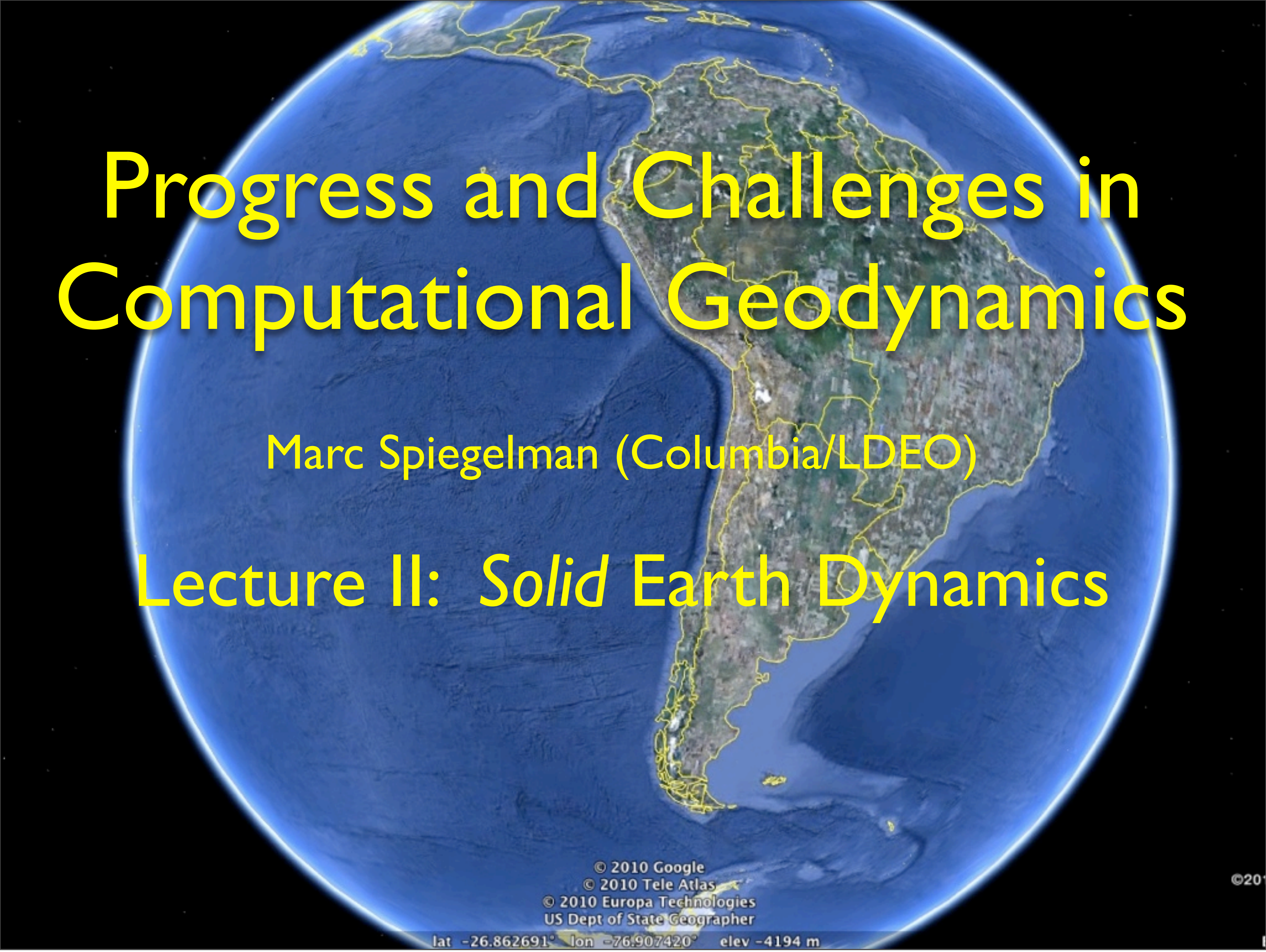
Progress and Challenges in Computational Geodynamics

Marc Spiegelman (Columbia/LDEO)

Lecture II: *Solid* Earth Dynamics



Lecture II


© 2010 Google
© 2010 Tele Atlas
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US Dept of State Geographer
lat -26.862691° lon -76.907420° elev -4194 m

Lecture II

- Quick Review of Plate Tectonics: Solid Mechanics problems

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    - Robust? solvers for variable viscosity Stokes flow

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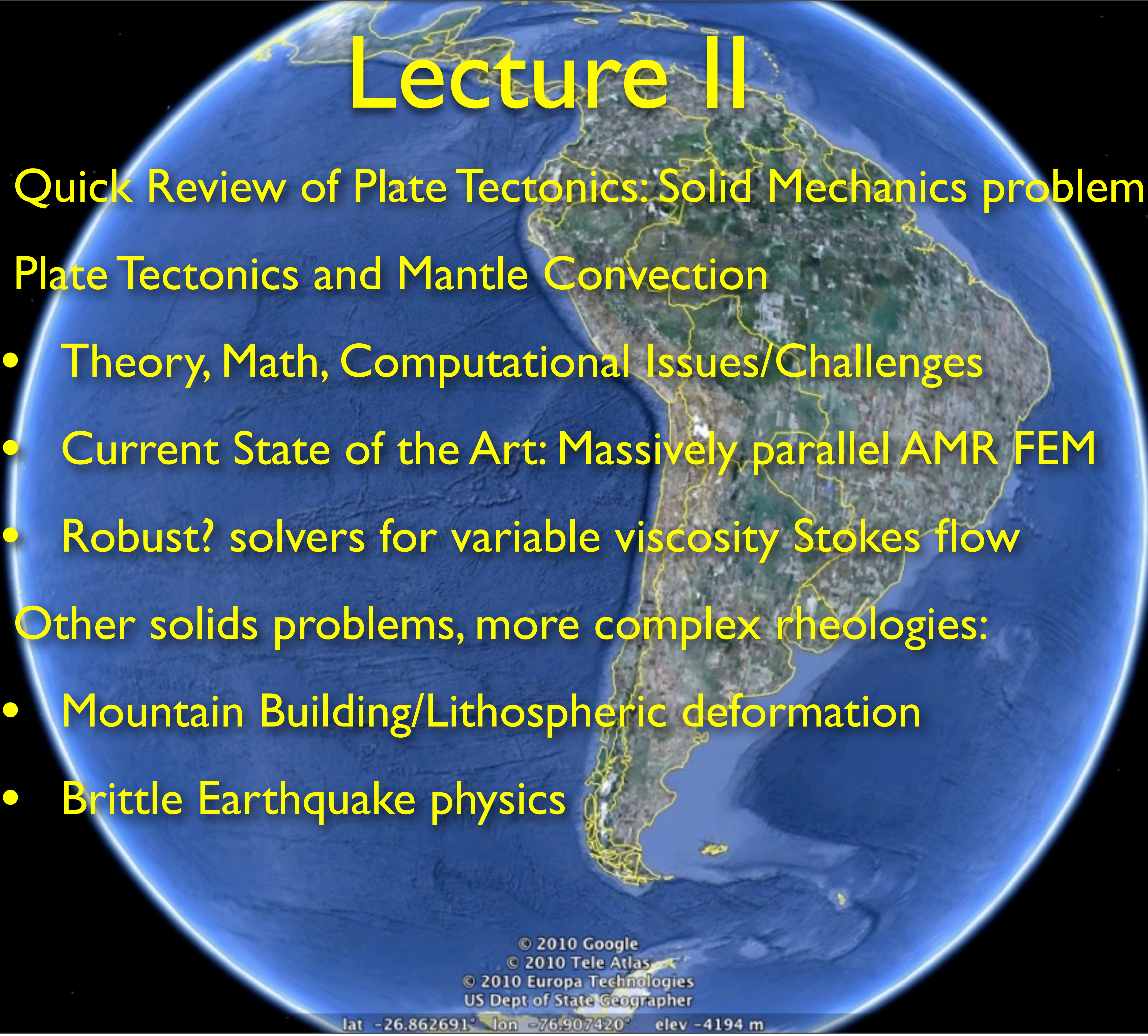

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- Future Challenges

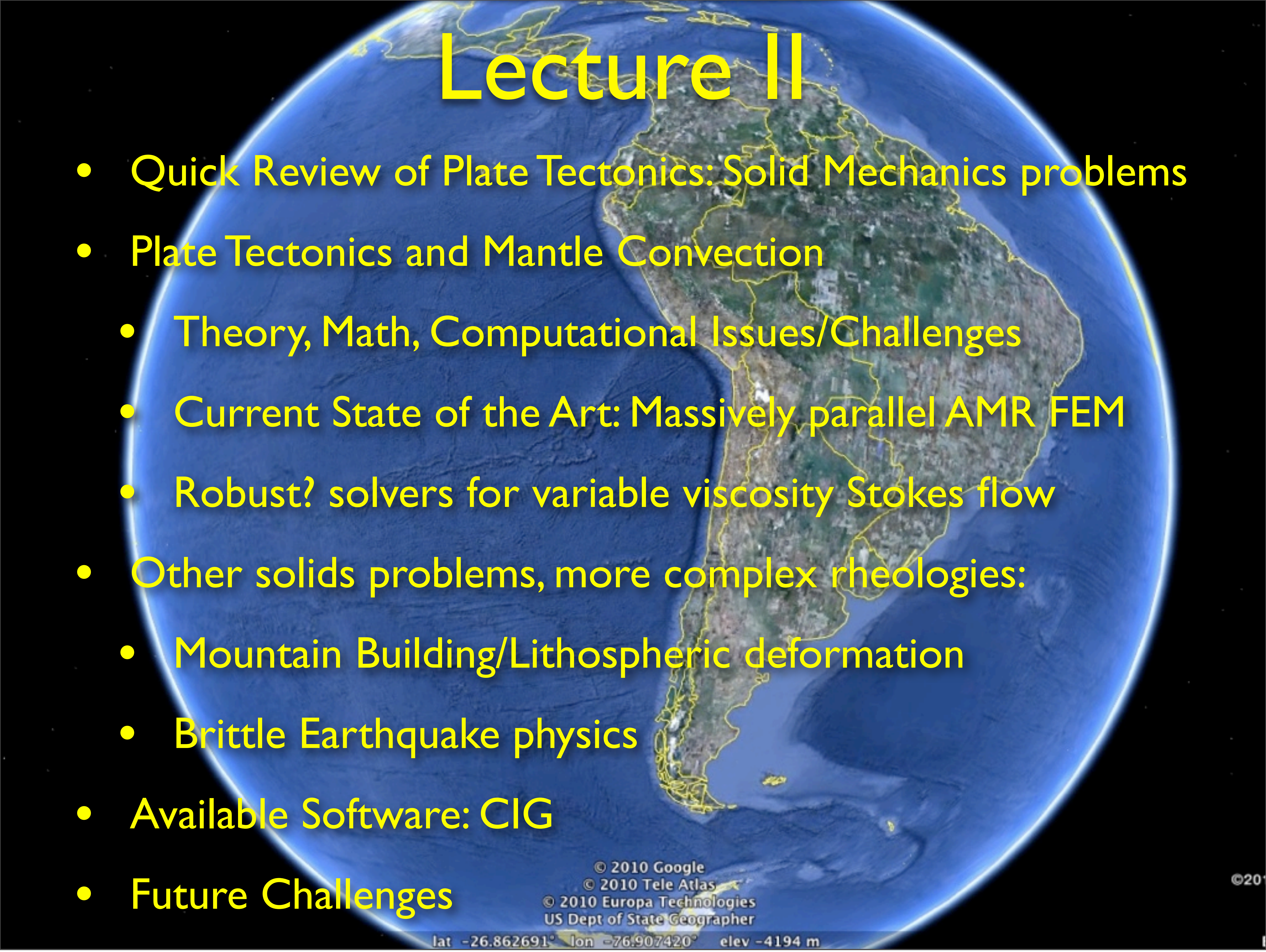
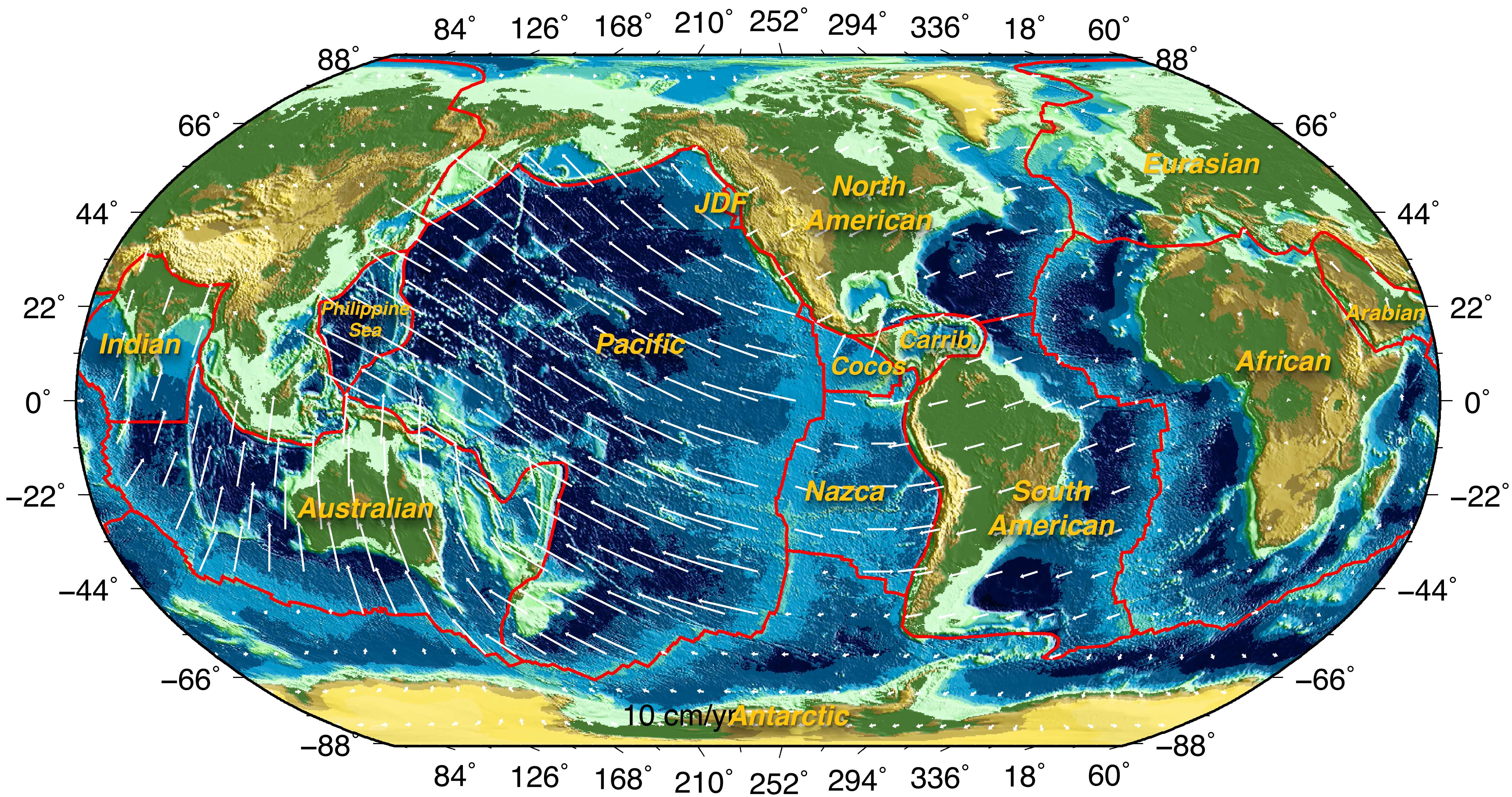


Plate Tectonics Review

Current plate velocities and boundaries



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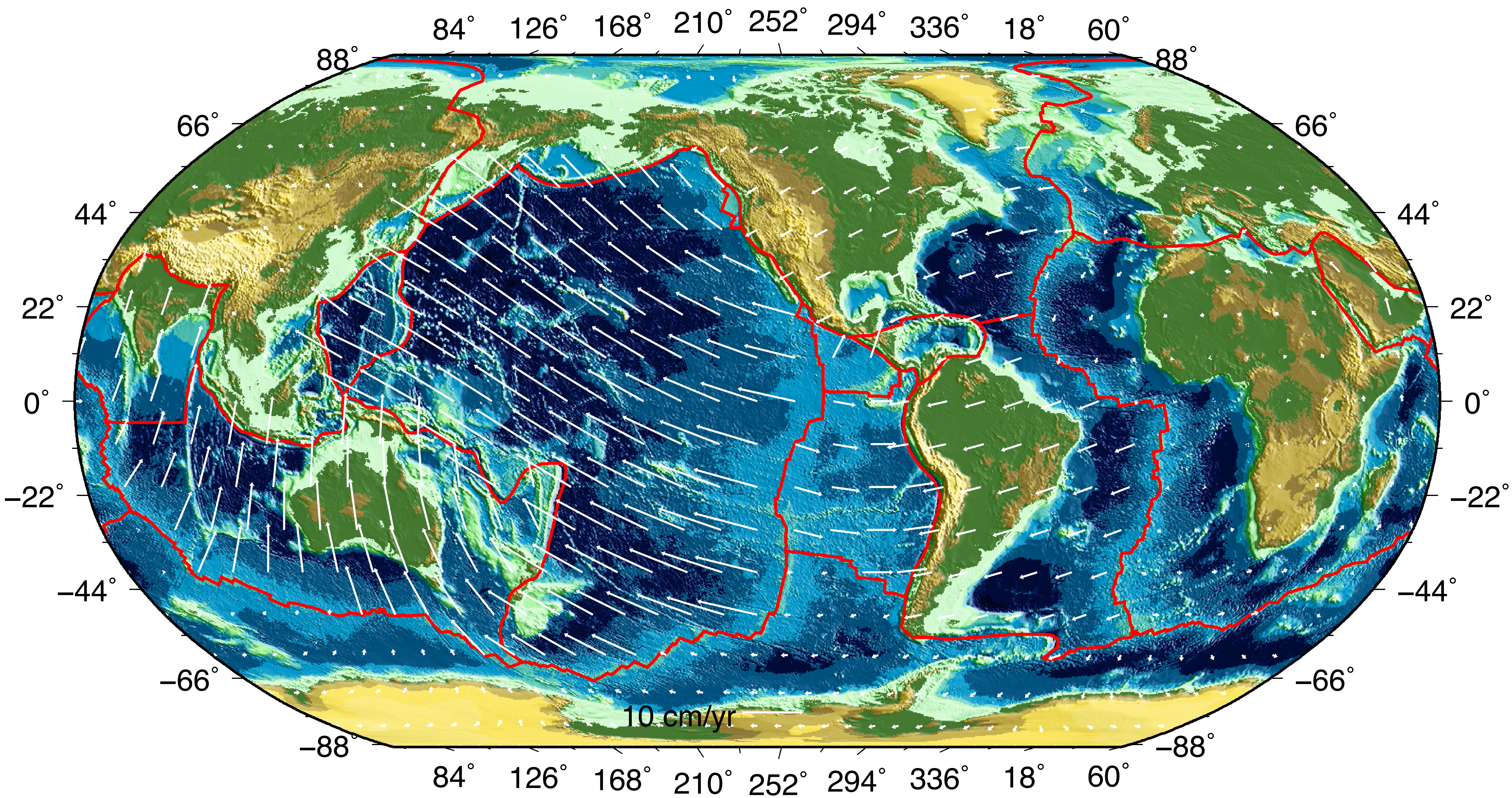

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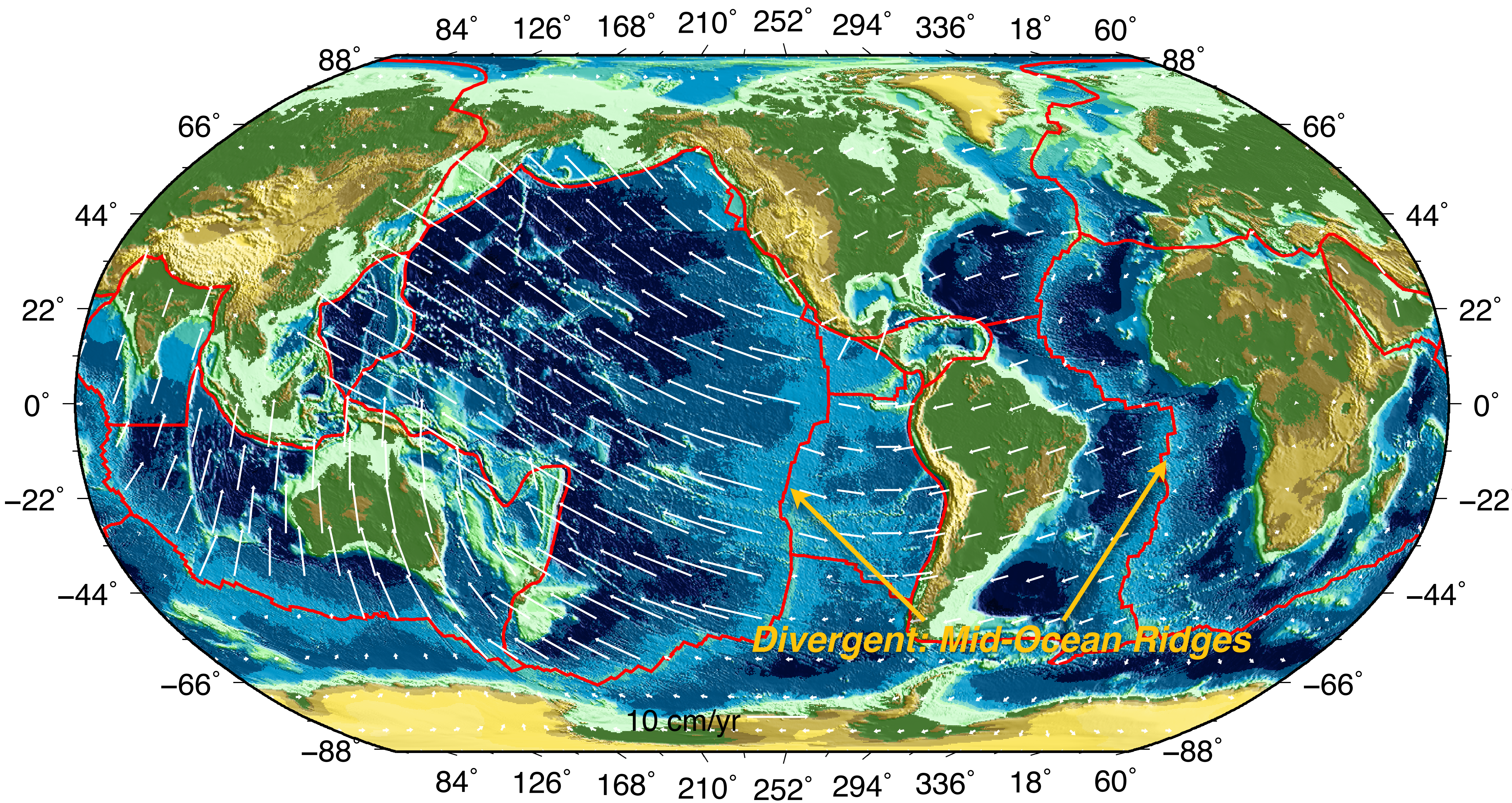

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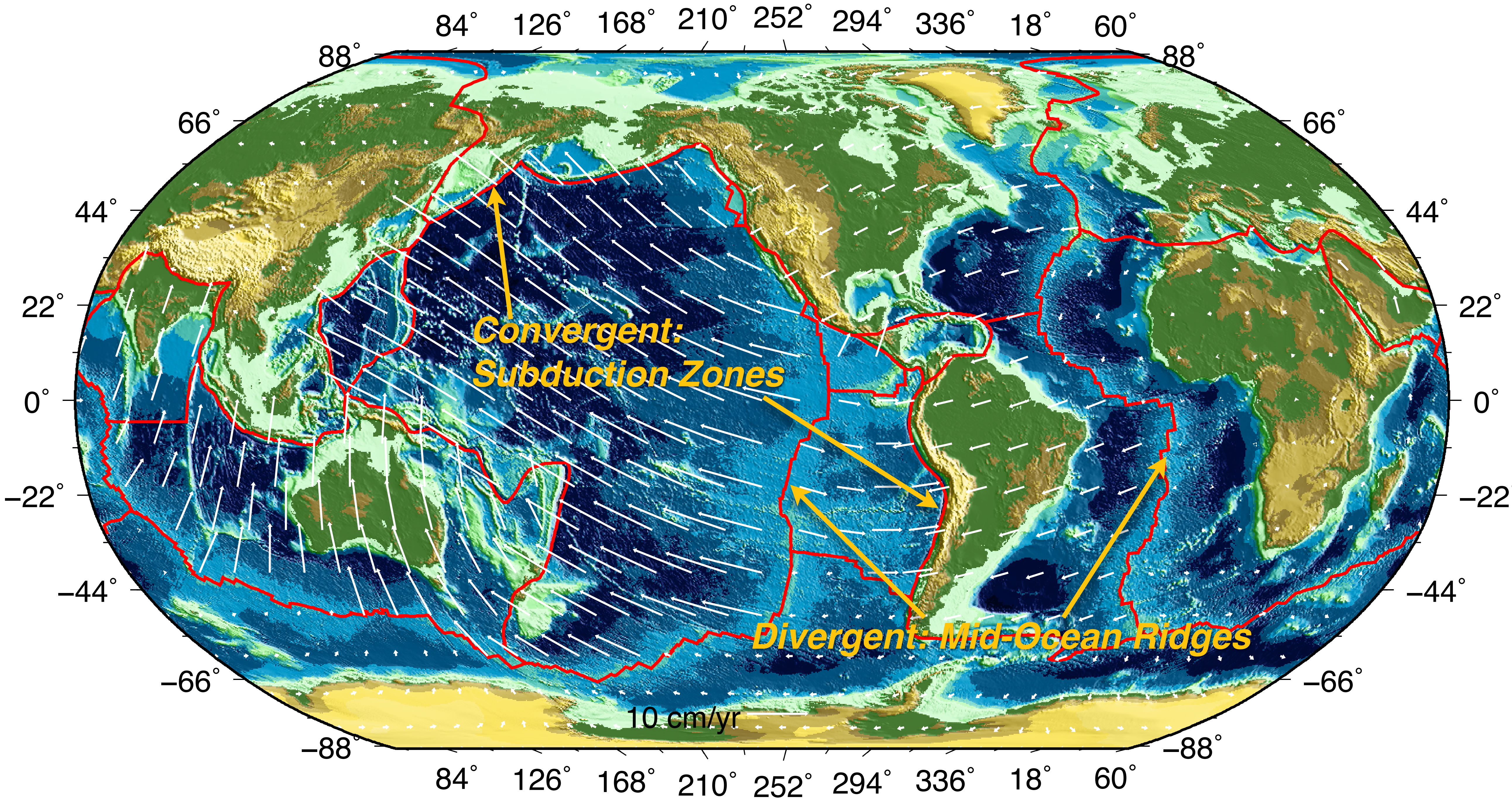

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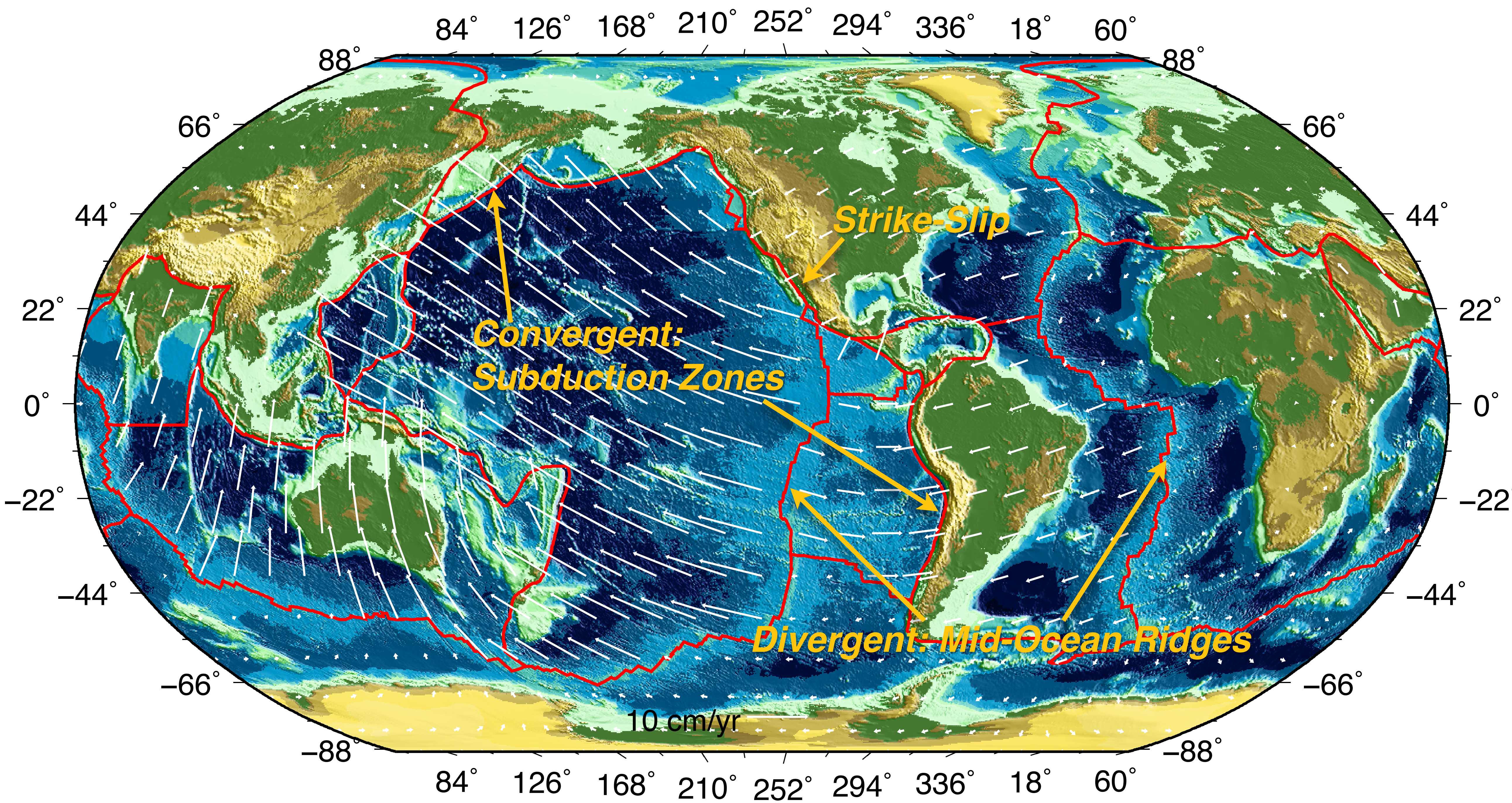


Plate Tectonics 101

Global Distribution of Volcanoes

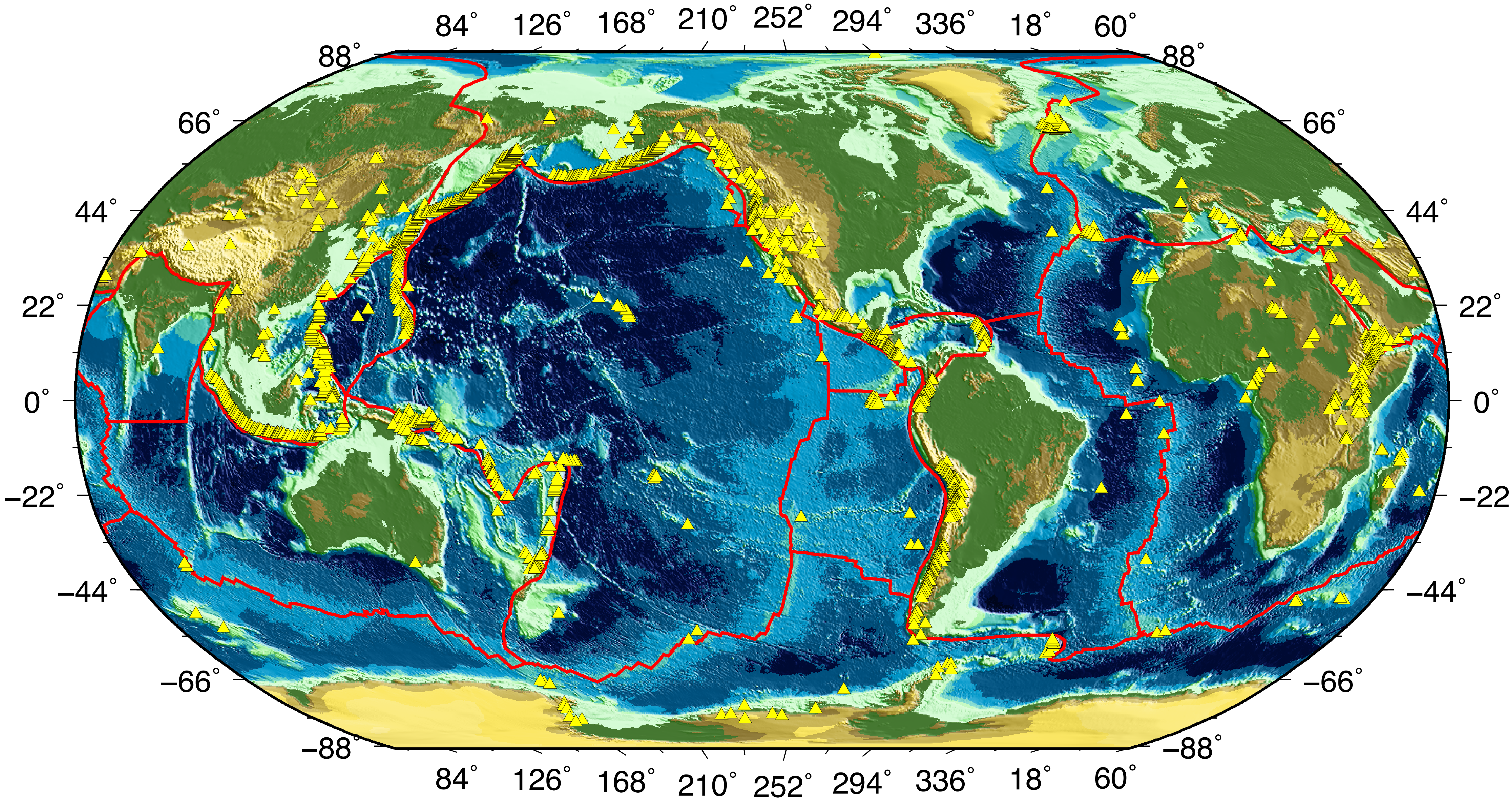
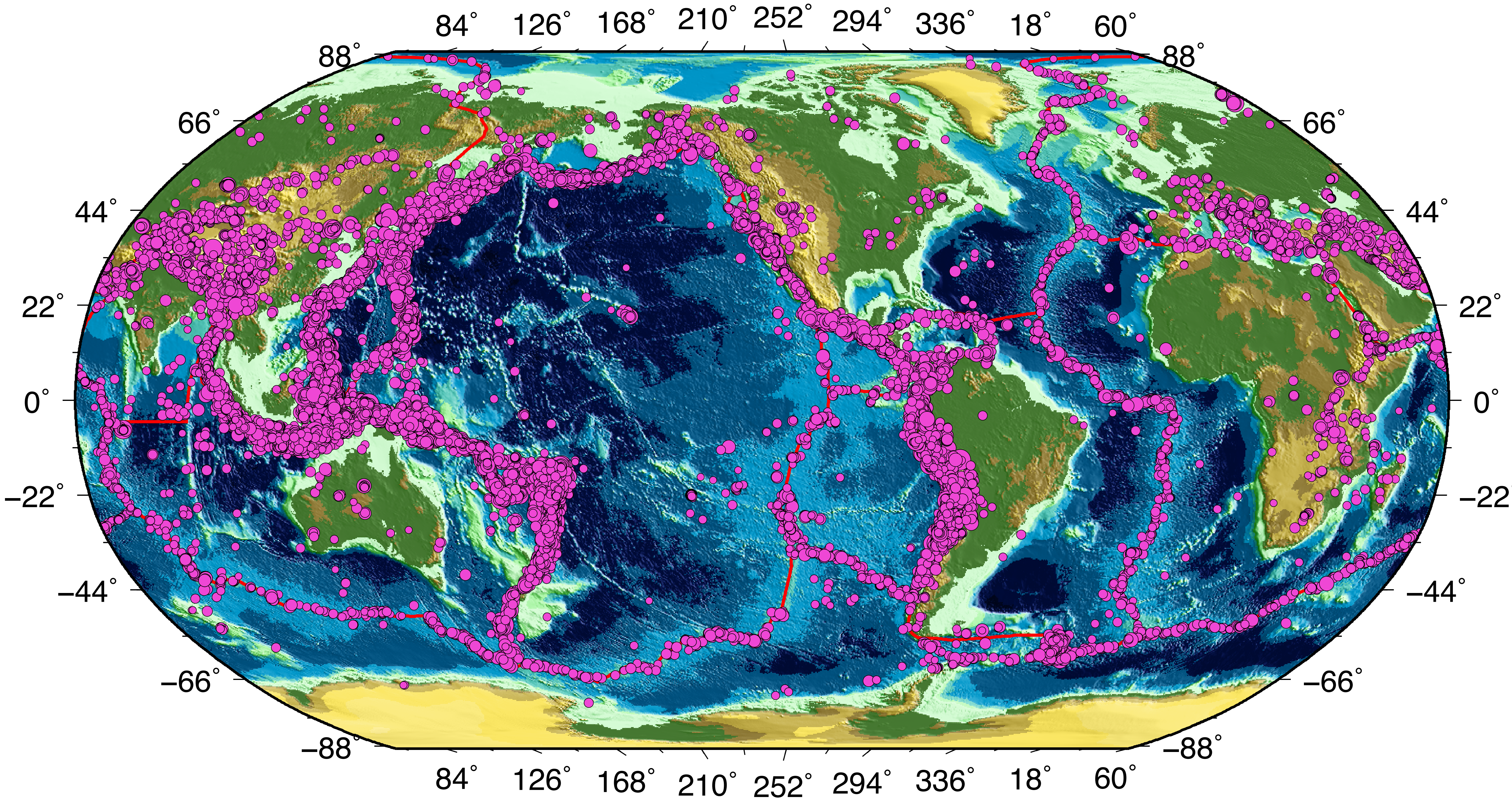


Plate Tectonics 101

Earthquakes > M5



Points

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- Plate tectonics is only a *kinematic* description of surface motions.

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- Convergent and Divergent margins imply 3-D circulation and ductile deformation of Earth's interior

Plate Tectonics Review

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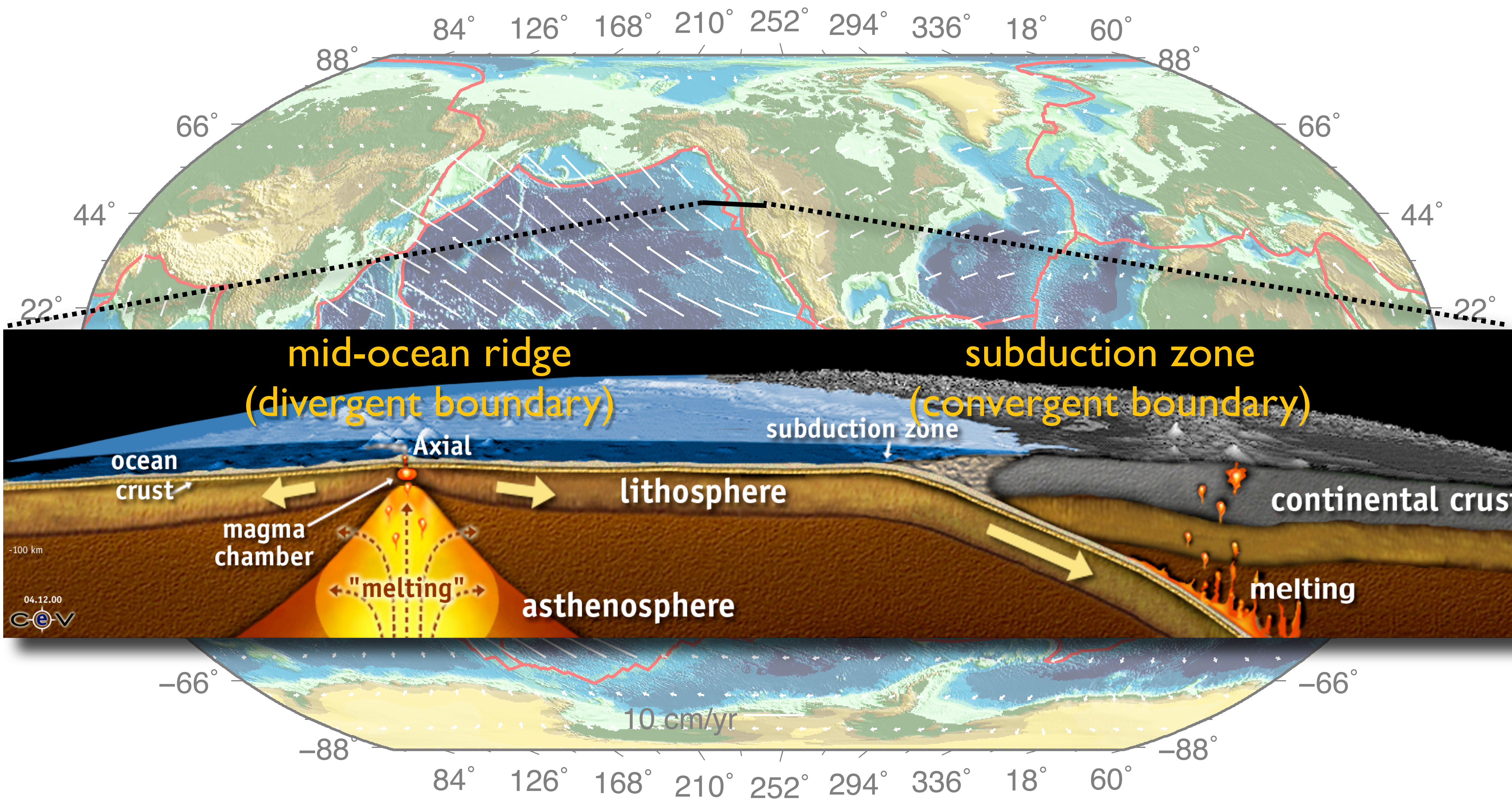
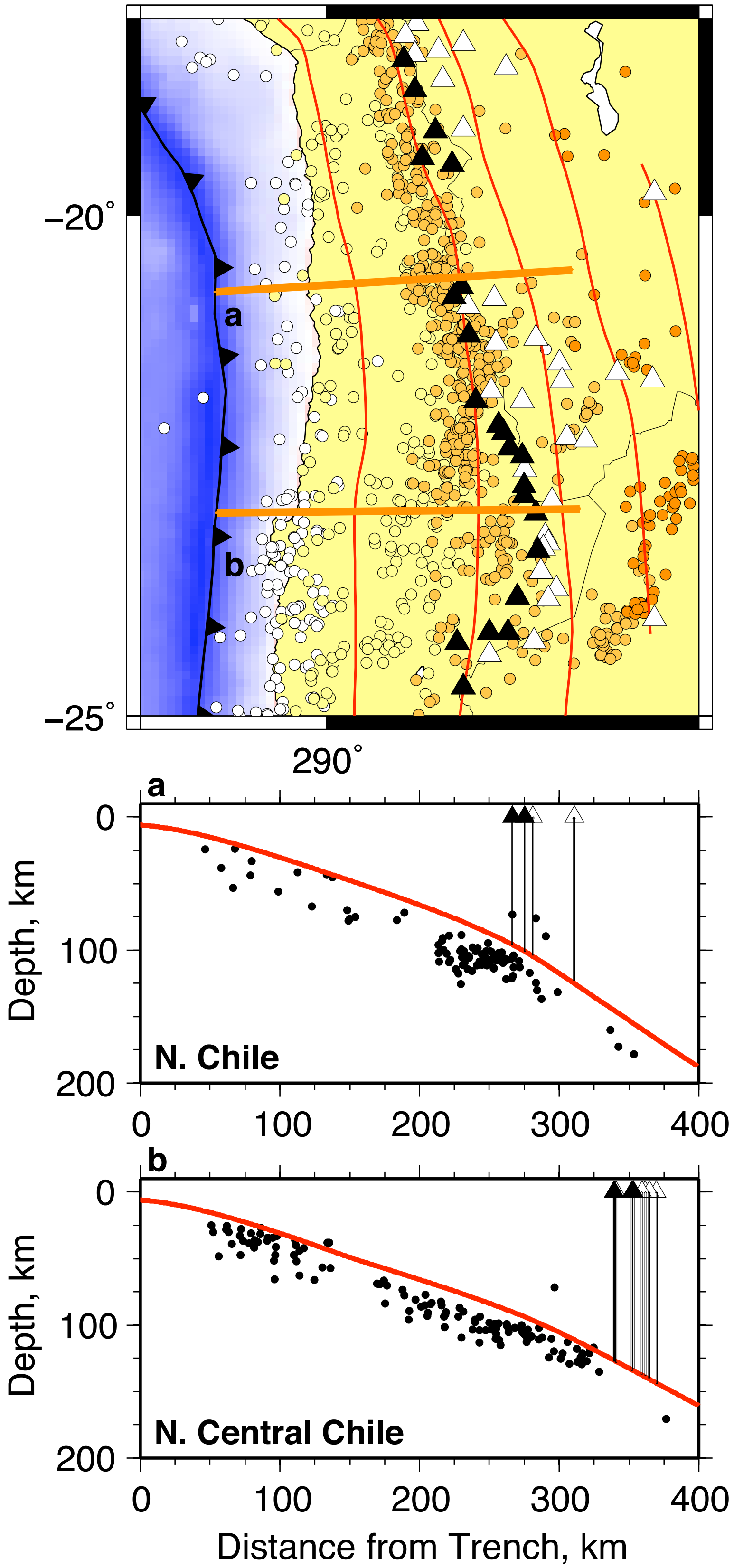


Seismicity of Subduction Zones

Geochemistry Geophysics Geosystems G3 SYRACUSE AND ABERS: ARC VOLCANO SLAB DEPTH 10.1029/2005GC001045



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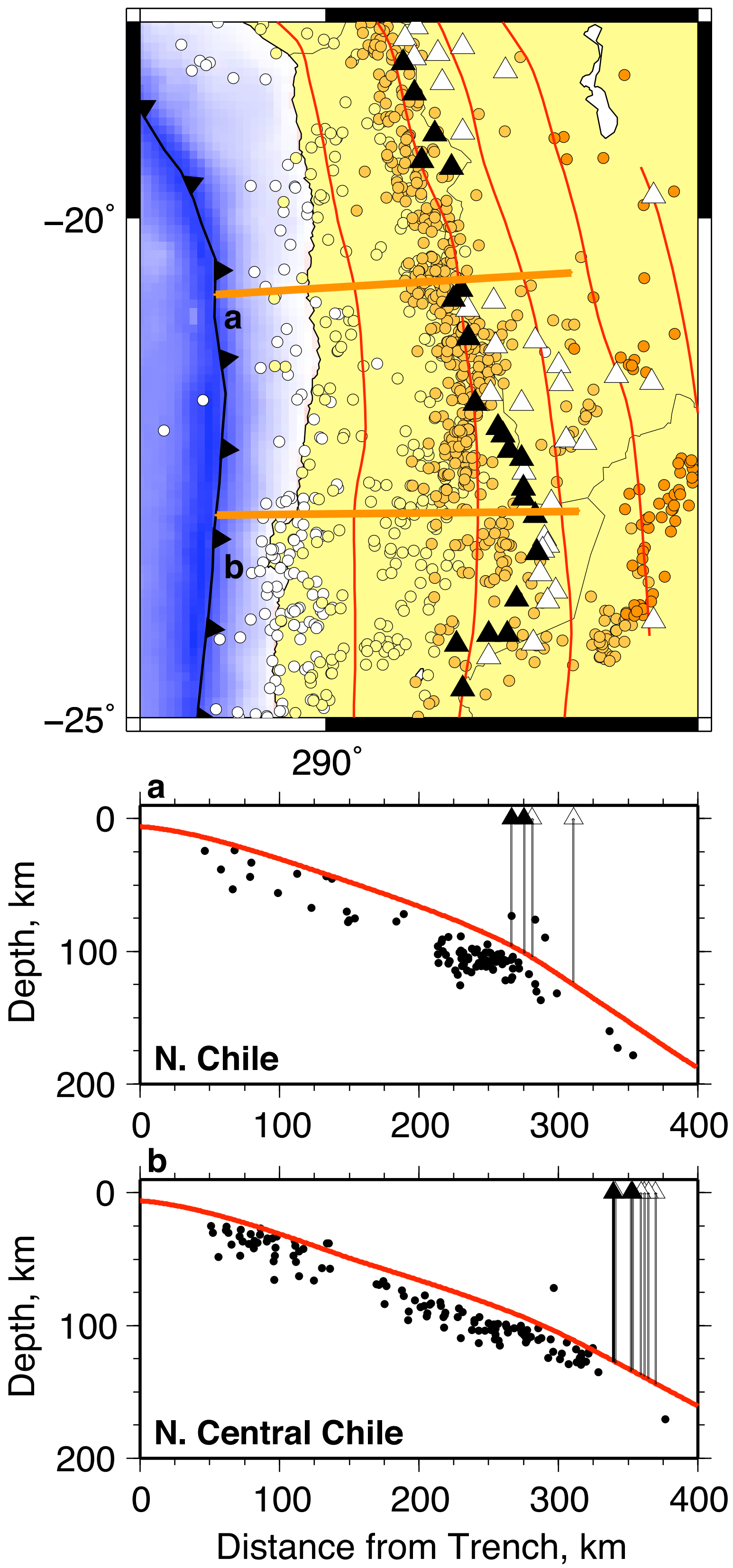

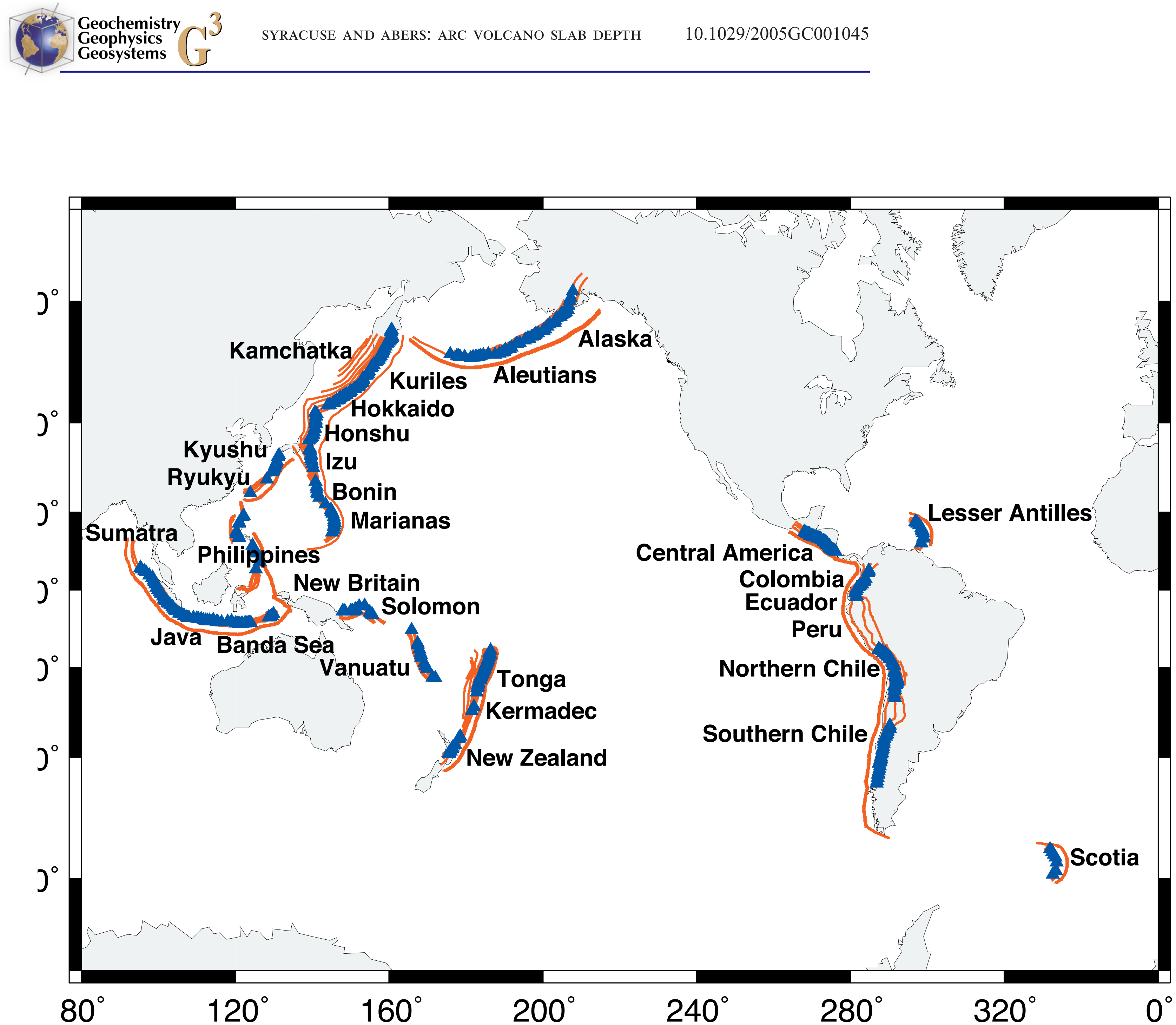

Global Slab Contours and Volcanoes

(Syracuse and Abers, 2006)

Open Questions

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- What is the structure of flow in the Earth's interior in space and time?

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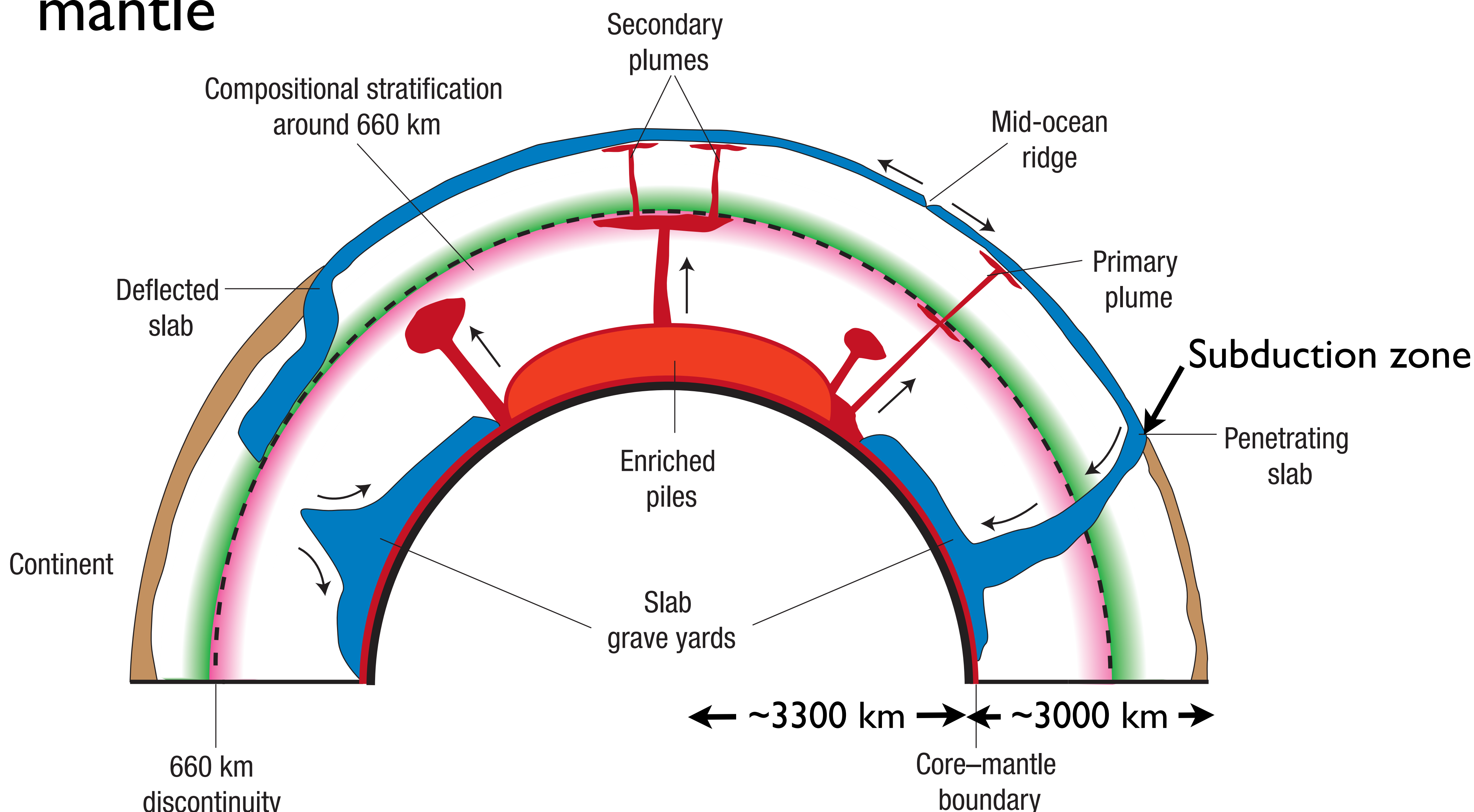
- What is the structure of flow in the Earth's interior in space and time?
- What is the state of stress in the planet (which affects Earthquake rupture and Volcanism)

Mantle Convection

- Reigning Hypothesis for Plate tectonics is *Solid State* Thermal-Chemical Convection of the Earth's mantle



Paul Tackley, News and Views

nature geoscience | VOL 1 | MARCH 2008 | www.nature.com/naturegeoscience

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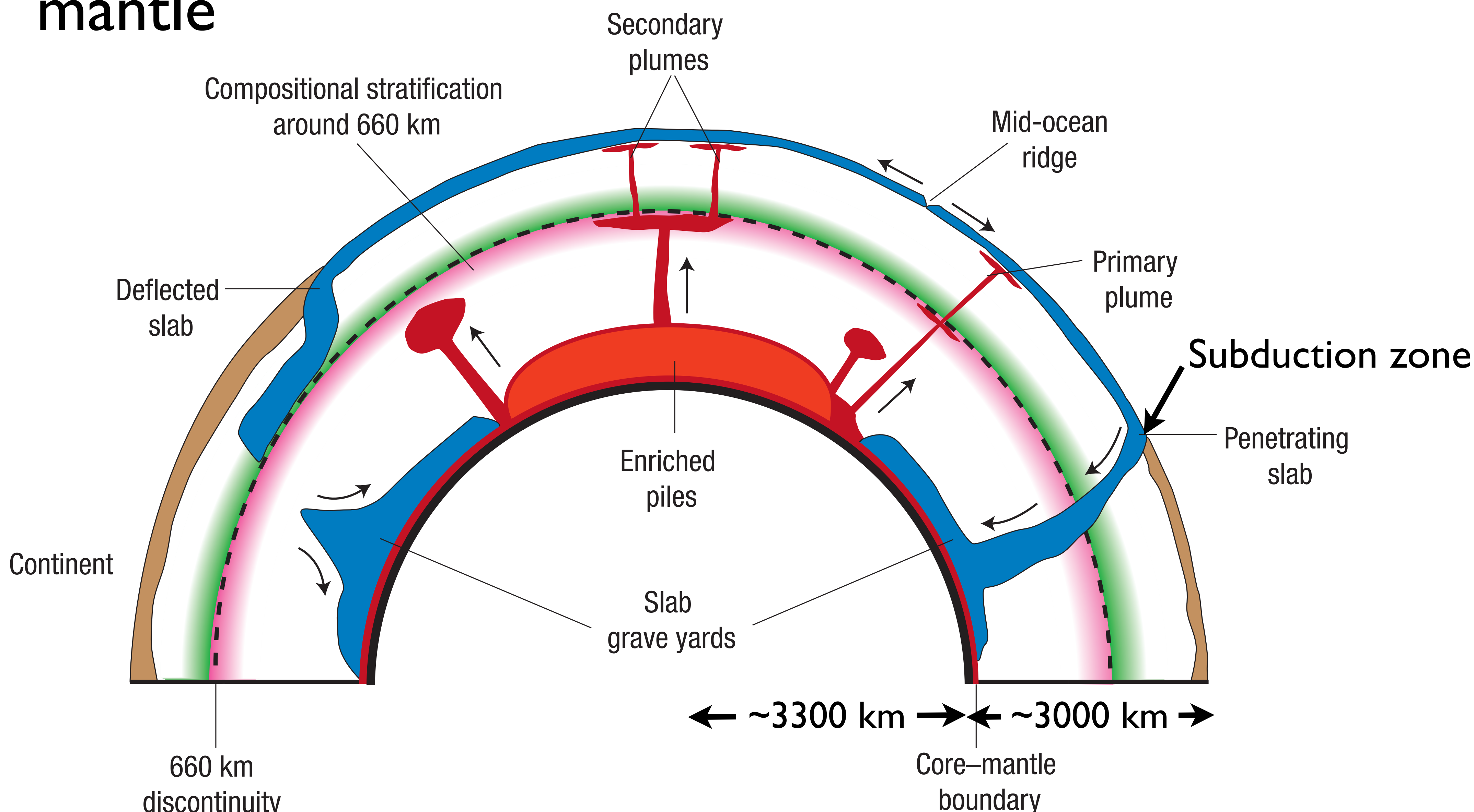


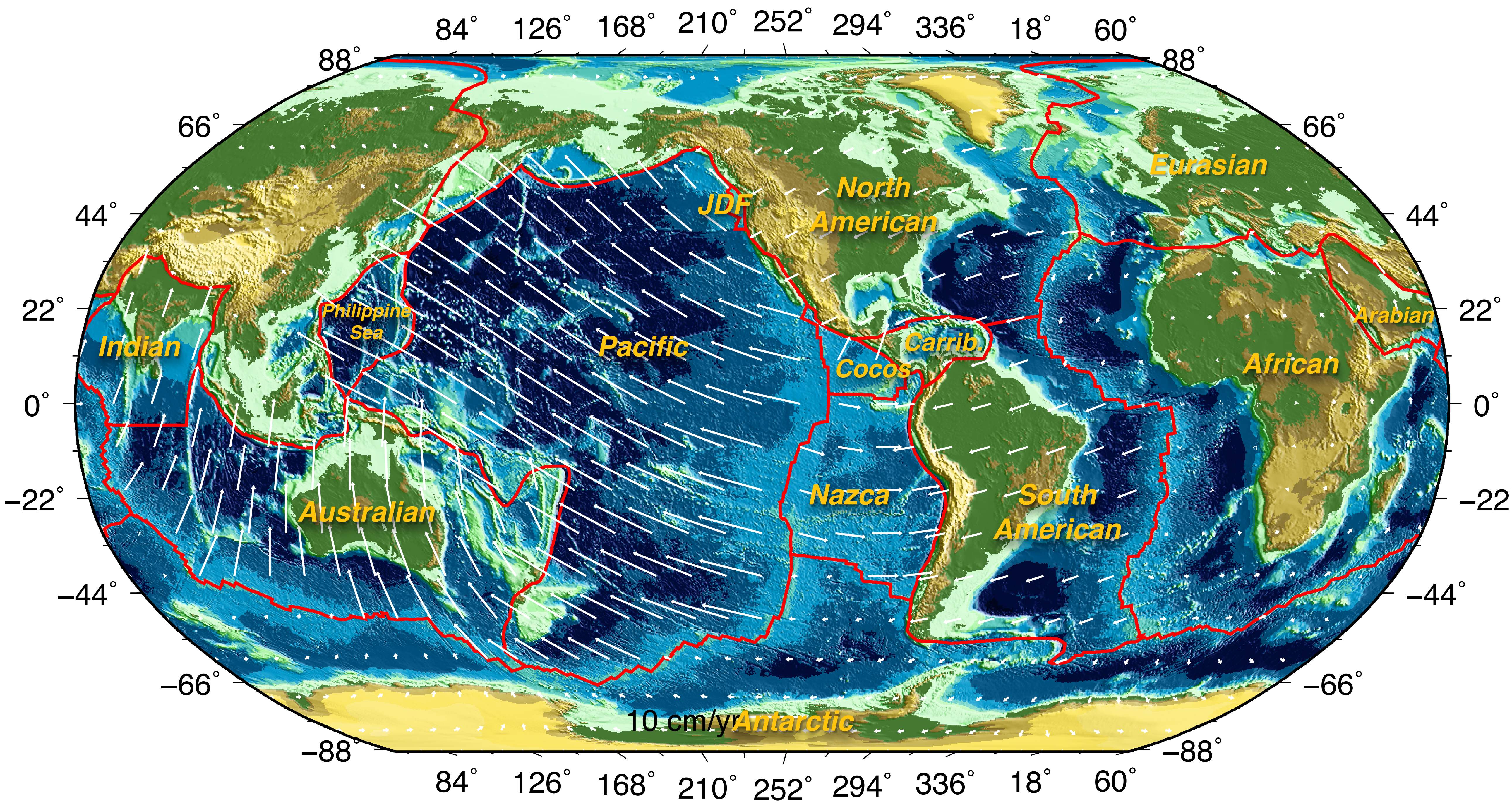
Paul Tackley, News and Views

nature geoscience | VOL 1 | MARCH 2008 | www.nature.com/naturegeoscience

- Plate tectonics *is* the zeroth-order scale of mantle convection

Plate Tectonics Review

Current plate velocities and boundaries



Mantle Convection: Basic Physics

- Principal Driving force is gravity acting on density variations $\rho(T, c)$
- The mantle convects in the *Solid* State
  - Propagation of elastic seismic waves shows that most of the planet is crystalline solid
  - Experiments show that Silicate rocks have ductile (if complex) rheologies at elevated Temperature and Pressure

Rheology of Silicate Rocks

http://rst.gsfc.nasa.gov/Sect2/eclogiteFoldsNordfjord.jpg

Rheology of Silicate Rocks

- Rocks are generally Visco-Elastic-Plastic
- On short time scales, they are essentially elastic ($G=10^{11}$ Pa)
- At sufficiently high P-T (but still sub-solidus) Rocks can be describe using a viscous rheology ($\eta \approx 10^{18} - 10^{23}\ \mathrm{Pa\ s}$)
- Maxwell time is $\tau = \frac{\eta}{G}$ ~ 4 months - 32000 yrs. Deformation on time-scales << shorter than τ behave elastically

Rheology of Silicate Rocks

General form of viscosity

$$\eta(T, \dot{\epsilon}) = C_1 \exp\left[\frac{C2}{T}\right] \dot{\epsilon}_{II}^{(n-1)/n}$$

where

$\dot{\epsilon}_{II} = \sqrt{\dot{\epsilon} : \dot{\epsilon}}$	2nd invariant of strain rate tensor
$\dot{\epsilon} = 1/2\left(\nabla\mathbf{v} + \nabla\mathbf{v}^T\right)$	strain rate tensor
$n \sim 1 - 5$	stress exponent (1 is Newtonian)

- At mantle (T, P), $\eta \sim 10^{18} - 10^{24}$ Pa s
- The viscosity of water is 10^{-3} Pa s!
- Mantle Reynolds Number $\mathrm{Re} = \frac{\rho U_0 d}{\eta} < 10^{-18}$

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Even at the scale of the planet: Inertia is negligible

Mathematical Description of Mantle Convection

Infinite Prandtl number thermal convection (Bouissinesq Approx)

- Conservation of Energy

$$\rho c_P \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} T \right) = \boldsymbol{\nabla} \cdot k \boldsymbol{\nabla} T$$

- Conservation of Momentum (no inertia)

$$-\boldsymbol{\nabla} \cdot \left[\eta \left(\boldsymbol{\nabla} \mathbf{v} + \boldsymbol{\nabla} \mathbf{v}^T \right) \right] + \boldsymbol{\nabla} P = \rho(T) \mathbf{g}$$

- Conservation of Mass (incompressible flow)

$$\boldsymbol{\nabla} \cdot \mathbf{v} = 0$$

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$$\nabla \cdot \mathbf{v} = 0$$

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Coupled, non-linear parabolic/elliptic system

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Stokes

- Coupled Multi-physics problem
- Two sources of coupling
  - advection and buoyancy (creates $\mathbf{v}(T)$)
  - constitutive relationships $\eta(T, \mathbf{v})$
- Time-dependence from Energy equations coupled to global Elliptic problem to be solved at every time step

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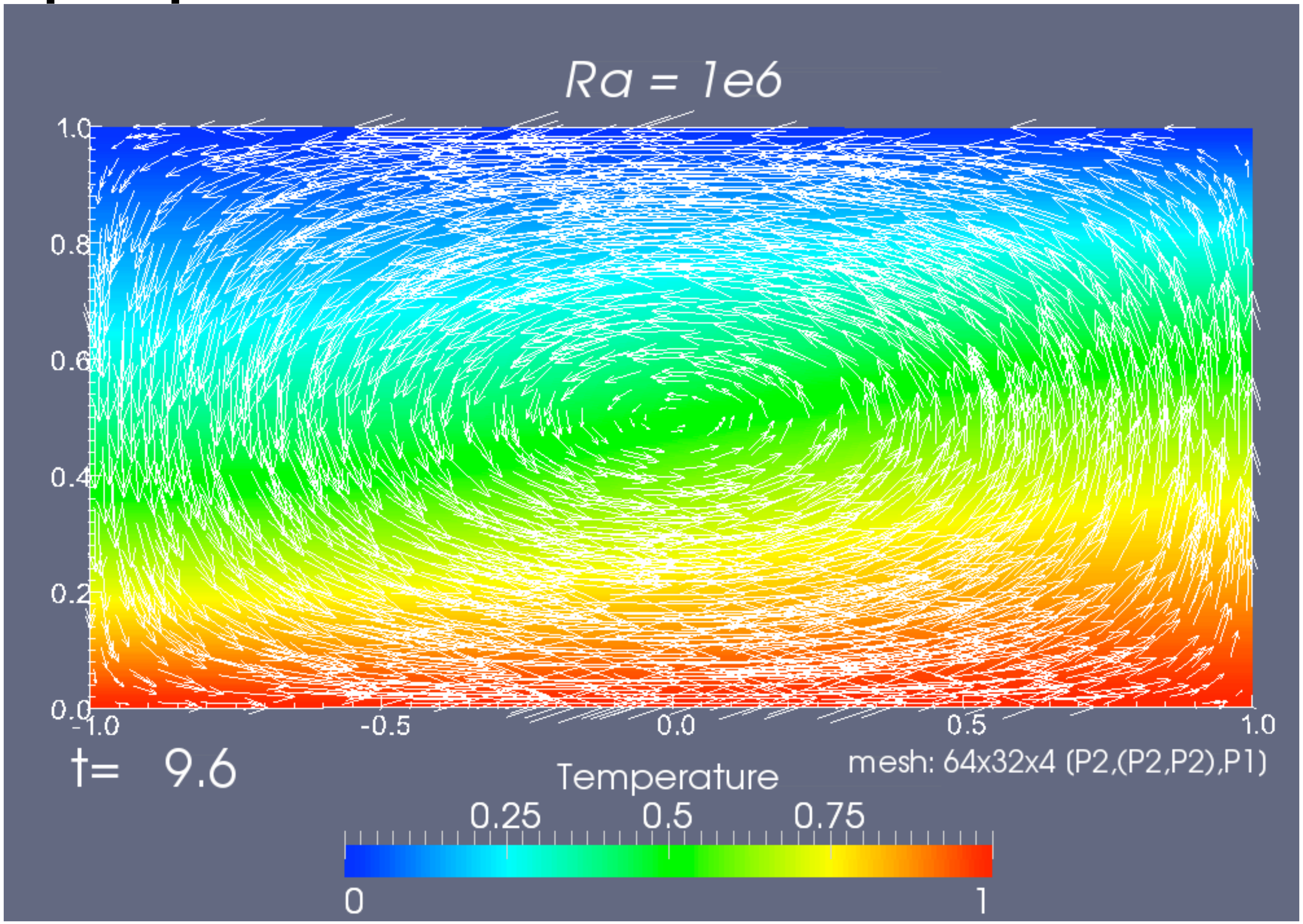
Simple problem: isoviscous 2-D convection



- Hybrid FEniCS/PETSc multi-physics codes
- Too “fluidy”, no Plates, 2-D
- Predicts lots of small scale structure
- Resolution requires resolving evolving boundary layers

Computational Issues

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- 3-D with strong localization

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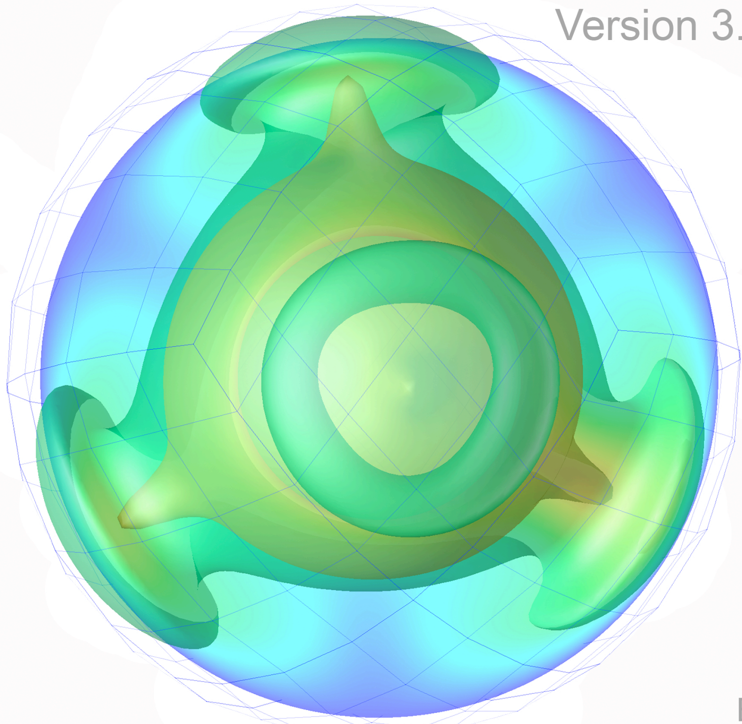
- *Much harder problem than Comp. Seismology*

Some Existing Computational Codes

Finite Element

CitcomS

User Manual
Version 3.1.1.1



Eh Tan
Michael Gurnis
Luis Armendariz
Leif Strand
Susan Kientz

www.geodynamics.org

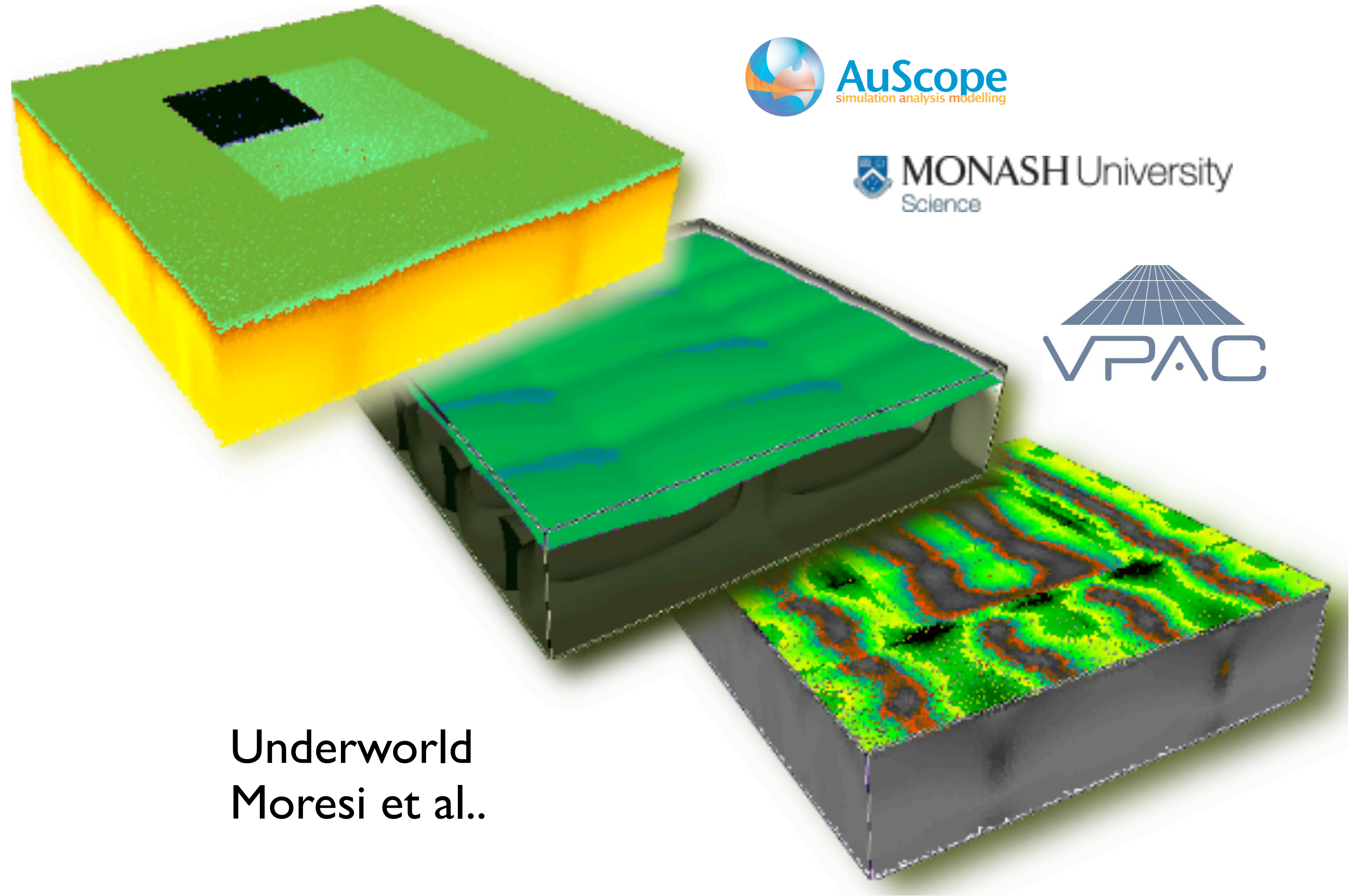
3-D Spherical Compressible convection
Low order Q1-P0 elements, on 12cap sphere mesh
Uzawa Scheme for Stokes
Well Benchmarked and Documented
Developed & Distributed by CIG
www.geodynamics.org

CIG COMPUTATIONAL INFRASTRUCTURE for GEODYNAMICS

AuScope simulation analysis modelling

MONASH University Science

VPAC



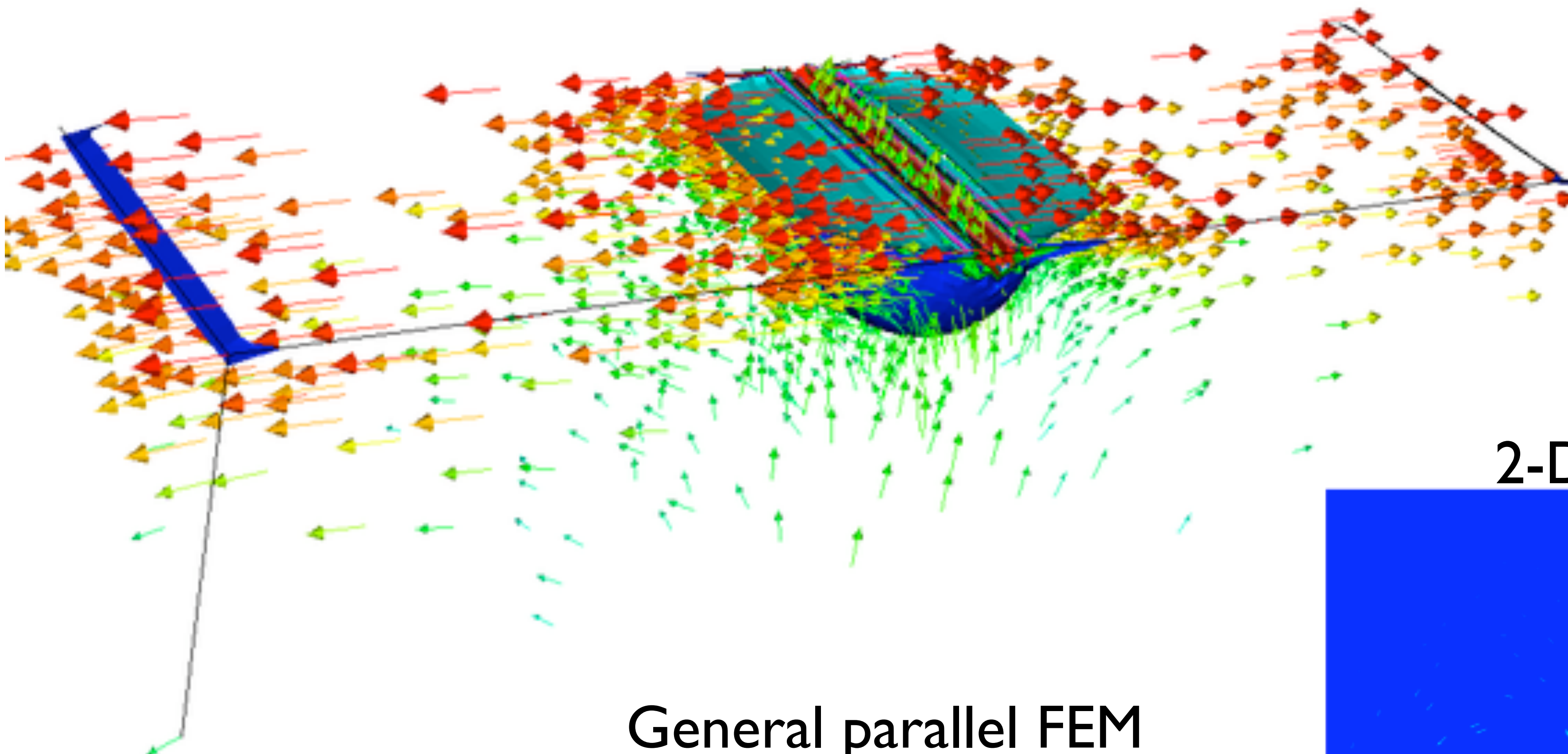
Underworld
Moresi et al..

3-D Cartesian incompressible convection
Low order Q1-P0, PIC code
Uzawa Scheme for Stokes
http://www.underworldproject.org/index.html

Some Existing Computational approaches

deall.ii www.dealii.org

Finite Element

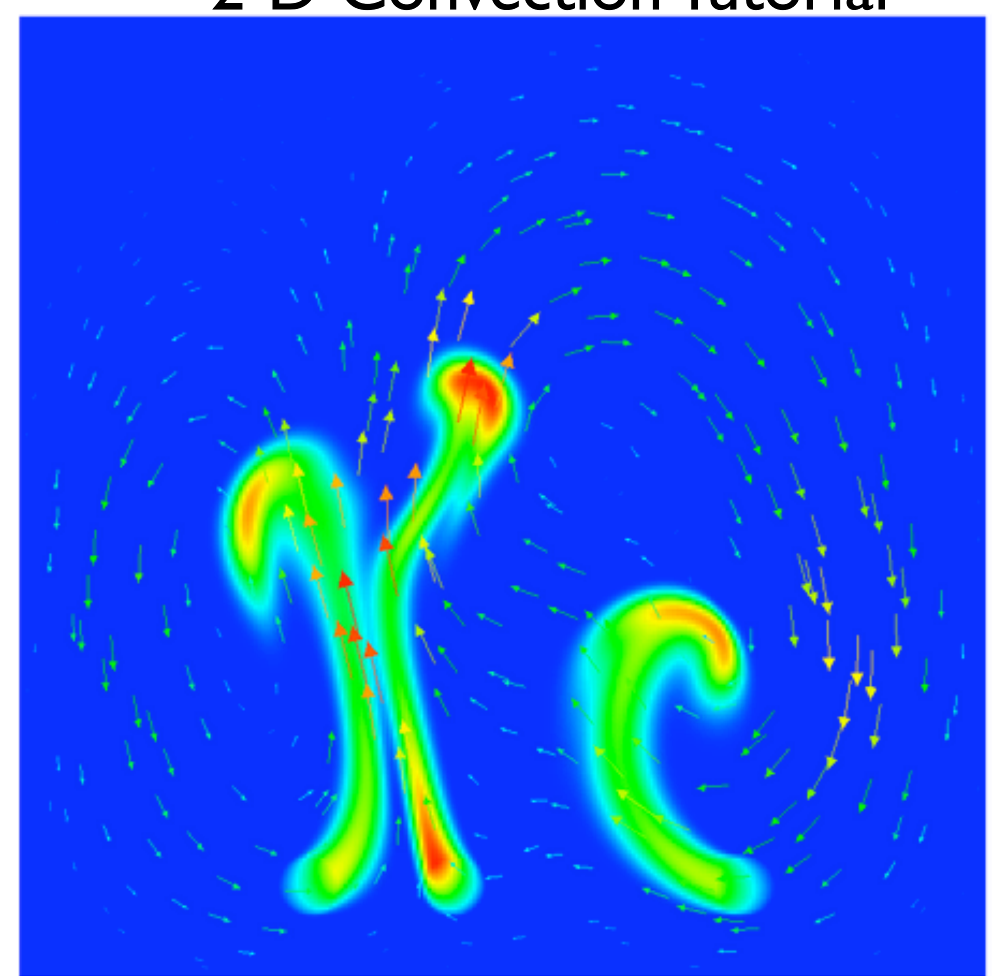


Tutorial Stokes solution for 3-D mid-ocean ridge spreading.

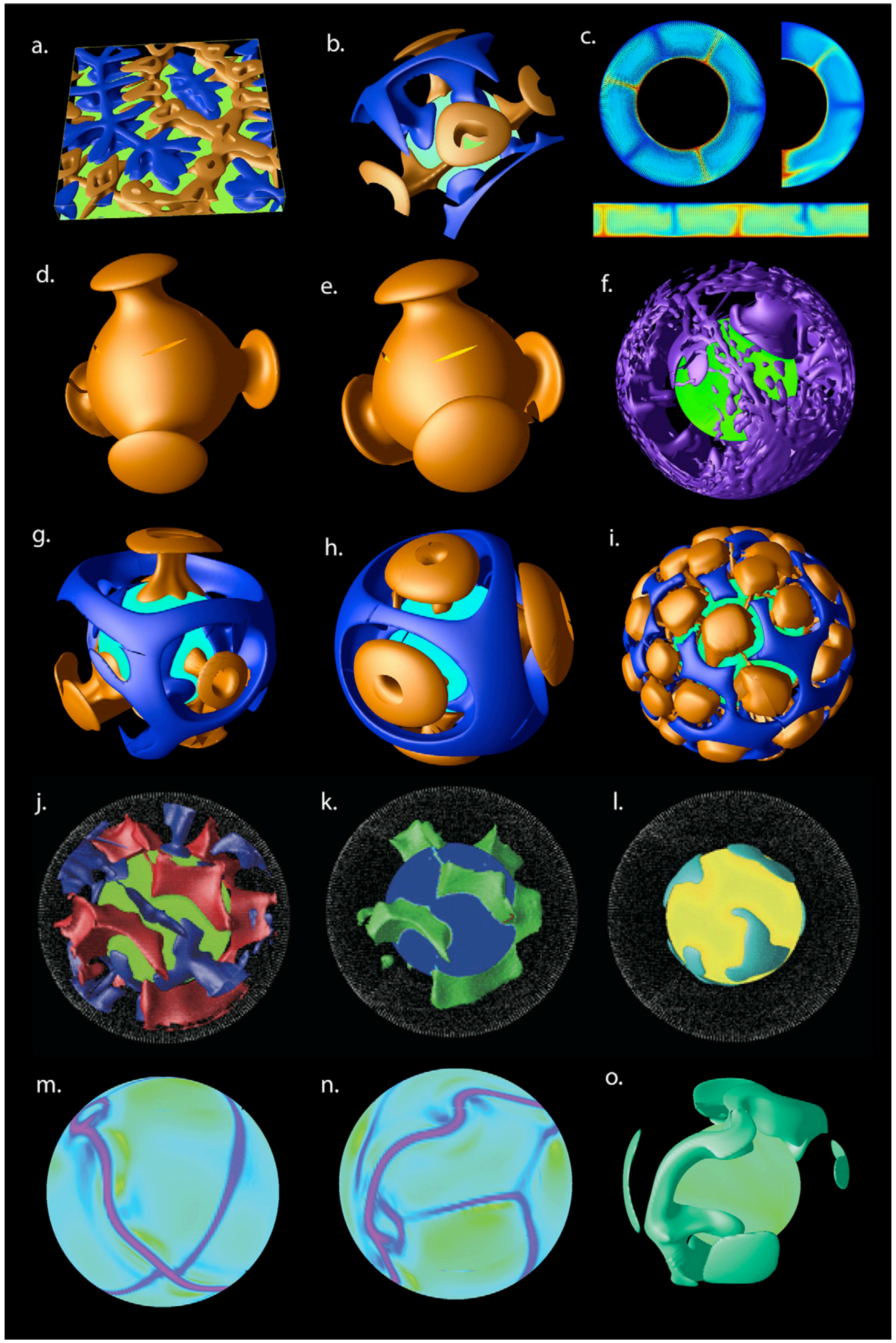
General parallel FEM Library for FEM solution on Forest of Octree, adaptive meshes.

Current release 6.3.1, QPL

2-D Convection Tutorial



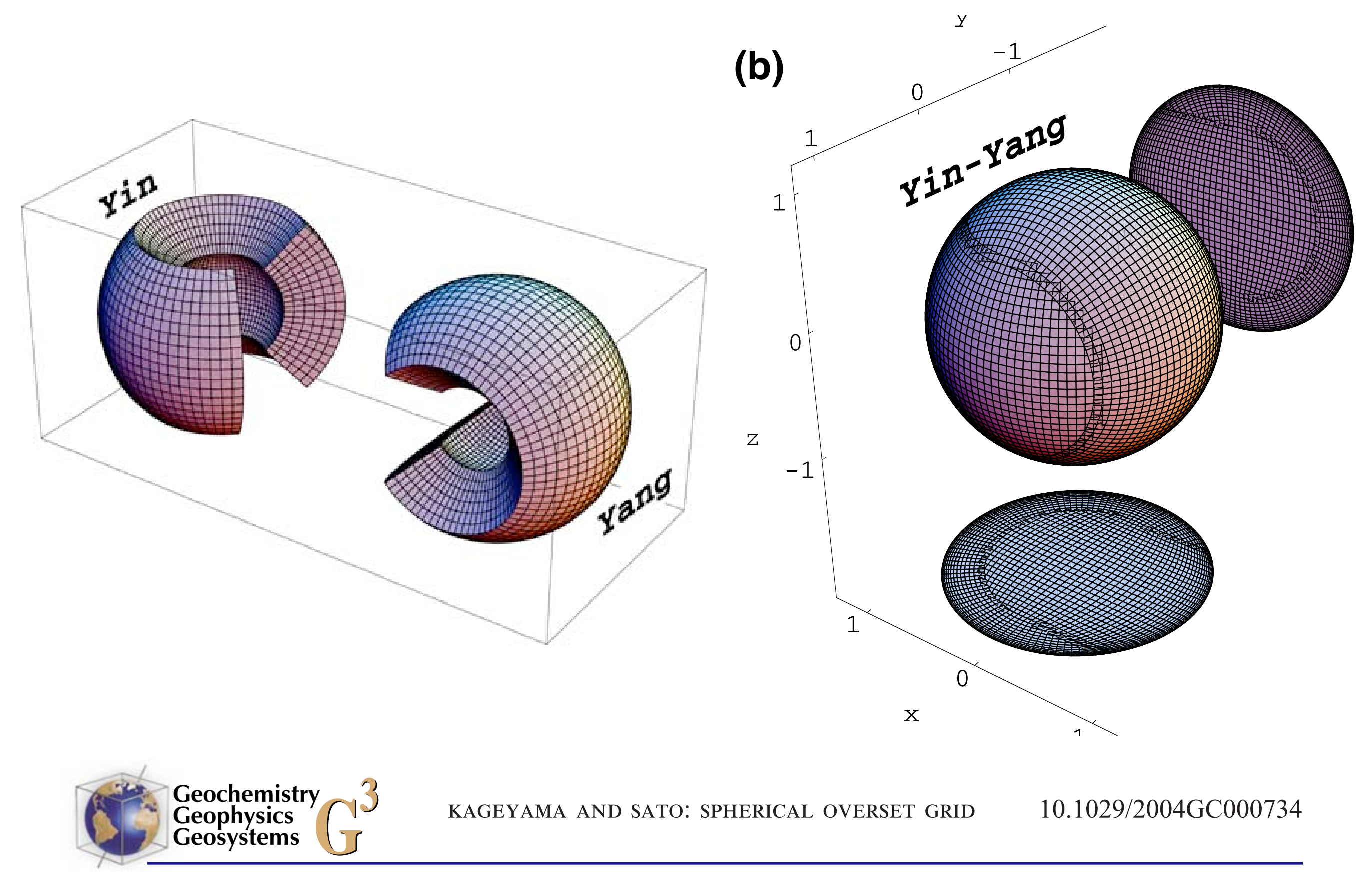
Some Existing Computational approaches



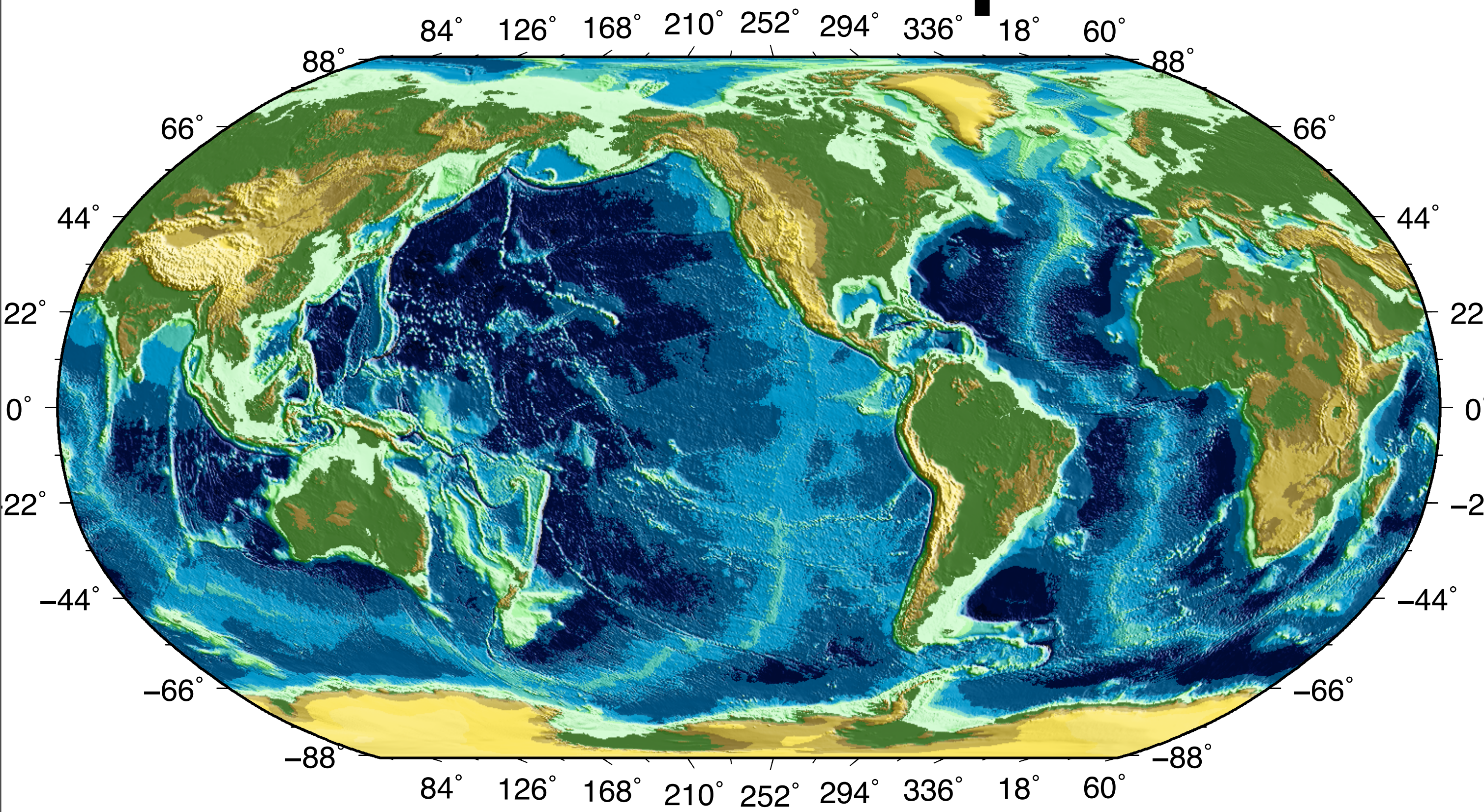
Tackley, P. J., Phys. Earth Planet. Inter, 171 (1-4), 7-18, doi: 10.1016/j.pepi.2008.08.005. PDF

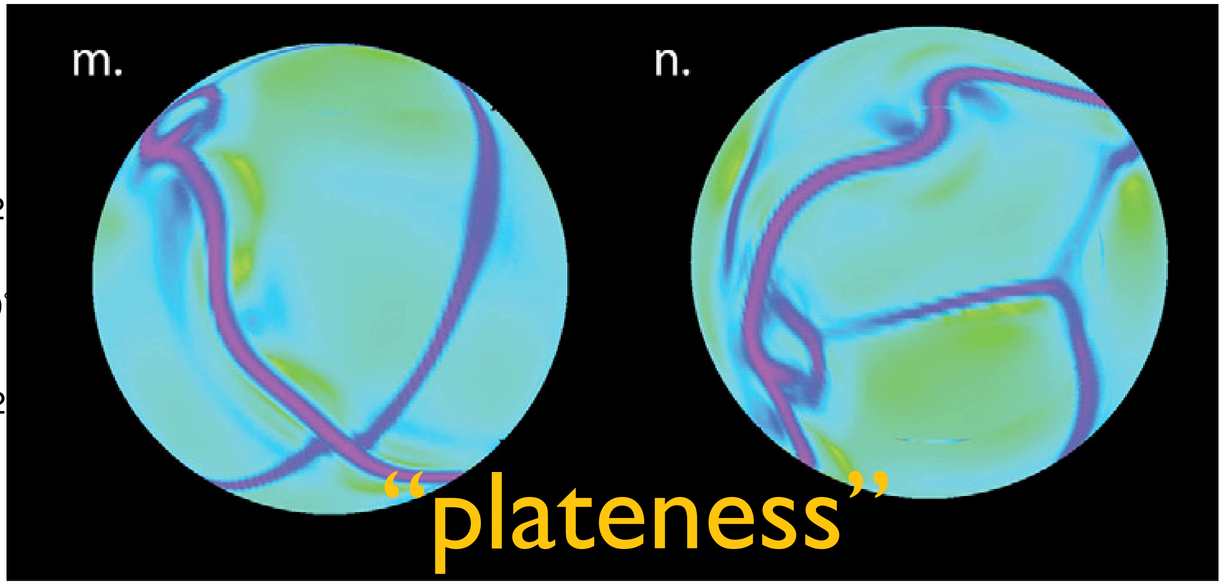
Finite Volume: STAG, STAGYY

- Paul Tackley, ETH
- Staggered Mesh, cartesian, "yin-yang" spherical
- 2nd order Geometric MG Stokes solver (custom) based on SIMPLER style projection
- MPDATA - corrected upwind advection scheme
- Proprietary research code



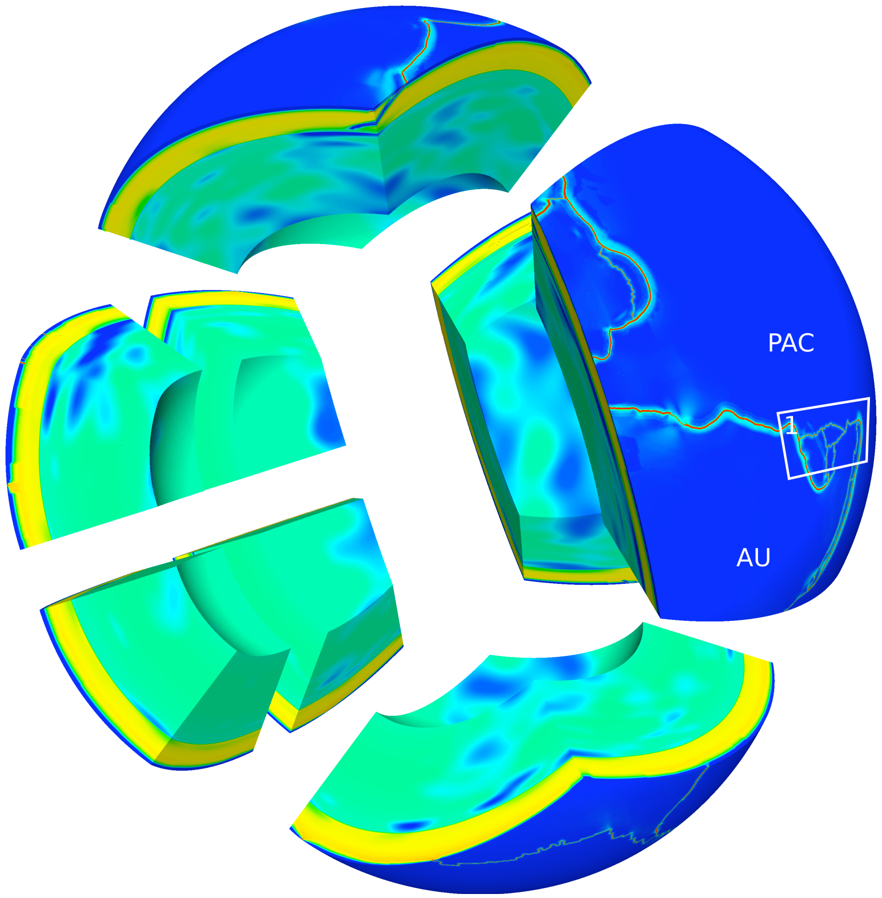
Computational Issues

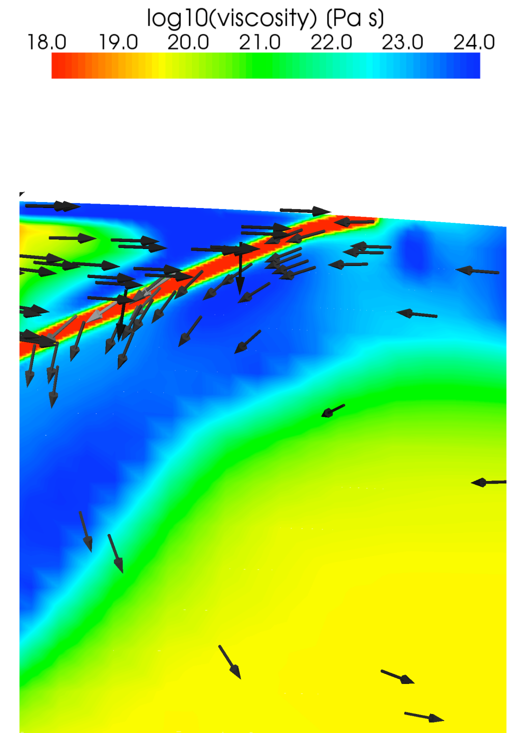


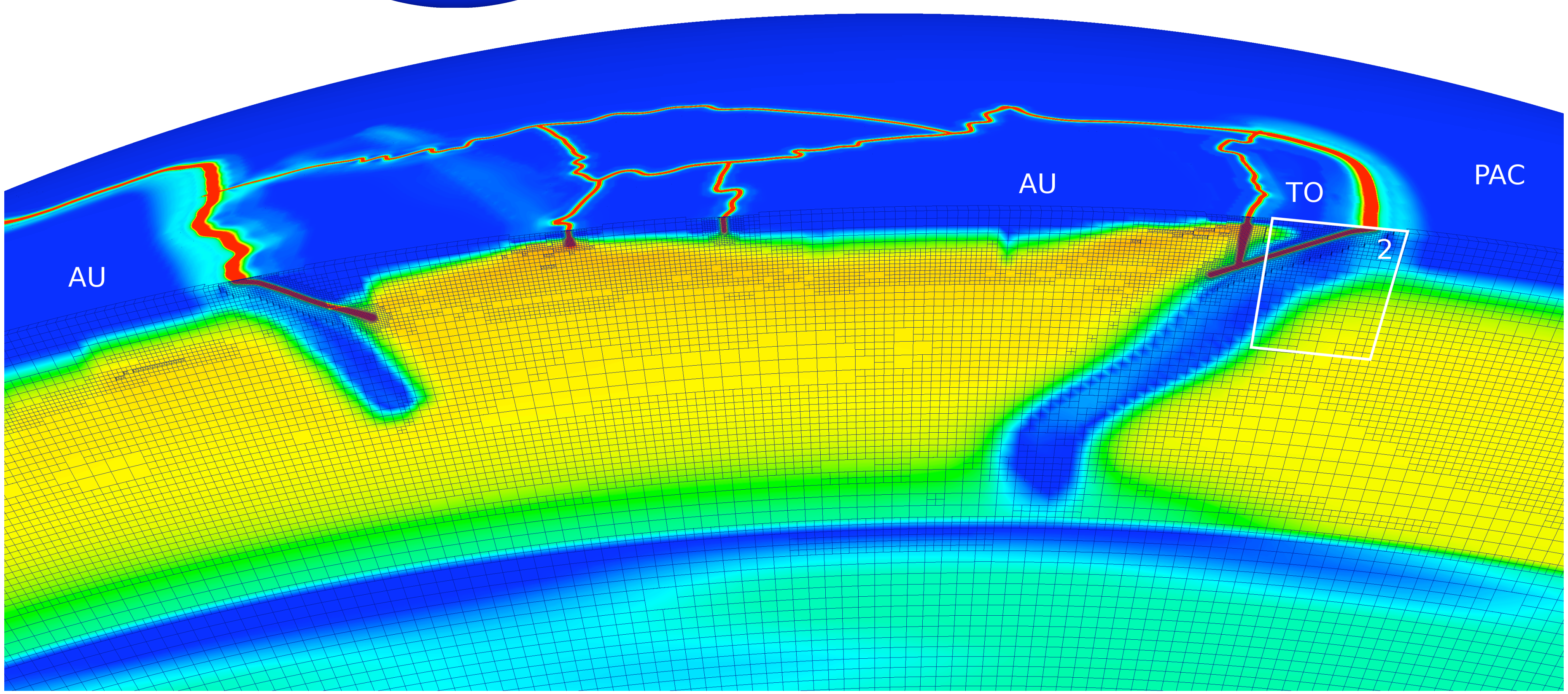


- 3-D Multi-scale elliptic problem
  - Plate boundaries are narrow-weak zones ~1km
  - But elliptic nature of flow field says global flow is sensitive to small scale weak features. (rigid vs. broken lid e.g.)

Petascale AMR FEM/Rhea







Global Convection code with parallel adaptive mesh refinement

- minimum mesh spacing ~1km resolves weak boundaries
- Adaptive refinement in weak/plastic regions
- Full refinement at h=1km ~ 10^{12} elements (exa-scale?)
- Can accomplish, goal oriented adaptation to convergence with 150-300 million elements (10^3-10^4) savings

The Gang from UT Austin

Carsten Burstedde

Georg Stadler

Lucas Wilcox

Some References: (from http://users.ices.utexas.edu/~carsten/)

•Carsten Burstedde, Lucas C. Wilcox, and Omar Ghattas, p4est: Scalable Algorithms for Parallel Adaptive Mesh Refinement on Forests of Octrees. Submitted to SIAM Journal on Scientific Computing (download revised preprint).

•Wolfgang Bangerth, Carsten Burstedde, Timo Heister, and Martin Kronbichler, Algorithms and Data Structures for Massively Parallel Generic Adaptive Finite Element Codes. Submitted to ACM Transactions on Mathematical Software (download preprint).

•Carsten Burstedde, Omar Ghattas, Michael Gurnis, Tobin Isaac, Georg Stadler, Tim Warburton, and Lucas C. Wilcox, Extreme-Scale AMR. Published in ACM/IEEE SC Conference Series, 2010 (download). Finalist paper for the Gordon Bell Prize 2010.

•Georg Stadler, Michael Gurnis, Carsten Burstedde, Lucas C. Wilcox, Laura Alisic, and Omar Ghattas, The Dynamics of Plate Tectonics and Mantle Flow: From Local to Global Scales. Published in Science 329 No. 5995 (August 27, 2010), pages 1033-1038 (doi: 10.1126/science.1191223, link, download, cover page, university newspaper).

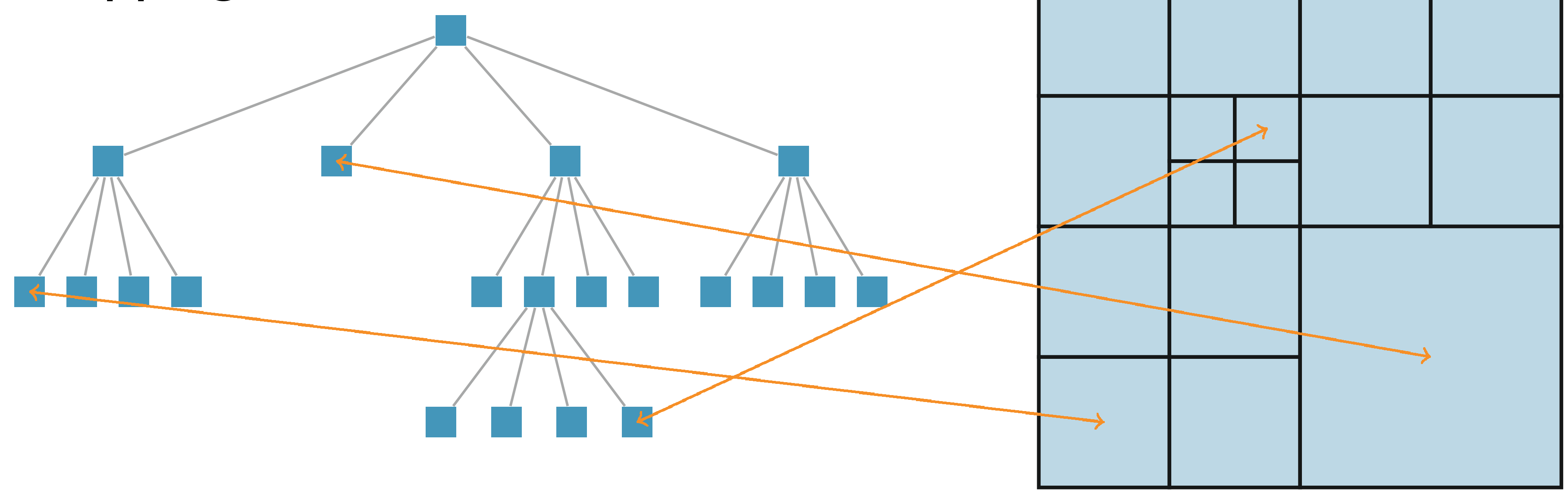
•Carsten Burstedde, Omar Ghattas, Georg Stadler, Tiankai Tu, and Lucas C. Wilcox, Parallel scalable adjoint-based adaptive solution for variable-viscosity Stokes flows. Published in Computer Methods in Applied Mechanics and Engineering 198 No. 21-26 (2009), pages 1691-1700 (doi: 10.1016/j.cma.2008.12.015, download preprint).

Extreme-Scale AMR for mantle convection components

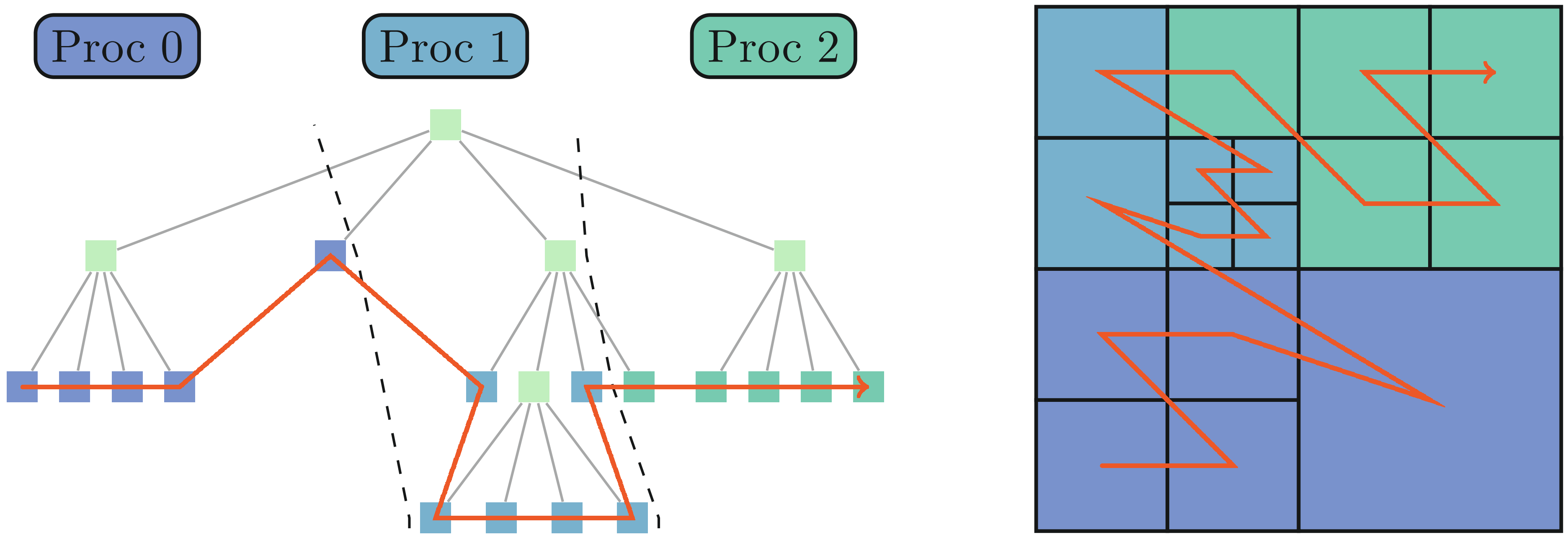
- p4est: Scalable mesh structure for forest of octree meshes
- mangll: general high order Element library for p4est meshes
- Massively parallel iterative solver for variable viscosity Stokes

Semi-structured parallel octree meshes (here quad-tree's for illustration)

mapping between tree and mesh



C. Burstedde et al. / Comput. Methods Appl. Mech. Engrg. 198 (2009) 1691–1700

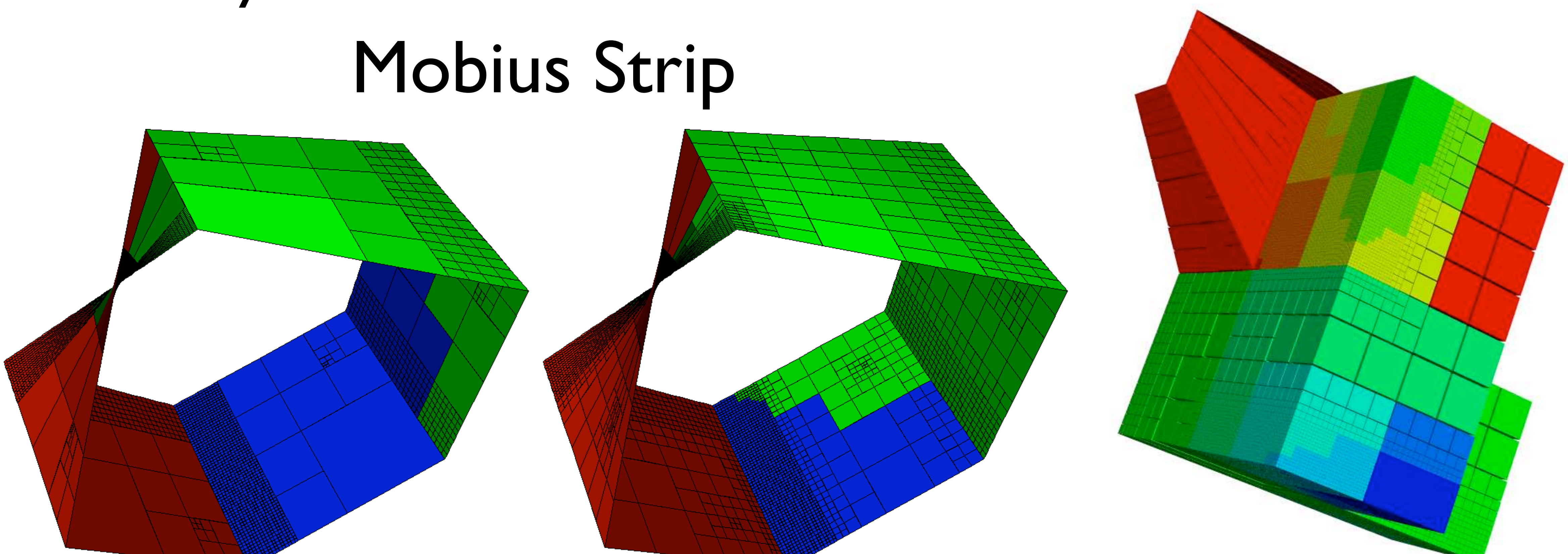


Leaf traversal yields unique ordering of elements through space filling Morton z-curve.
parallel partitioning/load balancing requires global array with 1 int / core

Semi-structured parallel *forest of octrees* meshes

- Octree meshes are easy to refine, but limited to domains that are topologically cubes.
- Forest of Octree's are unions of octree's which map to any arbitrary hexahedral mesh

Mobius Strip



The Borg ship?

FIG. 4.1. *Examples of forest-of-octree configurations where color encodes the process number. Left: 2D forest of five octrees that realize the periodic Möbius strip, here shown after initial calls to* `New` *and* `Refine`*. Middle: the same forest after* `Balance` *and* `Partition`*. Right: 3D forest composed of six cubes whose orientations are rotated against each other, with five octrees connecting through the horizontal central axis, after calls to* `New`, `Refine`, `Balance` *and* `Partition`.

Semi-structured parallel *forest of octrees* meshes

mapping between forest and mesh. 2 quadtrees

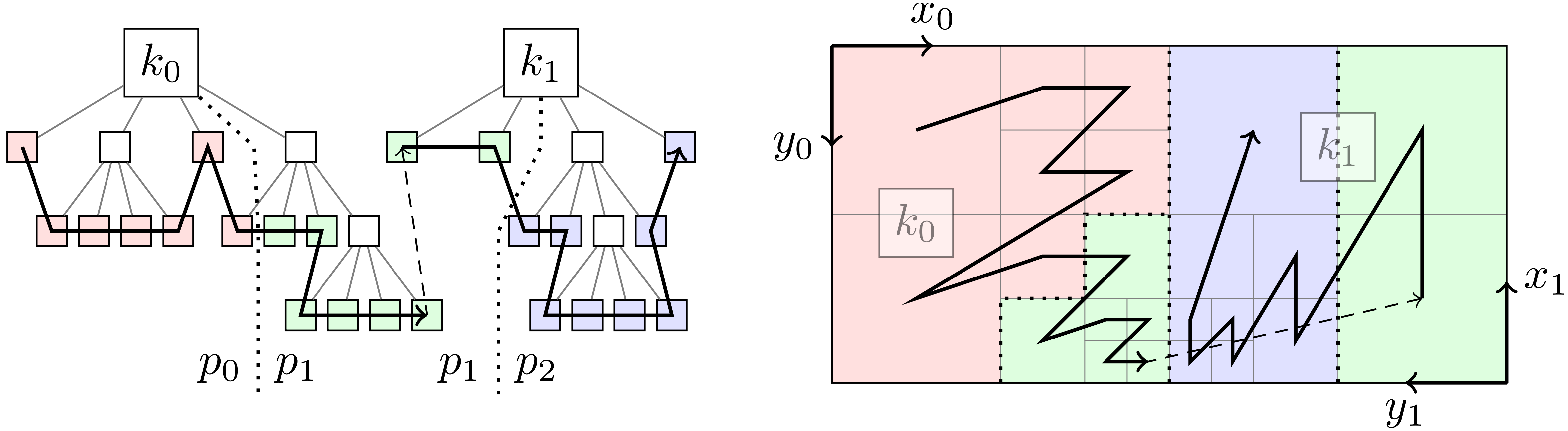


FIG. 2.1. *One-to-one correspondence between a forest of octrees (left) and a geometric domain partitioned into elements (right), shown for a 2D example with two octrees k_0 and k_1. The leaves of the octrees bijectively correspond to elements that cover the domain with neither holes nor overlaps. A left-to-right traversal of the leaves through all octrees creates a space-filling z-curve (black "zig-zag" line) that imposes a total ordering of all octants in the domain. For each octree the z-curve follows the orientation of its coordinate axes. In this example the forest is partitioned among three processes p_0, p_1 and p_2 by using the uniform partitioning rule (2.5). This partition divides the space-filling curve and thus the geometric domain into three process segments of equal (± 1) octant count.*

p4est Library

http://www.p4est.org/, GPL License

`New` Create an equi-partitioned, uniformly refined forest.

`Refine` Adaptively subdivide octants based on a refinement marker or callback function, once or recursively.

`Coarsen` Replace families of eight child octants by their common parent octant, once or recursively.

`Partition` Redistribute the octants in parallel, according to a given target number of octants for each process, or weights prescribed for all octants.

`Balance` Ensure at most 2:1 size relations between neighboring octants by local refinement where necessary.

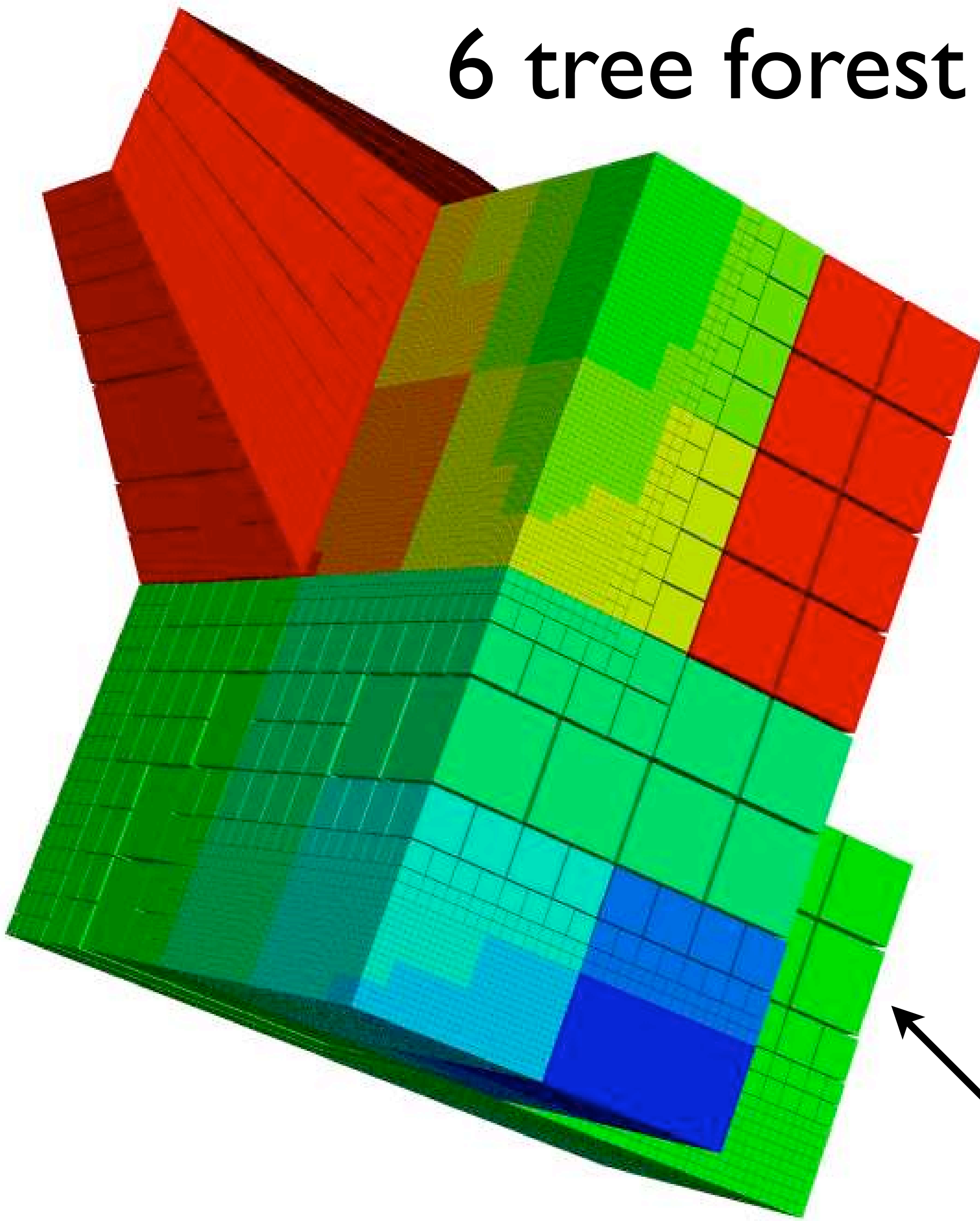
`Ghost` Collect one layer of off-process octants touching the process boundaries from the outside.

`Nodes` Create a globally unique numbering of the mesh nodes (i.e., the vertices at the corners of octants, not to be confused with octree nodes), taking into account the classification into "independent" and "hanging" nodes.

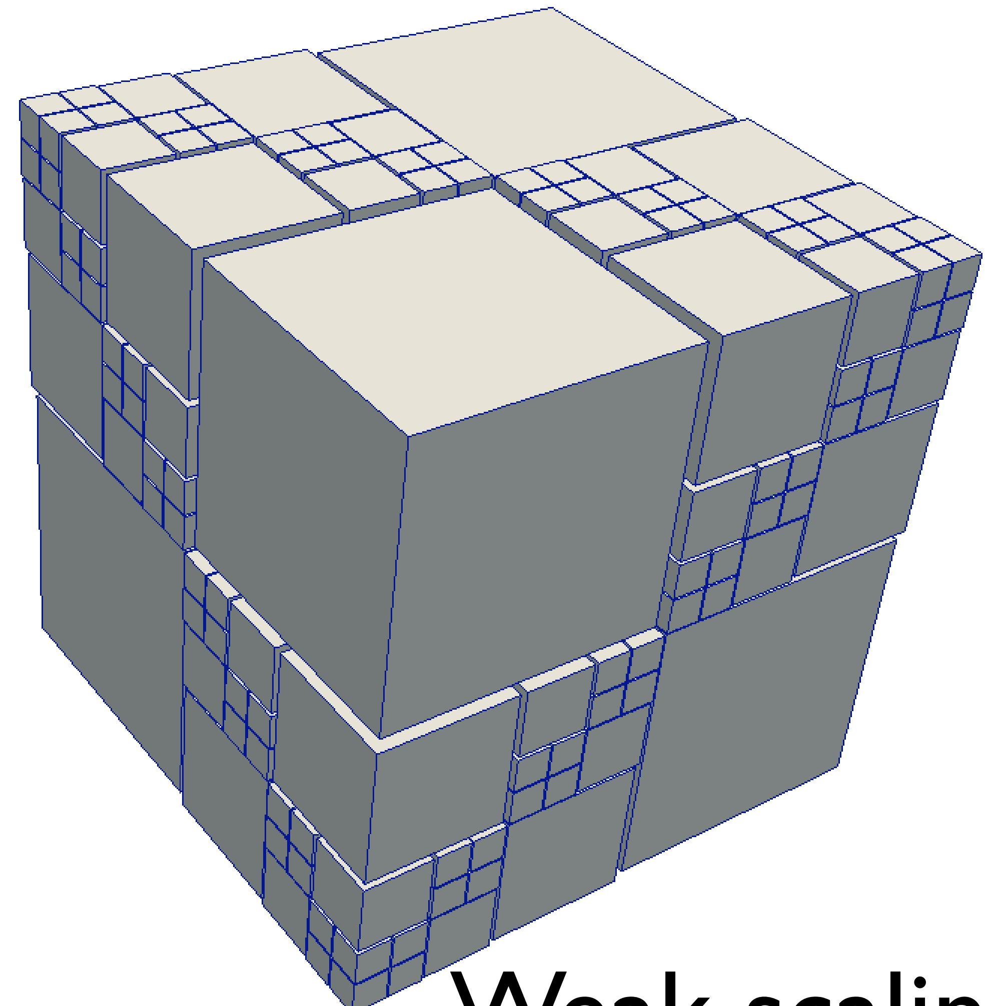
`Checksum` Compute a partition-independent integer "fingerprint" of a forest.

p4est Library
performance: weak scaling of pure mesh refinement

6 tree forest



4 level refinement



Weak scaling increases level by 1 and Nproc by 8

Recursive fractal refinement of using

p4est Library performance

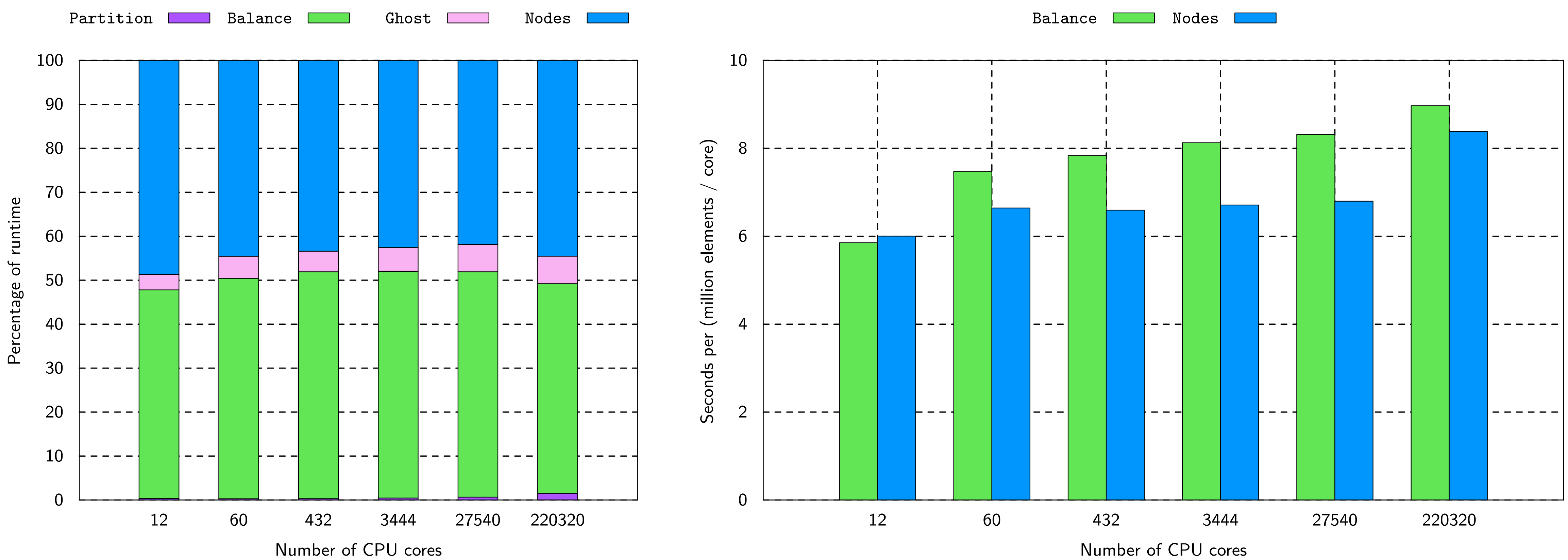


FIG. 4.2. *"Weak" scaling results up to 220,320 processes on* Jaguar. *The refinement is defined by choosing the same six-cube 3D connectivity as used on the right hand side of Figure 4.1, and recursively subdividing octants with child identifiers 0, 3, 5 and 6 while not exceeding four levels of size difference in the forest. This leads to a fractal mesh structure. To scale from 12 to 220,320 processes the maximum refinement level is incremented by one while the number of processes is multiplied by 8. Left: runtime is dominated by* `Balance` *and* `Nodes` *while* `Partition` *and* `Ghost` *together take up less than 10% (*`New` *and* `Refine` *are negligible and not shown). Right: performance assessed by normalizing the time spent in the* `Balance` *and* `Nodes` *algorithms by the number of octants per process which is held constant at approximately 2.3 million (ideal scaling would result in bars of constant height.) The largest mesh created contains over* 5.13×10^{11} *octants and is* `Balance`*'d in 21 seconds.*

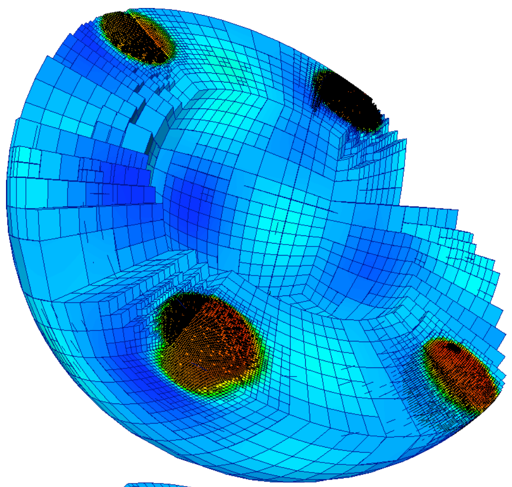
p4est Library performance
lightweight pure DG advection problem

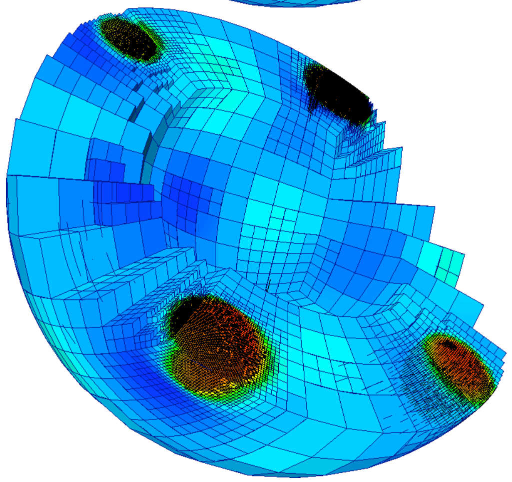
Solve

$$\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = 0$$

Using

- 3rd order Spectral DG elements (mangll) (diagonal mass matrix)
- Upwind nodal DG advection in space
- 5 stage 4th-order Runge-Kutta method in time
- On 24 octree spherical forest

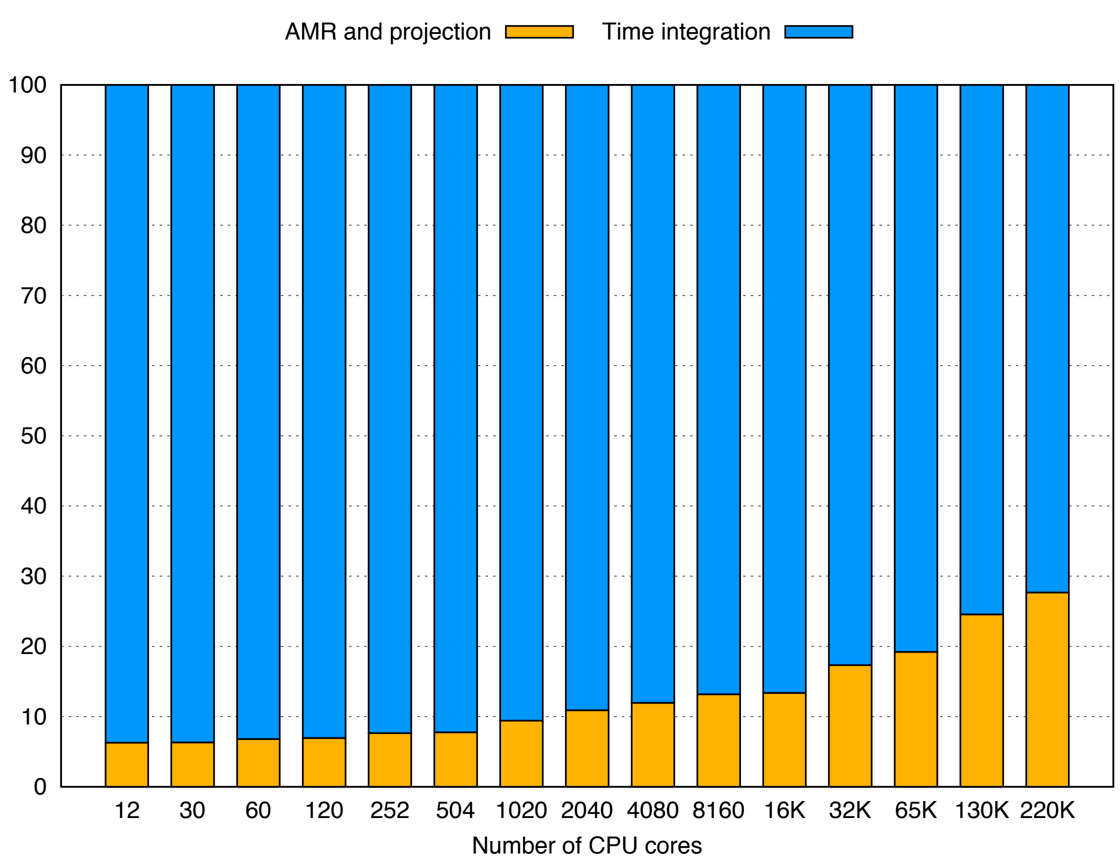




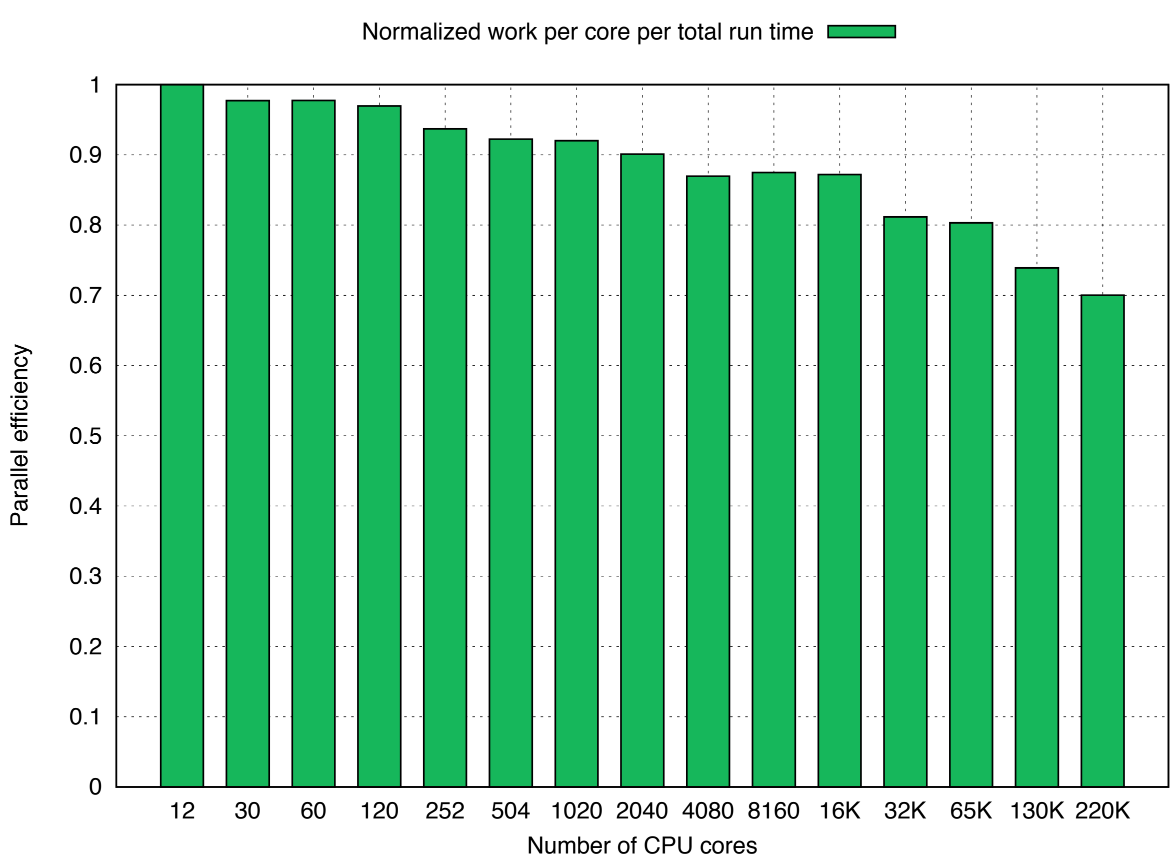
Two subsequent time steps
showing advected spherical inclusions

p4est Library performance
pure advection, weak scaling

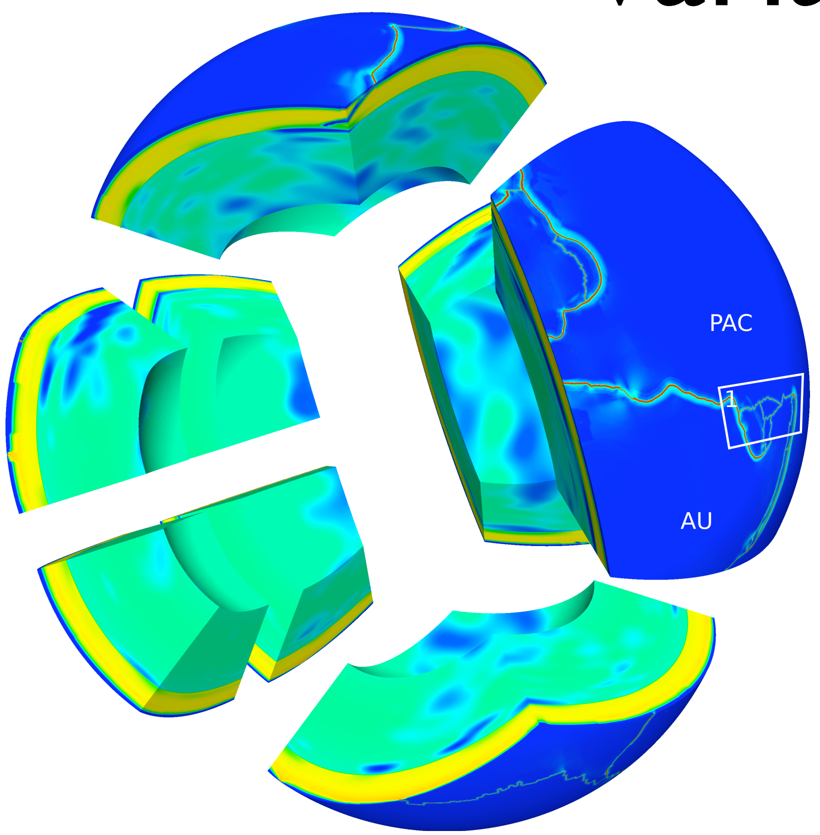
Fraction of time in AMR

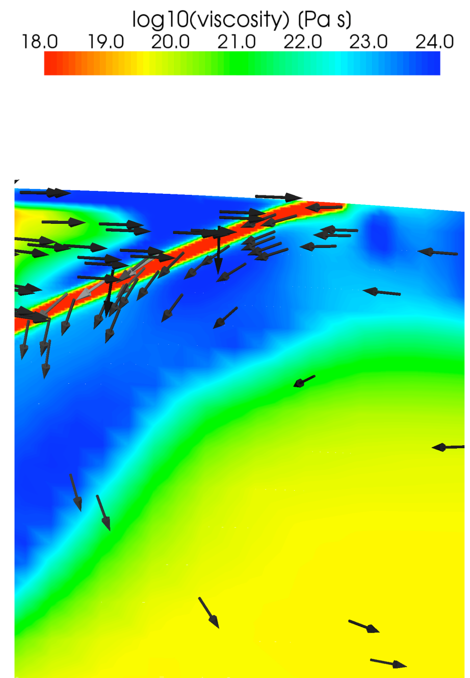


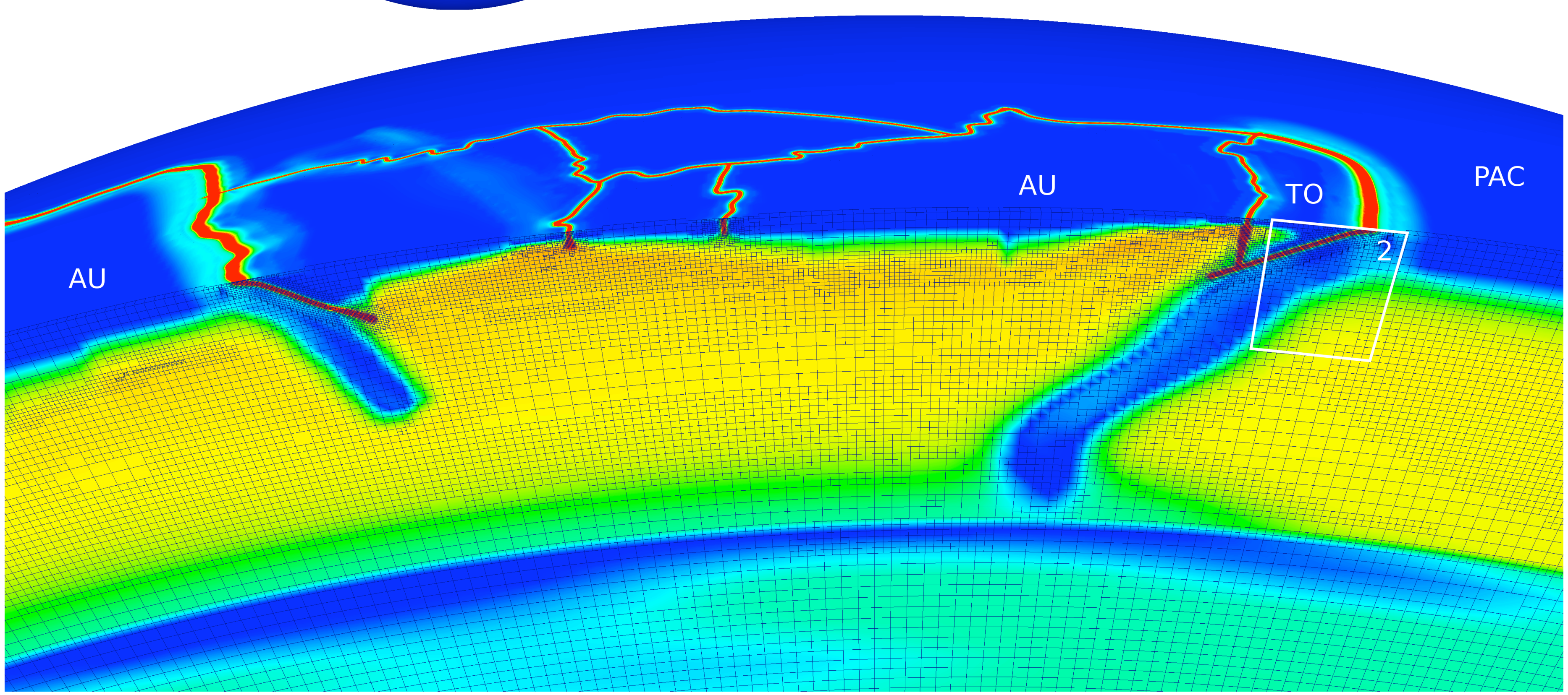
Parallel Efficiency



p4est Library performance
Variable Viscosity stokes



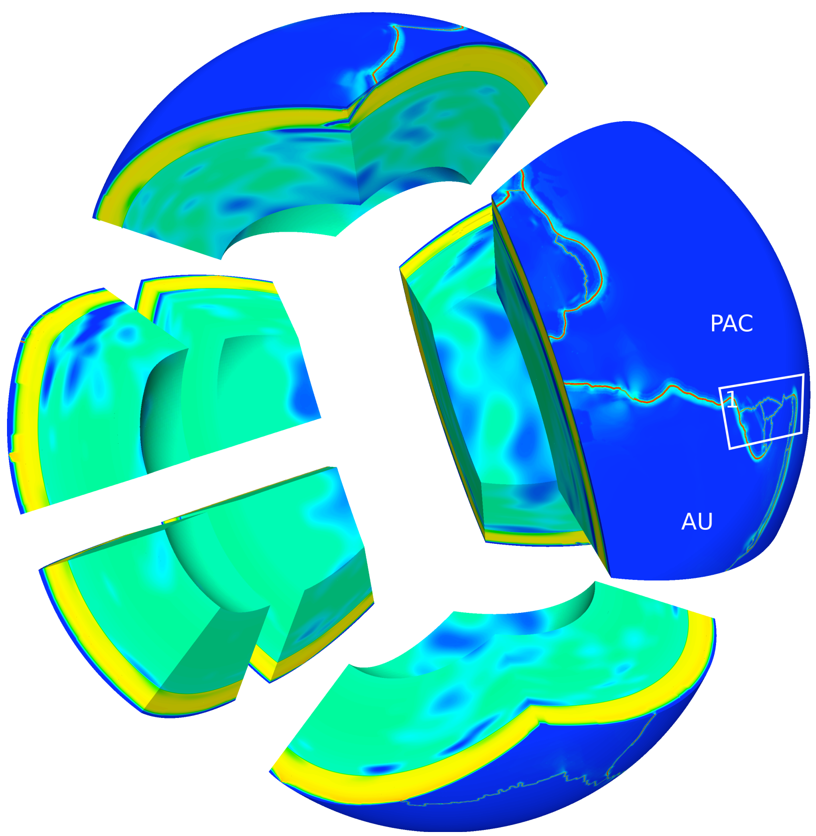


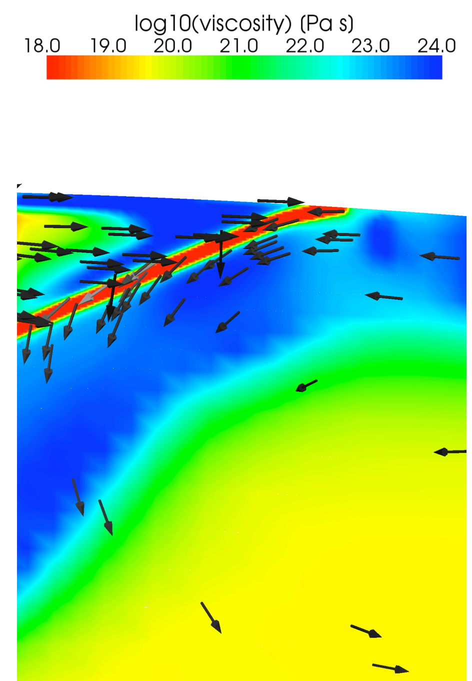


Single Stokes solve

- 24 octree forest on cubed sphere
- Q1-Q1 stabilized trilinear elements
- Imposed Temperature and Viscosity field
- Block Preconditioned MINRES Krylov solver
- AMR contributes < 0.12% of total run time

p4est Library performance
Variable Viscosity stokes





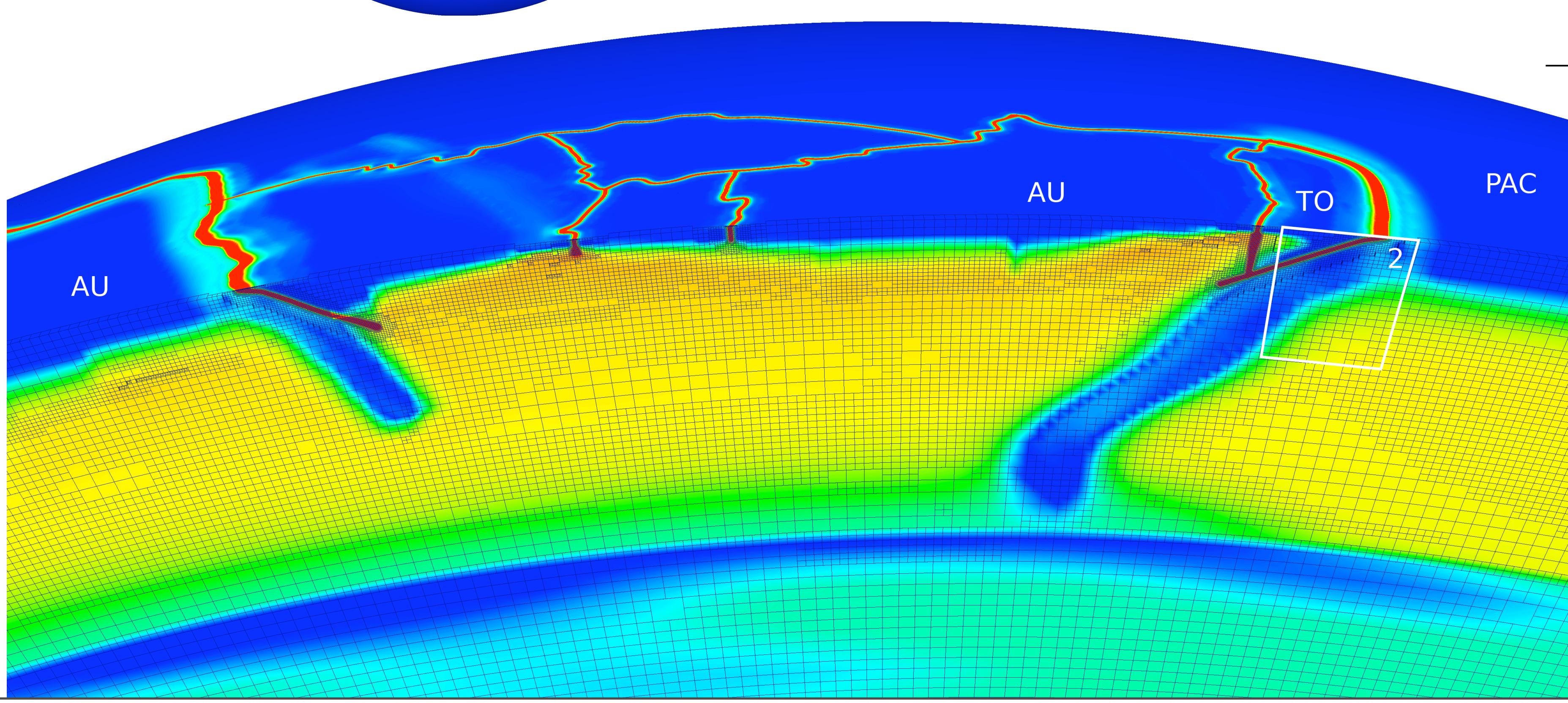
Actual timing

Statdler et al., Science, 2010 (supplement)
8000 Cores of Ranger (TACC)
"Typical Run"

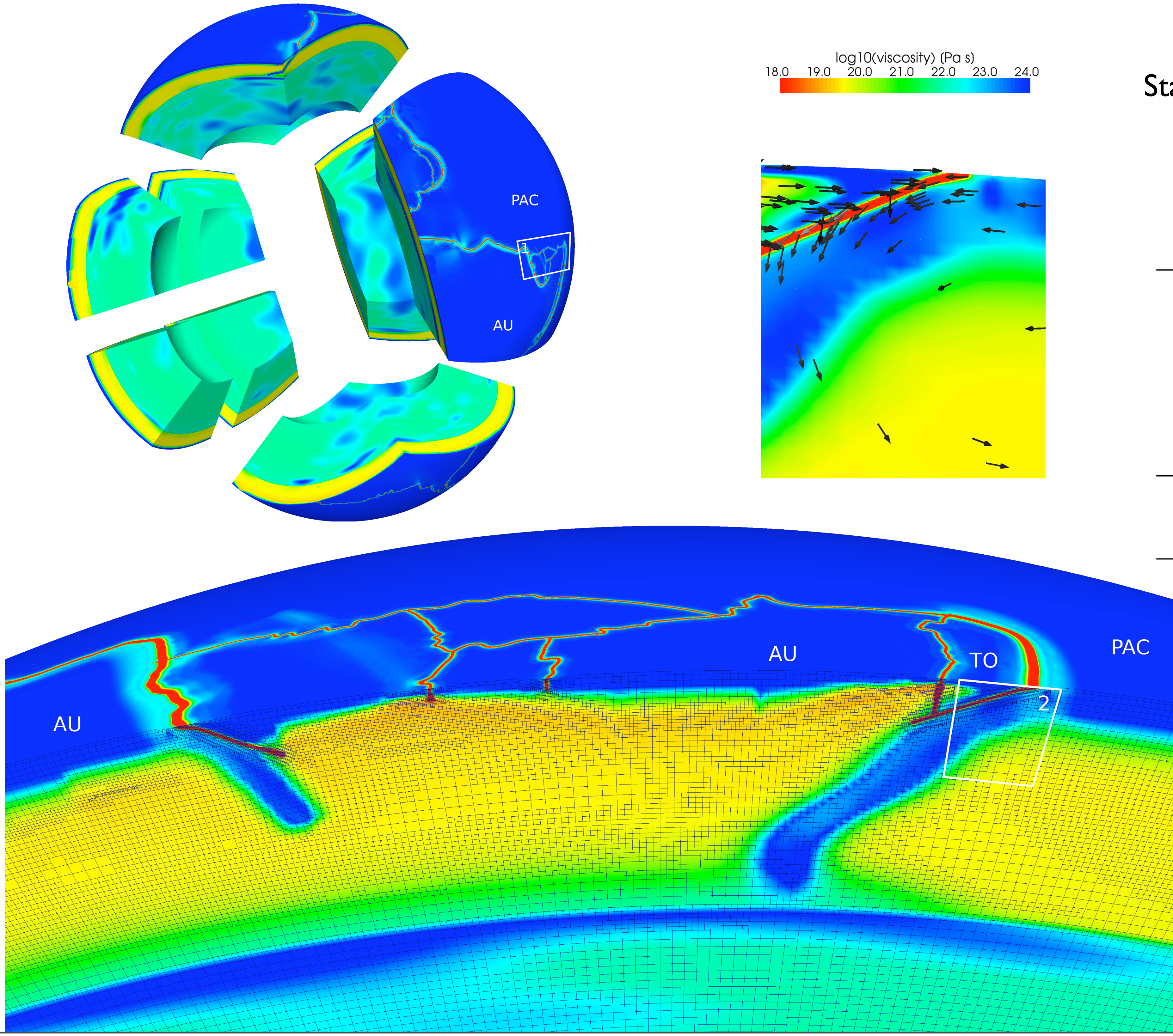
B

AMR algorithm	time (s)
Node Numbering	8.12
Mesh Partition	2.18
2:1 Balance	5.27
Refine/Coarsen	0.32
Error Indicator	1.52
overall AMR	17.41
overall solve	32,613.25
percentage AMR/solve	0.05%

D



p4est Library performance
Variable Viscosity stokes



Actual timing

Statdler et al., Science, 2010 (supplement)
8000 Cores of Ranger (TACC)
“Typical Run”

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D

Rhea Results
Stadtler et al, Science, 2010

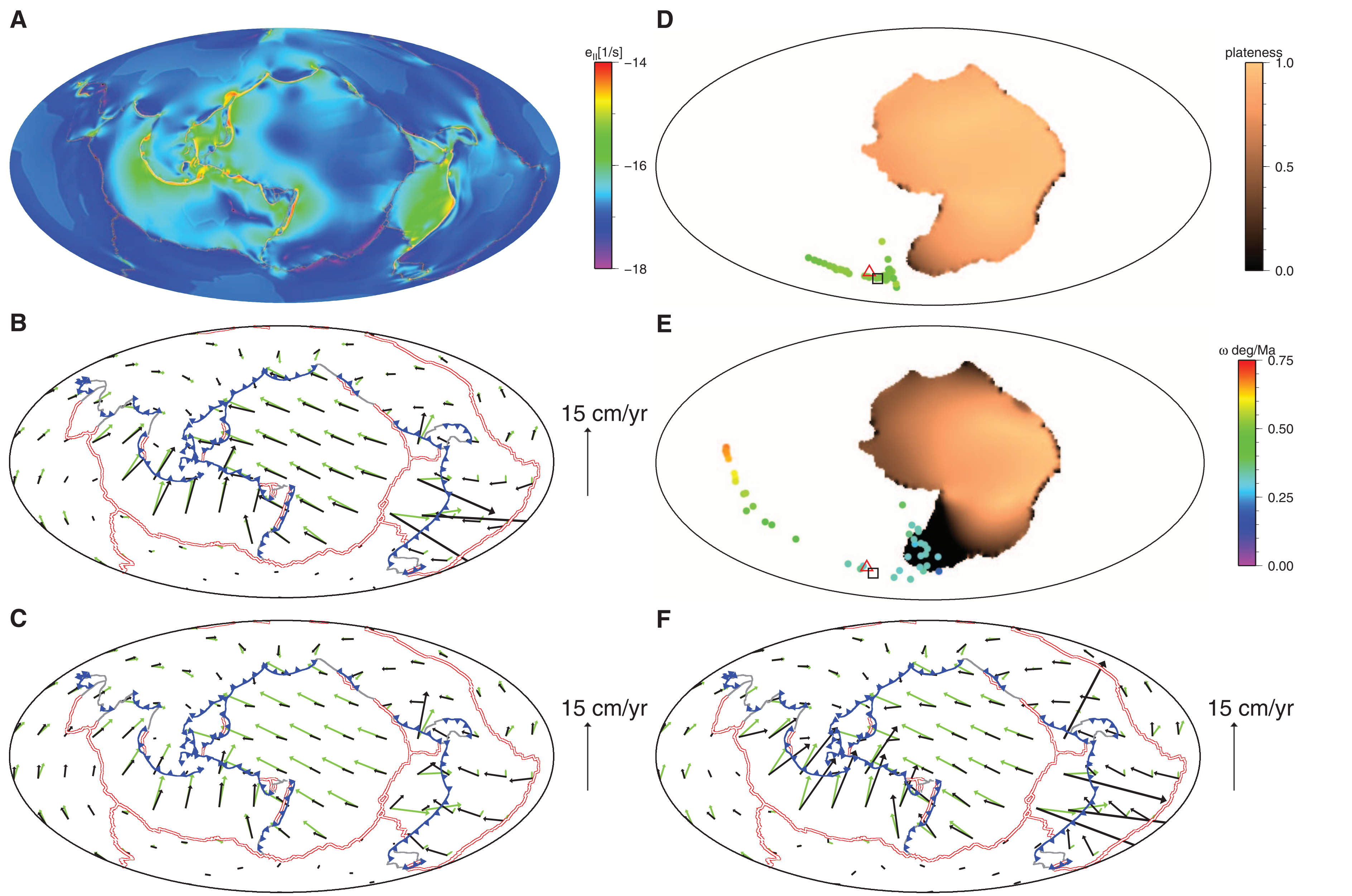


Fig. 2. Strain rate, plate velocities, and plateness for three cases centered at 180°W. (**A**, **B** and **D**) Case 1, with only plate cooling and upper mantle slabs. (**C**) Case 2, identical to case 1 except for lower-mantle lateral structure. (**E** and **F**) Case 4, similar to case 2, except that $n = 3.5$. (A) Second invariant of strain rate. (B), (C), and (F) Plate motions in a NNR from (*27*) as green arrows and predicted velocities as black arrows; actual plate margins are shown as red, gray, and blue symbols. (D) and (E) Plateness for PAC shown in two ways: vector difference between computed velocity and velocity from best-fitting Euler pole, P_2 (*22*), as a raster field with color palette shown to the right of (D); and individually inferred Euler poles within spherical caps (radius 20°) with magnitude of rotation (ω) denoted with color of pole [palette shown to the right of (E)]. The Nuvel1-NNR pole position is shown as a red triangle and best-fitting pole for all computed velocities within PAC as a black square.

Saturday, January 8, 2011

Parallel AMR for seismic wave Propagation dGea

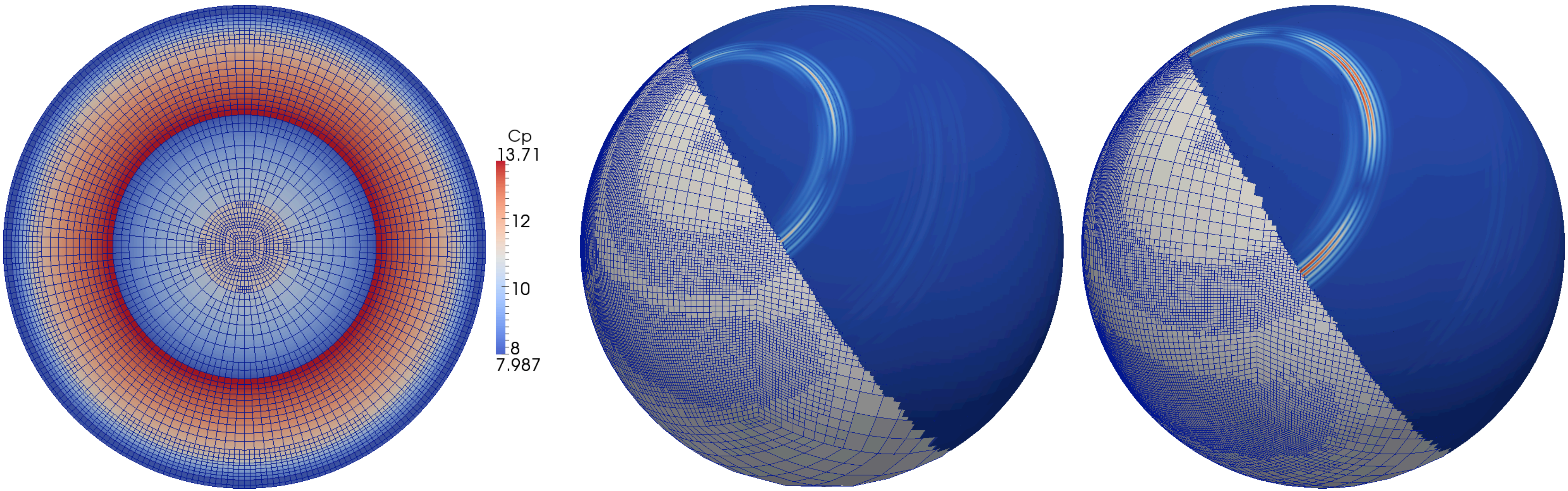
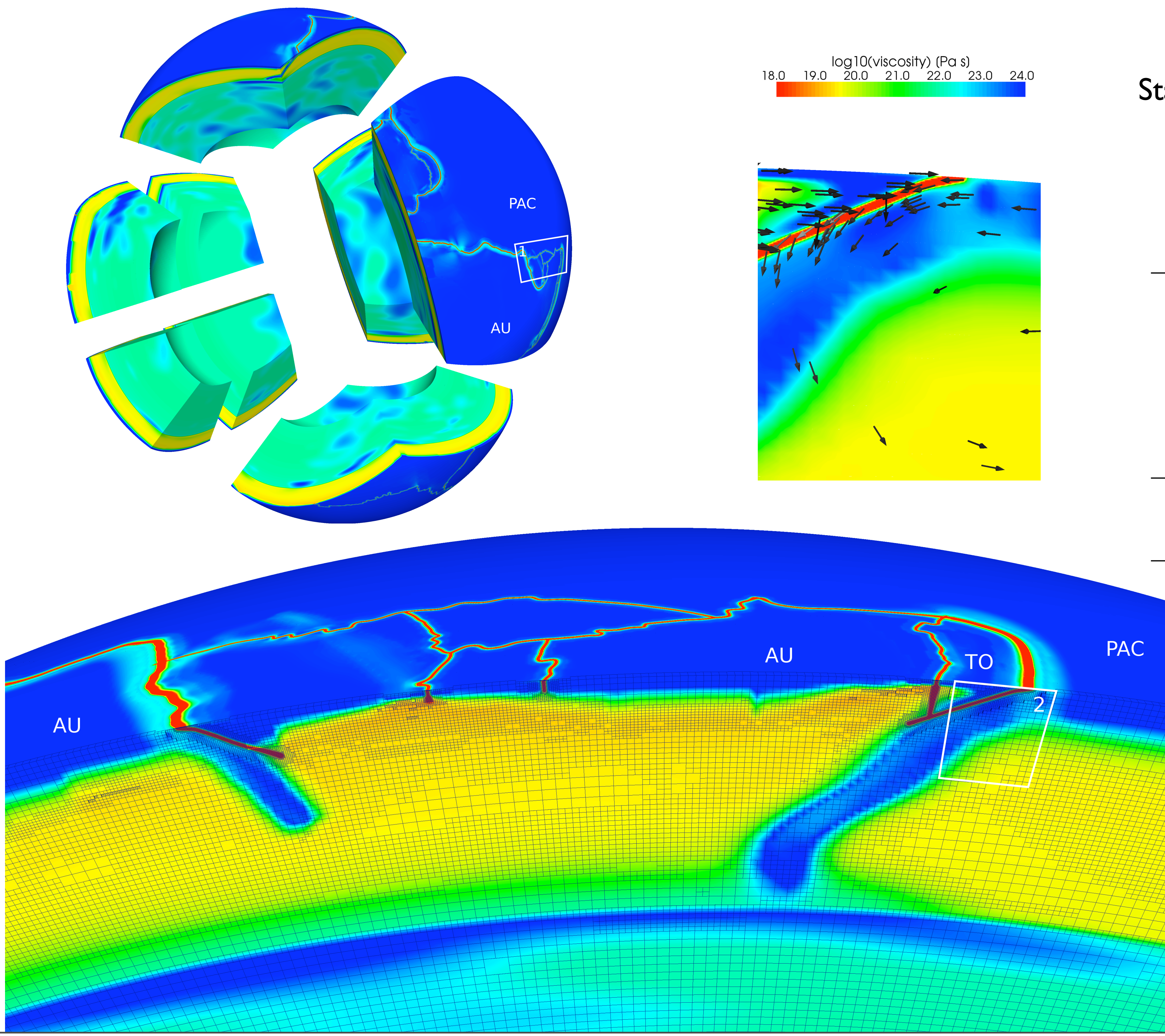


Fig. 8. Left: Section through mesh that has been adapted lcoally according to the size of spatially-variable wavelengths; low frequency used for illustrative purposes. The color scale corresponds to the primary wave speed in km/s. The mesh aligns with discontinuities in wave speed present in the PREM (Preliminary Reference Earth Model) model used [44]. Middle and right: Two snapshots of waves propagating from an earthquake source; the mesh is adapted dynamically to track propagating wavefronts.

p4est Library performance
Variable Viscosity stokes



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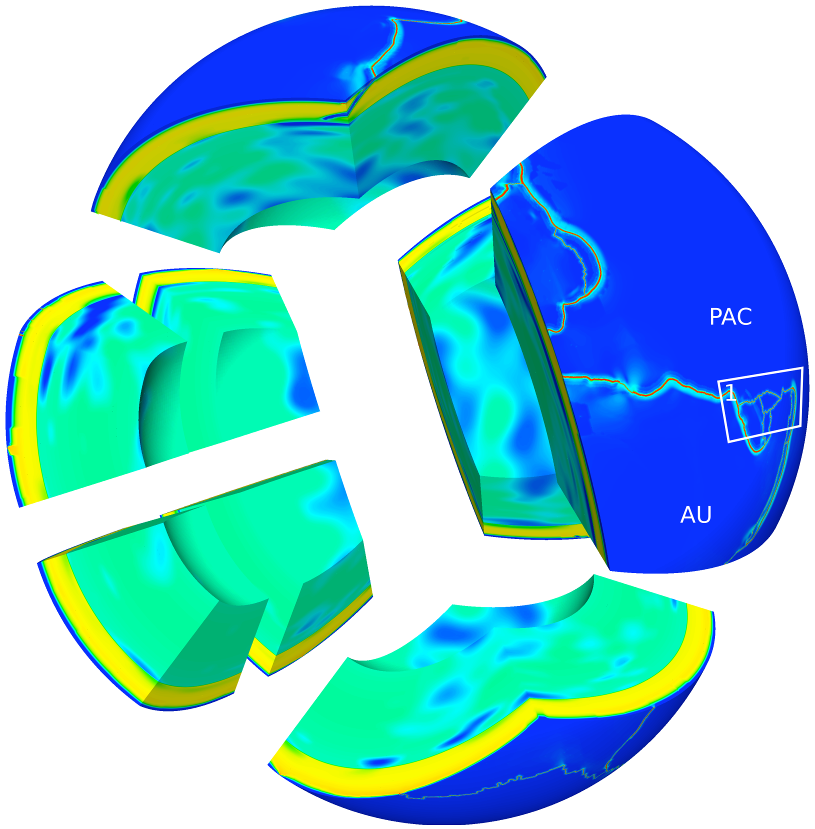
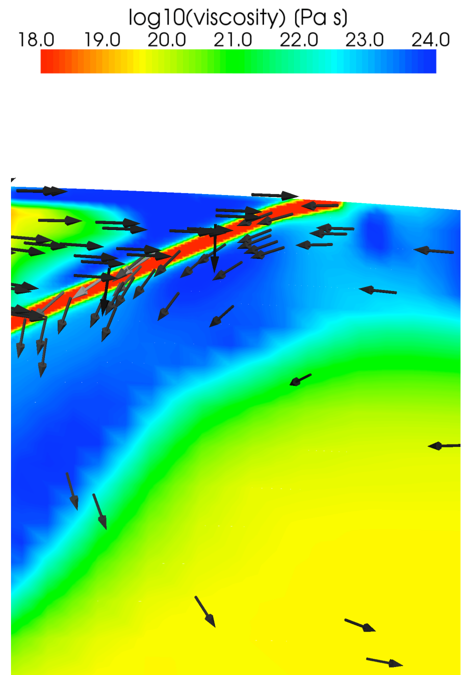
p4est Library performance
Variable Viscosity stokes





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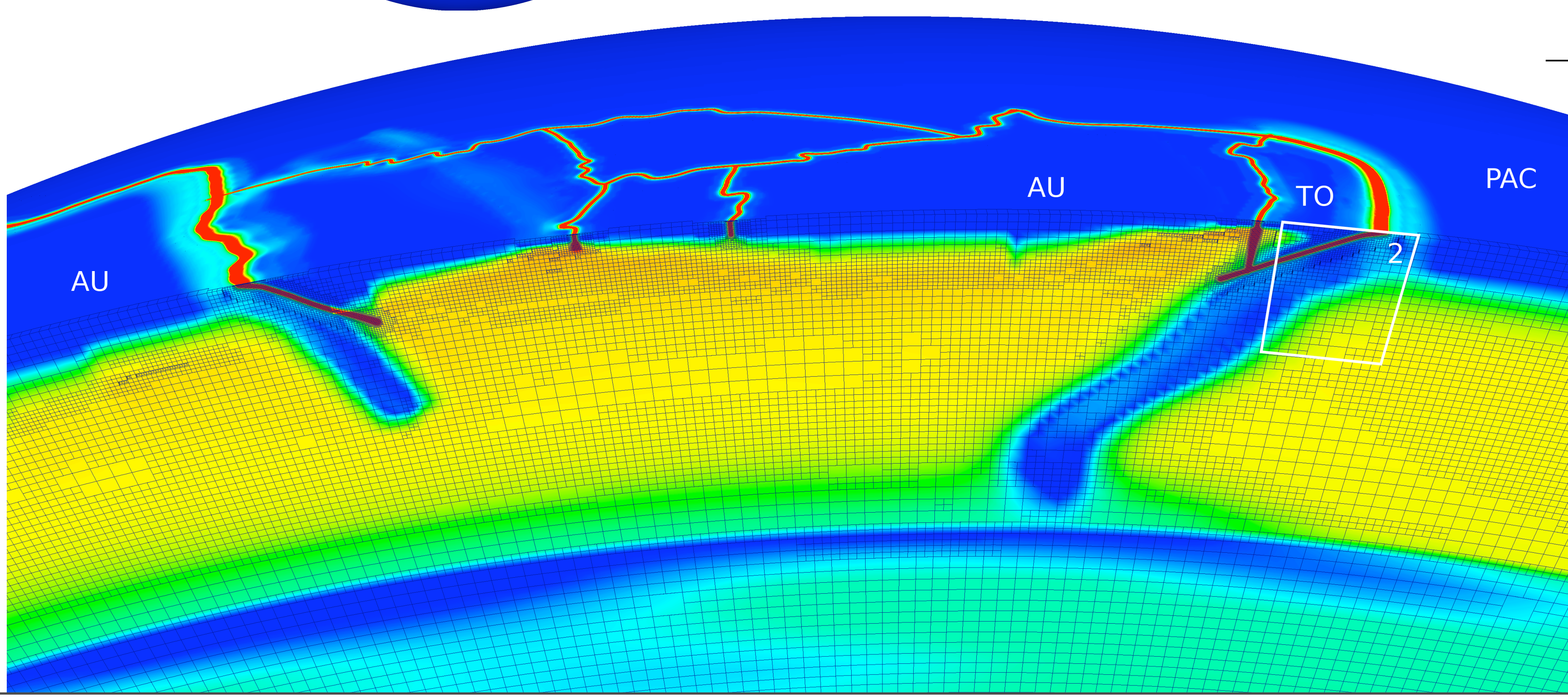

Variable Viscosity Stokes

the central problem in Solid Earth Geodynamics

Physics of the Earth and Planetary Interiors 171 (2008) 33–47

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Preconditioned iterative methods for Stokes flow problems arising in computational geodynamics

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Comput. Methods Appl. Mech. Engrg. 198 (2009) 1691–1700

Contents lists available at ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Parallel scalable adjoint-based adaptive solution of variable-viscosity Stokes flow problems

Carsten Burstedde[a], Omar Ghattas[a,b,*], Georg Stadler[a], Tiankai Tu[a], Lucas C. Wilcox[a]

[a] *Institute for Computational Engineering & Sciences (ICES), The University of Texas at Austin, Austin, TX 78712, USA*
[b] *Jackson School of Geosciences and Department of Mechanical Engineering, The University of Texas at Austin, Austin, TX 78712, USA*

Variable Viscosity Stokes

the central problem in Solid Earth Geodynamics

Weak Form

$$\begin{array}{rcl} \int_\Omega \eta(T, V^*)\nabla\mathbf{u} : \nabla\mathbf{v} dV \quad + \int_\Omega p\nabla\cdot\mathbf{u} dV & = & \int_\Omega \mathbf{u}\cdot T\mathbf{g} dV \\ \int_\Omega q\nabla\cdot\mathbf{v} dV & = & 0 \end{array}$$

Which for a stable mixed element (e.g. Q2-Q1, Taylor Hood) assembles to

Discrete Saddle-Point System

$$\begin{bmatrix} A(T,\mathbf{v}^*) & G \\ G^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}$$

Variable Viscosity Stokes
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The operator can be block factorized as

Factorization

$$\begin{bmatrix} A & G \\ G^T & -C \end{bmatrix} = \begin{bmatrix} I & 0 \\ -G^T A^{-1} & I \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} I & -G^T A^{-1} \\ 0 & I \end{bmatrix}$$

where

$$S = -(G^T A^{-1} G + C)$$

is the Schur Complement

Variable Viscosity Stokes

the central problem in Solid Earth Geodynamics

Block Diagonal Preconditioner

$$P = \begin{bmatrix} A & 0 \\ 0 & S \end{bmatrix}$$

or even more approximate preconditioner

$$P = \begin{bmatrix} \hat{A} & 0 \\ 0 & \mu^{-1} Q \end{bmatrix}$$

where

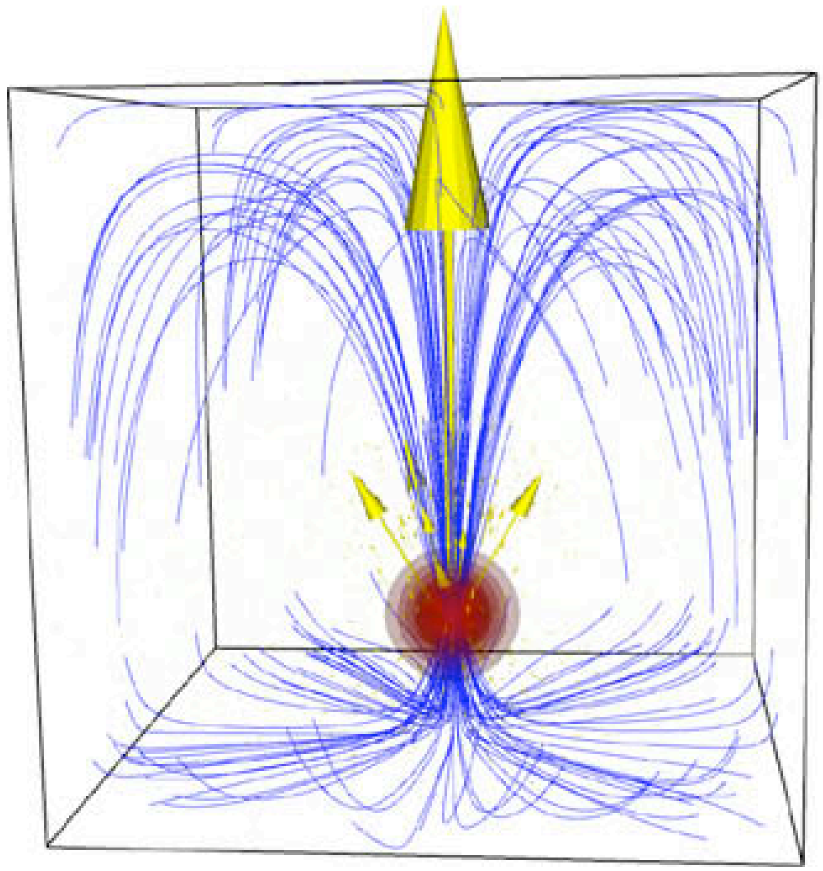
$$\hat{A} = \begin{bmatrix} L_1 & 0 & 0 \\ 0 & L_2 & 0 \\ 0 & 0 & L_3 \end{bmatrix} \quad L_i = \int_\Omega \mu \nabla u_i \cdot \nabla v_i dV$$

and Q is the pressure mass matrix.

All based on ideas nicely laid out for iso-viscous Stokes in H.C. Elman, D.J. Silvester, A.J. Wathen, *Finite Elements and Fast Iterative Solvers with Applications in Incompressible Fluid Dynamics*, Oxford University Press, Oxford, 2005.

Variable Viscosity Stokes
the central problem in Solid Earth Geodynamics

Note: if viscosity is constant, this PC can shown to be optimal (the problem is that viscosity is highly variable)



Test problem: Buoyancy driven flow from a Gaussian blob of excess Temperature

Variable Viscosity Stokes

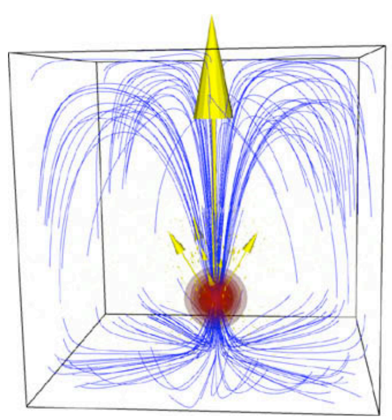
the central problem in Solid Earth Geodynamics

Timing: weak scaling on Ranger

# Cores	Solver time	Error estimate	Mark & refine	Extract mesh	Balance tree	Interp. & transfer	Partition tree	$\frac{\text{AMR time}}{\text{solve time}}$ (%)
1	345.6	1.78	0.08	2.05	0.12	0.13	0.00	1.2
8	374.8	2.29	0.22	3.38	0.27	0.16	1.77	2.2
64	497.6	2.66	0.36	6.21	1.00	0.22	2.51	2.6
512	696.5	2.89	0.84	9.64	2.05	0.43	3.26	2.8
4096	1095.8	3.04	1.41	10.44	2.39	0.64	10.92	2.6

Convergence: weak scaling on Ranger

# Cores	# Dofs	MINRES # iterations	AMG setup (s)	MINRES matvec (s)	AMG V-cycle (s)	η_A	η_I	η_V	η
1	403K	63	8.2	174.8	49.9	1.00	1.00	1.00	1.00
8	3.3M	66	14.8	215.2	78.1	0.95	0.85	0.67	0.76
64	26.8M	75	20.6	240.2	143.9	0.84	0.87	0.41	0.58
512	216M	90	28.4	295.4	222.2	0.70	0.85	0.32	0.43
4096	1.7B	106	50.2	349.5	378.2	0.59	0.84	0.22	0.34



- MINRES Iterations reasonably constant independent of system size
- Scaling of AMG (Hypre) degrades overall scalability

Variable Viscosity Stokes

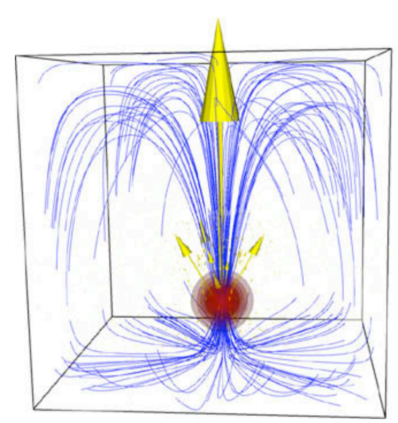
the central problem in Solid Earth Geodynamics

Performance with respect to viscosity variation

Table 4
Performance of Stokes solver for varying viscosity given by (14) for α and β as given in the table. As in Table 3 we use a mesh that has undergone three cycles of refinement, and examine only the final Stokes solve (which is initialized with a zero solution). The table reports the minimum and maximum viscosity values (μ_{min} and μ_{max}), the maximum viscosity gradient norm $\|\nabla\mu\|_{max}$, the number of MINRES iterations, the AMG setup time, and the average time per MINRES iteration. Each case has approximately 216M degrees of freedom and is solved on 512 cores.

α	β	μ_{min}	μ_{max}	$\|\nabla\mu\|_{max}$	# MINRES iterations	AMG setup time (s)	Solve time per iteration (s)
0	–	1.00e–0	1.00	0.00e+0	86	25.29	5.82
3	200	4.98e–2	1.00	2.05e+1	80	28.02	5.80
7.5	20	5.53e–4	1.00	8.33e+0	75	25.26	5.62
7.5	200	5.53e–4	1.00	2.63e+1	90	28.44	5.75
7.5	2000	5.53e–4	1.00	8.28e+1	91	26.97	5.35
12	200	6.14e–6	1.00	2.89e+1	95	28.42	5.70
15	200	3.06e–7	1.00	3.14e+1	93	31.35	6.46

~216 M dofs
512 cores



Variable Viscosity Stokes

the central problem in Solid Earth Geodynamics

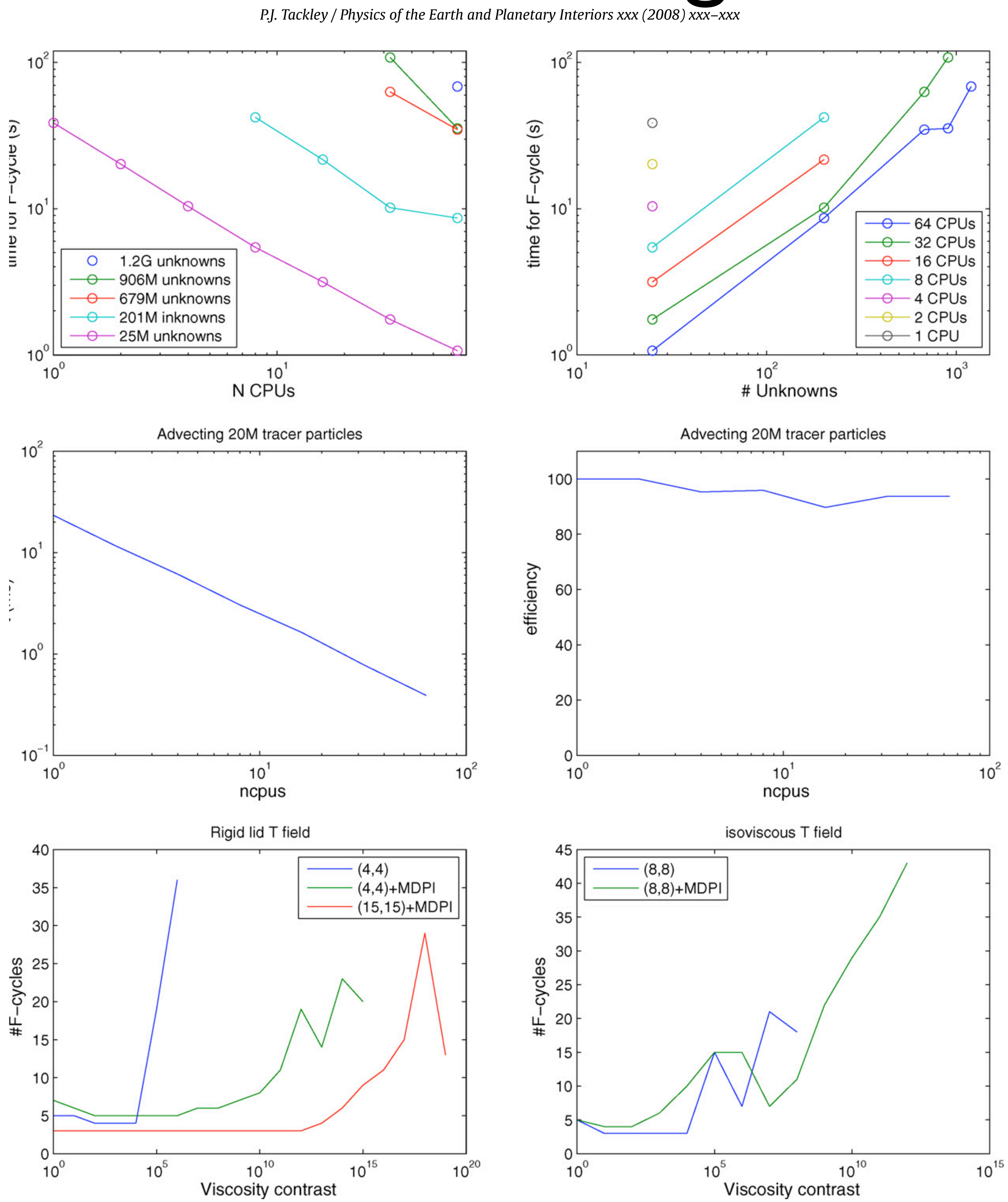
Some *speculations* on improving VV Stokes performance/ convergence

- Use a more coupled pre-conditioner, Vector Laplace is too weak
- Consider Newton over Picard for non-linear viscosity
- Consider GMG on Octree vs AMG

Anyone who can develop a robust and fast method for VV Stokes will be a real hero but....

- VV Stokes is not actually well defined (infinite number of A operators, some hard, some trivial.
- We need a VV - Stokes-off! (Serious benchmarking)

Performance/scaling of Finite Volume STAGYY



F-cycle
full Geometric Multi-grid with regular stencil

Tackley, P. J., Phys. Earth Planet. Inter, 171 (1-4), 7-18, doi: 10.1016/j.pepi.2008.08.005. PDF

Other Solid Mechanics problems

Mountain building/Lithospheric Deformation (Stokes + Plasticity)

Compression

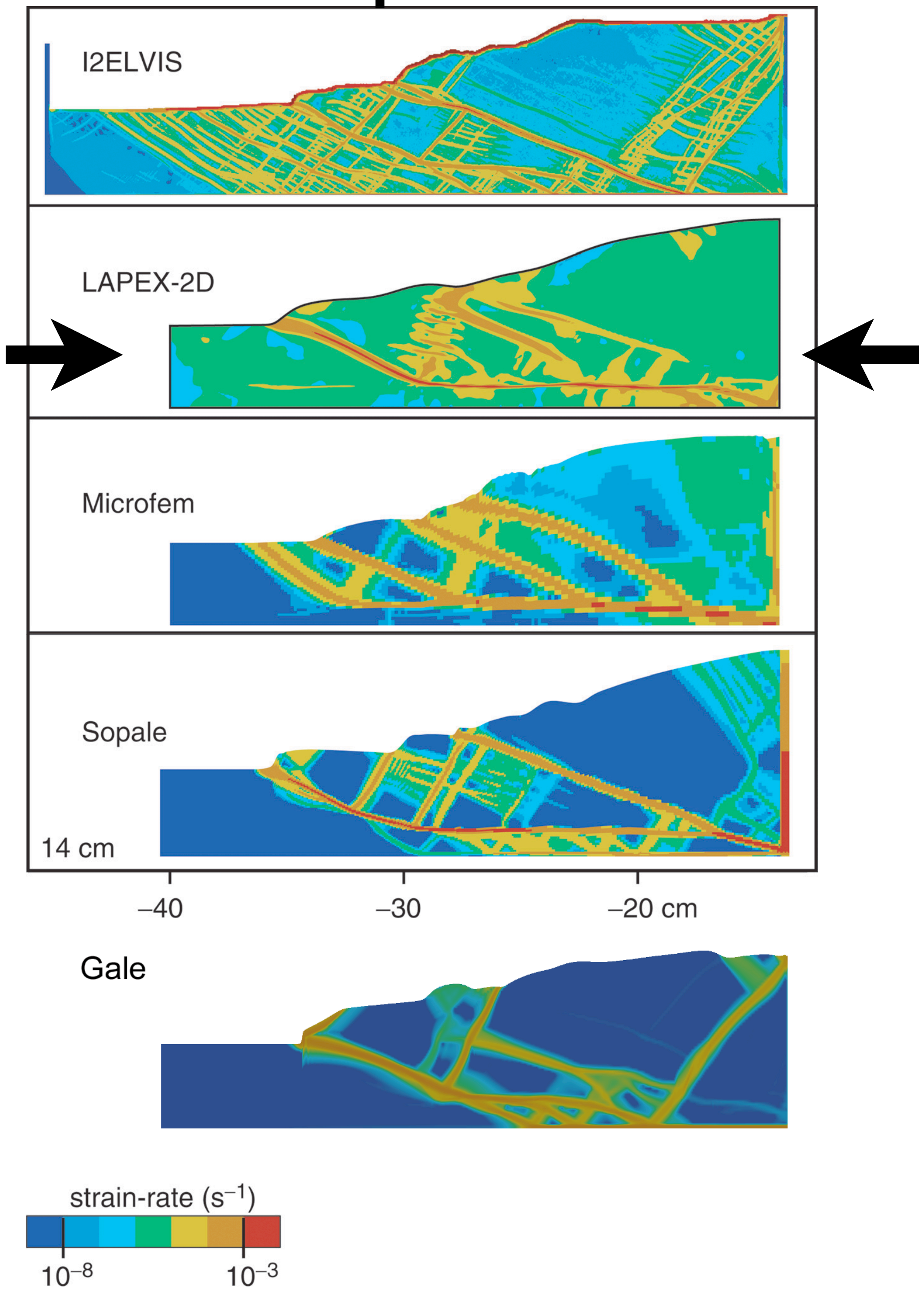


Figure C.24: Strain rate invariant for the numerical shortening models after 14 cm of shortening. The resolutions of the various models are: I2ELVIS: 900×75, LAPEX-2D: 351×71, Microfem: 201×36, Sopale: 401×71, Gale: 512×128. The upper portion of the figure is reproduced, with permission, from Buiter et al. [11].

Extension

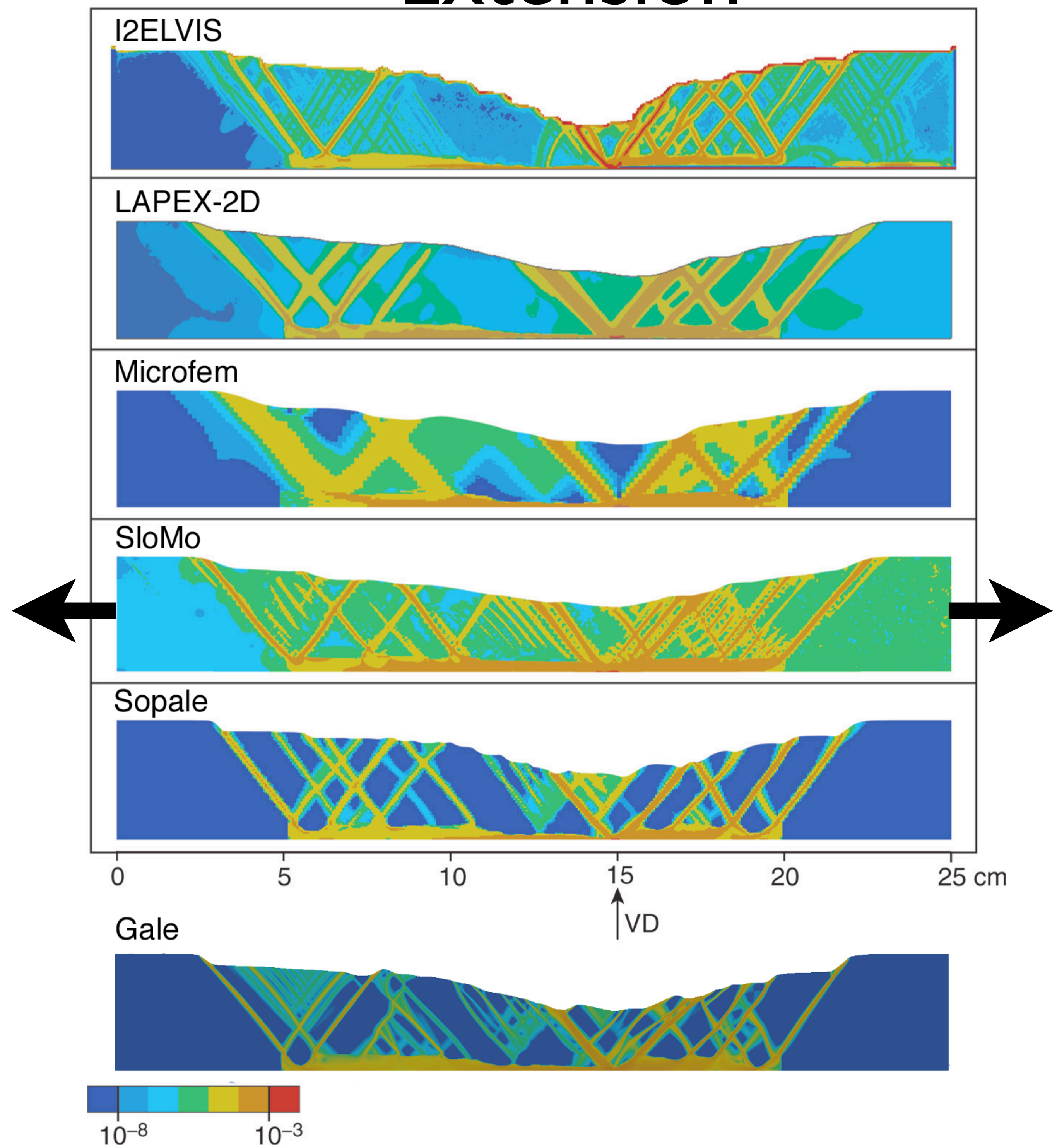


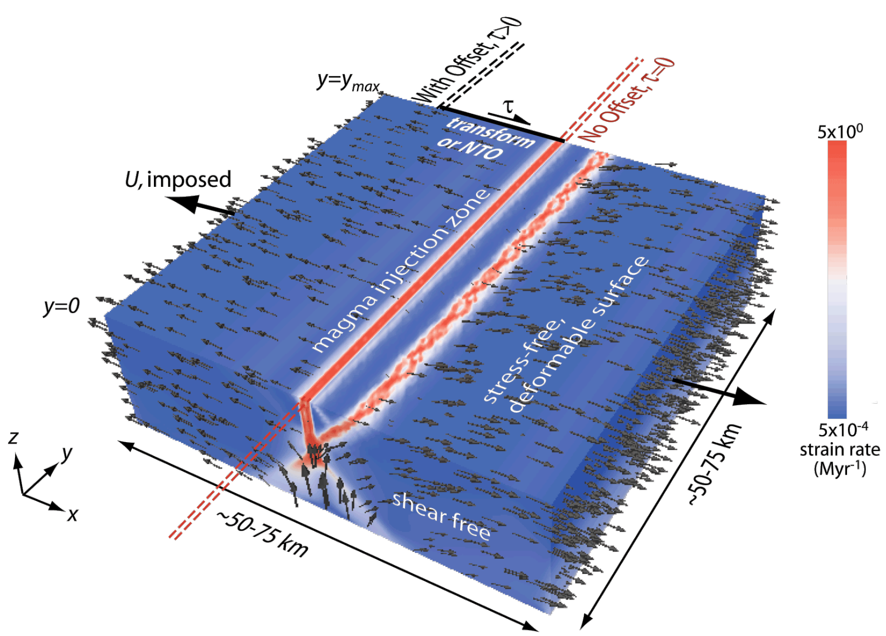
Figure C.22: Strain rate invariant for the numerical extension models after 5 cm of extension. The resolutions of the various models are: I2ELVIS: 400×75, LAPEX-2D: 301×71, Microfem: 201×61, SloMo: 401×71, Sopale: 401×71, Gale: 1024×128. Upper images reproduced, with permission, from Buiter et al. [11].

Available Software

COMPUTATIONAL INFRASTRUCTURE FOR GEODYNAMICS (CIG)
VICTORIA PARTNERSHIP FOR ADVANCED COMPUTING (VPAC)
MONASH UNIVERSITY

Gale

User Manual
Version 1.6.1

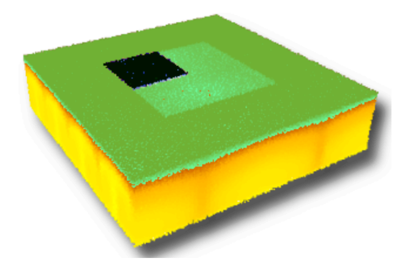


Walter Landry
Luke Hodkinson
Susan Kientz

www.geodynamics.org

Underworld User Manual

for Version 1.5



Created by

VPAC VICTORIAN PARTNERSHIP FOR ADVANCED COMPUTING

MONASH University Science

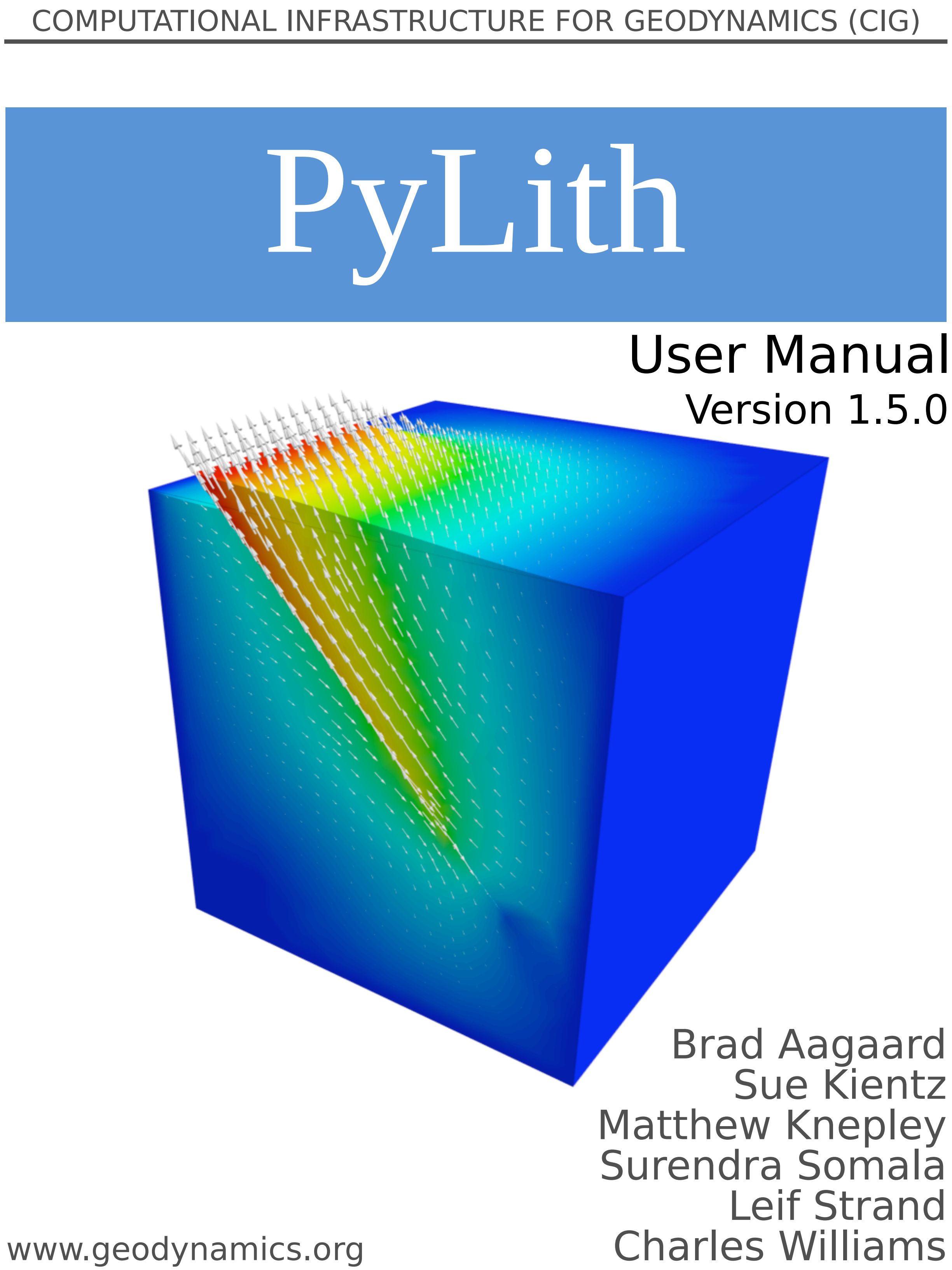
for

AuScope simulation analysis modelling

Editor:
Kathleen Humble

November 4, 2010

Earthquake Physics (adding faulting)



- fully Unstructured FEM with ability to include discrete faults, and earthquake rupture.
- Visco-elastic-plastic bulk plus faults.
- Challenge, model entire multi-scale earthquake cycle (fast rupture, and slow deformation between events)

Solid Mechanics

Summary

Solid Mechanics
Summary

- Solid Earth Dynamics is dominated by Variable Viscosity stokes

Solid Mechanics
Summary

- Solid Earth Dynamics is dominated by Variable Viscosity stokes
- Solid Mechanics is a *much* harder problem than Wave propagation
  - Convection/Tectonics is multi-scale with severe localization
  - can benefit from adaptive meshing
  - Multi-physics & non-Linear
  - More difficult to parallelize, Likely to be highly memory bandwith limited. Open question as to gains from gpu's?

Solid Mechanics
Summary

- Solid Earth Dynamics is dominated by Variable Viscosity stokes
- Solid Mechanics is a *much* harder problem than Wave propagation
  - Convection/Tectonics is multi-scale with severe localization
  - can benefit from adaptive meshing
  - Multi-physics & non-Linear
  - More difficult to parallelize, Likely to be highly memory bandwith limited. Open question as to gains from gpu's?
- Output is harder to compare to data

Solid Mechanics

Challenges and future directions

Solid Mechanics

Challenges and future directions

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- Just when you thought it was bad, it's going to get worse (aka more fun), when we add fluids....see you monday.