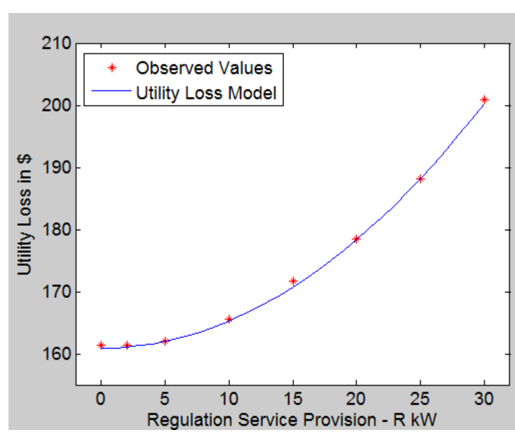

Fig. 2: Utility Loss - RS Provision Relationship, $\chi = 160.9$, $\gamma = 0.0437$

C. Normal Distribution Approximation to Tracking Error p.d.f.

Figure 3 shows that the normal distribution approximation to the tracking error p.d.f. proposed in Section V-B holds with a reasonable accuracy, supporting the veracity of our Hour Ahead optimization problem formulation. Although not shown in the figure, the standard deviation of the normal distribution approximation increases linearly in R. Namely, we have $\sigma_e = 4.008$ for $R = 15$ kW, $\sigma_e = 4.067$ for $R = 20$ kW and $\sigma_e = 4.113$ for $R = 25$ kW.

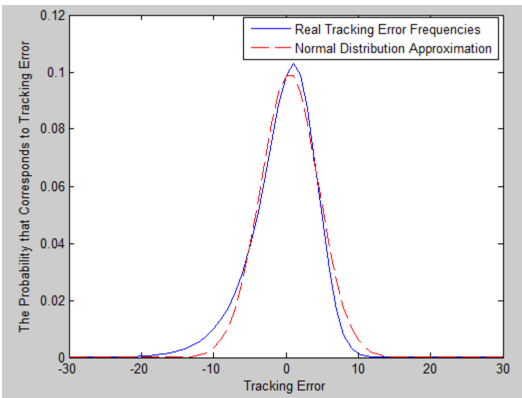

Fig. 3: The Normal Distribution Approximation to Tracking Error for $R = 15$ kW

D. Optimization of the RS Offer

Using the approximation parameters obtained in Sections VI-B and VI-C, namely $\chi = 160.9$, $\gamma = 0.0437$, $\varpi = 3.843$ and $\nu = 0.011$, we now solve the optimization problem to determine the optimal amount RS offer, R, to the Hour Ahead Market. The results show that there is a threshold value of π, beyond which it is profitable for the SBO to offer RS reserve. On the other hand, in this problem setting, the amount of RS offer is limited by the SBO's tracking capability of a downward RS signal ramp as imposed by Inequality (16). Moreover, the optimal RS management price policy exhibits a bang-bang type of structure as shown in Figure 4. This behavior is parameter choice specific and does not occur for different parameter selections. Due to space limitations additional numerical experience is deferred to future work.

VII. Conclusion

We have investigated the ability of Smart Buildings to provide Regulation Service for broad consumption characteristics of flexible appliance loads, and used it to propose an optimization model assisting SBOs to determine optimal RS offers to the Hour Ahead Market. Probabilistic constraints were developed by studying the look ahead probability distribution of active appliances under a fixed price control and its implications on the speed with which it converges to a steady state. Broad building characteristics were summarized as the determinants of the convergence speed.

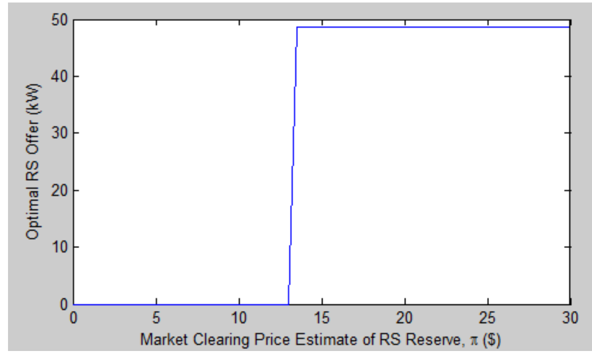

Fig. 4: Optimal RS Provision Policy ($\kappa = 1$)

References

[1] S. G. Whitley, "New Challenges Facing System Operators", *Past Present and Future of the Power Grid, A Cornell University Workshop*, August 2012, New York.

[2] *Manual 2: Ancillary Services Manual*, v. 3.26, NYISO, Rensselaer, NY, 2013.

[3] NYISO, "Growing Wind", *Final Report of the NYIS0 2010 Wind Generation Study*, Sept. 2010.

[4] PJM (2013). Market-Based Regulation [Online]. Available: http://pjm.com/markets-and-operations/ancillary-services/mkt -based-regulation.aspx

[5] M. C. Caramanis, I.C. Paschalidis, C.G. Cassandras, E. Bilgin, E. Ntakou, "Provision of Regulation Service Reserves by Flexible Distributed Loads", in *51st IEEE Conference on Decision and Control*, Maui, HI, Dec. 2012, pp. 3694-3700.

[6] B. Zhang, J. Baillieul, "A Packetized Direct Load Control Mechanism for Demand Side Management", in *51st IEEE Conference on Decision and Control*, Maui, HI, Dec. 2012, pp. 3658-3665.

[7] I.C. Paschalidis, B. Li, M. C. Caramanis, "Demand-Side Management for Regulation Service Provisioning through Internal Pricing", *IEEE Trans. Power Systems*, vol. 27, no. 3, pp. 1531-1539, Aug. 2012.

[8] I.C. Paschalidis, B. Li, M. C. Caramanis, "Market-Based Mechanism for Providing Demand-Side Regulation Service Reserves", in *50th IEEE Conference on Decision and Control*, Orlando, FL, Dec. 2011, pp. 21-26.

[9] A. Keech, *PJM Manual 12: Balancing Operations*, 26th ed., PJM, Valley Forge, PA, 2012.

[10] *PJM Manual 15: Cost Development Guidelines*, 20th ed., PJM, Valley Forge, PA, 2012.

[11] *PJM Manual 11: Energy & Ancillary Services Market Operations*, 55th ed., PJM, Valley Forge, PA, 2012.

[12] *PJM Manual 28: Operating Agreement Accounting*, 56th ed., PJM, Valley Forge, PA, 2012.

[13] W. Feller, *An Introduction to Probability Theory and Its Applications*, 3rd ed. New York, NY: John Wiley, 1967, vol. I.

[14] W. Stadje and P.R. Parthasarathy, "On the Convergence to Stationarity of the Many Server Poisson Queue", *J. of Appl. Probability*, vol. 36, pp. 546-557, 1999.

[15] P. Coolen-Schrijner and E. A. Van Doorn, "On the Convergence to Stationarity of Birth-Death Processes", *J. of Appl. Probability*, vol. 38, no. 3, pp. 696-706, 2001.

[16] D.P. Bertsekas, *Dynamic Programming and Optimal Control*. Belmont, MA: Athena Scientific, 1995, vol. II.