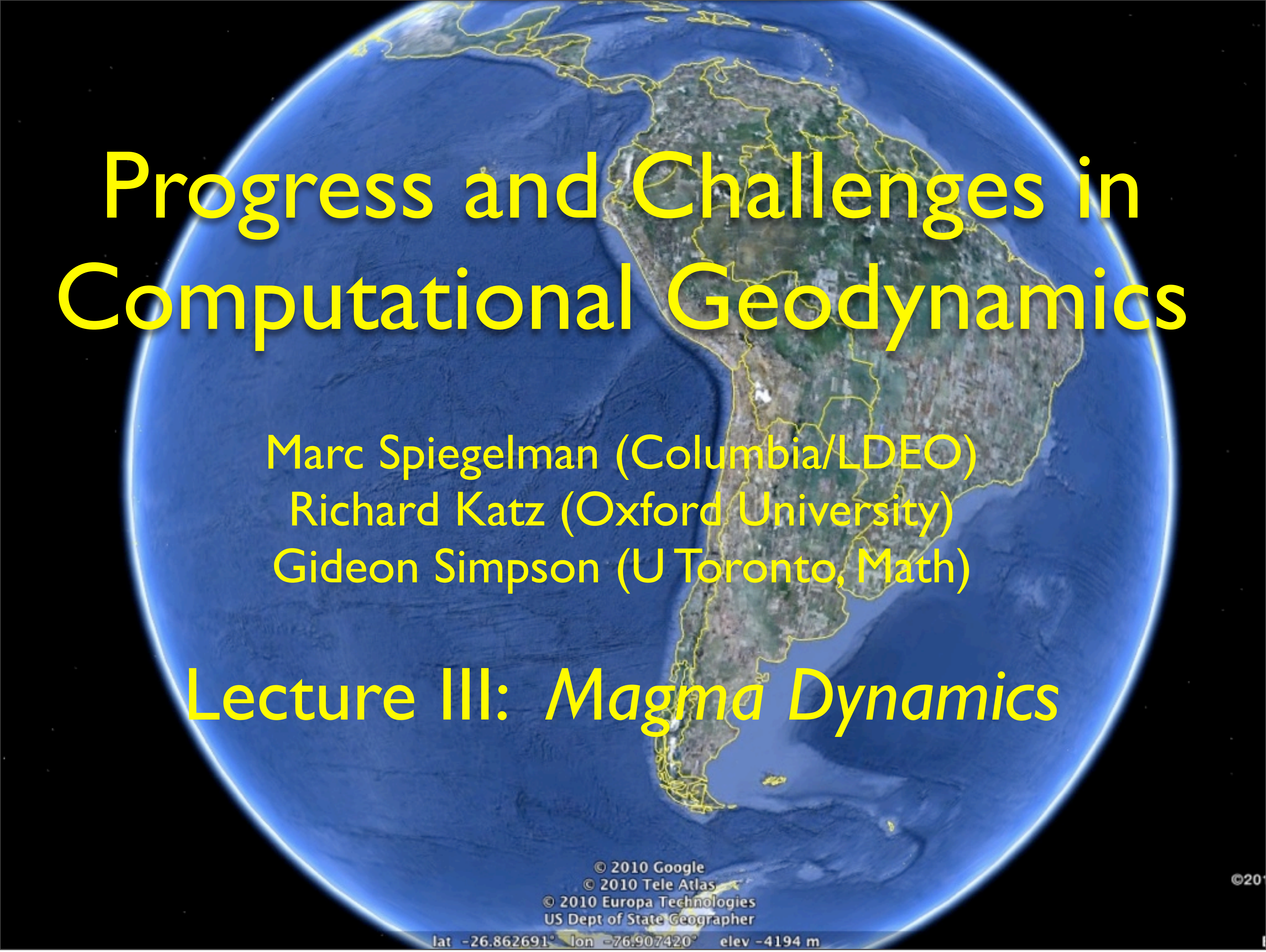

Progress and Challenges in Computational Geodynamics

Marc Spiegelman (Columbia/LDEO)
Richard Katz (Oxford University)
Gideon Simpson (U Toronto, Math)

Lecture III: *Magma Dynamics*

Outline

- Overview and Motivation
- Equations for coupled fluid-solid dynamics
- Basic Physics: Localization phenomena
  - non-linear waves
  - Shear band formation, reactive flow
- Geodynamics Applications : Mid Ocean Ridges and Subduction Zones
- Open Questions/Future Directions

The Take Away...

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lat -26.862691° lon -76.907420° elev -4194 m

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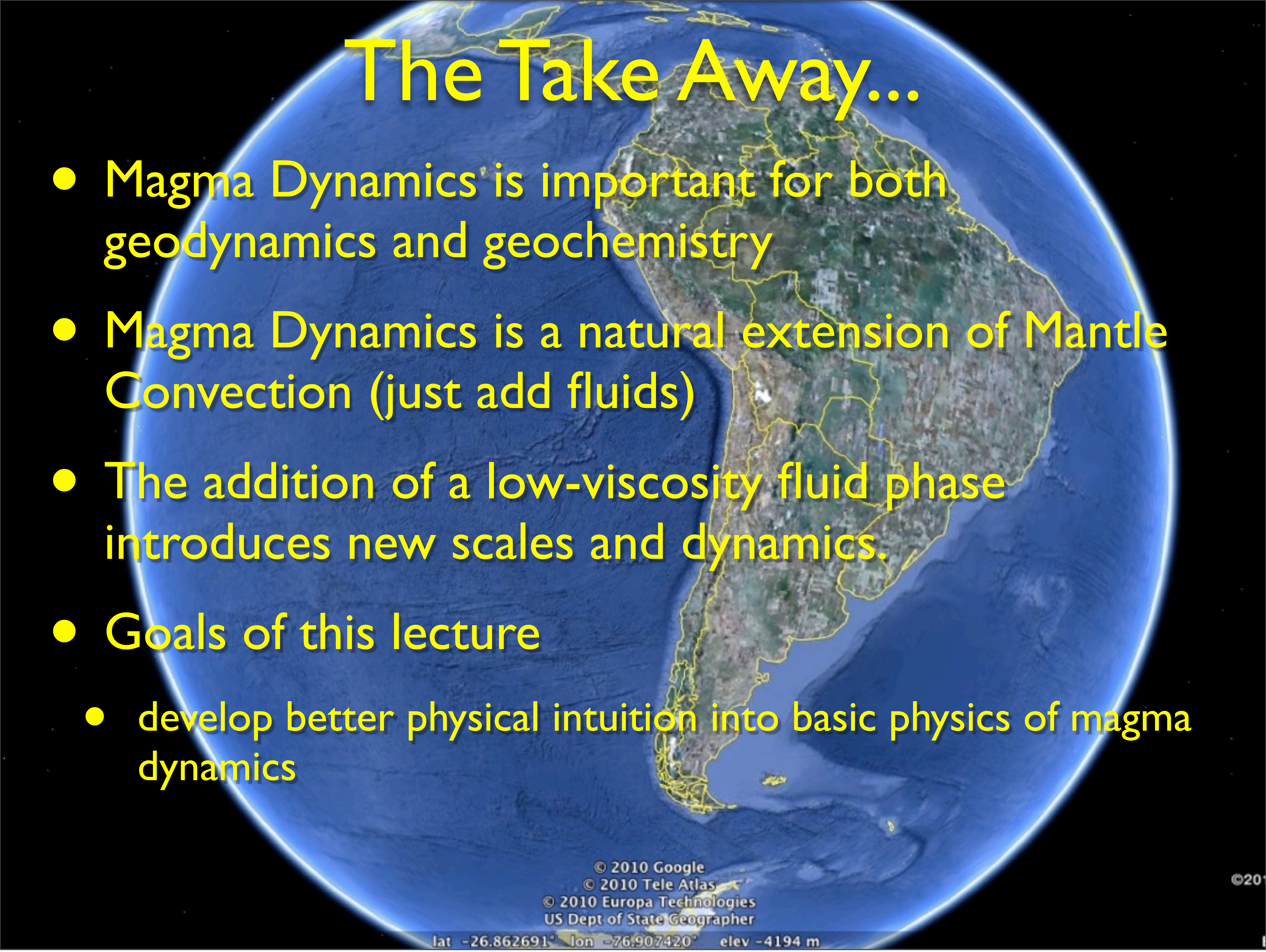
  - develop better physical intuition into basic physics of magma dynamics



The Take Away...

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- The addition of a low-viscosity fluid phase introduces new scales and dynamics.
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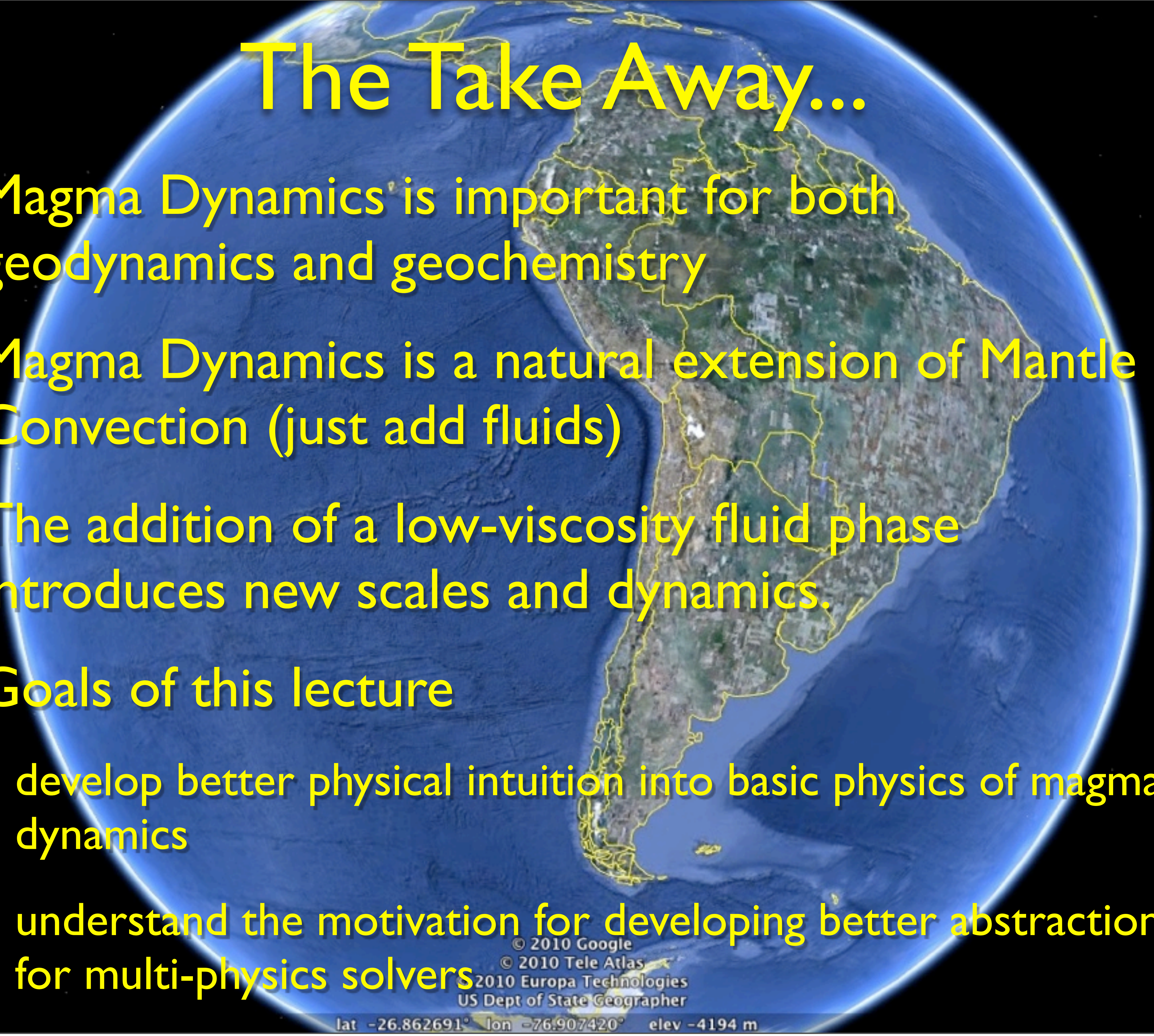
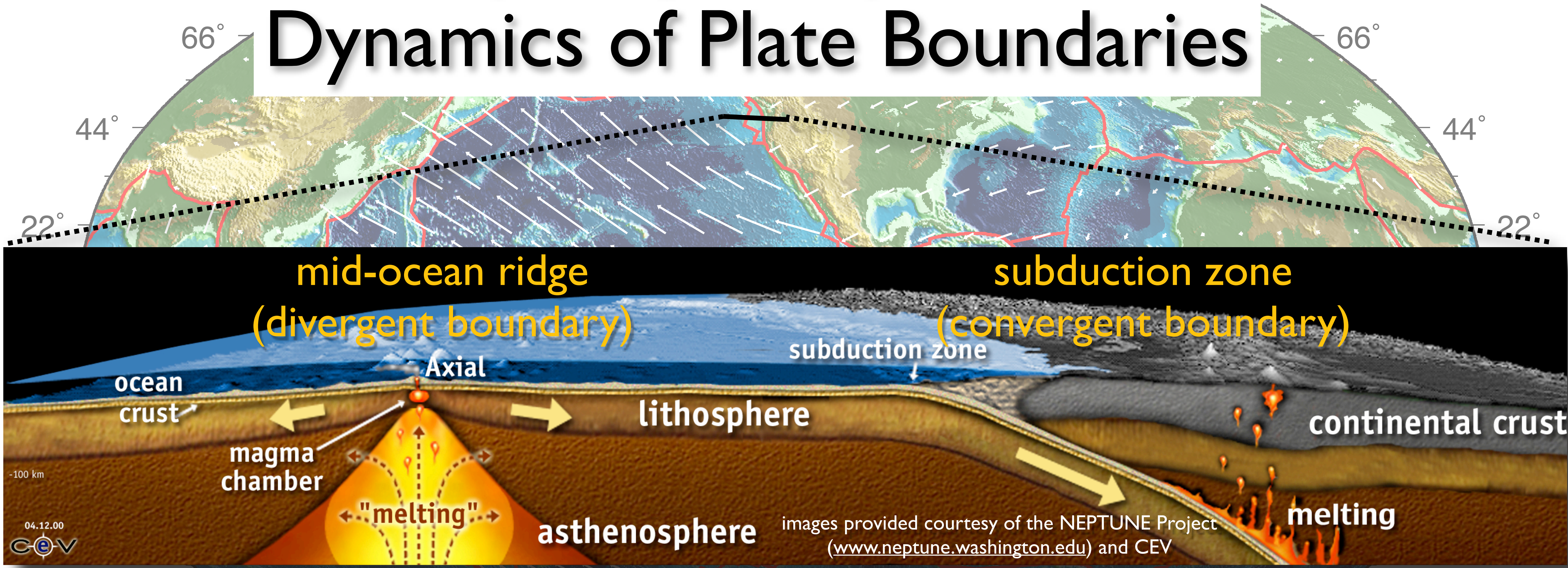
  - develop better physical intuition into basic physics of magma dynamics
  - understand the motivation for developing better abstractions for multi-physics solvers

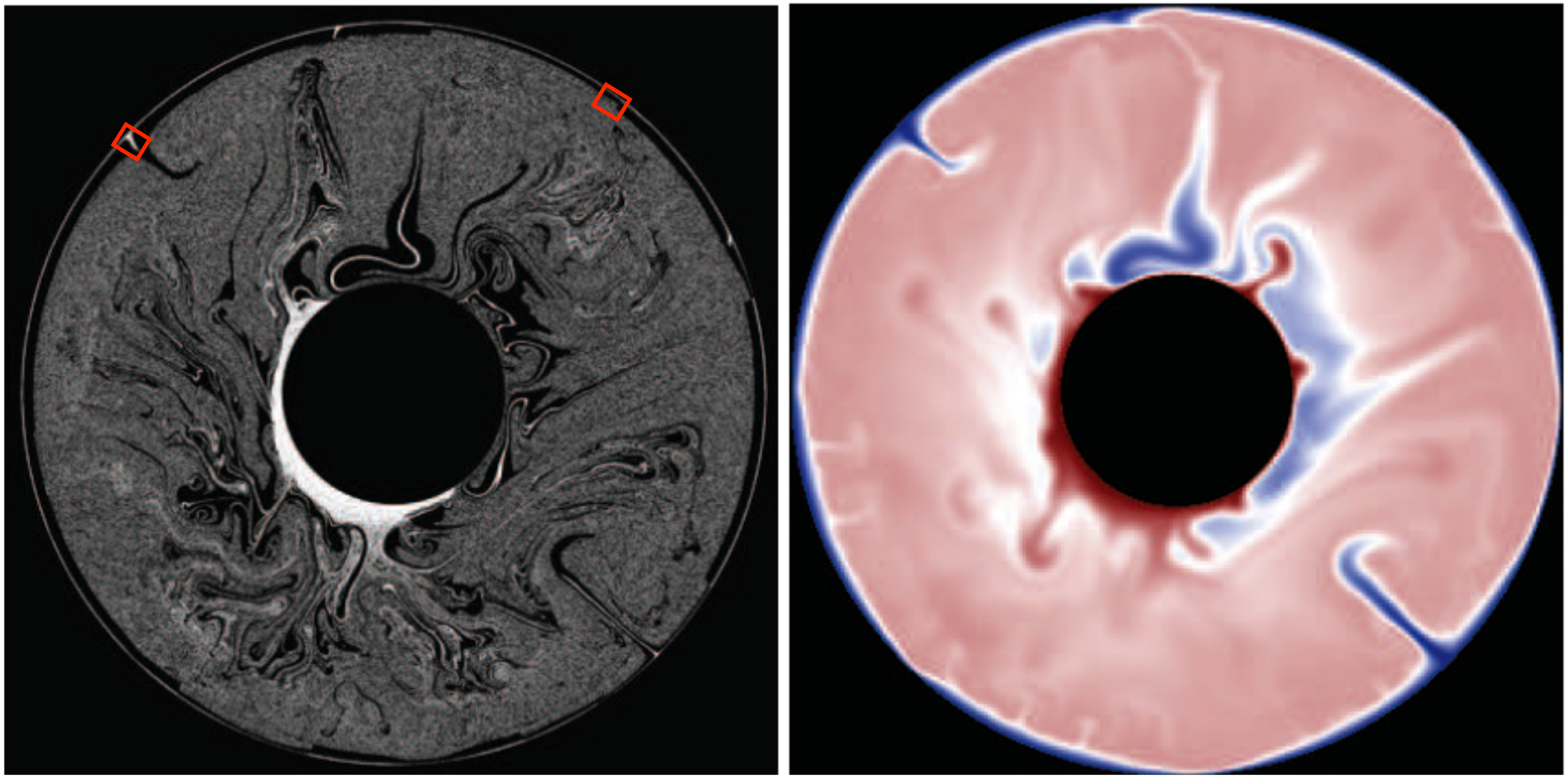


Why Magma Dynamics?
Dynamics of Plate Boundaries



- Mantle convection = Convection ***with Plates***
- Plates are defined by their ***weak*** boundaries.
- Convergent and Divergent Boundaries are fundamentally magmatic
- How does magmatism affect the dynamics and structure of plate boundaries and global mantle convection?

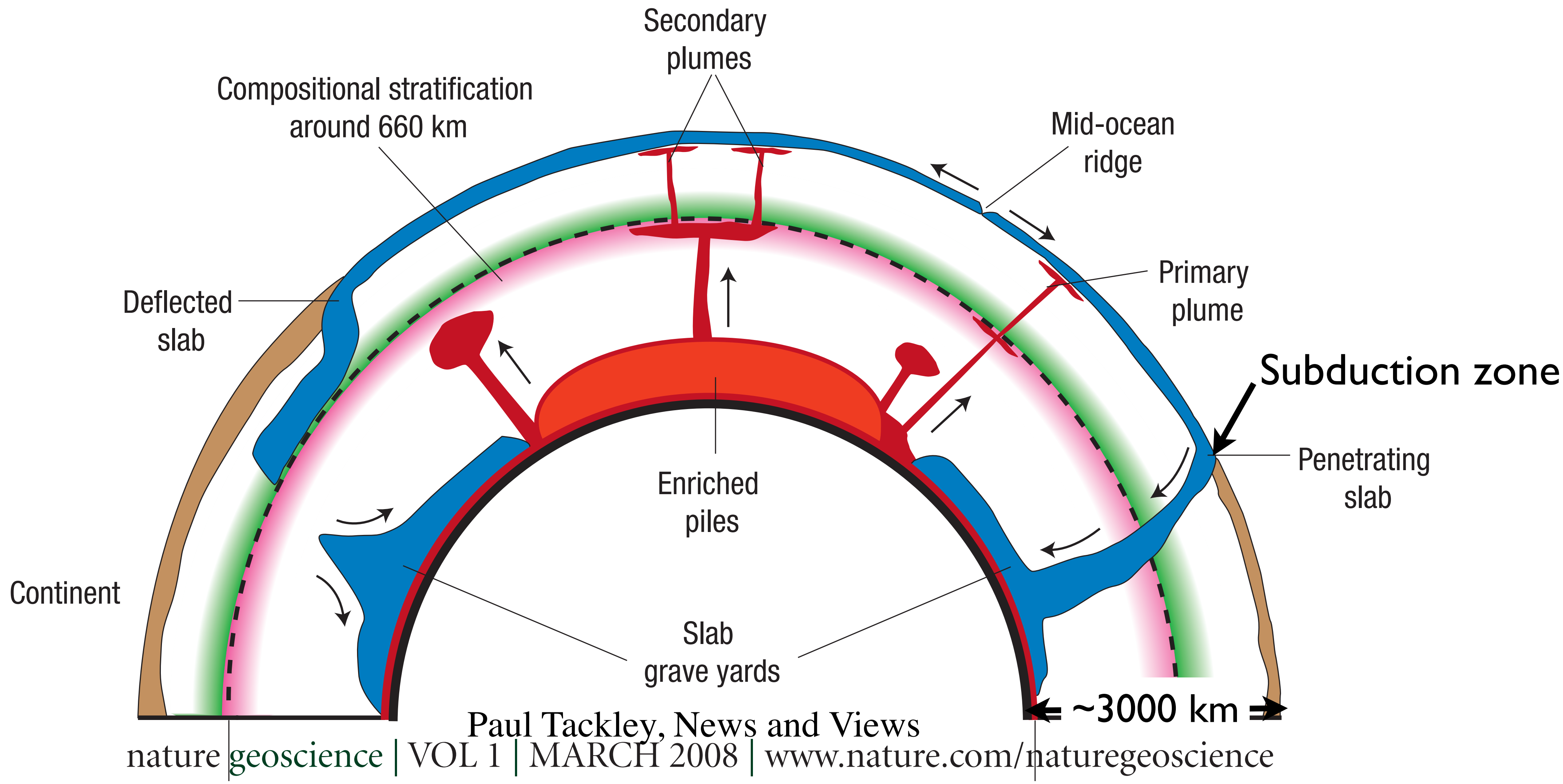
Why Magma Dynamics?
Global Geochemical Evolution



Brandenburg et al, EPSL 2008, 2-D Cylindrical High Ra convection calculation

- Solid State Convection primarily *stirs*
- Chemical Fractionation, mixing and sampling of the mantle requires a *mobile* liquid phase
- Can we use variation in composition of erupted lavas to infer rate and efficiency of convecting stirring in Earth?

Multi-Scale, Multi-Physics nature of Mantle Convection



Paul Tackley, News and Views
nature geoscience | VOL 1 | MARCH 2008 | www.nature.com/naturegeoscience

- Large Scale Deformation of the Earth is in the solid-state
- Most melting occurs in small scale regions near plate boundaries, but may affect global flow and plate tectonics
- How do we understand the basic physics and interactions across scales and constrain it with chemical data?

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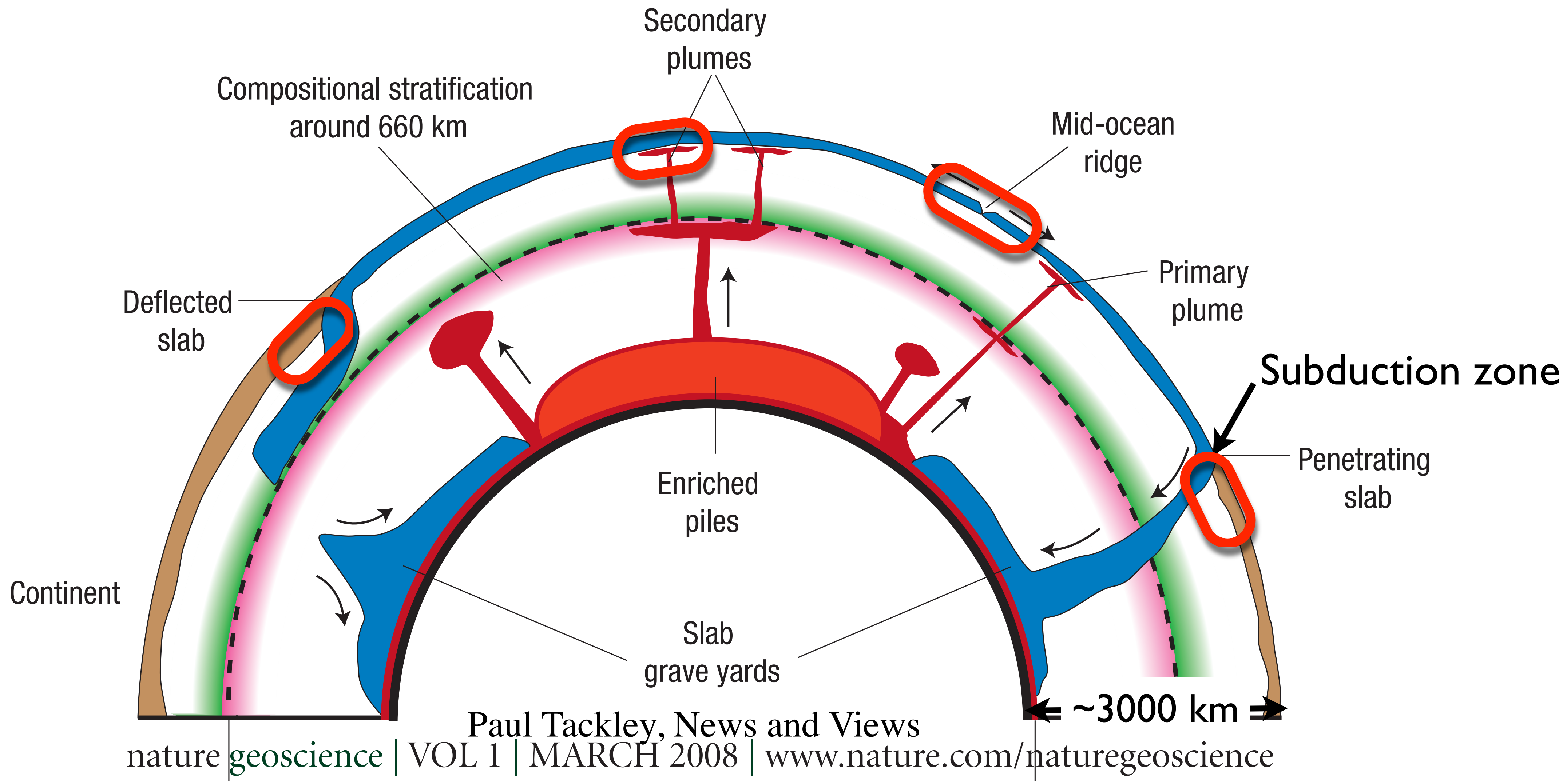

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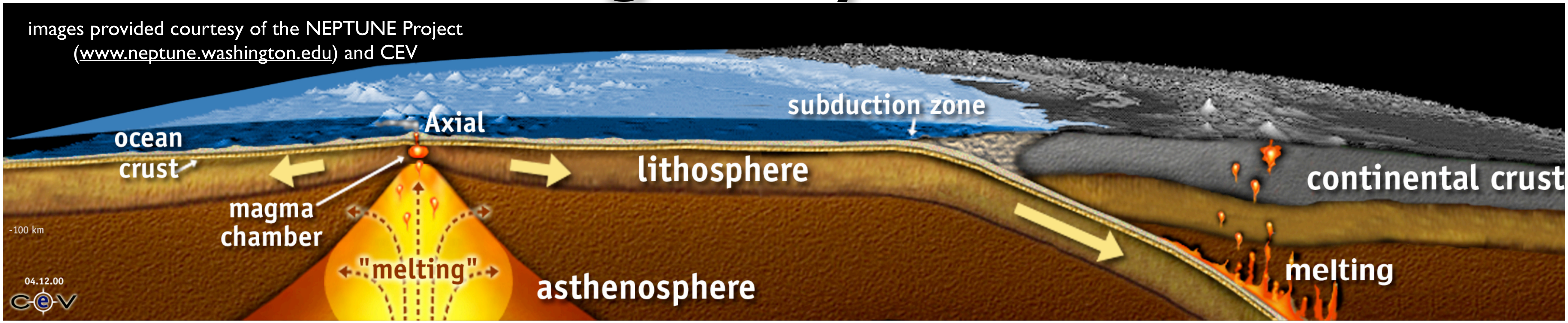
Ingredients for a consistent theory of magma dynamics



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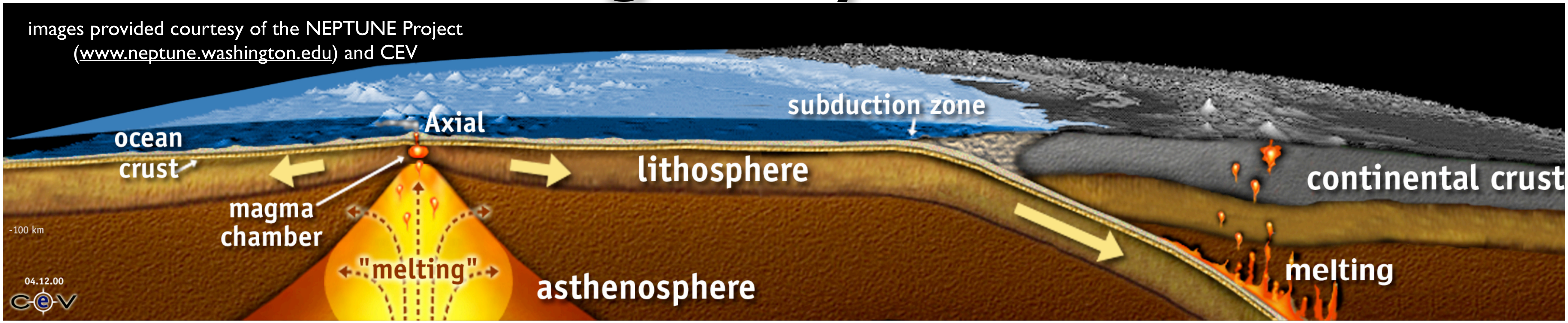

- At least two phases (solid & liquid)

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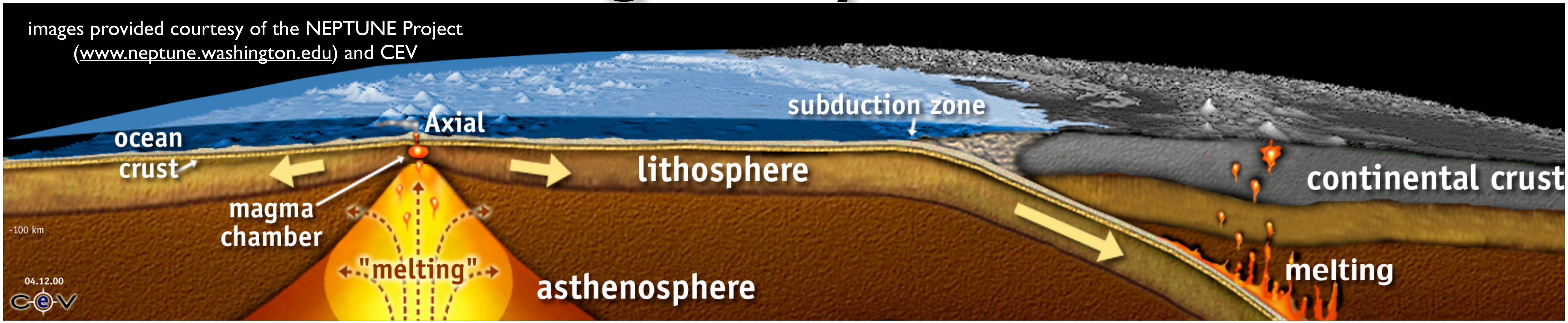

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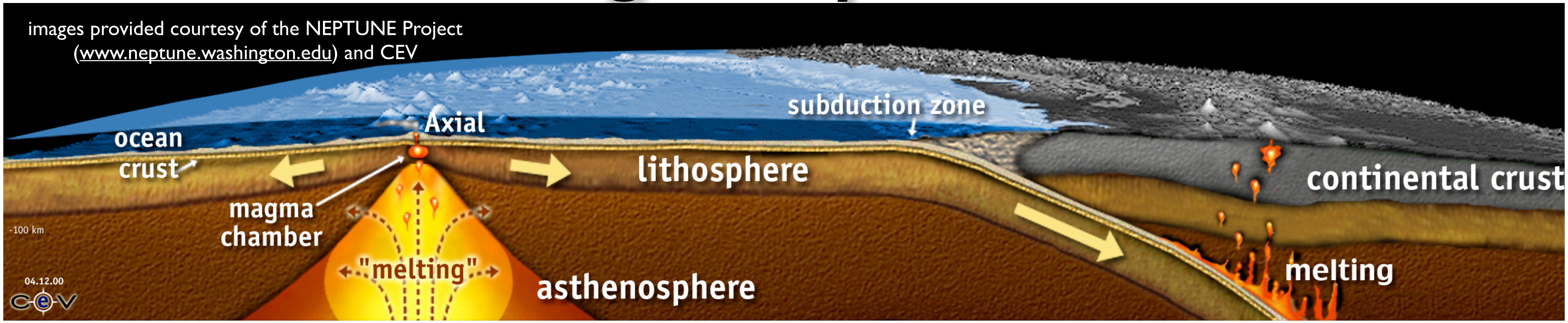

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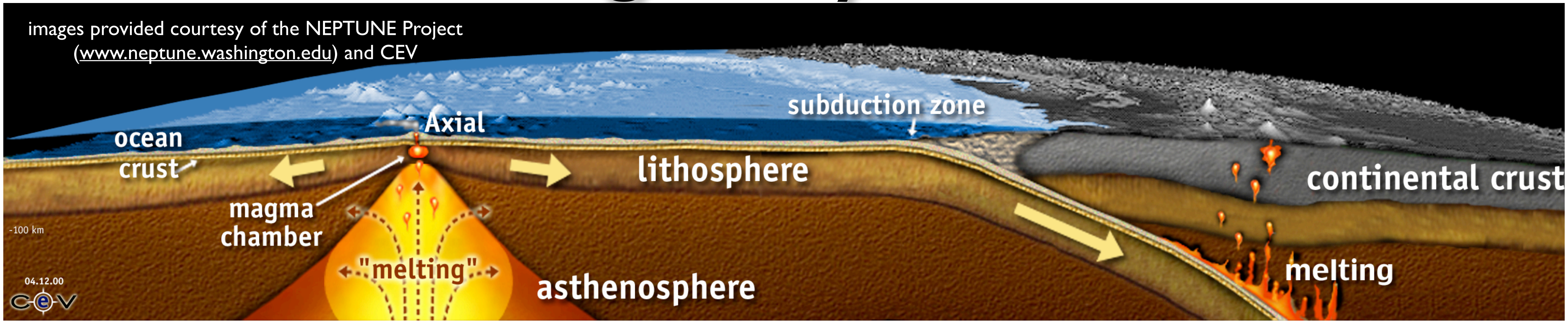

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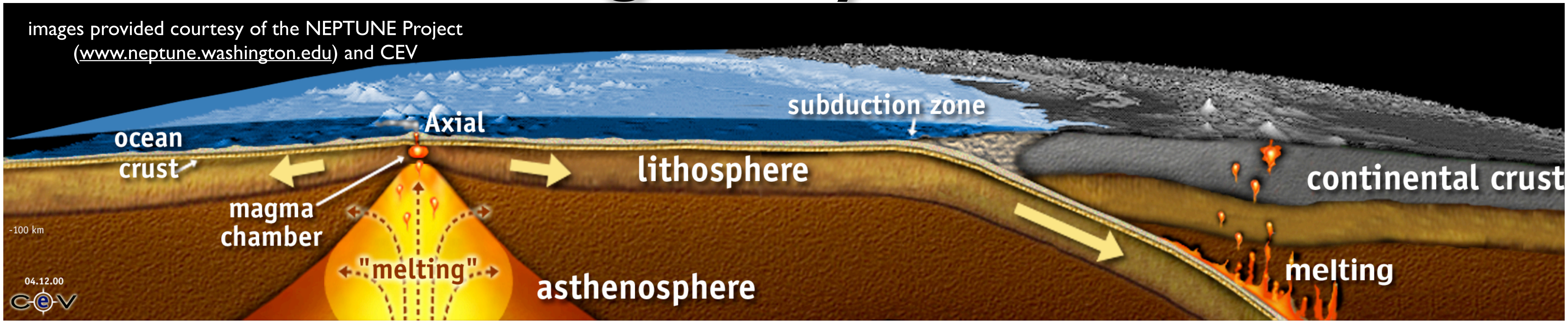

- At least two phases (solid & liquid)
- Significant mass-transfer between phases (melting/crystallization)
- The solid must be ***permeable*** at some scale
- In the absence of solid flow should look like porous media flow.
- In the absence of liquids, the system must be consistent with mantle convection (viscously deformable)

Governing Equations

(McKenzie, 1984, JPet; Scott & Stevenson, 1984,1986, JGR; Bercovici,Ricard et al., 2001,2003; Simpson et al, 2010 JGR)

Conservation of Mass: Fluid

$$\frac{\partial(\rho_f \phi)}{\partial t} + \boldsymbol{\nabla}\cdot(\rho_f \phi \mathbf{v}) = \Gamma$$

Conservation of Mass: Solid

$$\frac{\partial[\rho_s(1-\phi)]}{\partial t} + \boldsymbol{\nabla}\cdot[\rho_s(1-\phi)\mathbf{V}] = -\Gamma$$

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Coupled through pressure

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Coupled through Constitutive Relations

A Better (?) Formulation

(McKenzie Tutorial Notes @ CIG, Katz et al, 2007 Pepi)

Conservation of Momentum for Solid (viscous rheology)

$$\nabla P = \bar{\rho}\mathbf{g} + \nabla\left(\zeta - \frac{2\eta}{3}\right)\nabla\cdot\mathbf{v} + \nabla\cdot\eta\left(\nabla\mathbf{v} + \nabla\mathbf{v}^T\right)$$

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Buoyancy

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Buoyancy

Volumetric Strain

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Buoyancy

Volumetric Strain

Shear Strain

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(McKenzie Tutorial Notes (CIG/bSpace), Katz et al, 2007 Pepi)

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Decompose the pressure into 3 terms

$$P = P_l + \mathcal{P} + P^*$$

with

- Lithostatic Pressure, $P_l = \rho_s^0 g z$
- "Compaction Pressure", $\mathcal{P} = (\zeta - 2\eta/3)\boldsymbol{\nabla}\cdot\mathbf{V}$
- Dynamic Pressure, P^*

A Better (?) Formulation

(McKenzie Tutorial Notes (CIG), Katz et al, 2007 Pepi)

Compressible Flow

$$\frac{D\phi}{Dt} = (1-\phi)\frac{\mathcal{P}}{\xi} + \Gamma/\rho_s$$

$$-\boldsymbol{\nabla}\cdot\frac{K}{\mu}\boldsymbol{\nabla}\mathcal{P} + \frac{\mathcal{P}}{\xi} = \boldsymbol{\nabla}\cdot\frac{K}{\mu}\left[\boldsymbol{\nabla}P^* + \Delta\rho\mathbf{g}\right] + \Gamma\frac{\Delta\rho}{\rho_f\rho_s}$$

"Incompressible" Flow

$$\boldsymbol{\nabla}\cdot\mathbf{V} = \frac{\mathcal{P}}{\xi}$$

$$\boldsymbol{\nabla}P^* = \boldsymbol{\nabla}\cdot\eta\left(\boldsymbol{\nabla}\mathbf{V} + \boldsymbol{\nabla}\mathbf{V}^T\right) - \phi\Delta\rho\mathbf{g}$$

with

- $\xi = (\zeta - 2\eta/3) = \eta\left(\frac{1}{\phi} - \frac{2}{3}\right) \approx \eta/\phi$
- $\Delta\rho = \rho_s - \rho_f$

Comparison to Thermal Convection

(McKenzie Tutorial Notes (CIG/bSpace), Katz et al, 2007 Pepi)

"Magma"

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Stokes Eq.

Thermal Convection

$$\frac{DT}{Dt} = \nabla^2 T$$

$$\boldsymbol{\nabla}\cdot\mathbf{V} = 0$$

$$\boldsymbol{\nabla}P = \boldsymbol{\nabla}\cdot\eta\left(\boldsymbol{\nabla}\mathbf{V} + \boldsymbol{\nabla}\mathbf{V}^T\right) - \mathrm{Ra}T\mathbf{g}$$

Non-linear wave equations for porosity

(Scott & Stevenson, Nature, 1984, Spiegelman, JFM 1993, Simpson & Spiegelman, JSC 2010)

Compressible Flow

$$\frac{D\phi}{Dt} = (1-\phi)\frac{\mathcal{P}}{\xi} + \Gamma/\rho_s$$

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Intrinsic length Scale: The compaction length

(McKenzie, JPet, 1984, Scott & Stevenson, Nature, 1984, Spiegelman, JFM 1993)

The Compaction Length

$$\delta = \sqrt{\frac{K(\phi)\zeta(\phi)}{\mu}}$$

Permeability	$K(\phi) \propto \phi^n$
Solid Bulk Viscosity	$\zeta(\phi) \propto \eta/\phi^m$
melt Shear Viscosity	μ

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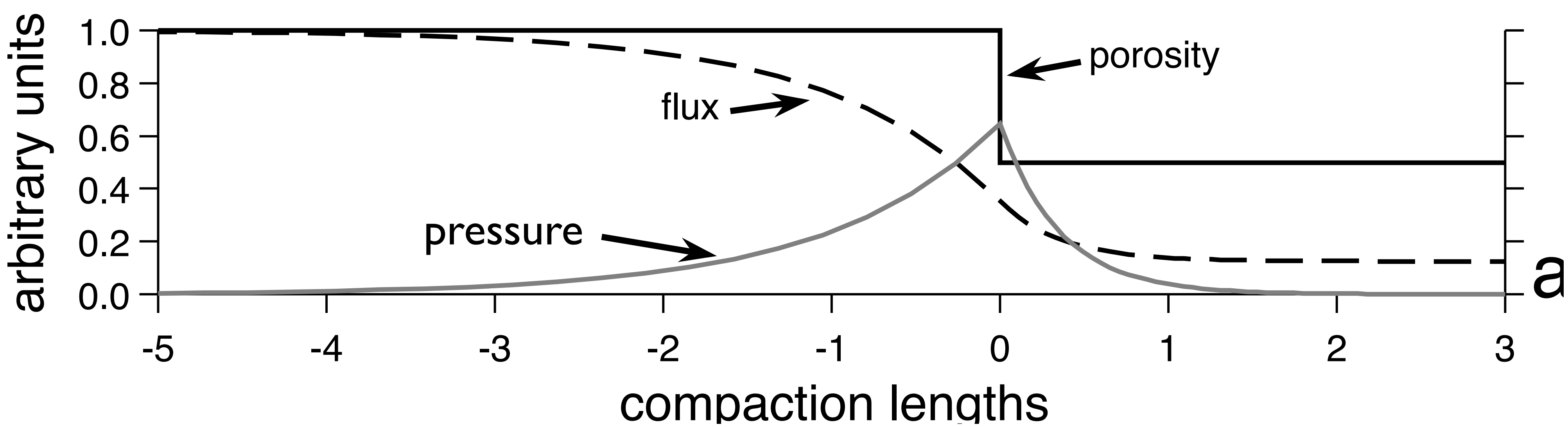
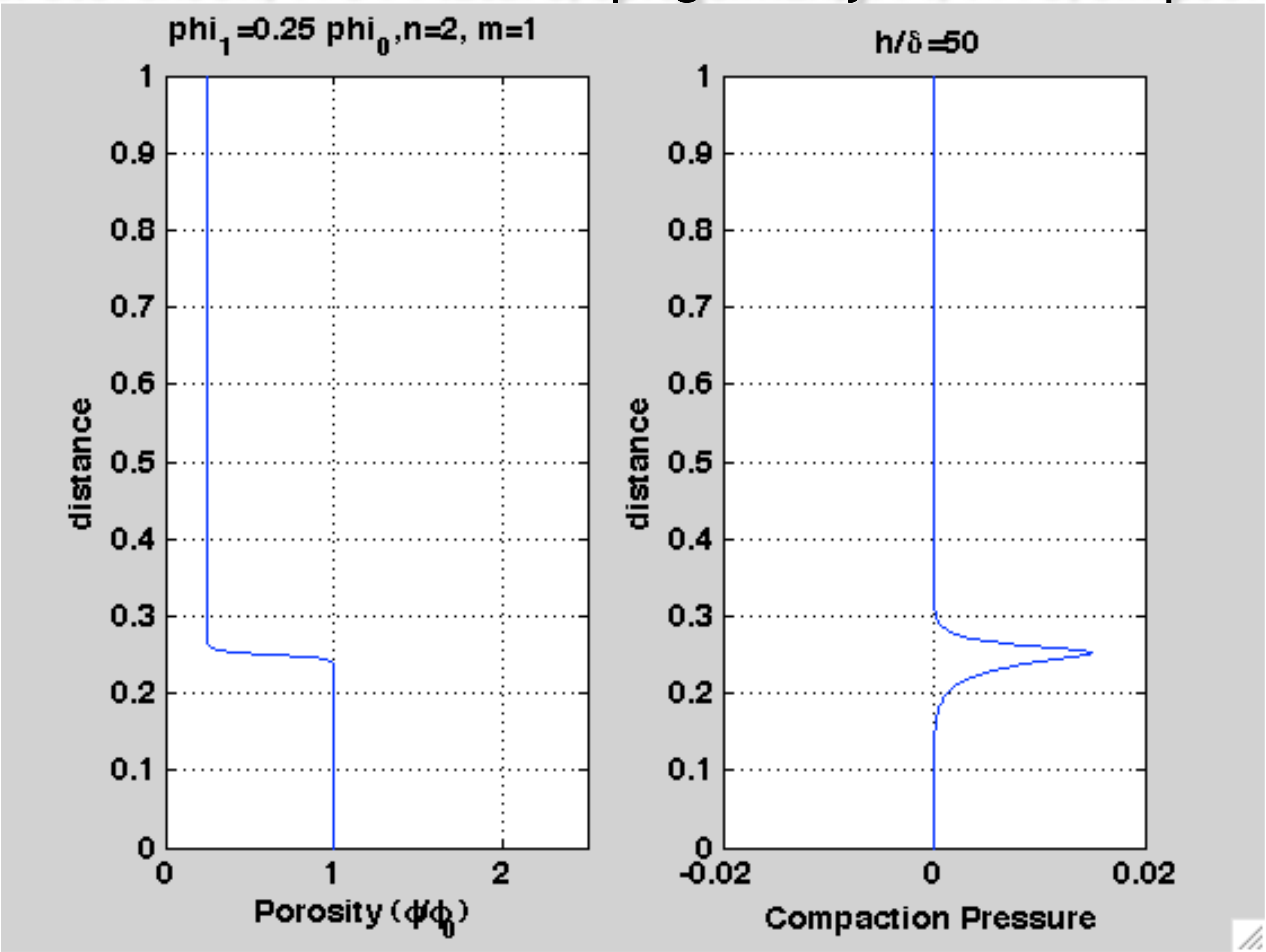
Length scale of pressure variations due to a change in flux.



Non-linear porosity waves

(Scott & Stevenson, 1984 Nature, Spiegelman, JFM, 1993, Simpson et al 2008)



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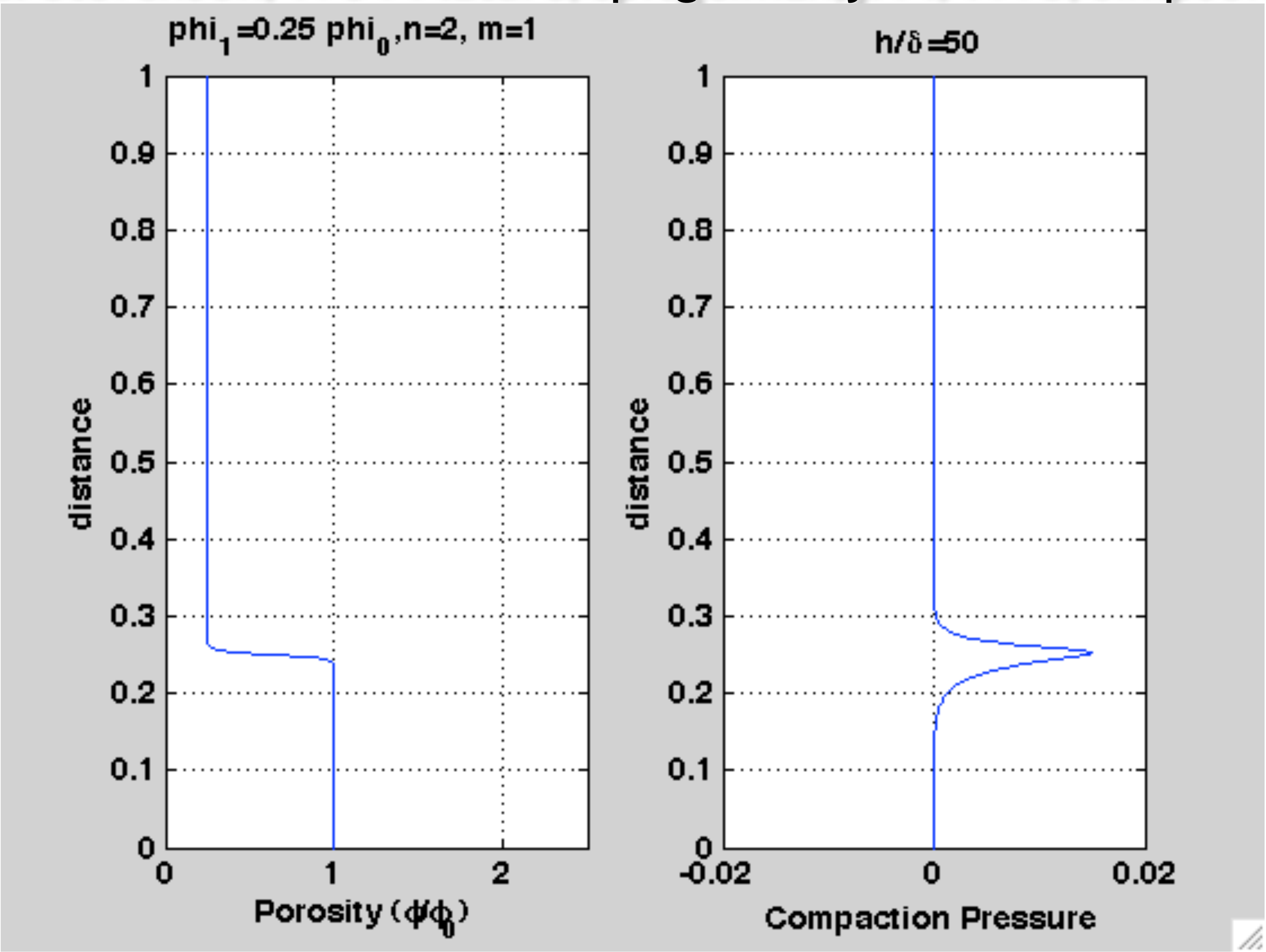

- Variations in melt flux propagate as *non-linear* porosity waves

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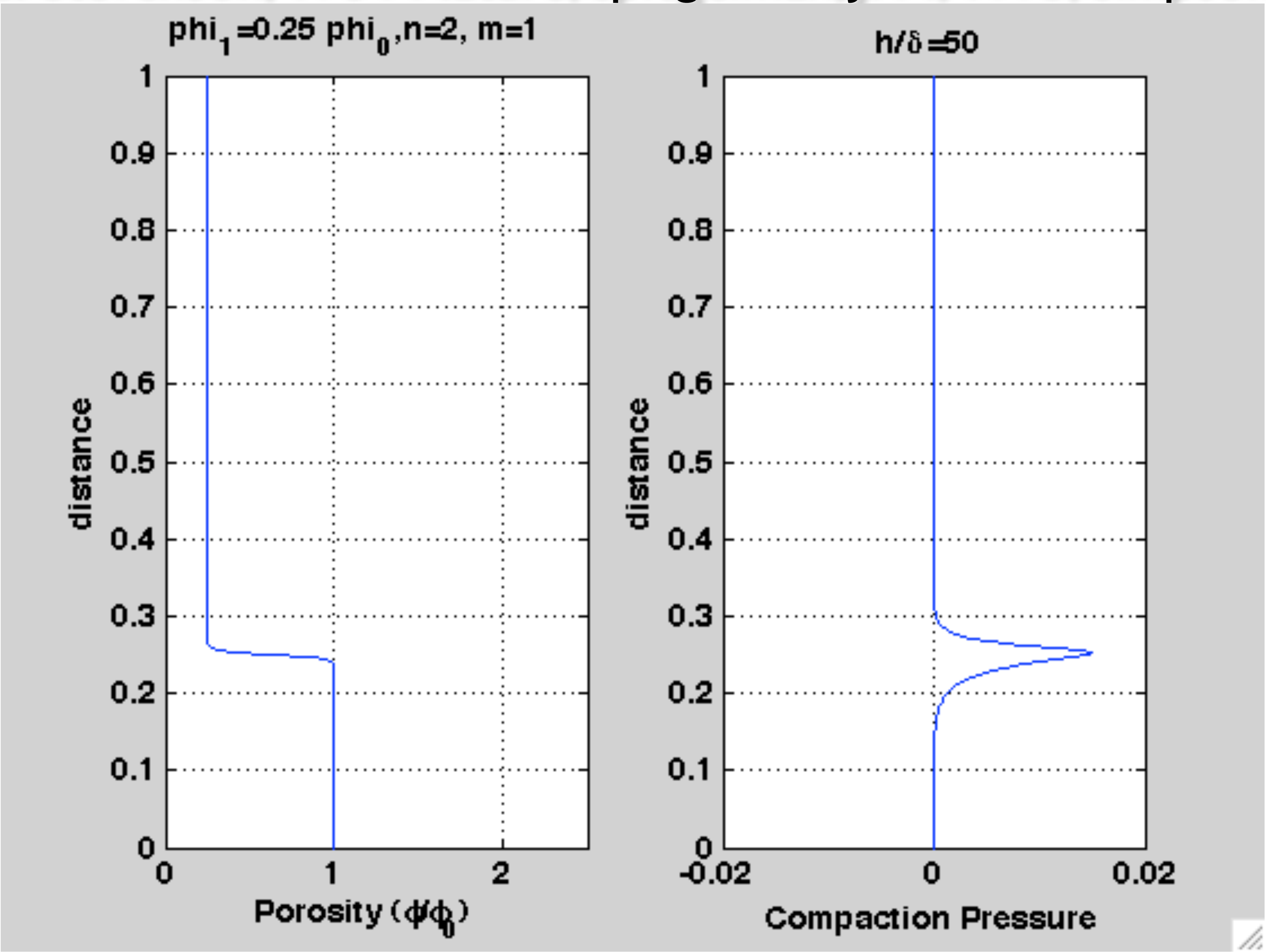
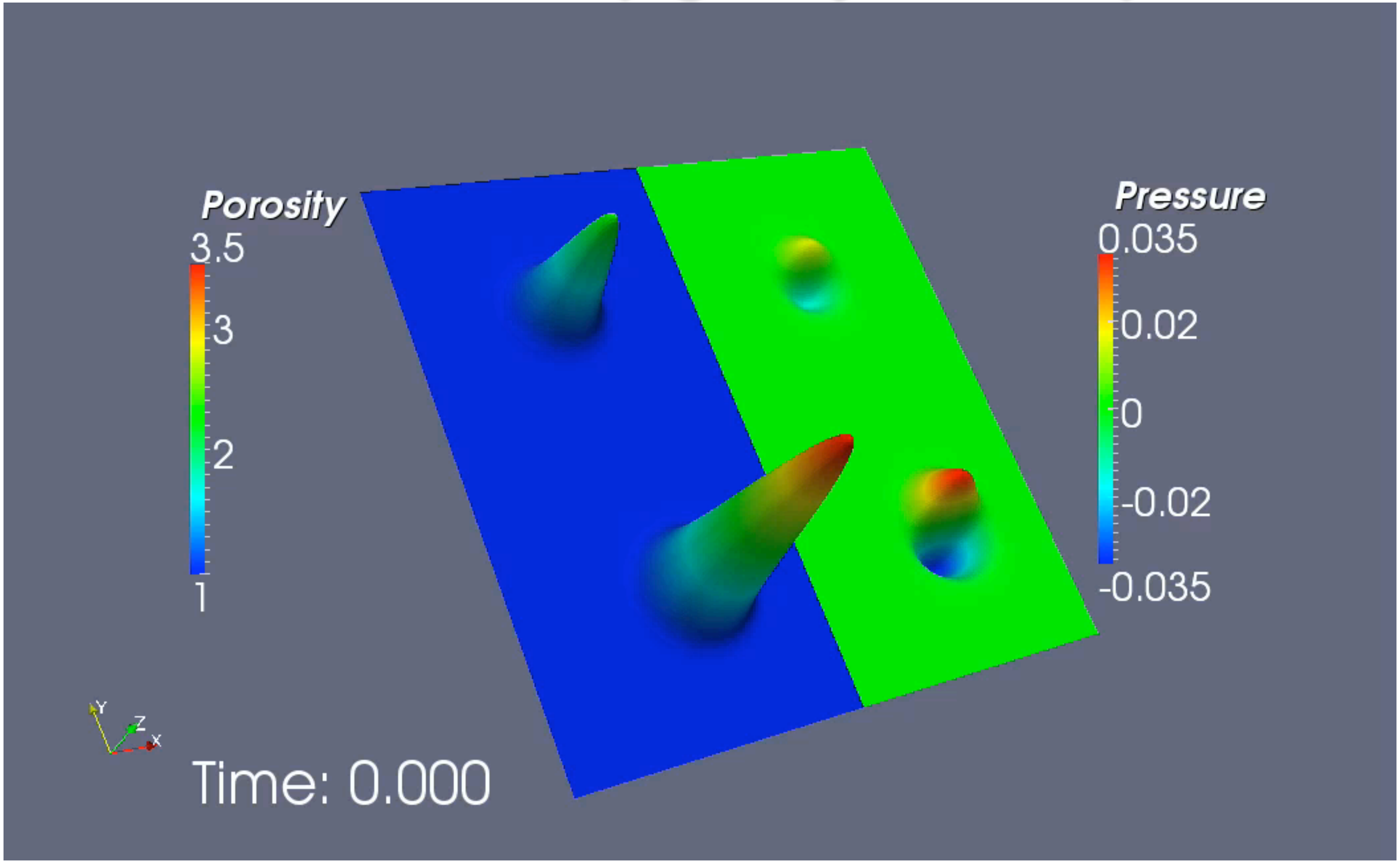


- Variations in melt flux propagate as *non-linear* porosity waves
- Speed and structure of porosity waves depends on *permeability* and *solid rheology*

Non-linear porosity waves

(Scott & Stevenson, 1984 Nature, Spiegelman, JFM, 1993, Simpson et al 2009,2010)



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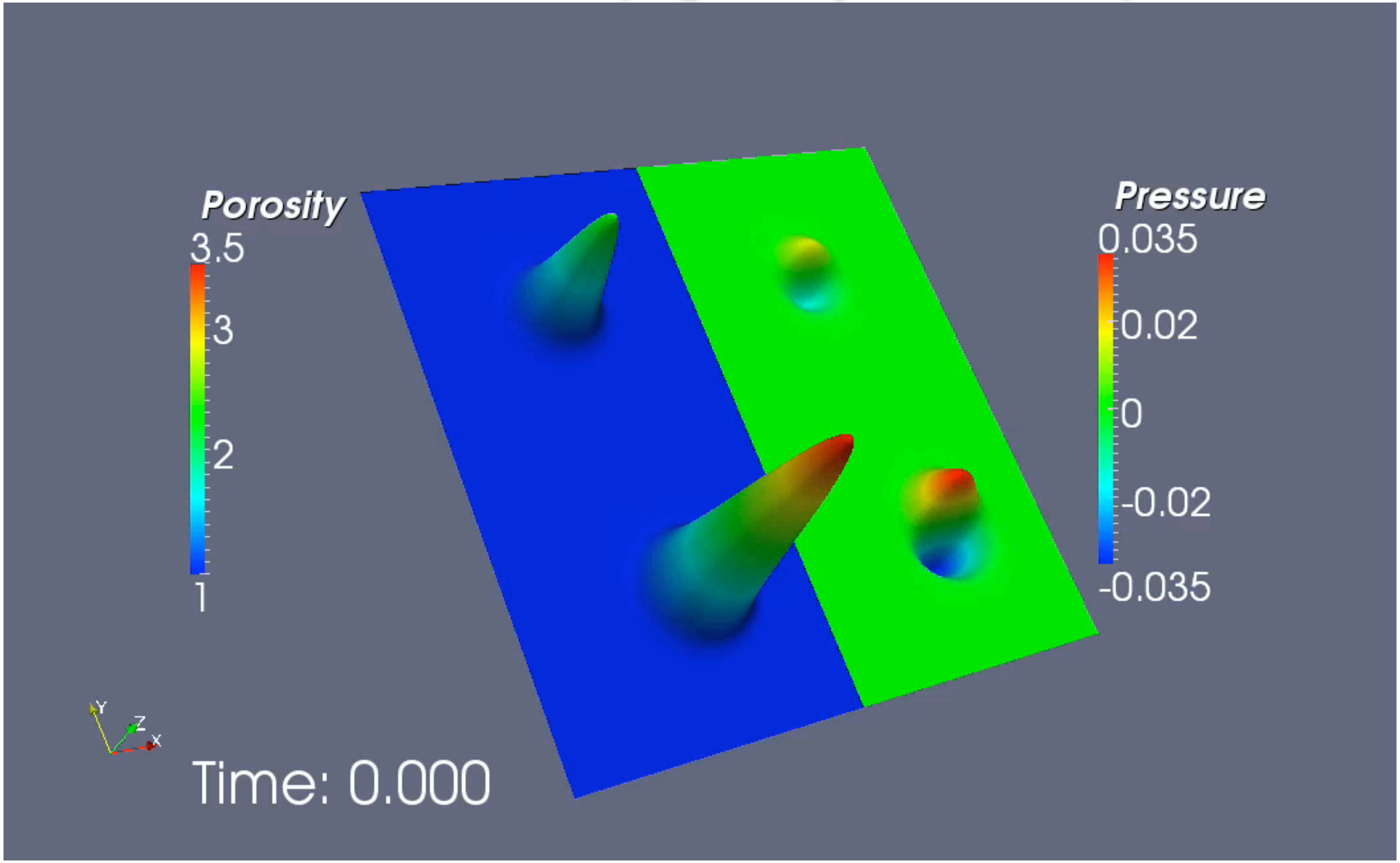
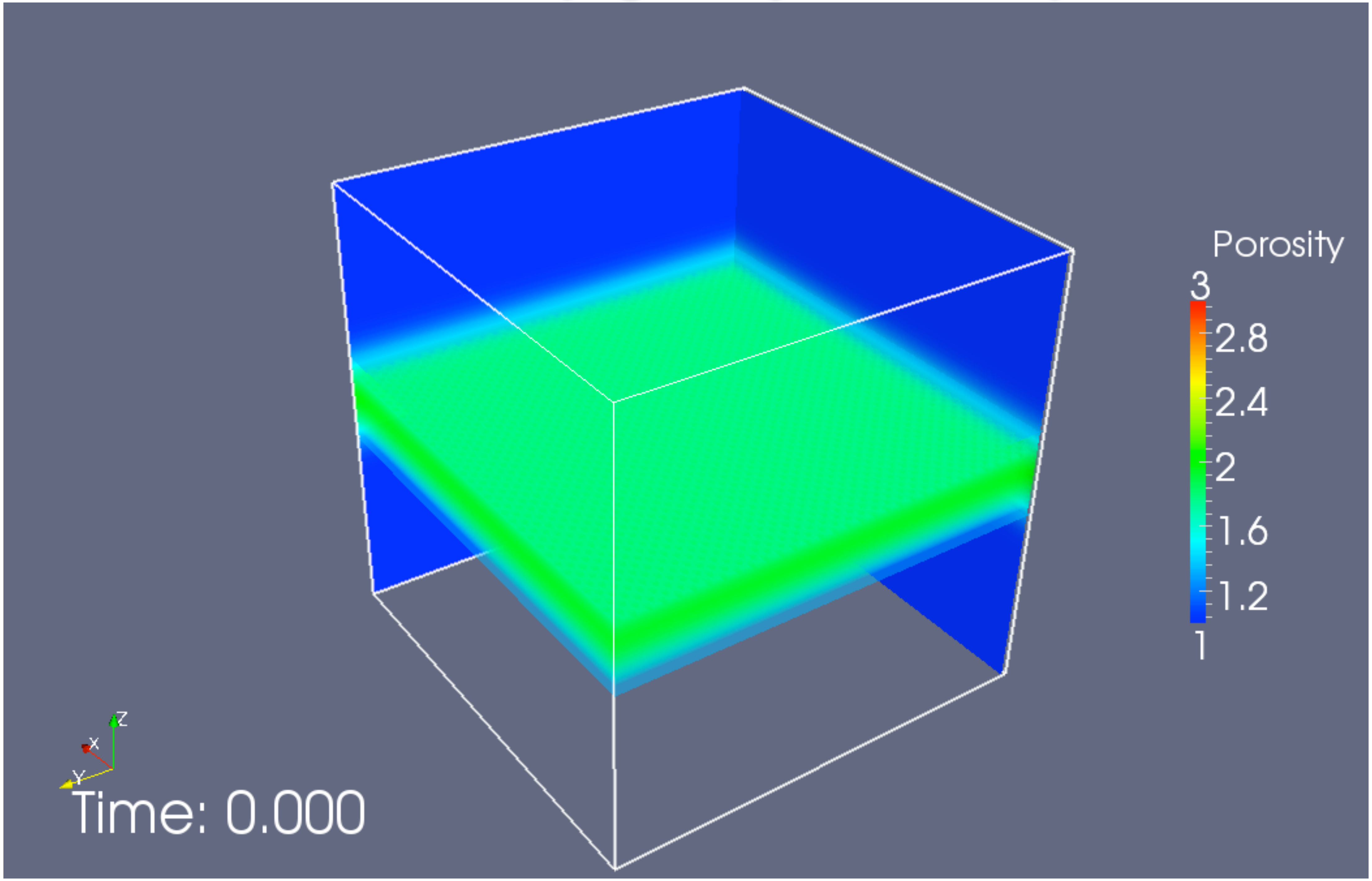


- Collision of 2, 2D-porosity waves. P2-P2 FEM with Semi-Lagrangian 2nd-order time stepping. Hybrid FEniCS/PETSc codes.

Non-linear porosity waves

(Scott & Stevenson, 1984 Nature, Spiegelman, JFM, 1993, Simpson et al 2009,2010)



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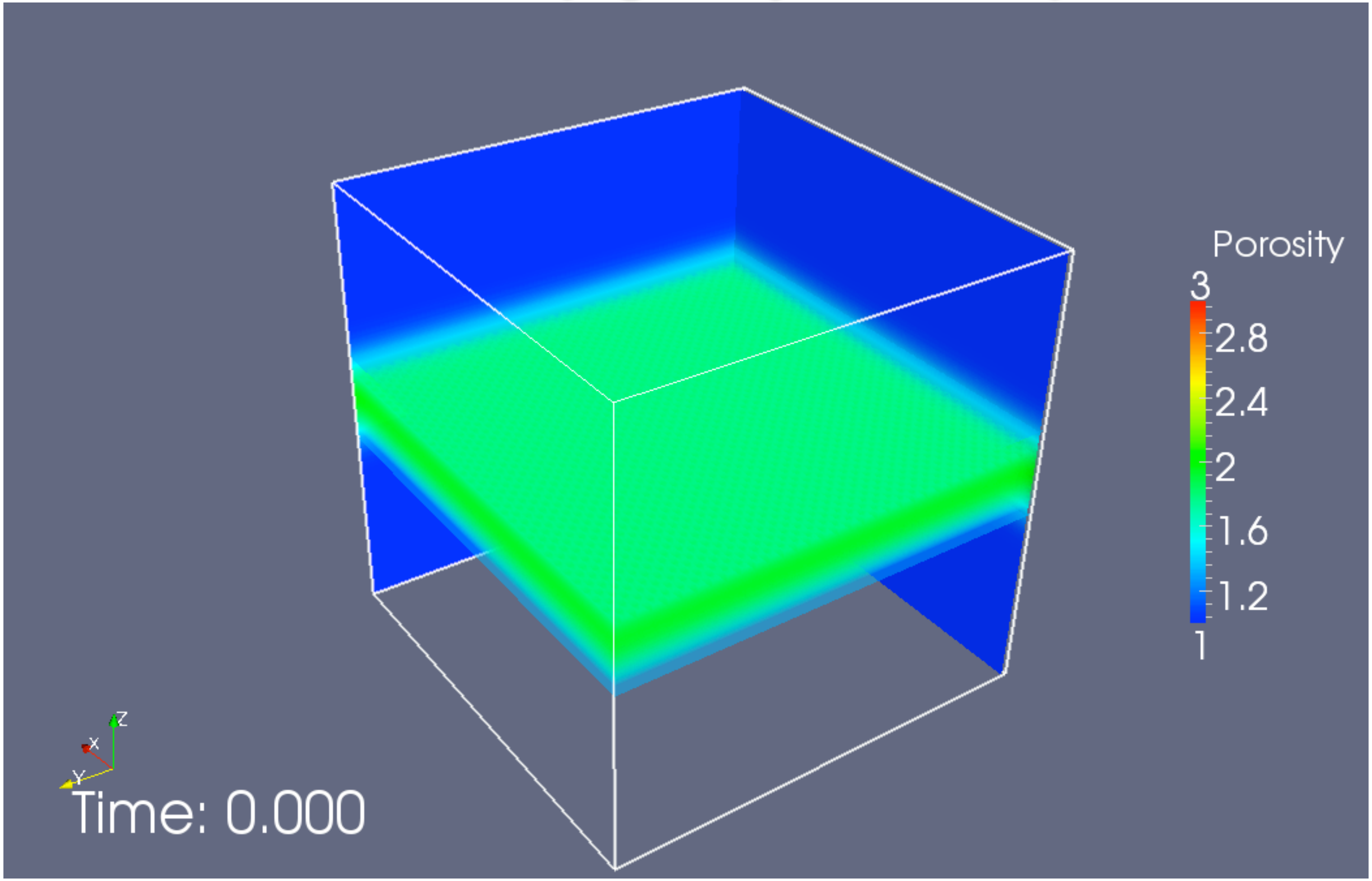

- Instability of 1D- 3-D waves. 3-D mixed finite elements. Hybrid FEniCS/PETSc codes. (CIG)

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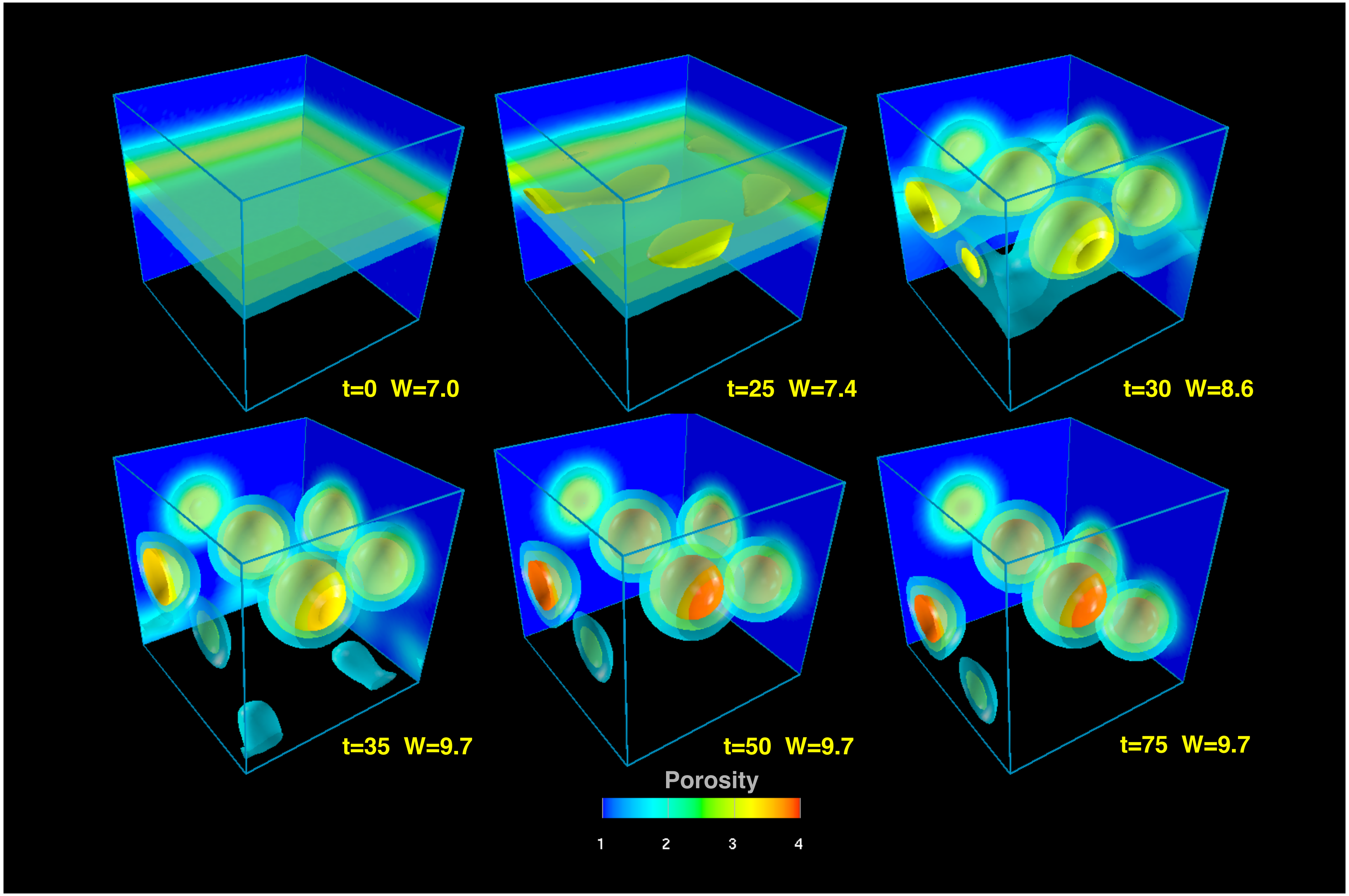
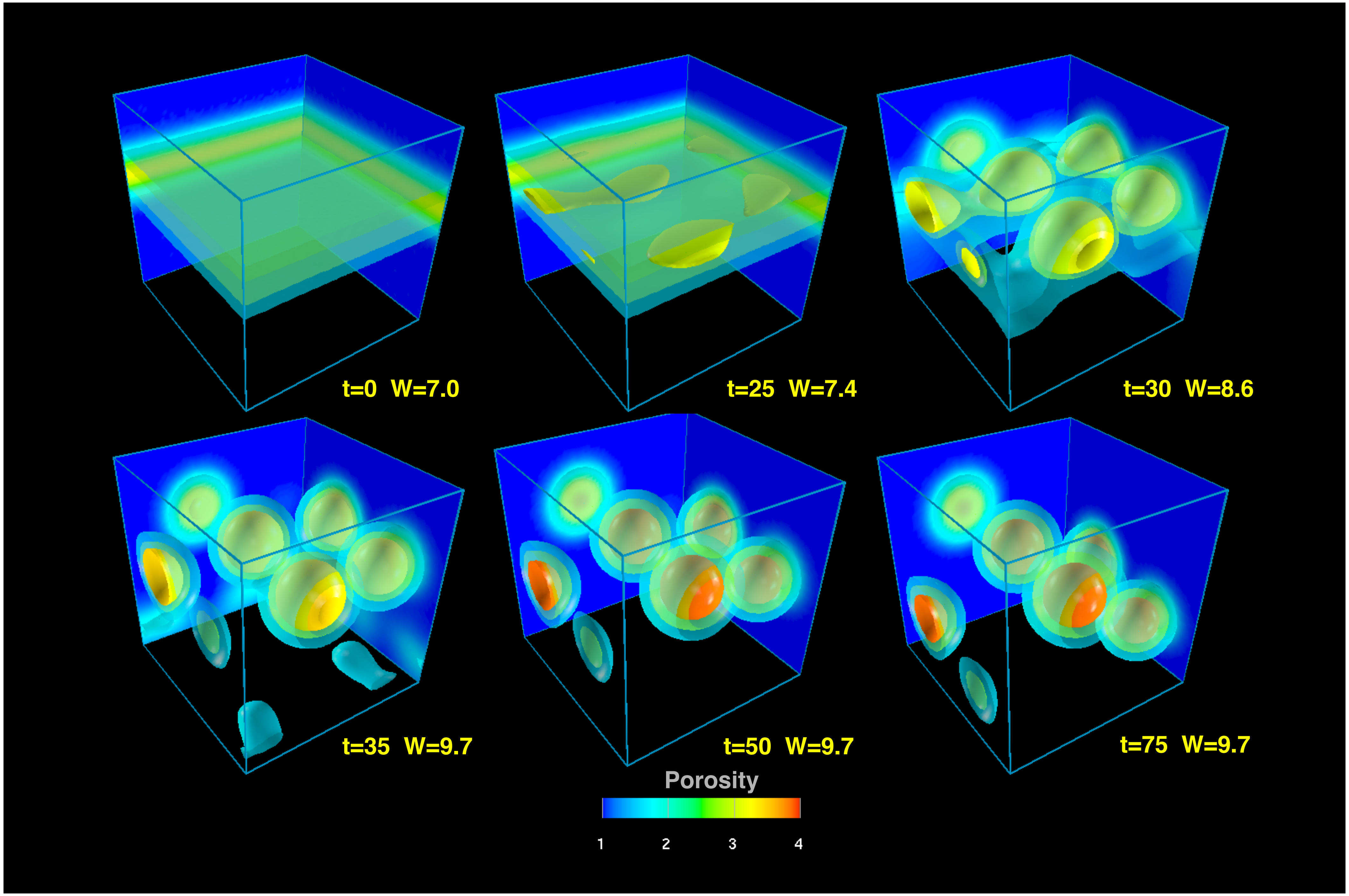


Non-linear porosity waves

(Scott & Stevenson, 1984 Nature, Spiegelman, JFM, 1993, Simpson et al 2009,2010)



- Instability of 1D- 3-D waves. Spiegelman and Wiggins, 1994, GRL.

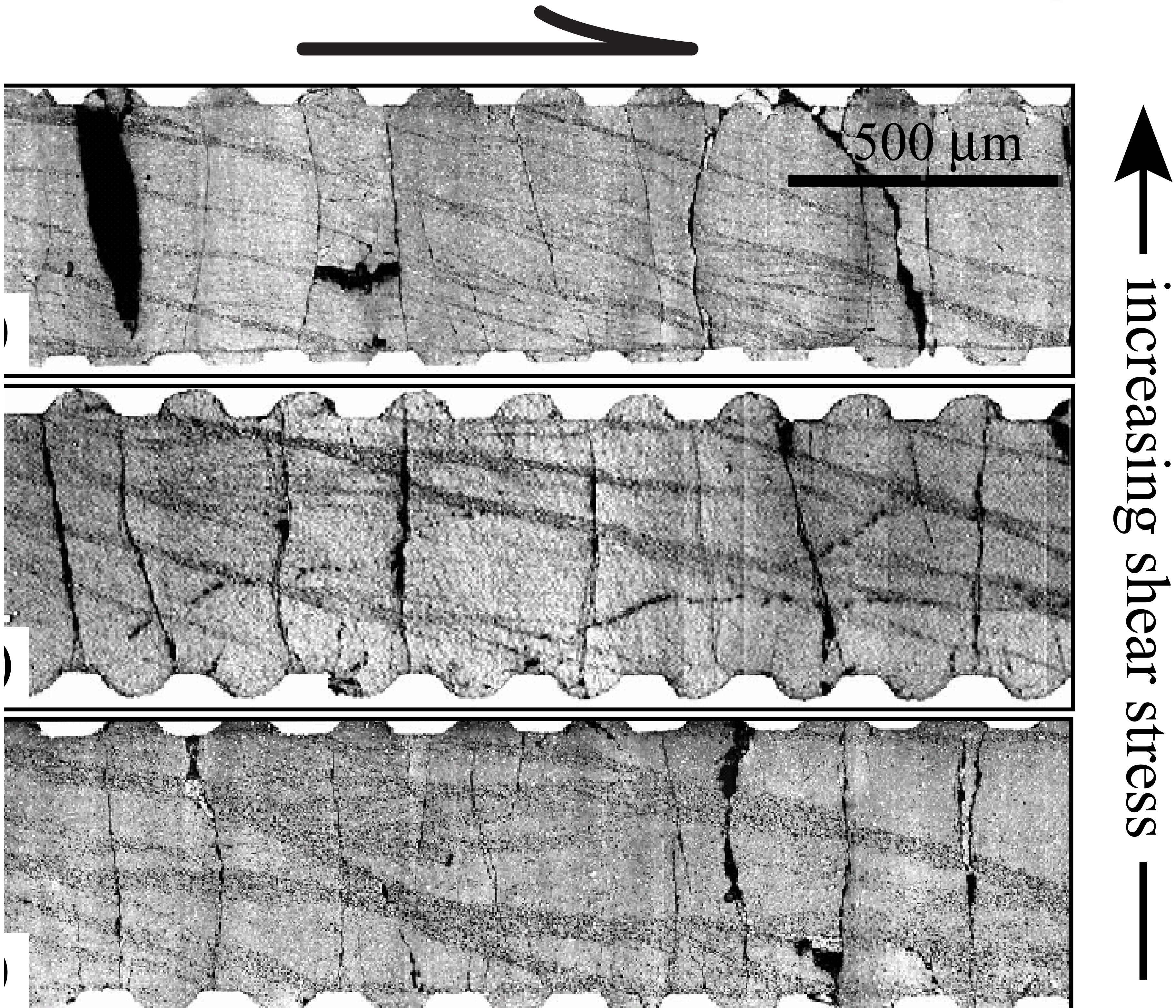
FV geometric multi-grid code.

Non-linear porosity waves

- Wave behavior is the natural consequence of non-linearity of flux with porosity and viscous deformation of the solid.
- Waves are generated by obstructions in the flux.
- Implies that magma dynamics is highly time dependent
- Solitary waves provide an excellent non-linear benchmark for space-time codes.
- Simpson and Spiegelman, JSC, 2010 provides sinc-codes for calculating spectrally accurate wave profiles in 1, 2 and 3-D.

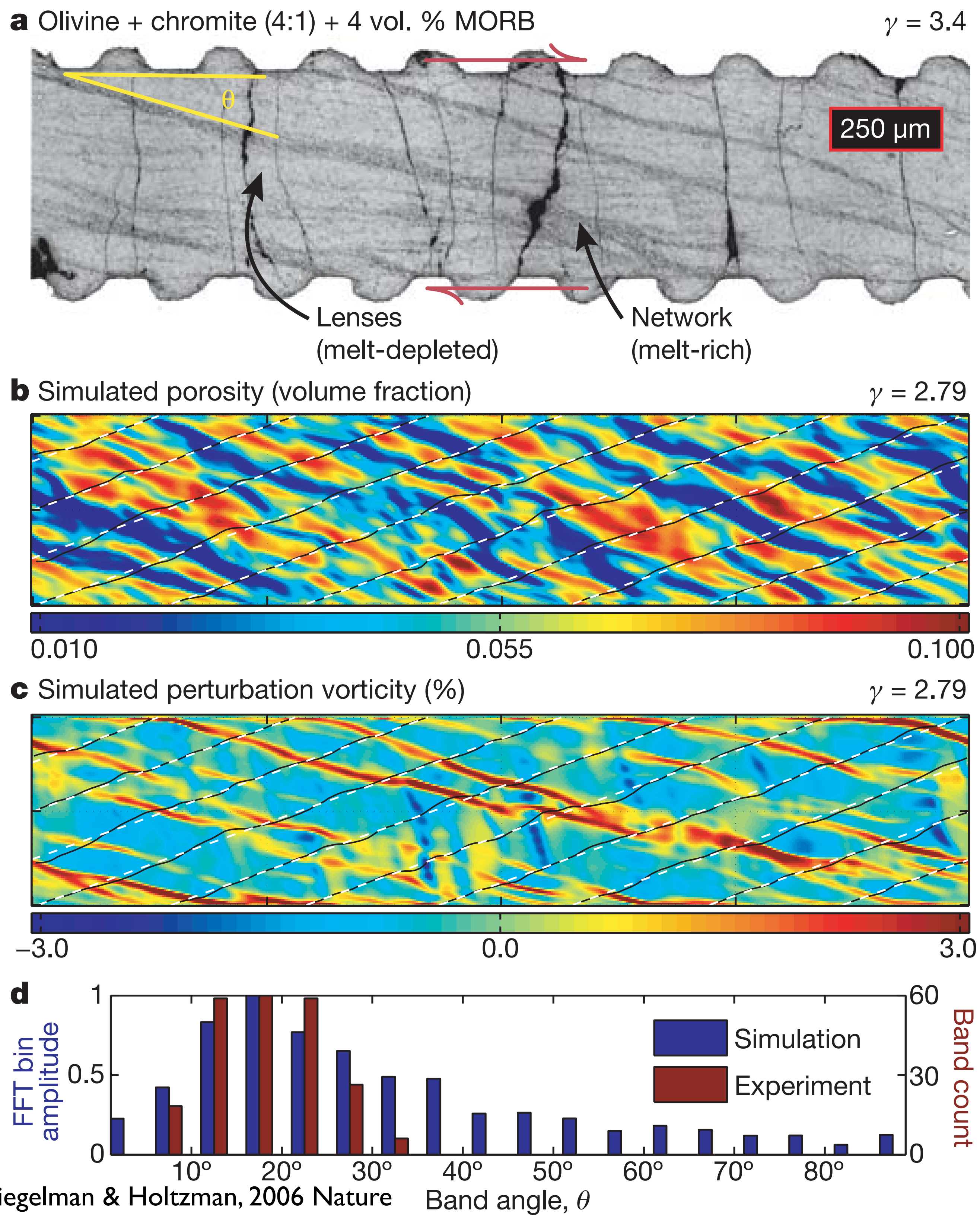
Other Localization instabilities

Mechanical shear band formation, experiments



Kohlstedt and Holtzmann, Ann Rev Geophys., 2009

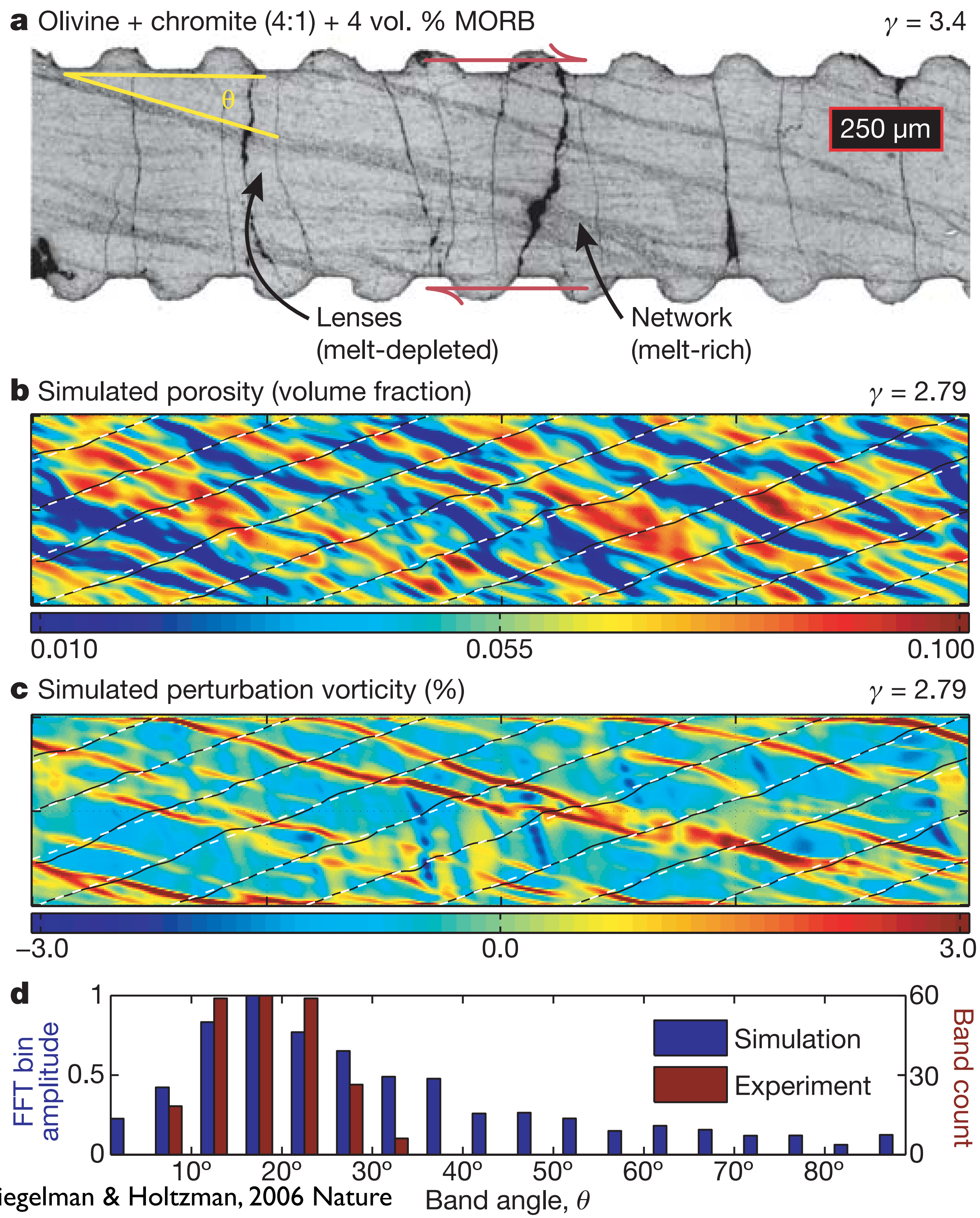
Mechanical shear band instability
(Katz et al, 2006 Nature)



Katz, Spiegelman & Holtzman, 2006 Nature

- Full equations with porosity weakening shear viscosity $\eta(\phi, \mathbf{V})$
- Neglect gravity (at lab scale)
- PETSc codes with segregated SNES
- Spontaneously develops shear band instability

Mechanical shear band instability
(Katz et al, 2006 Nature)



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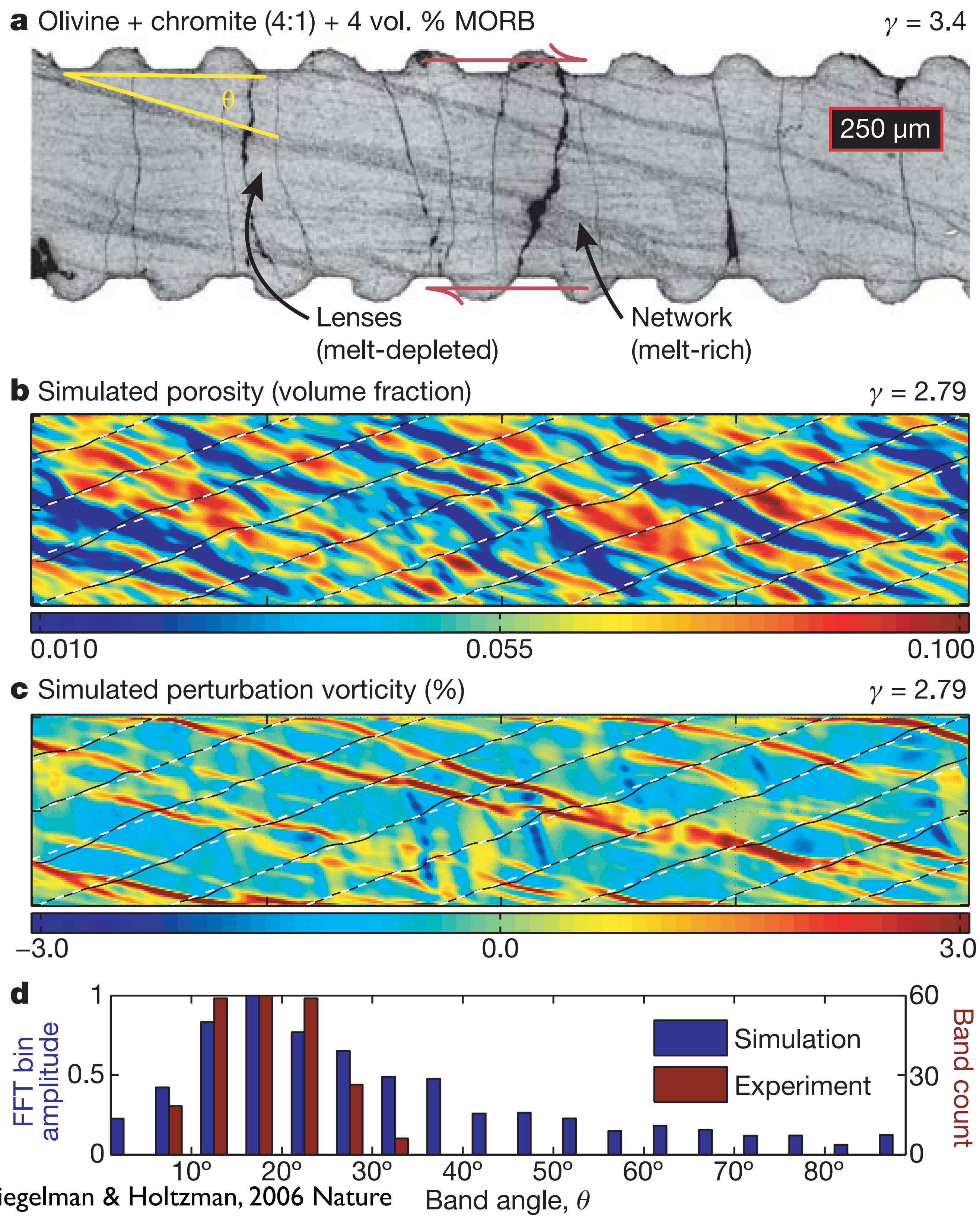

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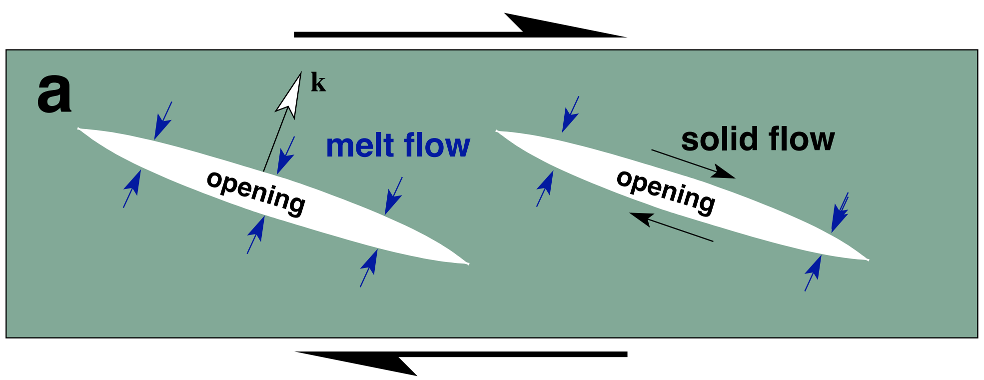
a Cartoon



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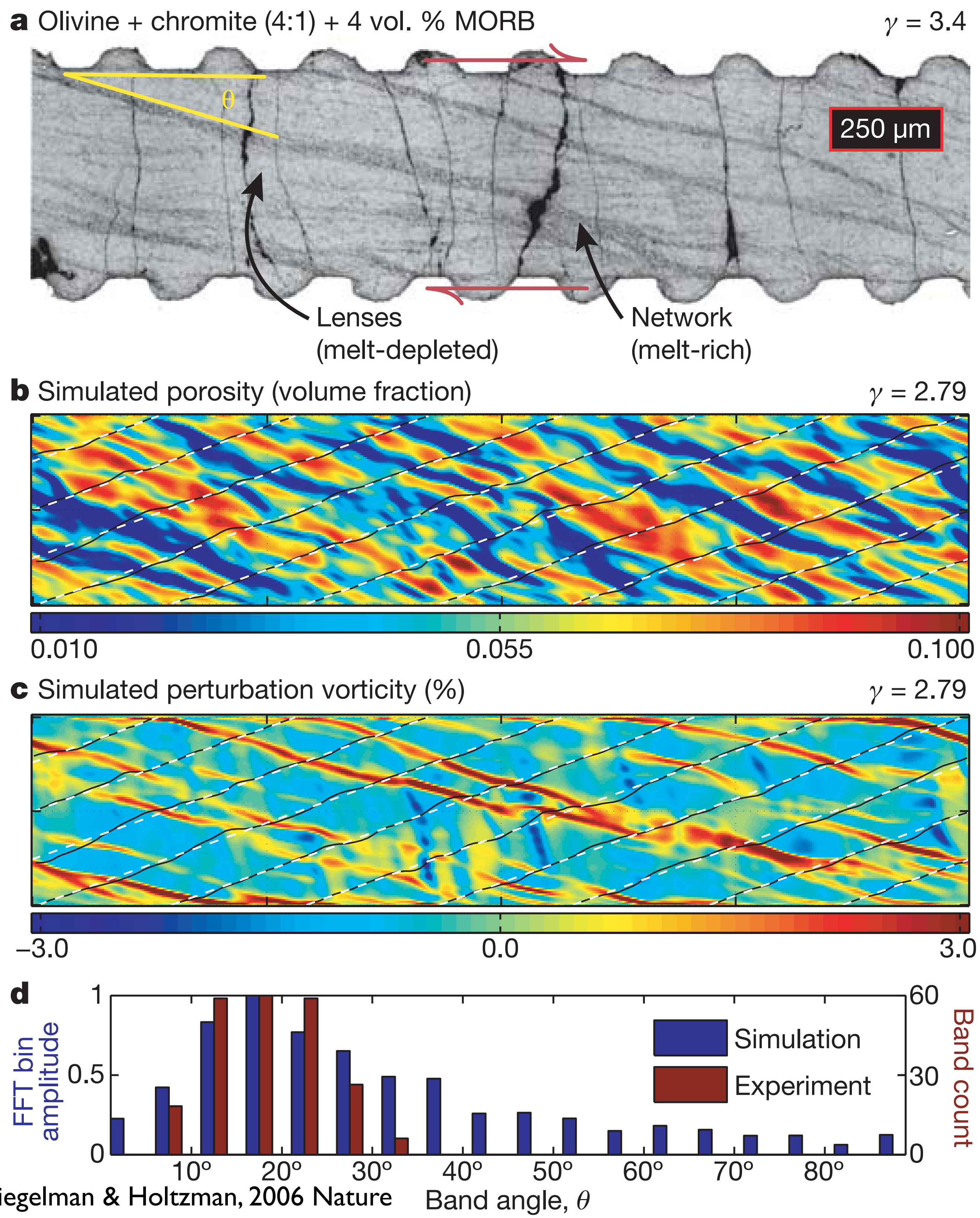

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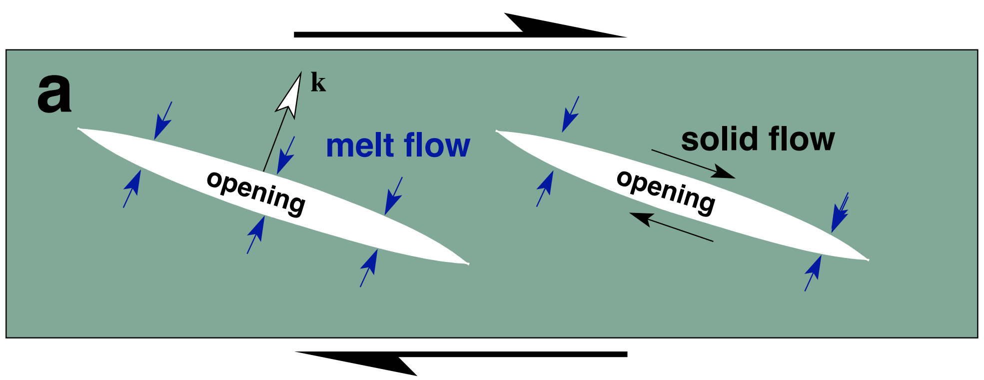
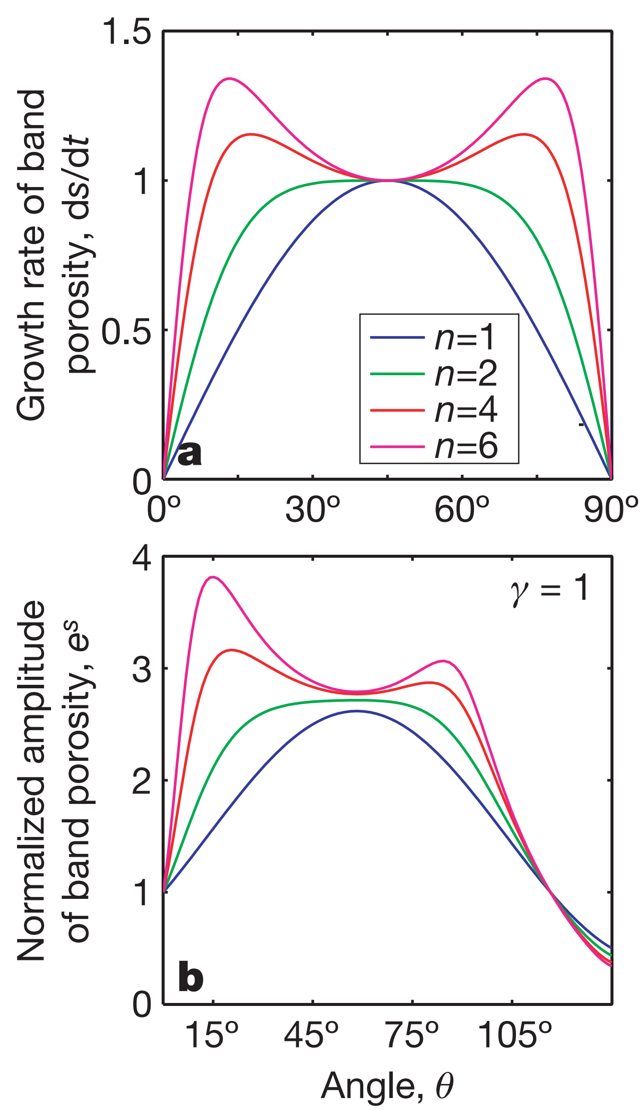




Linear Analysis

Other sources of melt channelization

Reactive infiltration instability

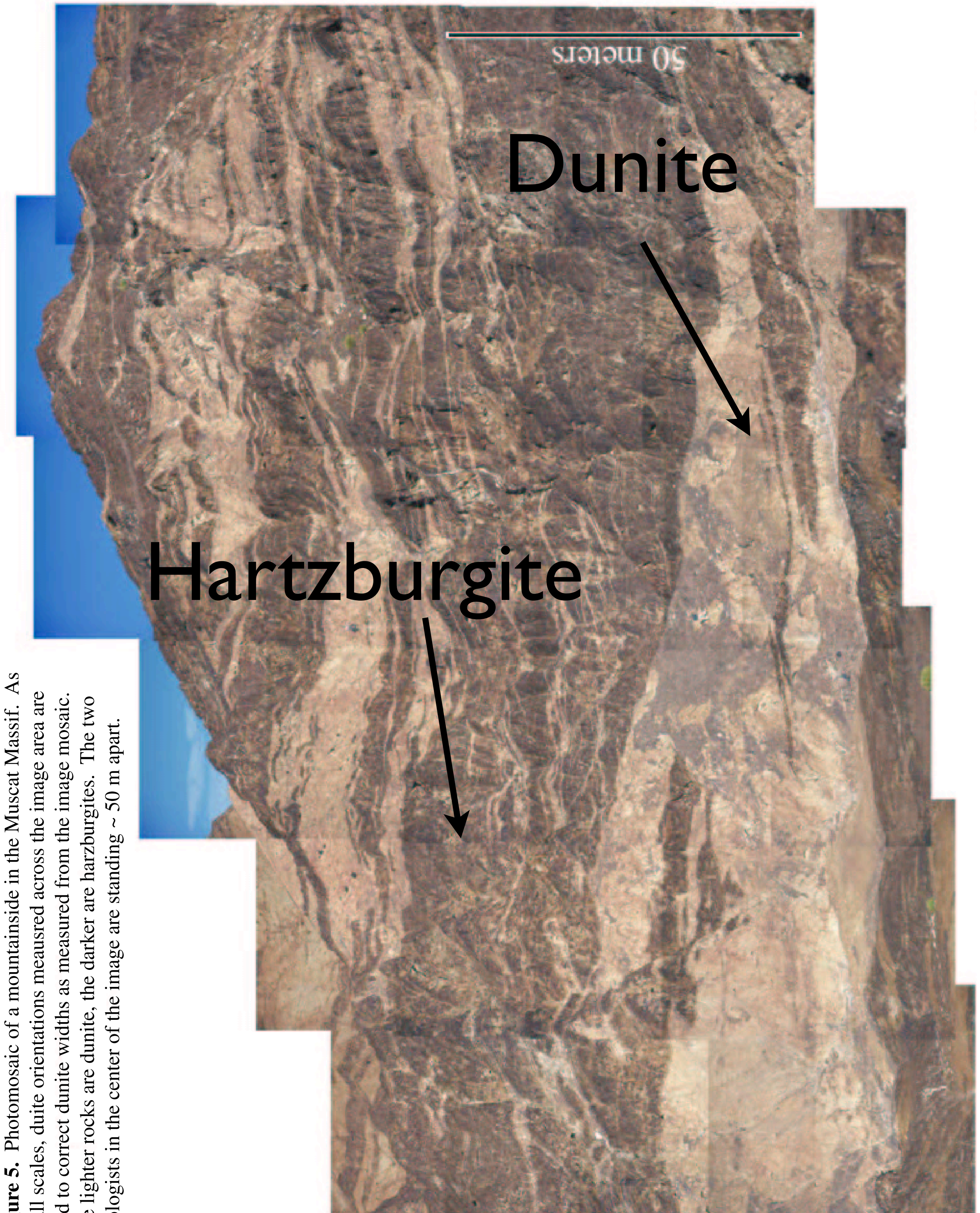
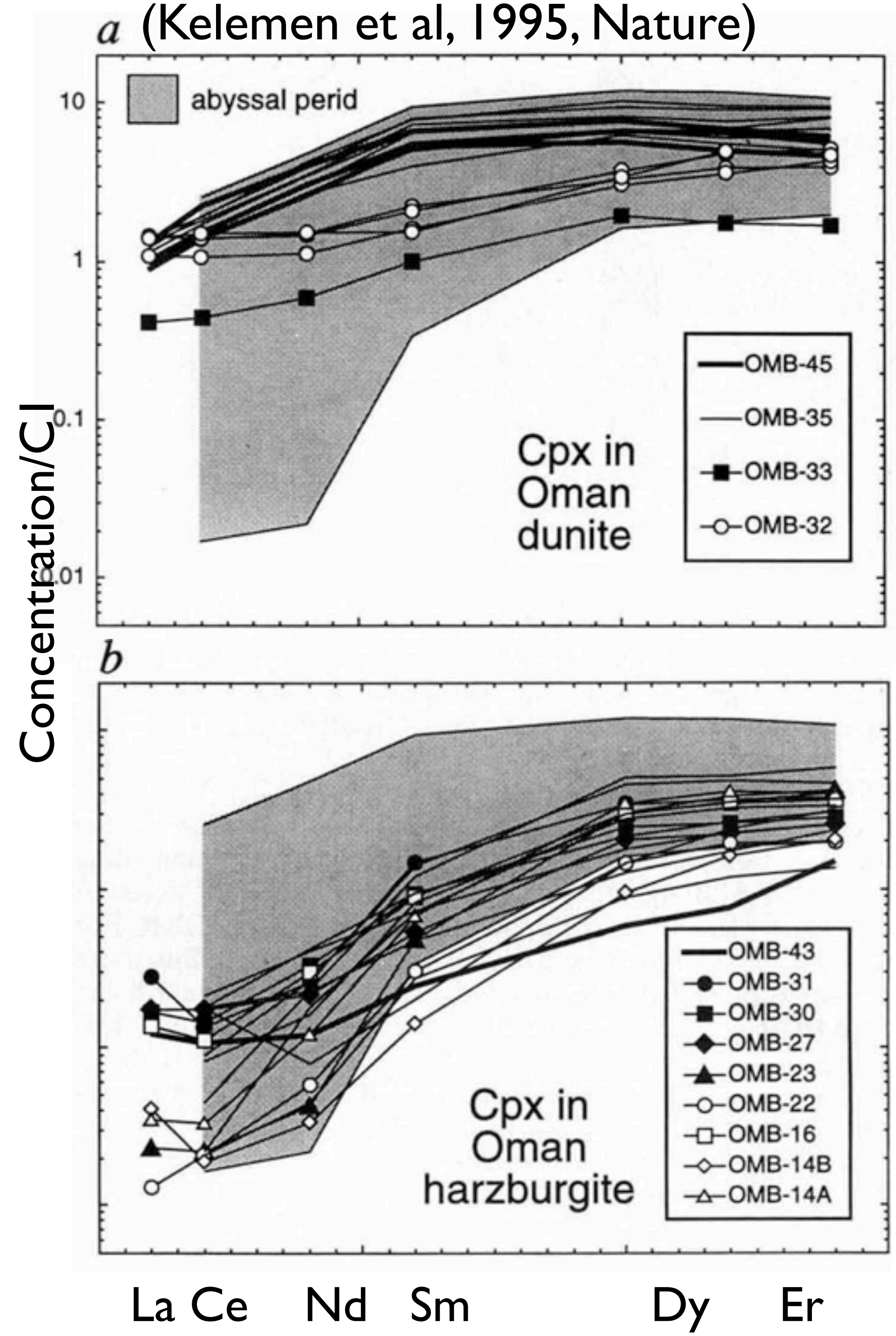


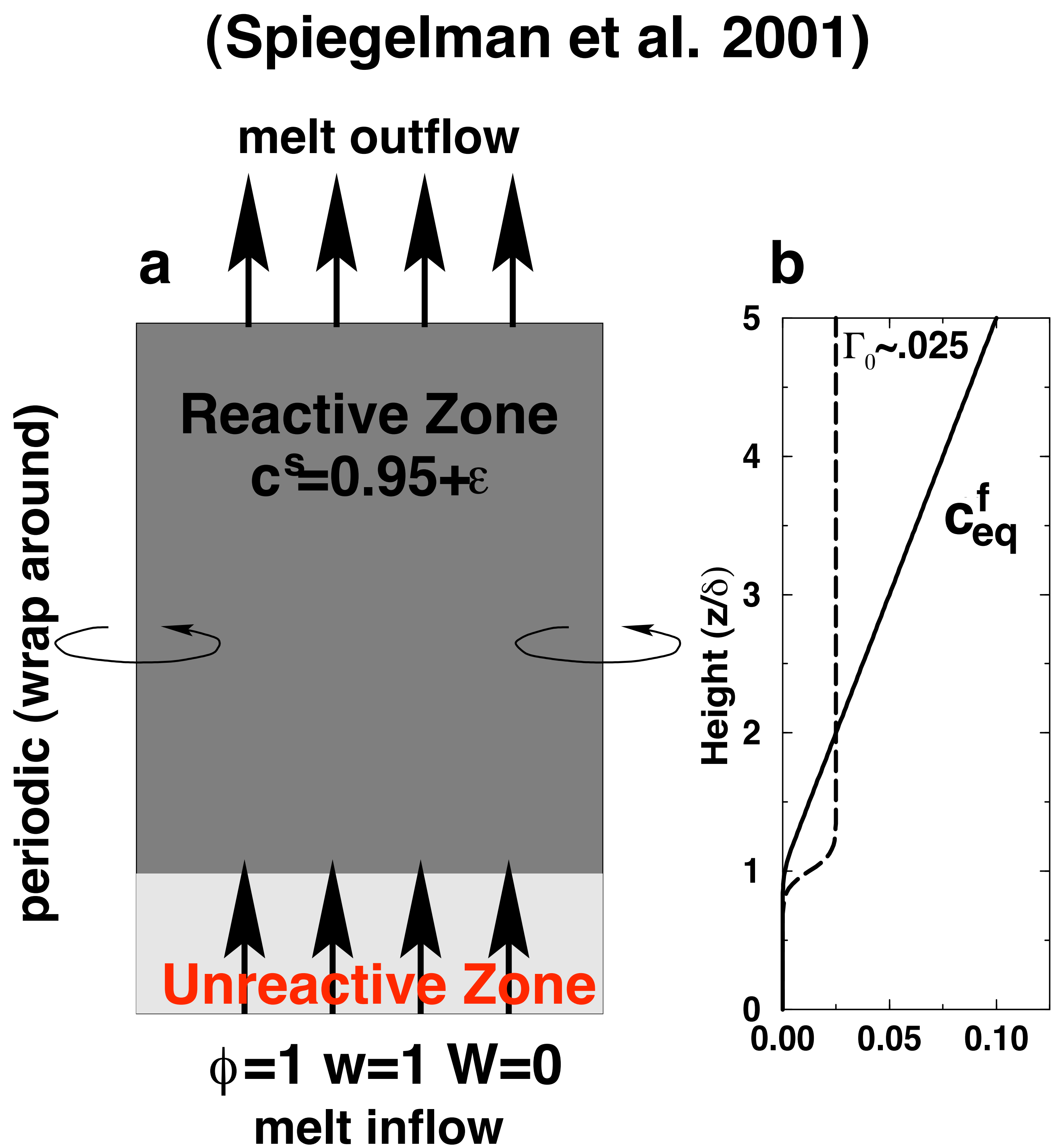
Figure 5. Photomosaic of a mountainside in the Muscat Massif. As at all scales, duite orientations meausred across the image area are used to correct dunite widths as measured from the image mosaic. The lighter rocks are dunite, the darker are harzburgites. The two geologists in the center of the image are standing ~ 50 m apart.

Chemistry

(Kelemen et al, 1995, Nature)

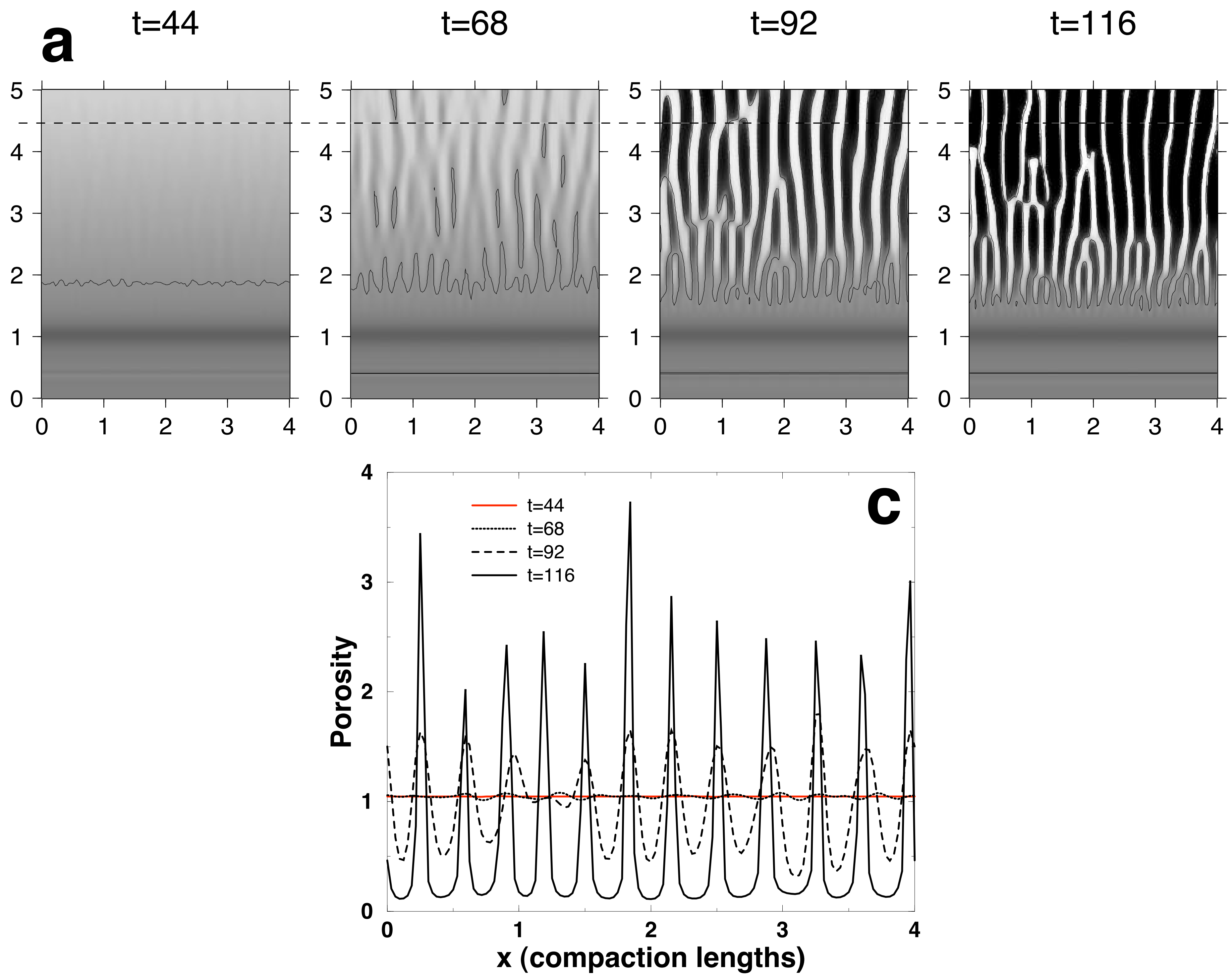


Reactive Infiltration Instability



Reactive Infiltration Instability

(Spiegelman, Kelemen and Aharonov, JGR 2001)



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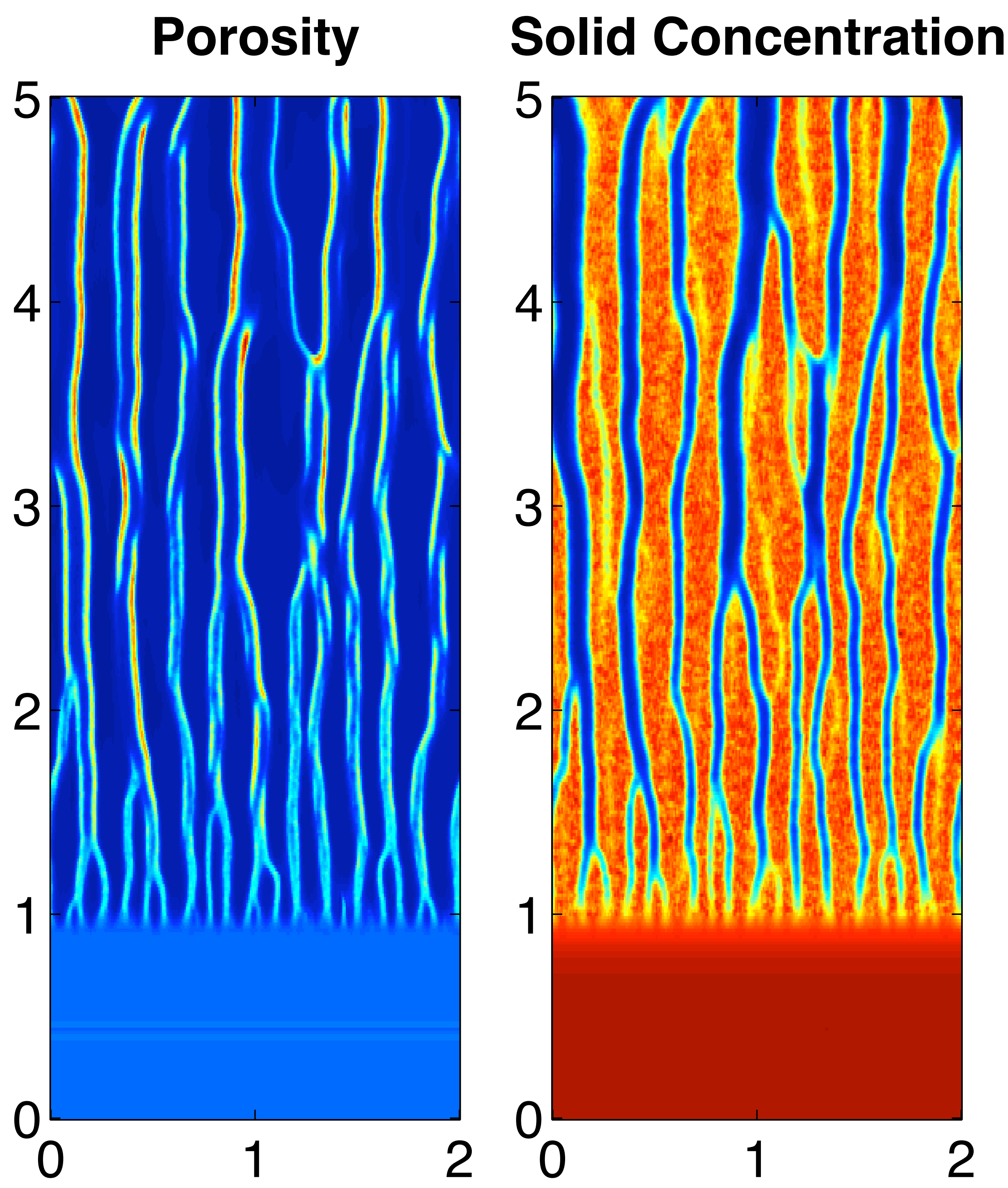
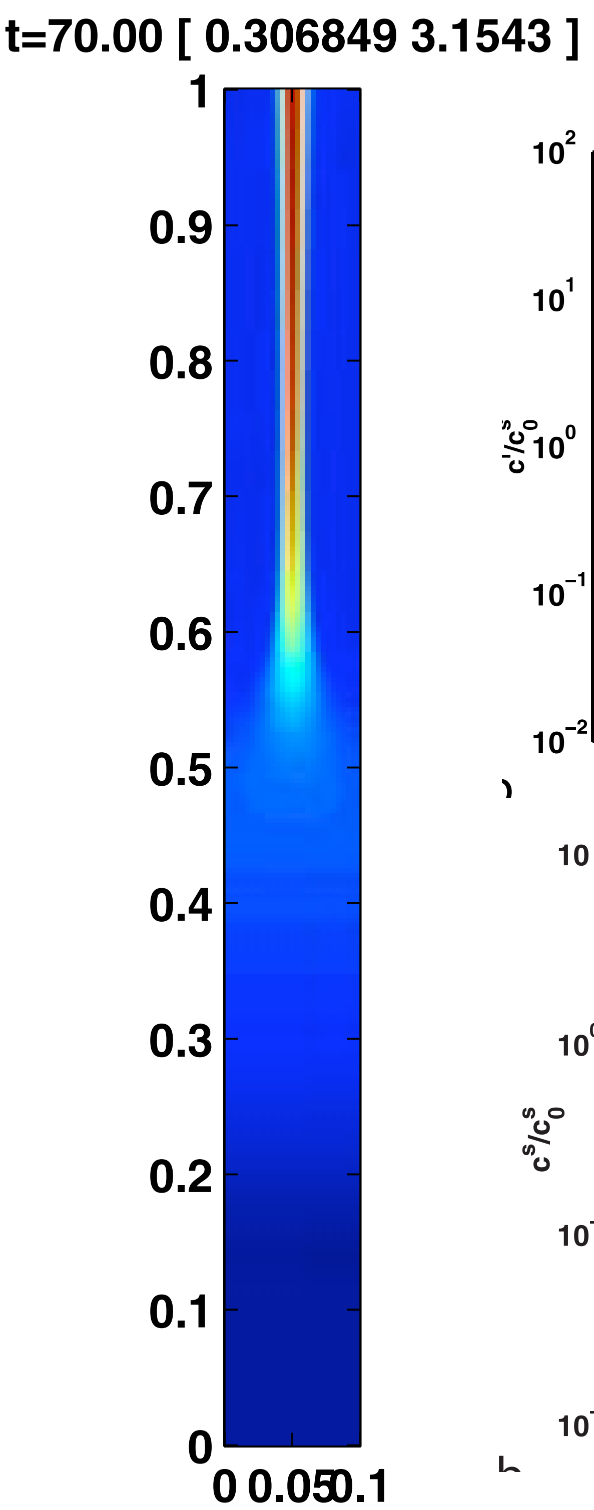
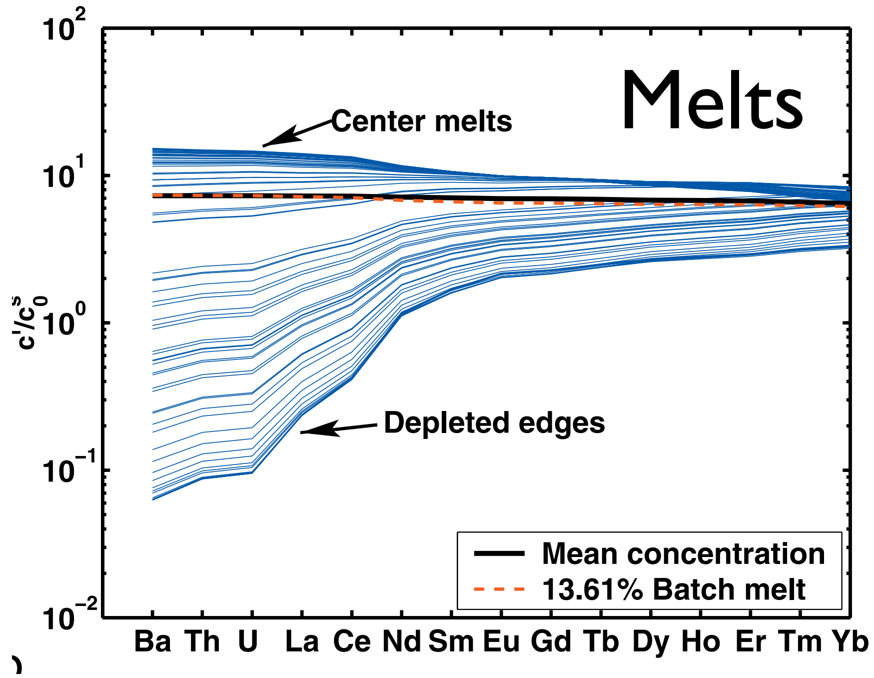


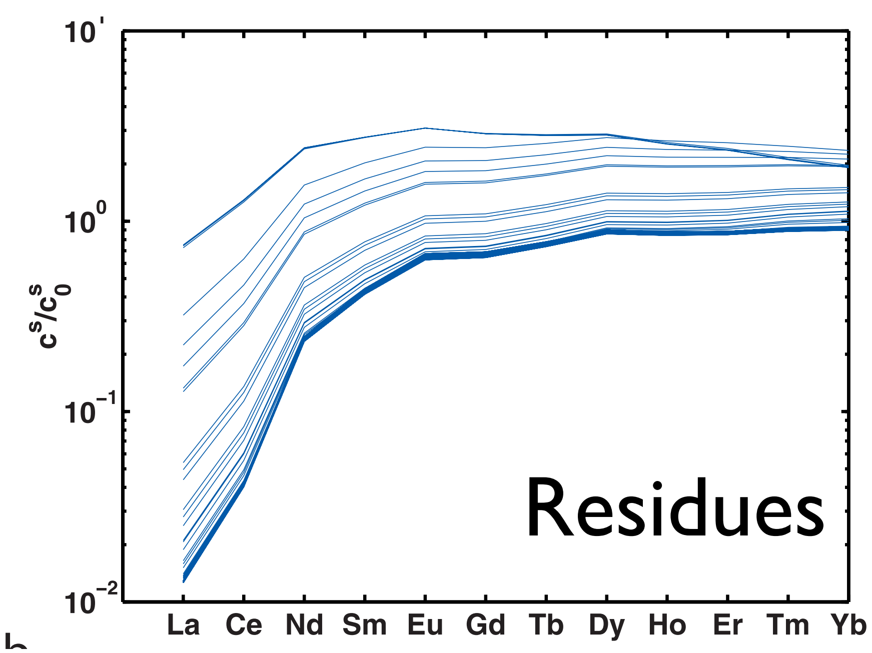
Chemical Consequences of Melt Channeling

(Spiegelman & Kelemen, 2003, G3)

Calculations







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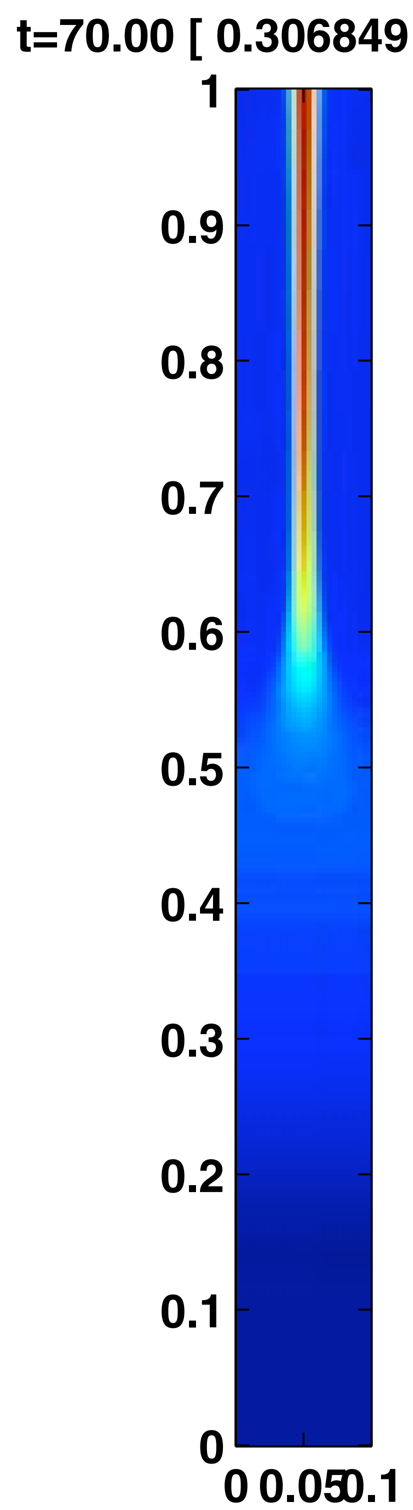

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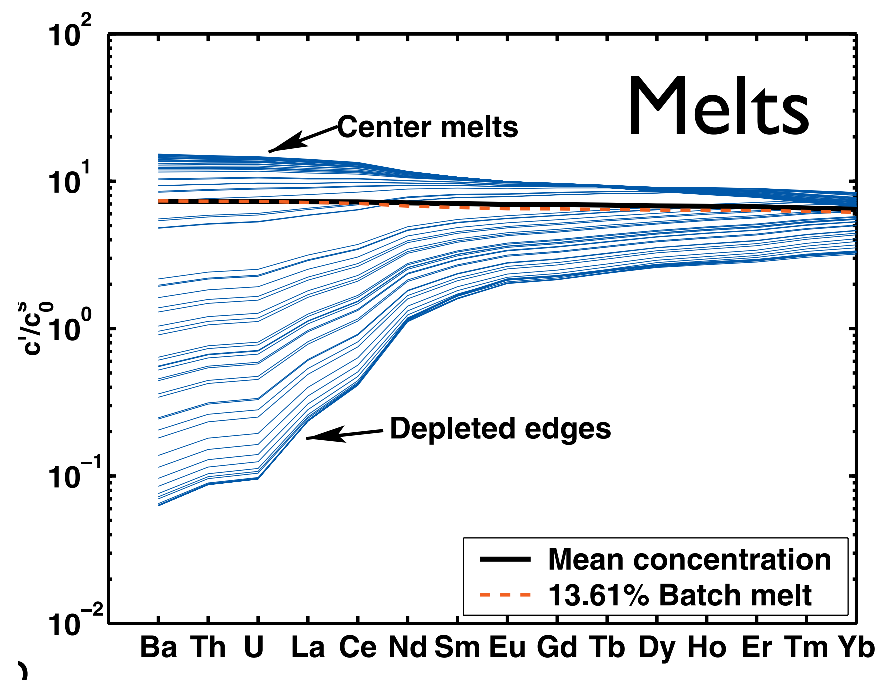
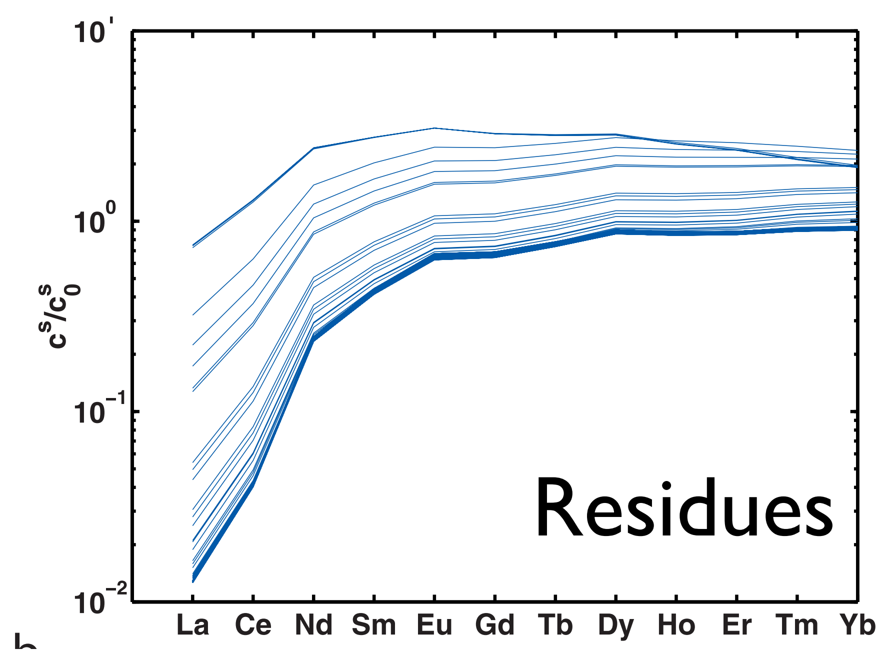
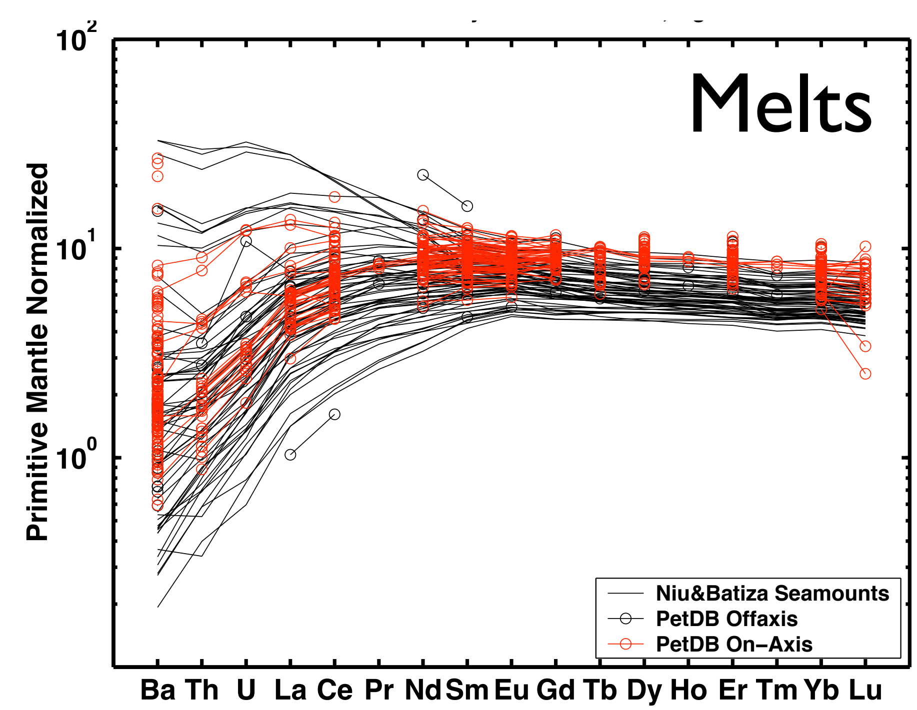
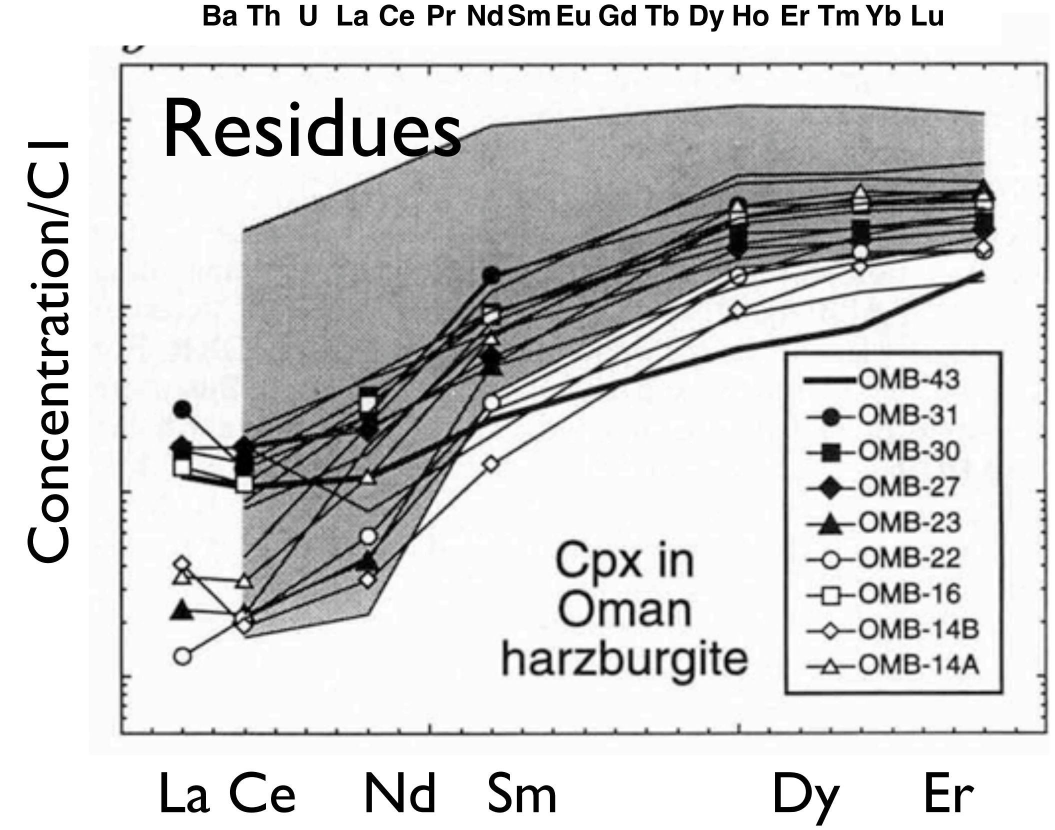




Observations





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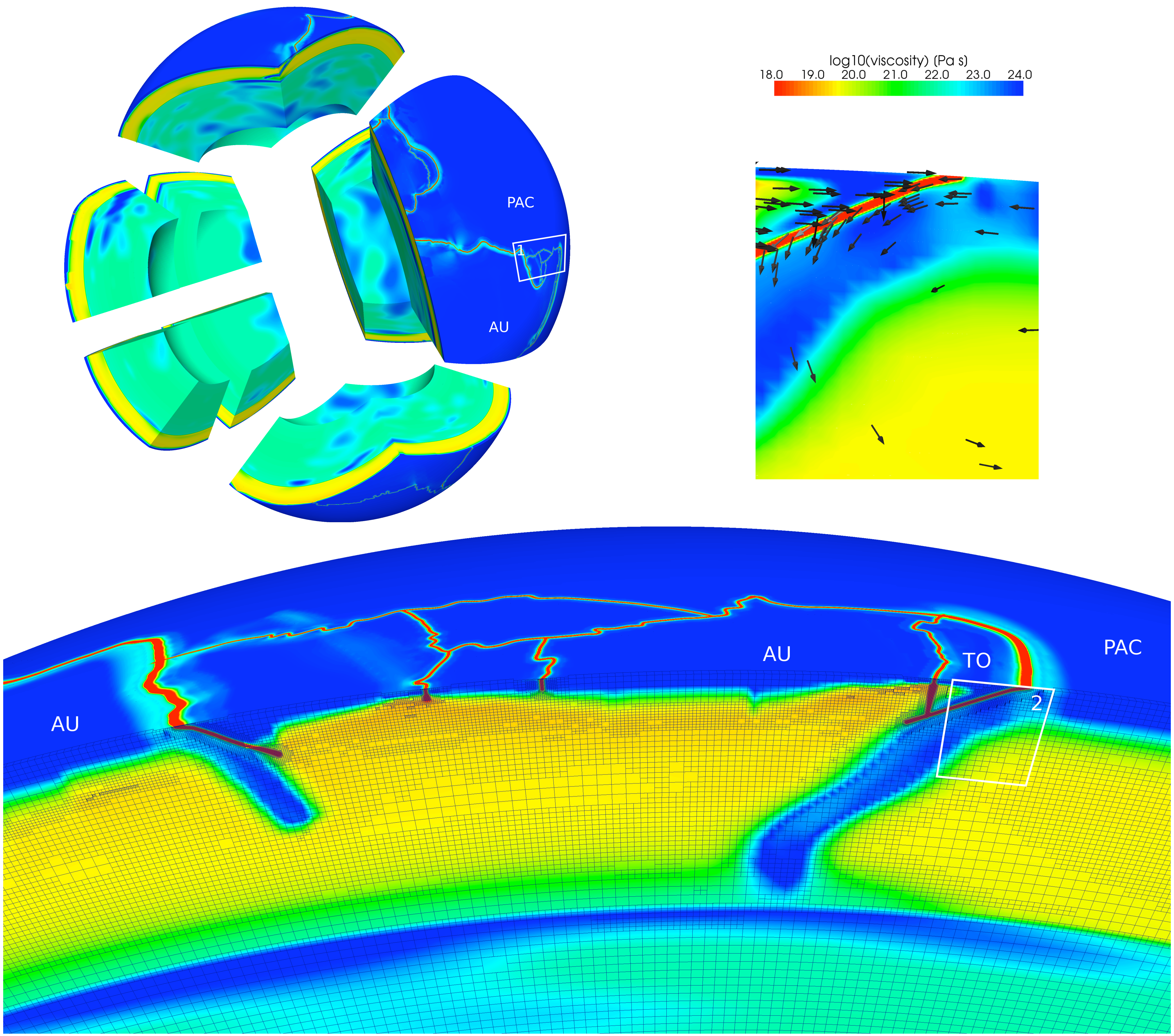
- How/do small scale physics influence large scale mantle dynamics?

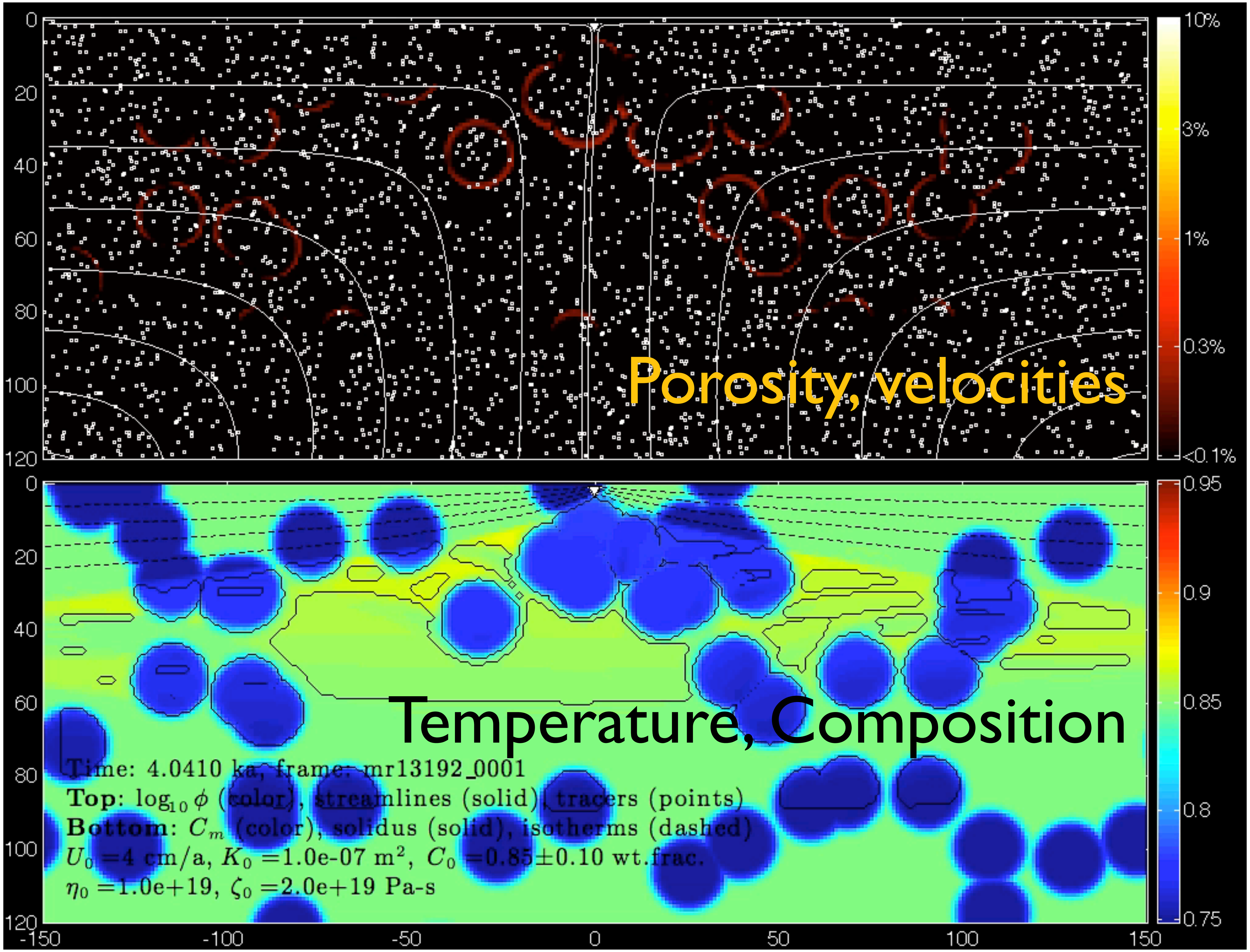
Petascale AMR FEM/Rhea



Global Convection code with parallel adaptive mesh refinement

- minimum mesh spacing ~1km resolves weak boundaries
- Adaptive refinement in weak/ plastic regions
- Full refinement at h=1km ~ 10^{12} elements (exa-scale?)
- Can accomplish, goal oriented adaptation to convergence with 150-300 million elements (10^3-10^4) savings

Geo problems: mid-ocean ridge models
(Courtesy Richard Katz)

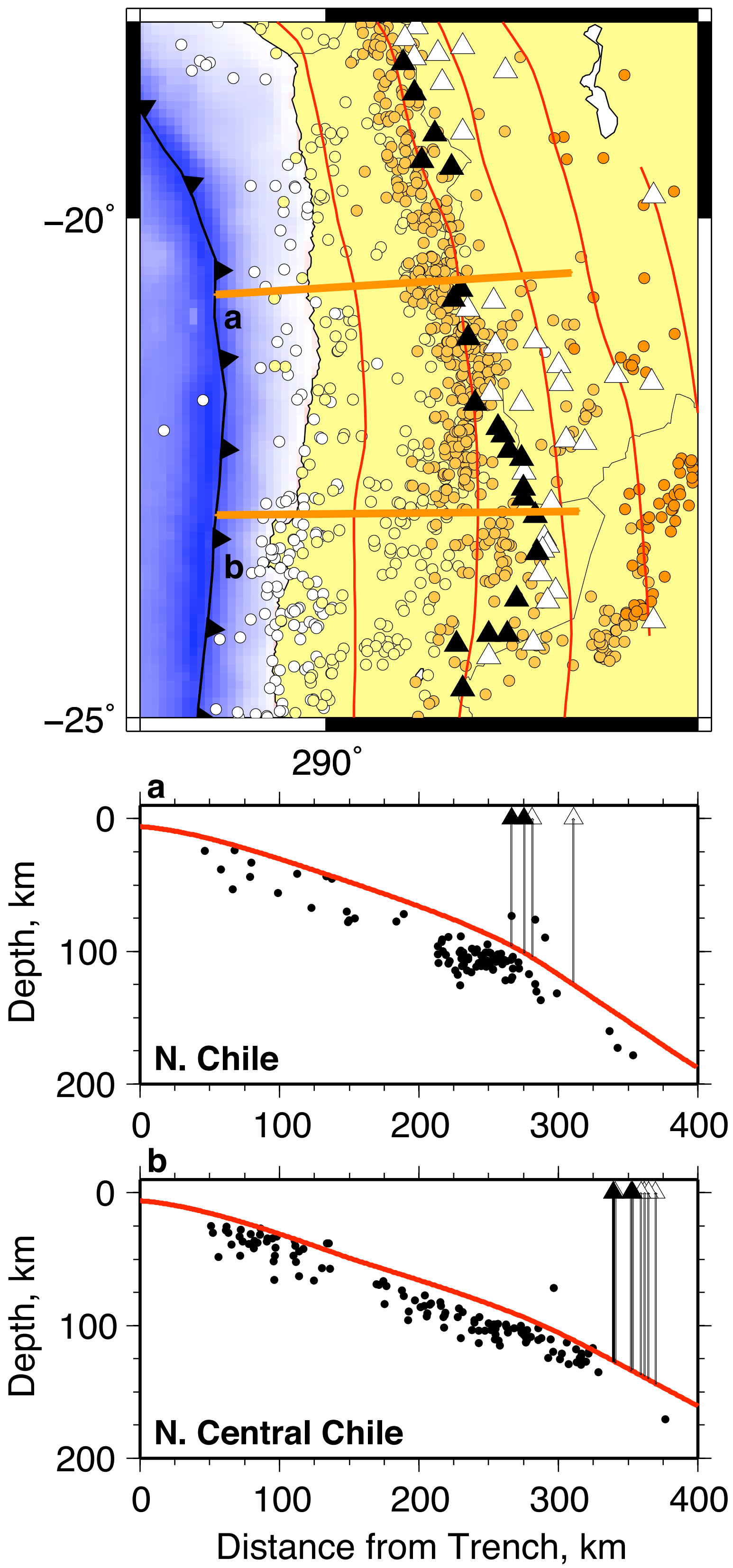


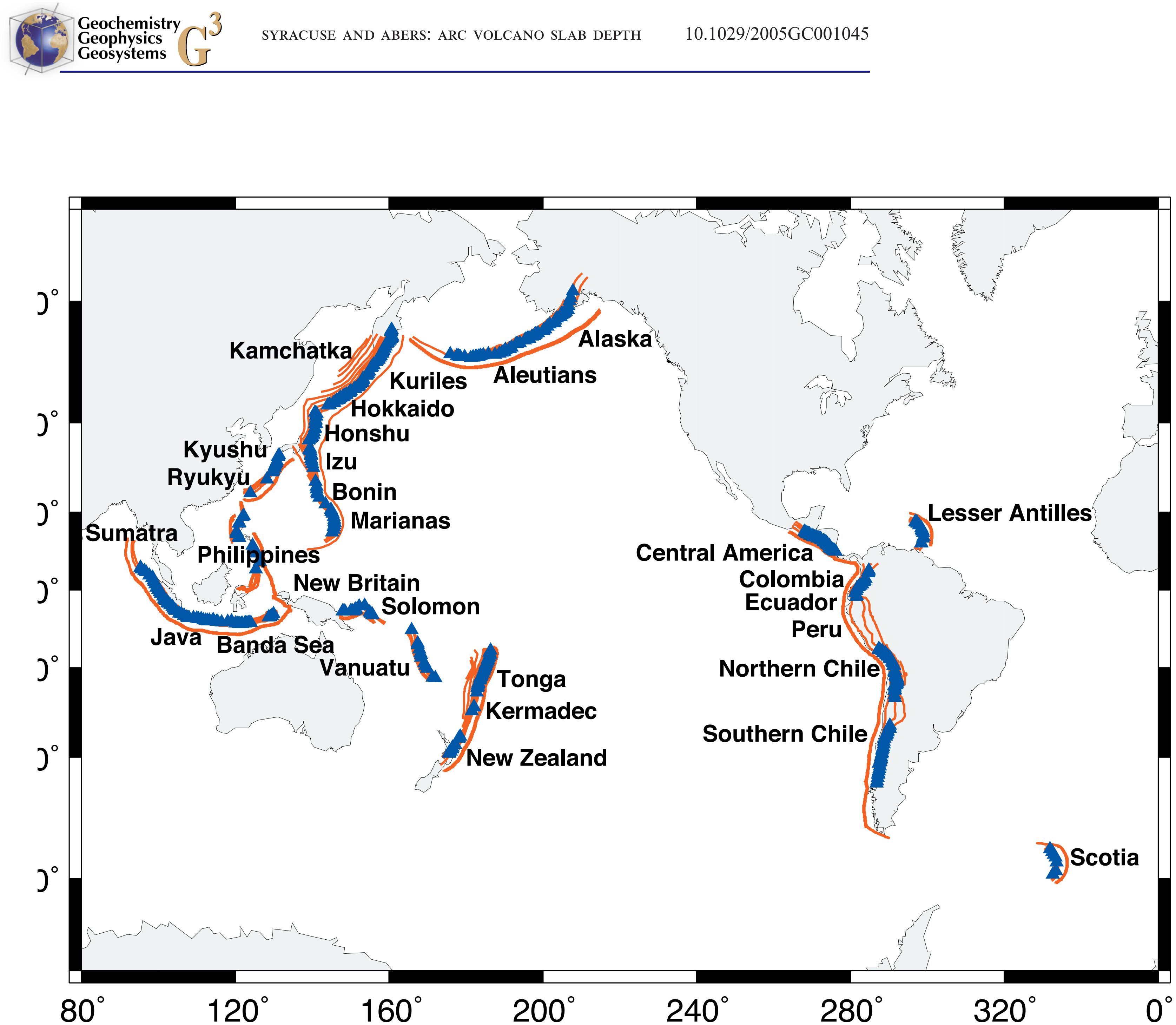
Melt and solid flow field for a heterogeneous melting mantle beneath a mid-ocean ridge

Full solution of magma dynamics using the "enthalpy method" Katz, J. Pet, 2008

PETSc parallel, structured FV code on staggered mesh.

Location of Volcanoes in Subduction Zones





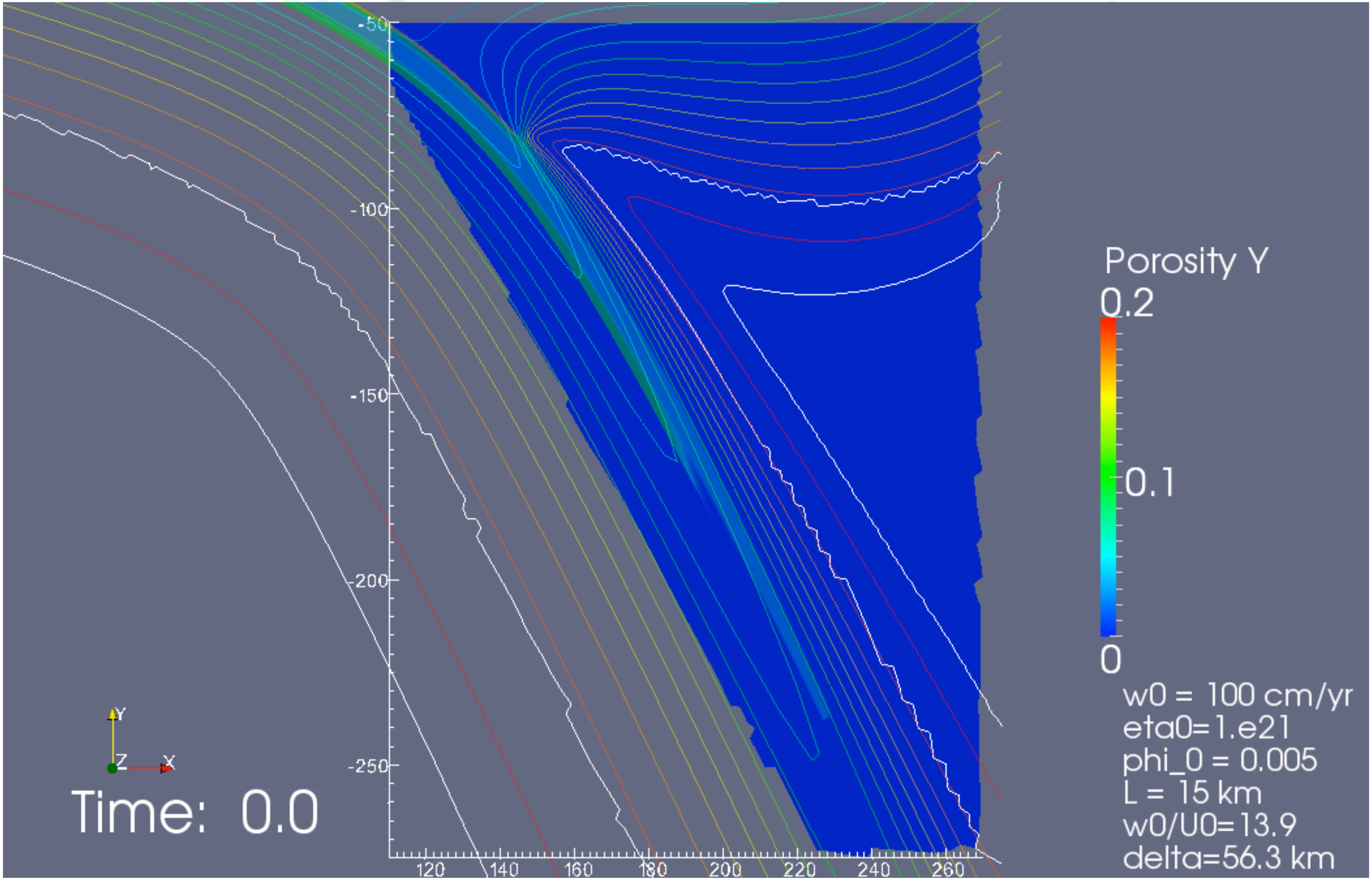
Global Slab Contours and Volcanoes

(Syracuse and Abers, 2006)

Monday, January 10, 2011

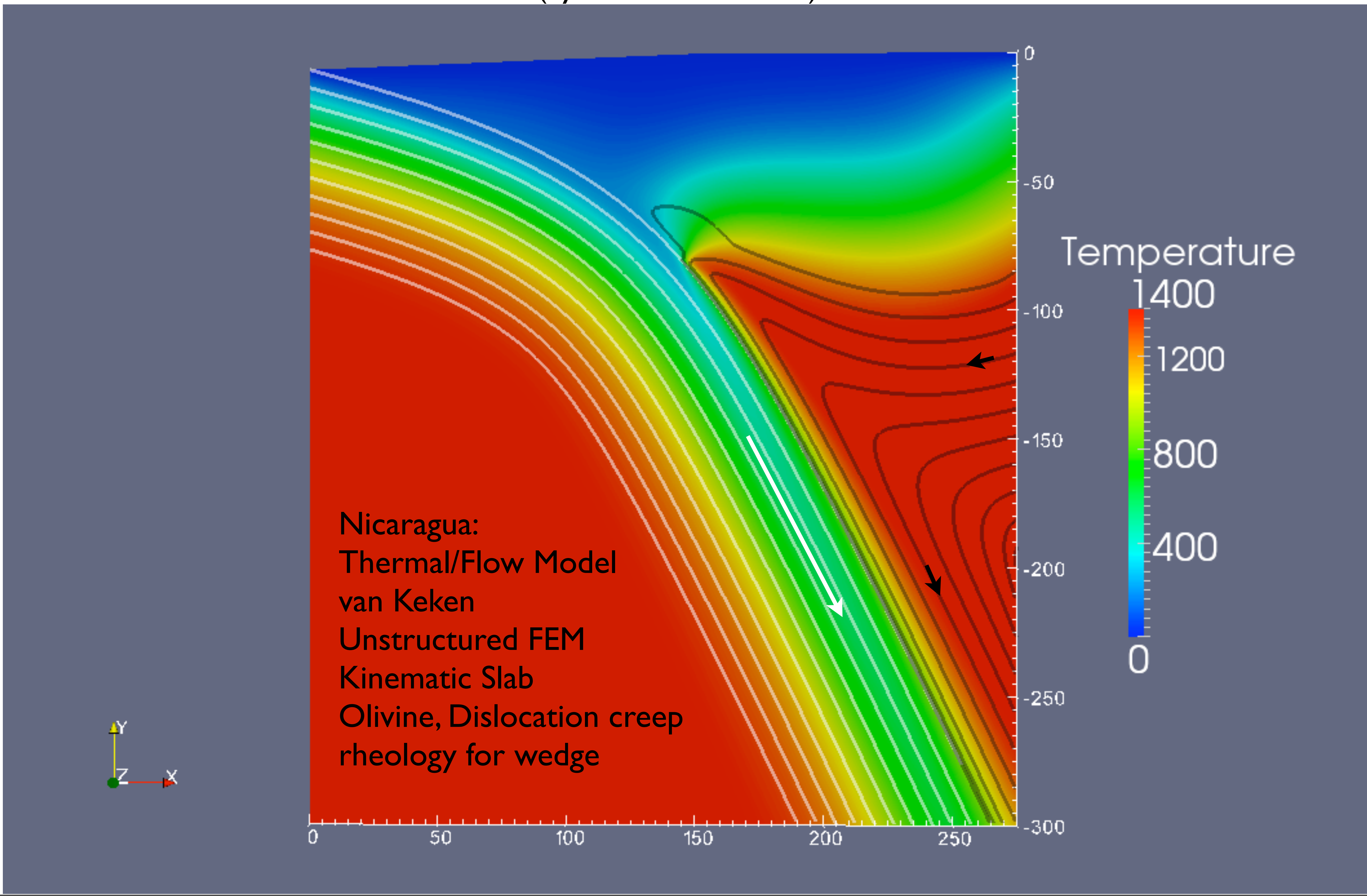
Geo problems: Subduction Zone models

(Spiegelman, van Keken, Hacker, 2009)



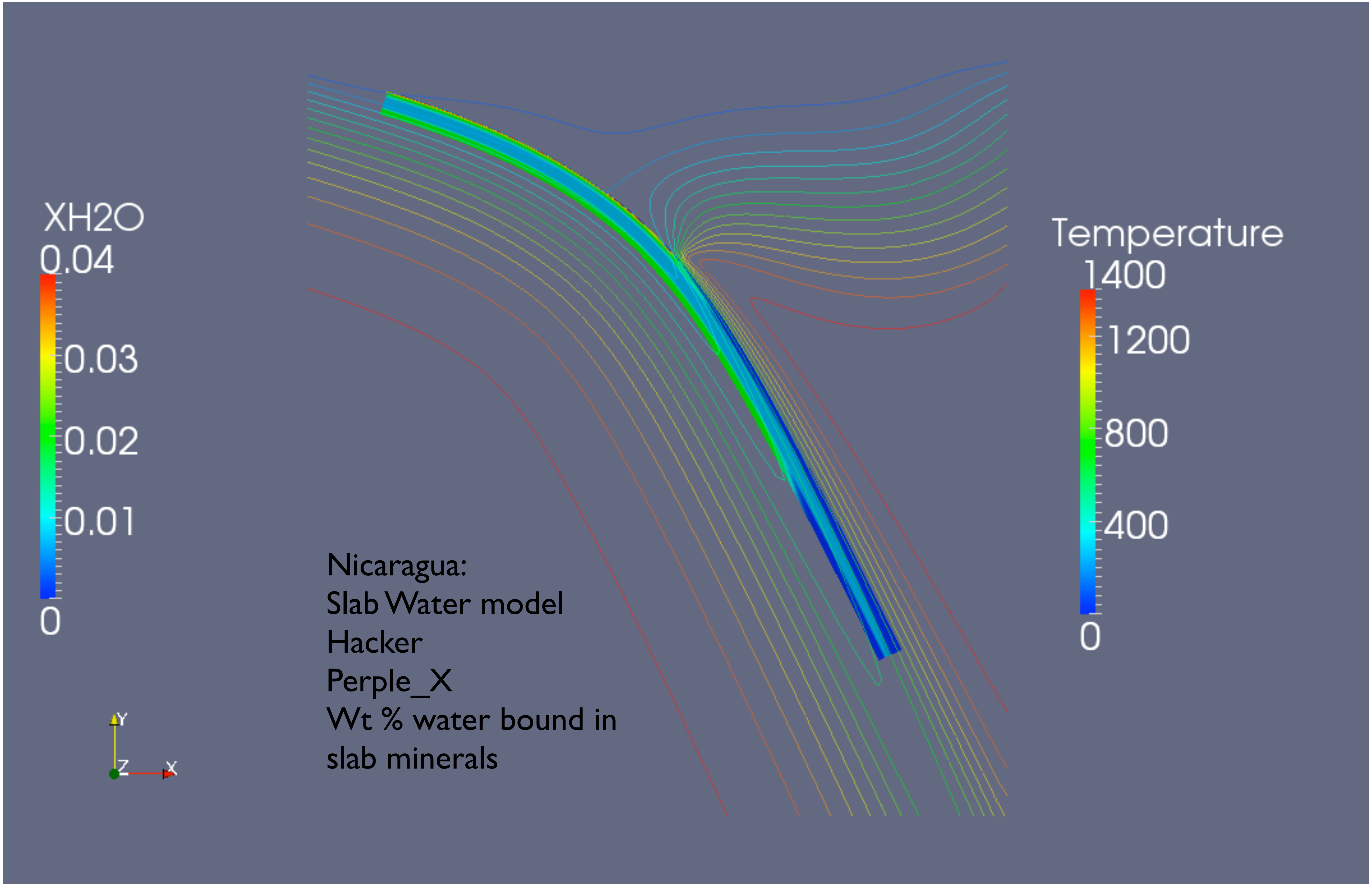
Nicaragua Model

(Syracuse et al, 2009)



Slab H_20 Model

(B Hacker, Perple_X)



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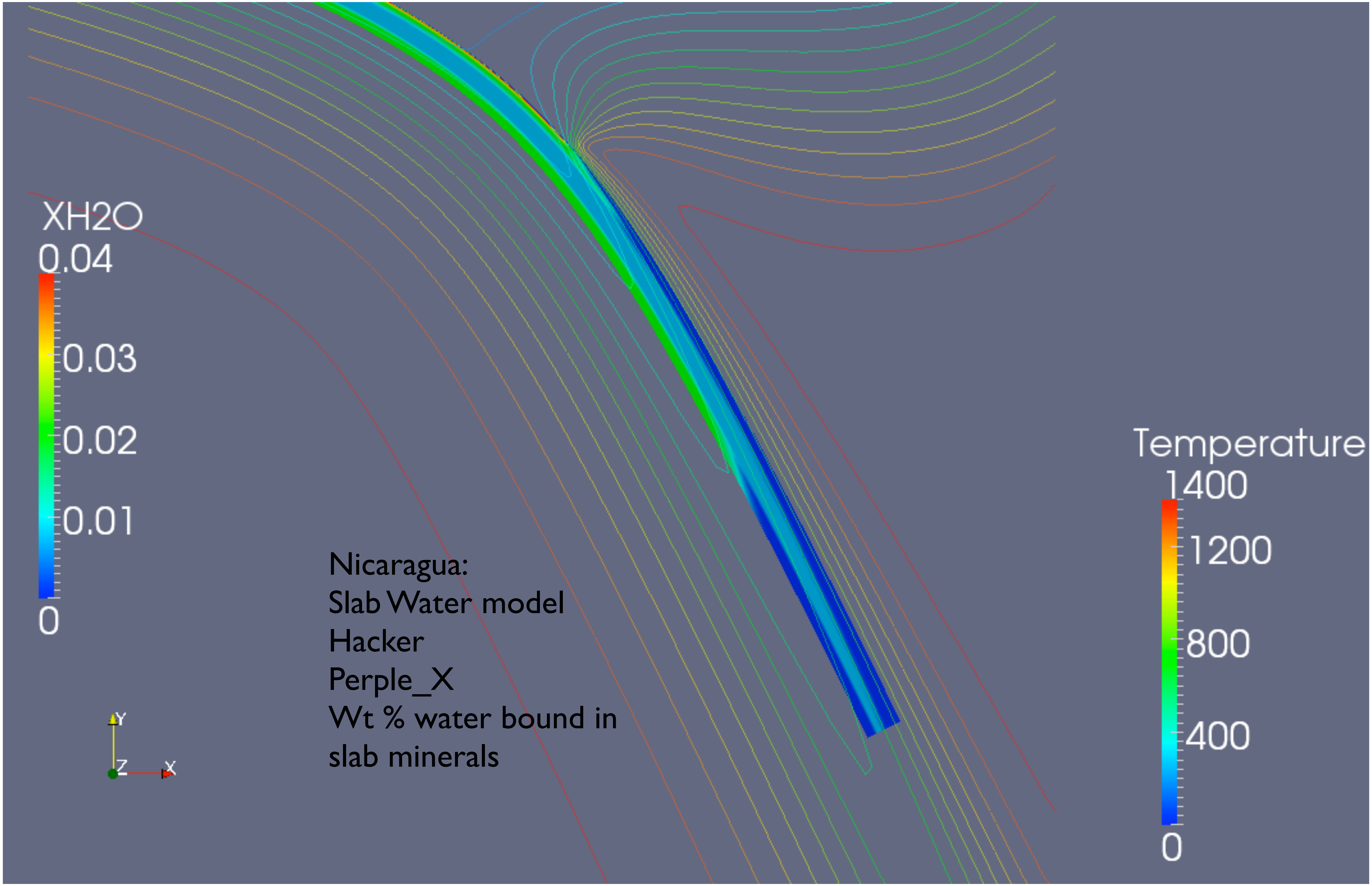
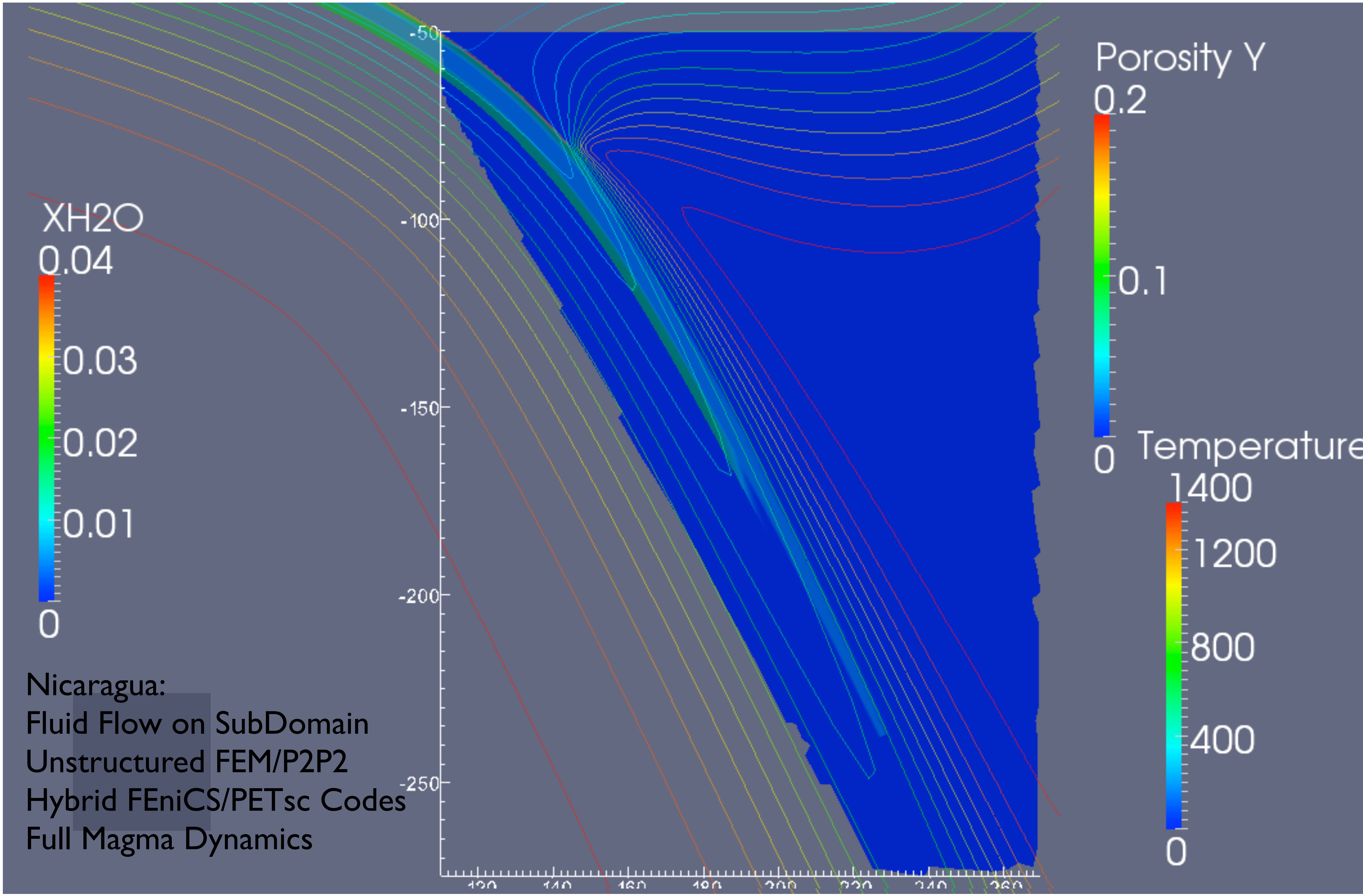


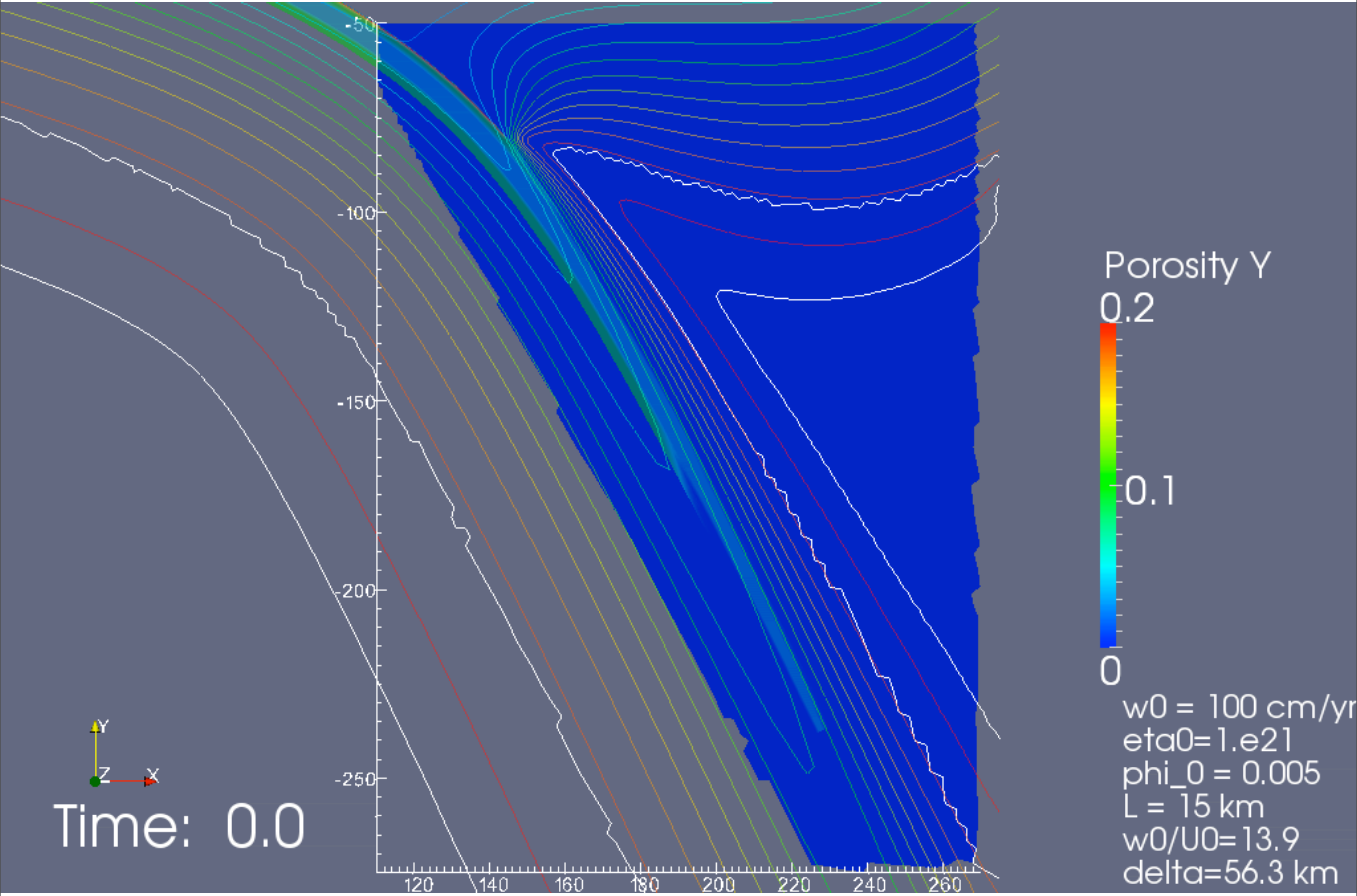
Permeable Flow model on subdomain

Spiegelman (MADDs-FP -- CIG)



Nicaragua:
Fluid Flow on SubDomain
Unstructured FEM/P2P2
Hybrid FEniCS/PETsc Codes
Full Magma Dynamics

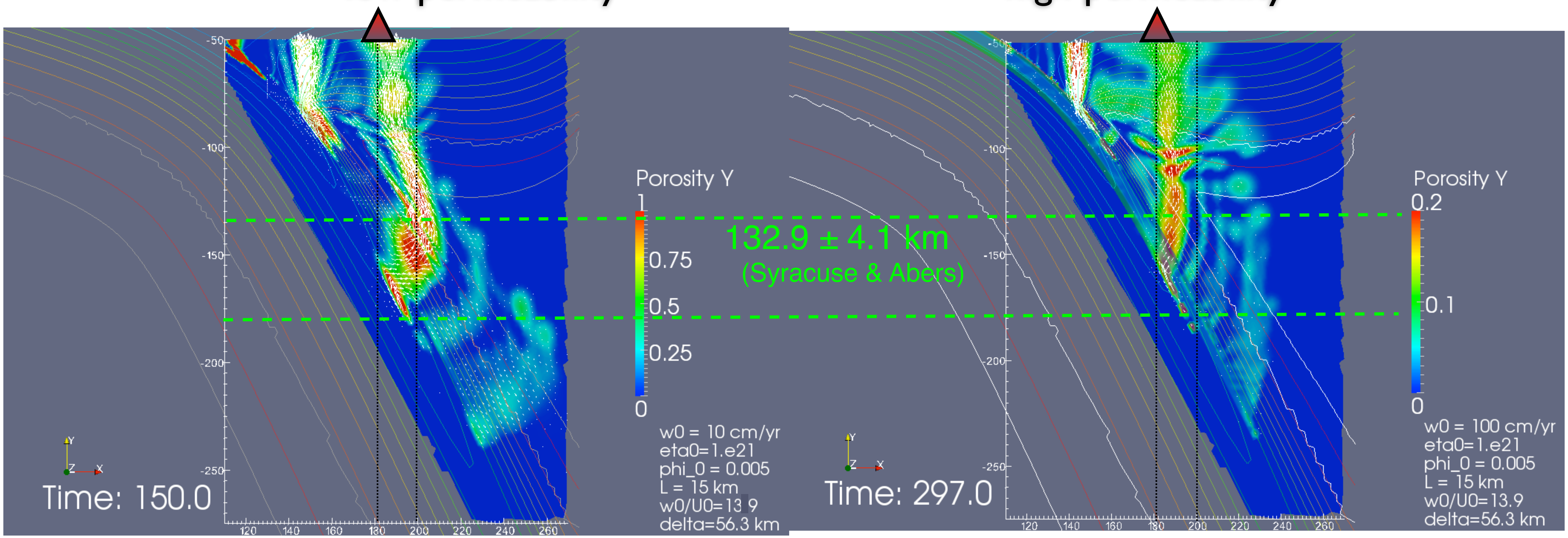
Fluid Flow Trajectories given dehydration rates

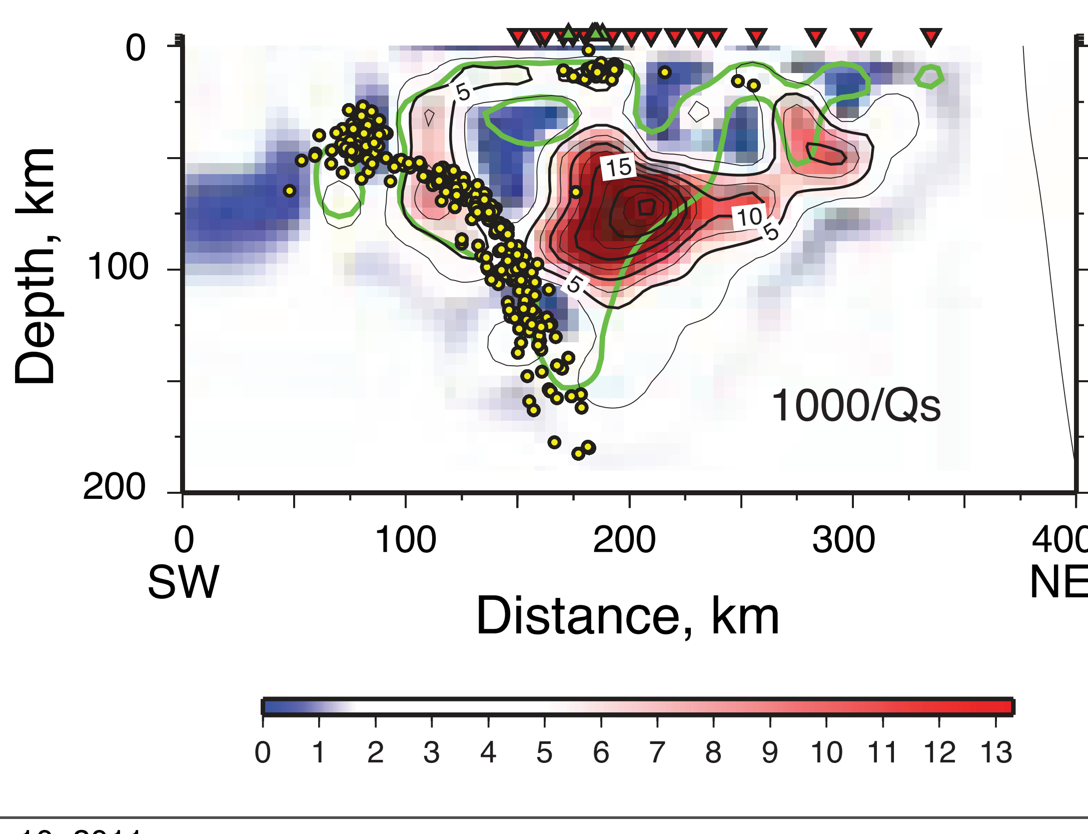


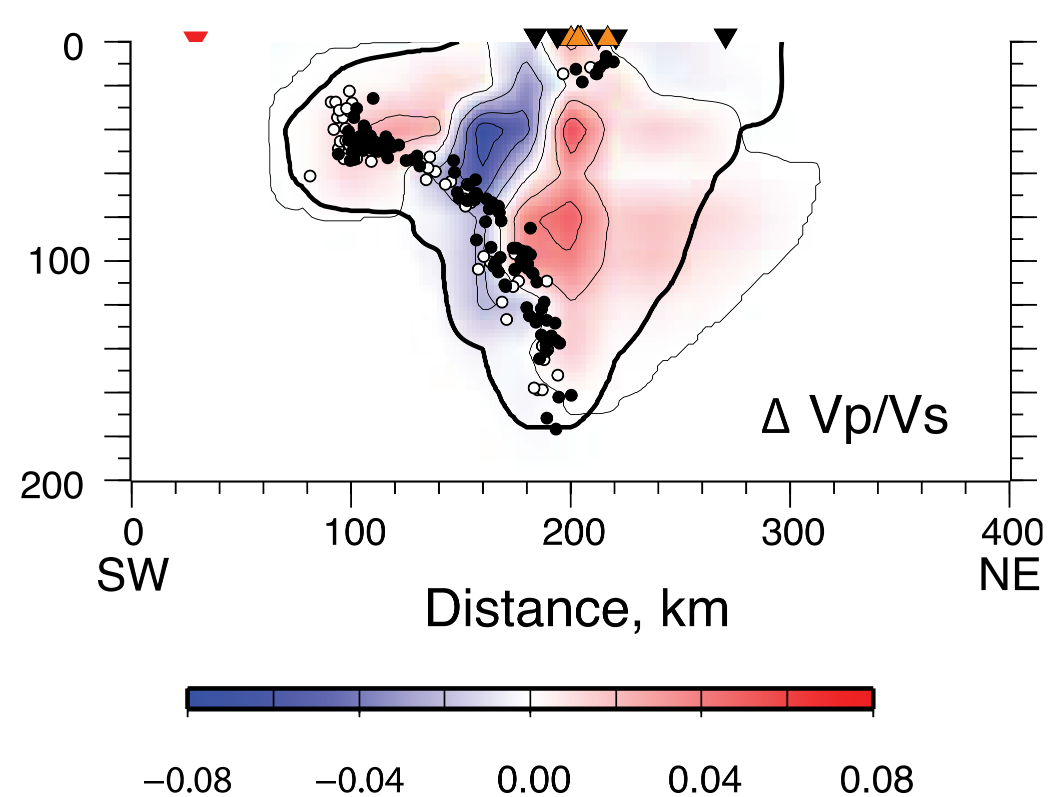
Comparison to TUCAN Data

low permeability

high permeability







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- addition of a melt phase introduces new dynamics and new length & time scales
- Many different mechanisms suggest some form of mesoscale organization into melt channels in the mantle which may have significant observational consequences.
- Small changes in couplings can significantly change the physics and computational requirements of these problems

Open Questions

- What are the interactions/dominant mechanism for localization at the meso-scale?
- What are the interaction between meso-scale and plate-boundary scale flow? Plate boundary dynamics and global mantle convection?
- What are the observational consequences of these processes and can important inferences be made from existing data on the structure and processes of partially molten regions?

Computational & Software issues

- Magma Dynamics is fundamentally a coupled multi-scale, multi-physics problem.
- How do we develop flexible, high-performance tools for more readily exploring the space of models and behavior?
- This is a completely different issue than finding/tuning a well understood problem (eg. Navier-Stokes, Seismic Wave tomography).
- Much of the essential software already exists (e.g. PETSc, FEniCS). Next time will detail how we can use it to develop some flexible and general approach to solving multi-physics models.

Philosophy of multi-physics PDE based models

- Overall Structure and Choices
- Software design for managing choices (PETSc, FEniCS)
- General abstractions of Non-linear multi-physics problem
- Examples in Hybrid FEniCS/PETSc codes.
- HPC issues...