The Riemann problem shallow water wave systems

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Plan

- ☑ Part I : Discussion of the Riemann solution for one-dimensional linear and nonlinear shallow water wave equations
- ☑ Part II : Approximate Riemann solvers in GeoClaw; discussion of accuracy, and extensions to higher dimensions; f-wave approach to well-balancing.
- ☑ Part III : Adaptive mesh refinement (AMR).

Shallow water wave equations

The shallow water wave equations, given by

$$
\begin{aligned}
h_t + (uh)_x &= 0 \\
(uh)_t + (hu^2 + \frac{1}{2}gh^2)_x &= 0
\end{aligned}
$$

is an example of a system of equations written in *conservative form.* More generally, we can write PDEs in conservative form as

$$q_t + f(q)_x = 0$$

These are typically derived from conservation laws for mass, momentum, energy, species, and so on.

GeoClaw

- Based on solving the conservative form of the shallow water wave equations using a finite volume method.
- At the heart of many finite volume methods is a *Riemann solver* which is used to compute numerical fluxes.
- In GeoClaw, these are stored in files like **`rpn2_geo.f`** and **`rpt2_geo.f`**

Finite volume method

Assume a conservation law of the form

$$q_t + f(q)_x = 0$$

Define cell averages over the interval $C_i = [x_{i-1/2}, x_{i+1/2}]$

$$Q_i^n = \frac{1}{\Delta x} \int_{C_i} q(x, t_n)\, dx$$

How does the average evolve?

$$\begin{aligned} \frac{d}{dt} \int_{C_i} q(x,t)\, dx &= -\int_{C_i} \frac{d}{dx} f(q(x,t))\, dx \\ &= f(q(x_{i-1/2}, t)) - f(q(x_{i+1/2}, t)) \end{aligned}$$

Finite volume method

Evolution of the cell average value :

$$\frac{d}{dt}\int_{C_i} q(x,t)\;dx = f(q(x_{i-1/2},t)) - f(q(x_{i+1/2},t))$$

Integrate in time

$$\begin{aligned}\int_{C_i} q(x,t_{n+1})\,dx \quad &= \quad \int_{C_i} q(x,t_n)\,dx \\ &+ \quad \int_{t_n}^{t_{n+1}} \left[f(q(x_{i-1/2},t)) - f(q(x_{i+1/2},t))\right]\;dt\end{aligned}$$

Finite volume method

Using numerical fluxes, we use the update formula :

$$Q_i^{n+1} \quad = \quad Q_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^n - F_{i-1/2}^n\right]$$

Written as

$$\frac{Q_i^{n+1} - Q_i^n}{\Delta t} + \frac{F_{i+1/2} - F_{i-1/2}}{\Delta x} = 0$$

this form resembles the conservation law :

$$q_t + f(q)_x = 0$$

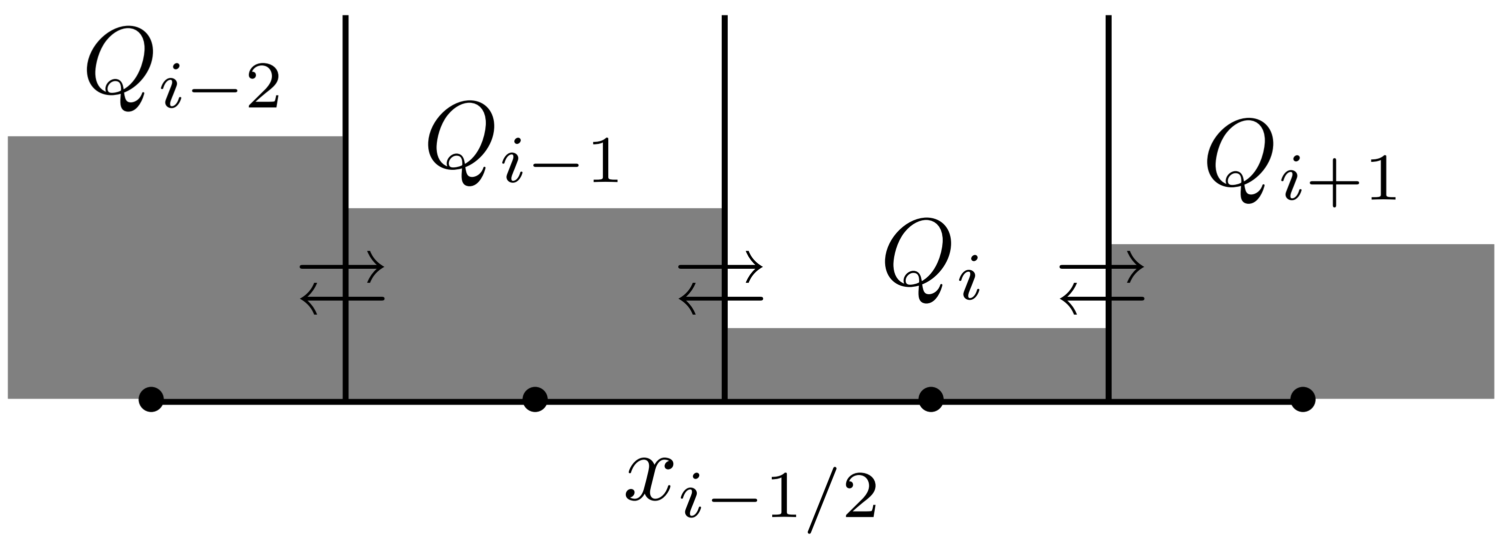
Numerical fluxes

We want to approximate the numerical flux.

$$F^n_{i-1/2} \approx \frac{1}{\Delta t} \int_{t_n}^{t_{n+1}} f(q(x_{i-1/2}, t))\, dt$$

For an explicit time stepping scheme, we try to find formulas for the flux of the form

$$F^n_{i-1/2} = \mathcal{F}(Q^n_i, Q^n_{i-1})$$



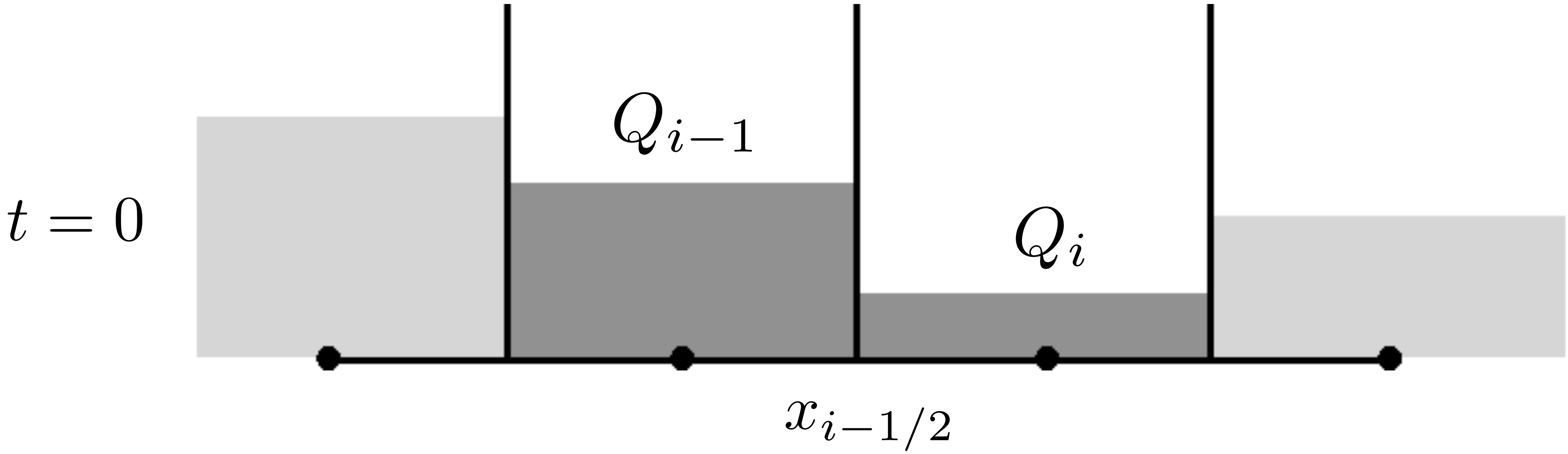
1d Riemann problem

At each cell interface, solve the hyperbolic problem with special initial data, i.e.

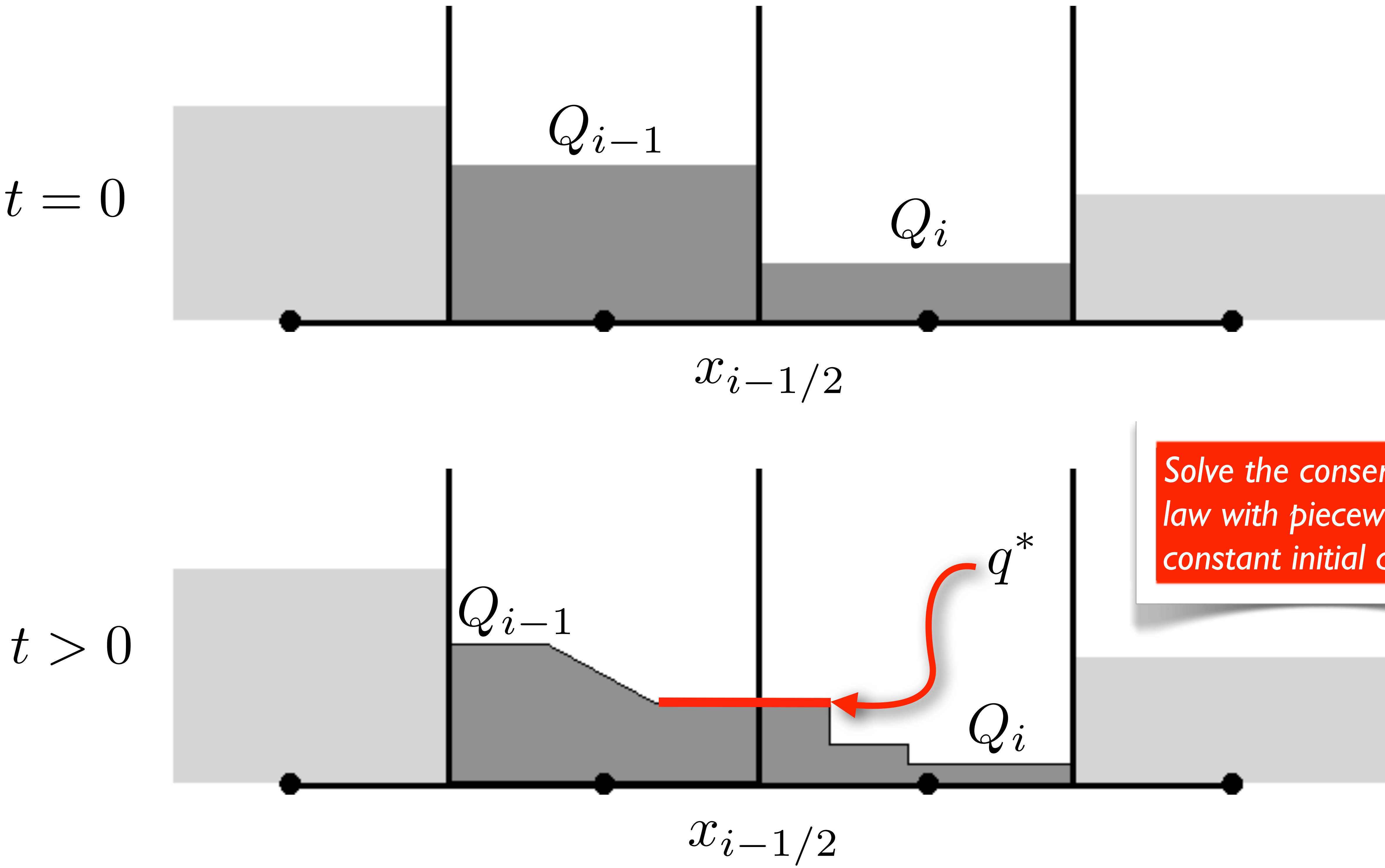
$$q_t + f(q)_x = 0$$

subject to

$$q(x,0) = \begin{cases} Q_{i-1} & x < x_{i-1/2} \\ Q_i & x > x_{x-1/2} \end{cases}$$

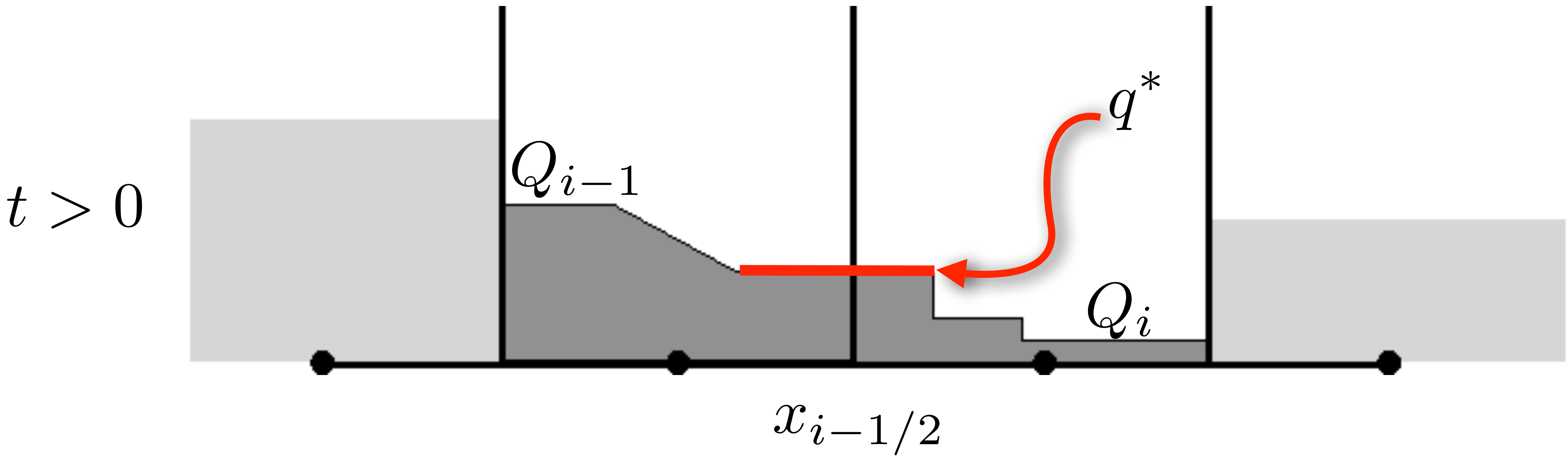


1d Riemann problem



Solve the conservation law with piecewise constant initial data

1d Riemann problem



Numerical flux at cell interface is then approximated by

$$F_{i-1/2} = f(q^*)$$

This is the classical Godunov approach for solving hyperbolic conservation laws.

- Resolves shocks and rarefactions

Conservation?

Integrating over entire domain, we have

$$\frac{d}{dt}\int_{x_a}^{x_b} q(x,t)\,dx = -\int_{x_a}^{x_b} (f(q))_x\,dx = f(q(x_a,t)) - f(q(x_b,t))$$

Discrete case

$$\begin{aligned}\sum_{i=1}^{M} Q_i^{n+1} &= \sum_{i=1}^{M} Q_i^n - \frac{\Delta t}{\Delta x}\sum_{i=1}^{M}\left(F_{i+1/2} - F_{i-1/2}\right)\\ &= \sum_{i=1}^{M} Q_i^n - \frac{\Delta t}{\Delta x}\left(F_{M+1/2} - F_{1/2}\right)\end{aligned}$$

Quantities are conserved up to fluxes at domain boundaries.

Scalar advection

Consider the constant initial value problem

$$q_t + \bar{u} q_x = 0$$

$$q(x,0) = \eta(x)$$

It is easy to verify that

$$q(x,t) = \eta(x - \bar{u}t)$$

solves the initial value problem.

Scalar advection

We can describe the problem in terms of how the solution behaves along curves in the x-t plane.

We might look for curves $\sigma = (X(t), t)$ along which the solution is constant or

$$\frac{d}{dt} q(X(t), t) = 0$$

Then we would get

$$\frac{d}{dt} q(X(t), t) = q_x(X(t), t) X'(t) + q_t(X(t), t) = 0$$

Characteristic curves

$$\frac{d}{dt}q(X(t),t) = q_x(X(t),t)X'(t) + q_t(X(t),t) = 0$$

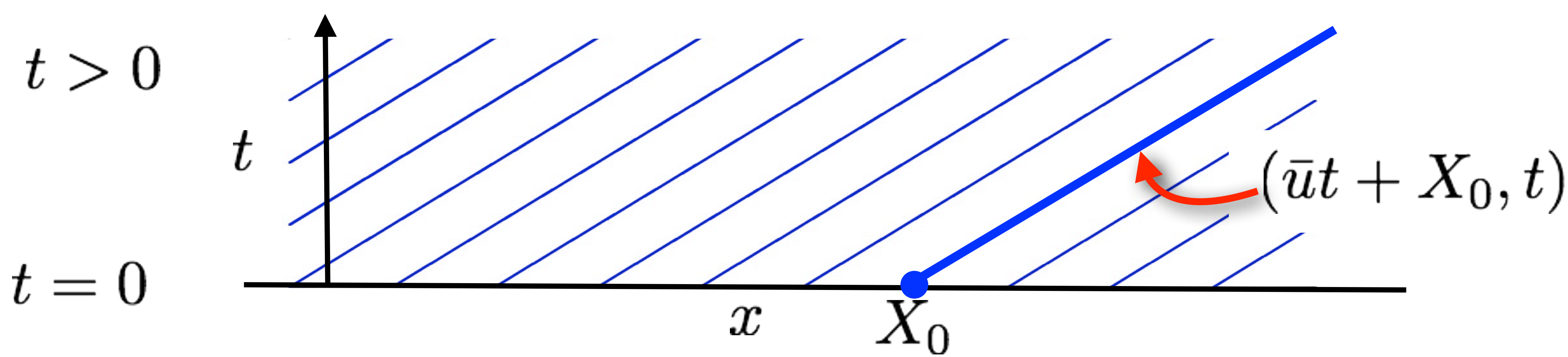
But this is true only if

$$X'(t) = \bar{u}$$

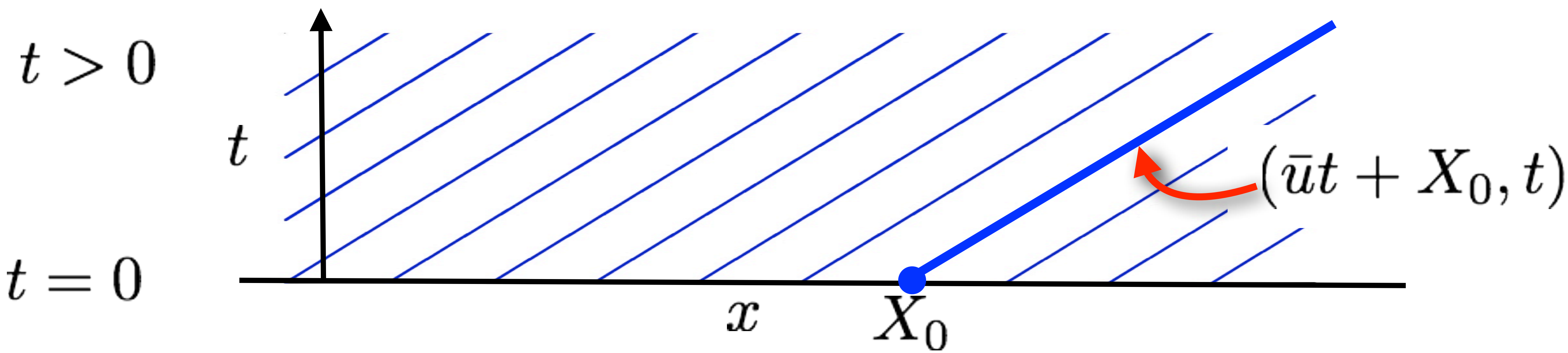
or

$$X(t) = \bar{u}t + X_0$$

Solution is constant along *characteristic curves*. *For* $\bar{u} > 0$,



Characteristic curves



The solution can be traced back along characteristics. That is, $q(x,t)$ can be found by determining the X_0 from which the solution propagated. Solve

$$x = \bar{u}t + X_0 \quad \rightarrow \quad X_0 = x - \bar{u}t$$

or

$$q(x,t) = q(X_0, 0) = q(x - \bar{u}t, 0)$$

Scalar advection

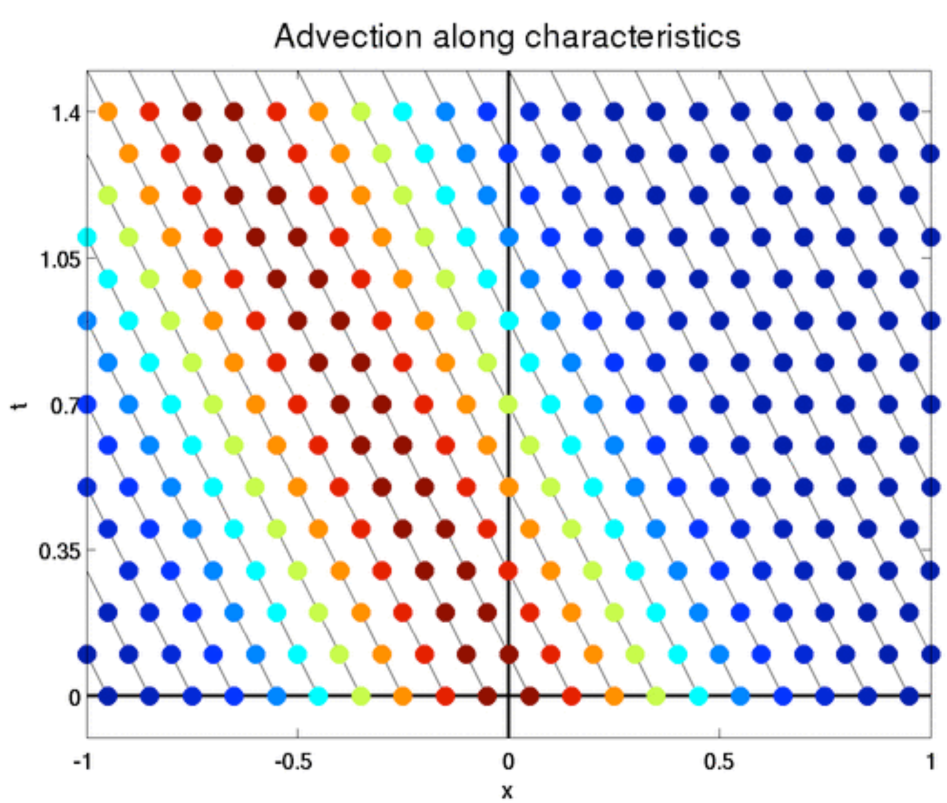
Consider the scalar advection equation :

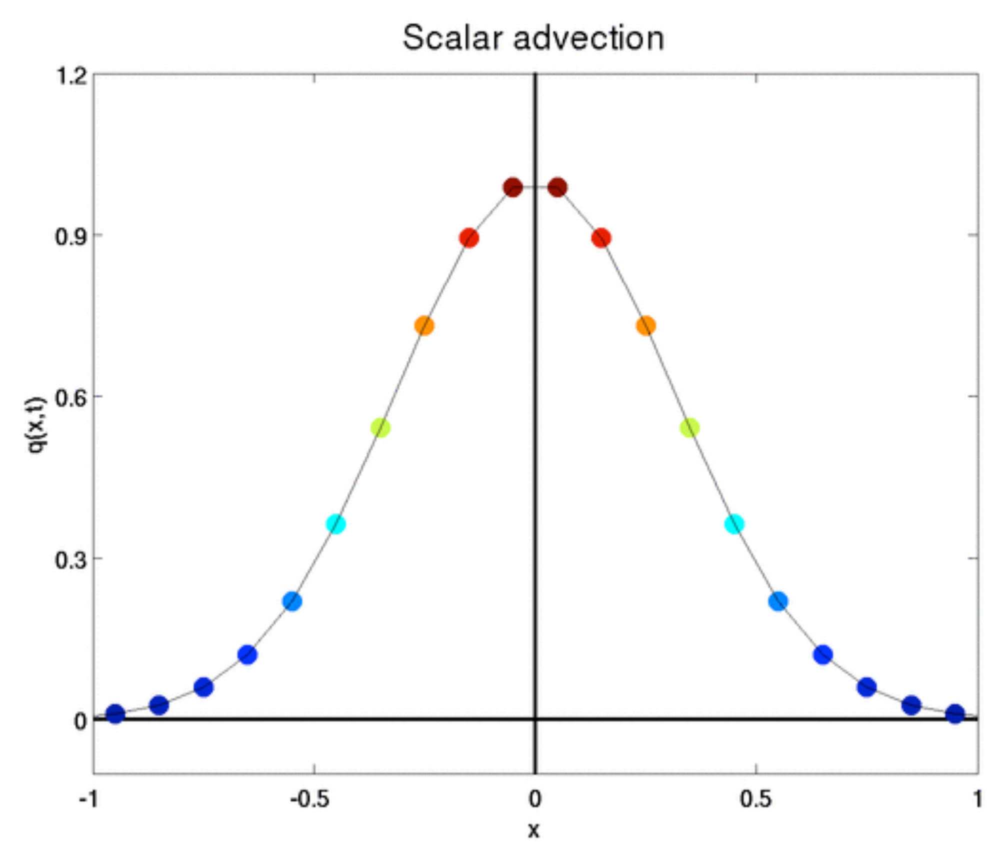
$$q_t + \bar{u} q_x = 0$$

The solution travels along characteristic rays in the (x,t) plane given by $(x - X_0)/t = \bar{u}$. For u < 0 :

$t = 1 \rightarrow$

$t = 0 \rightarrow$



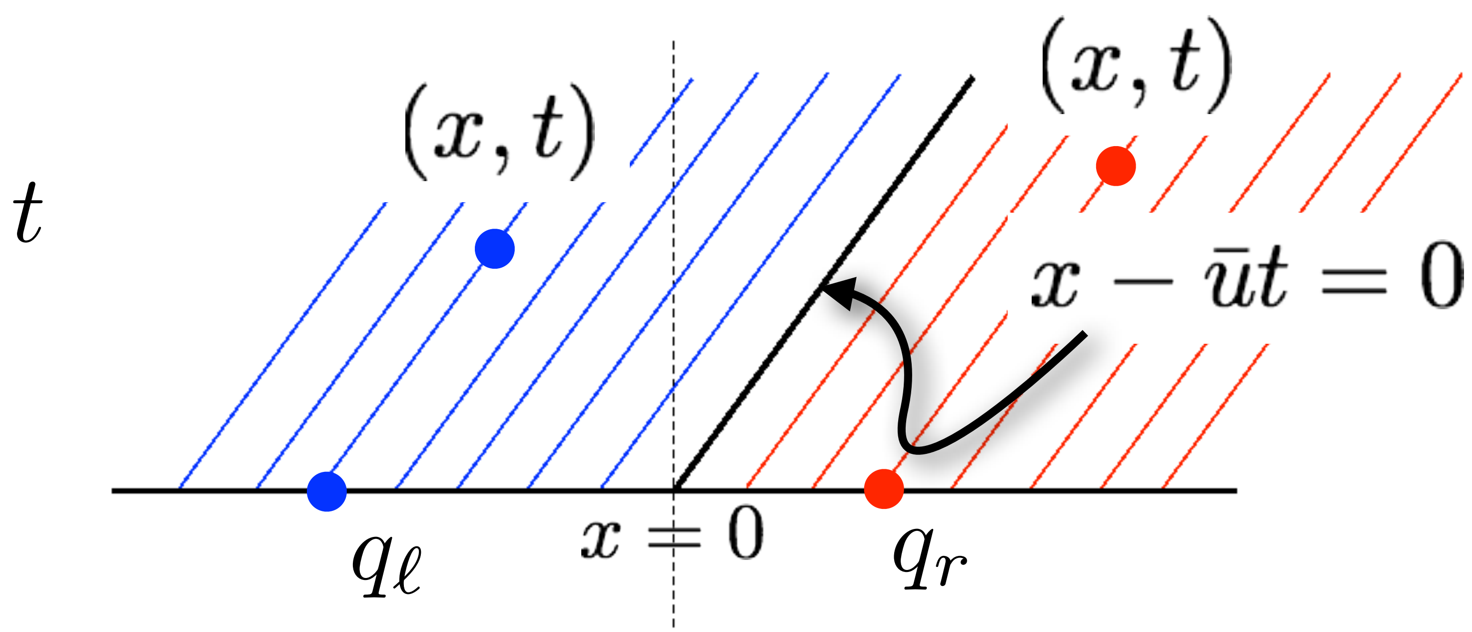


Riemann problem for scalar advection

$$q_t + \bar{u} q_x = 0$$

subject to particular initial conditions

$$q(x,0) = \begin{cases} q_\ell & x < 0 \\ q_r & x > 0 \end{cases}$$



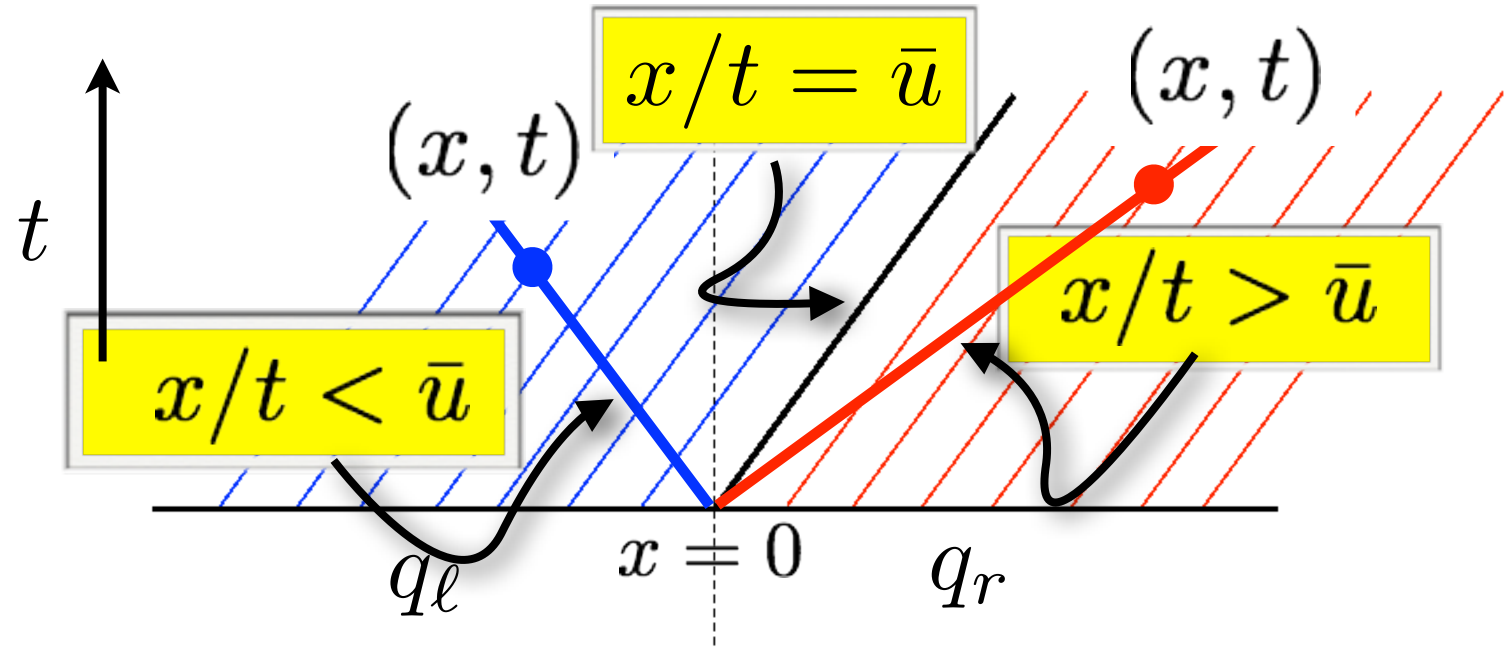
Solution : $$q(x,t) = \begin{cases} q_\ell & x - \bar{u}t < 0 \\ q_r & x - \bar{u}t > 0 \end{cases}$$

Scalar Riemann Problem

$$q_t + \bar{u} q_x = 0$$

subject to initial conditions

$$q(x,0) = \begin{cases} q_\ell & x < 0 \\ q_r & x > 0 \end{cases}$$

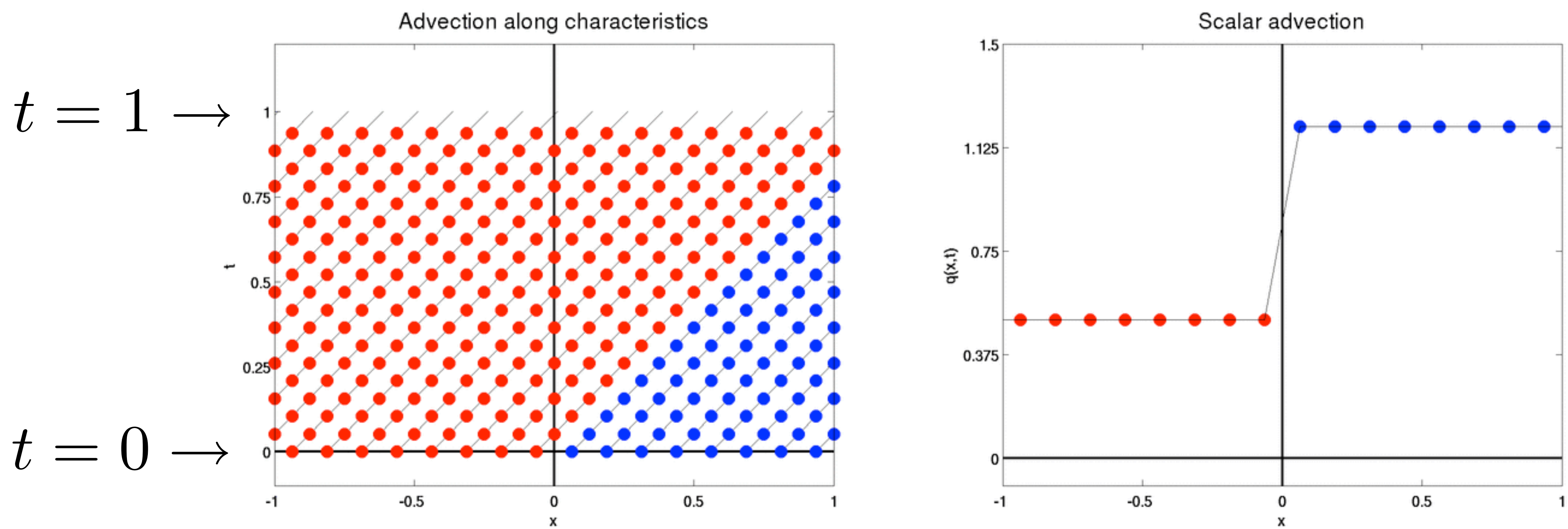


Solution to the Riemann problem

Solution : $$q(x,t) = \begin{cases} q_\ell & x/t < \bar{u} \\ q_r & x/t > \bar{u} \end{cases}$$

Scalar constant coefficient advection

Discontinuity propagates at speed $\bar{u}$ and has strength $q_r - q_\ell$



Solving constant coefficient linear systems

$$q_t + Aq_x = 0, \qquad A \in R^{m \times m}$$

We assume that A has a complete set of eigenvectors and real eigenvalues and so can be written as

$$A = R\Lambda R^{-1}$$

$$R = [r^1, r^2, \ldots r^m] \qquad \Lambda = \mathrm{diag}(\lambda^1, \lambda^2, \ldots \lambda^m)$$

Examples : Linearized shallow water wave equations, constant coefficient acoustics, ...

Solving a constant coefficient system

$$q_t + A\,q_x = 0 \quad \rightarrow \quad q_t + R\Lambda R^{-1}\,q_x = 0, \qquad A \in R^{m\times m}$$

Assume that A is diagonalizable

Define characteristic variables $\omega \in R^m$ as

$$\omega(x,t) = R^{-1}\,q(x,t), \qquad \omega(x,0) = R^{-1}\,q(x,0)$$

Characteristic equations decouple into m scalar equations :

$$\omega_t^p + \lambda^p \omega_x^p = 0, \qquad p = 1, 2, \ldots, m$$

Solution to characteristic equations are given by

$$\omega^p(x,t) = \omega^p(x - \lambda^p t, 0)$$

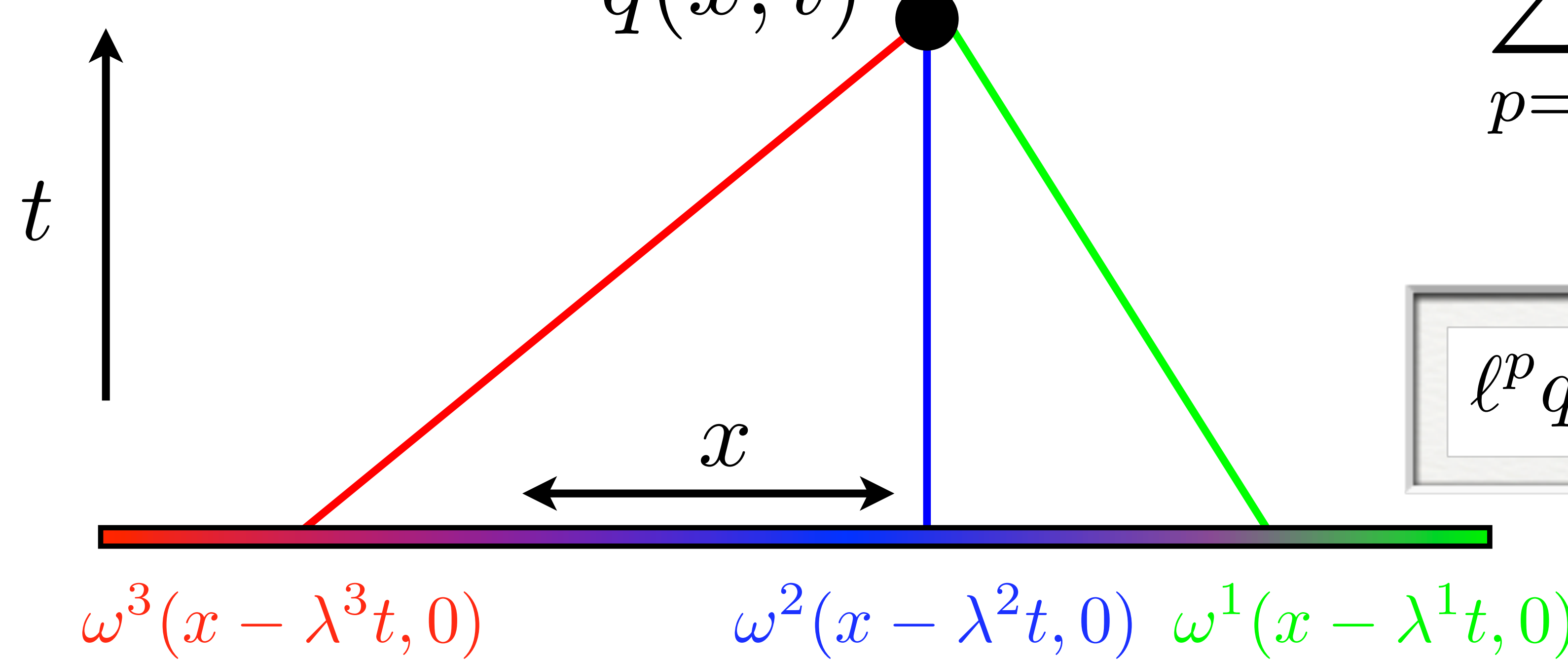
Solving a constant coefficient systems

$$q_t + A\,q_x = 0 \qquad \rightarrow \qquad \omega_t^p + \lambda^p \omega_x^p = 0$$

Solution for general initial conditions $q(x,0)$:

$$q(x,t) = R\,\omega(x,t) \quad = \quad \sum_{p=1}^{m} \omega^p(x,t)\, r^p$$

$$q(x,t) \quad = \quad \sum_{p=1}^{m} \omega^p(x - \lambda^p t, 0)\, r^p$$



$$\ell^p q(x - \lambda^p t, 0) = \omega^p(x - \lambda^p t, 0)$$

Riemann problem for systems

Assume a constant coefficient system :

$$q_t + A\,q_x = 0, \qquad q \in R^3$$

with piecewise constant initial data :

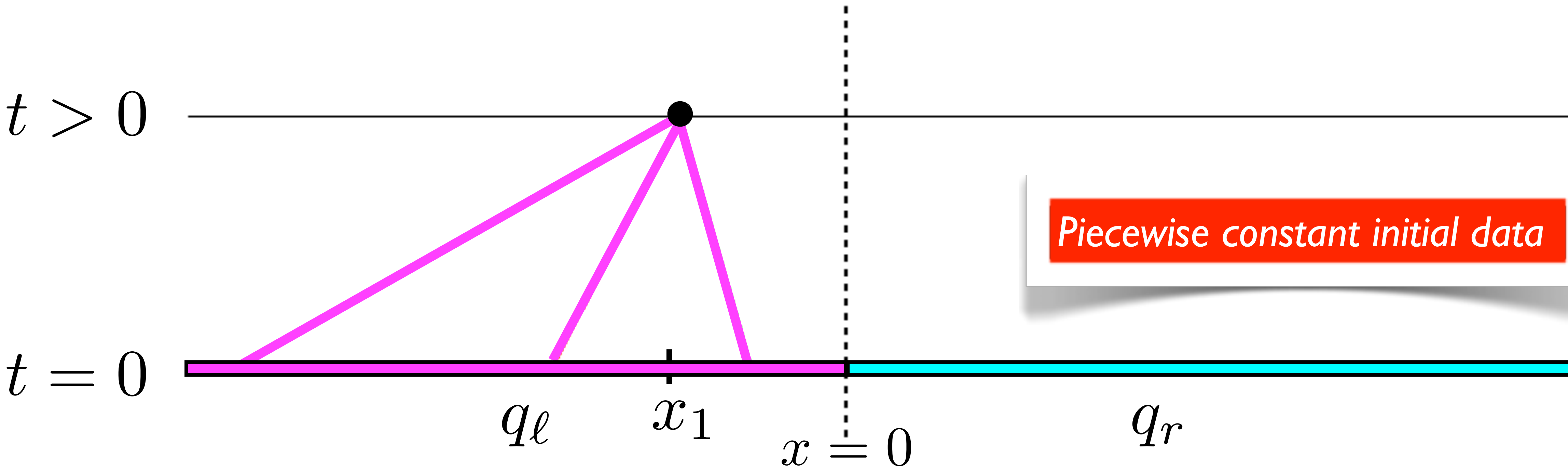
$$q(x,0) = \begin{cases} q_\ell & x < 0 \\ q_r & x > 0 \end{cases}$$

which can be decomposed as :

$$q_\ell = \sum_{p=1}^{3} \omega_\ell^p\, r^p \qquad q_r = \sum_{p=1}^{3} \omega_r^p\, r^p$$

Riemann problem for systems

$$q_t + A\,q_x = 0, \qquad q \in R^3$$



Piecewise constant initial data

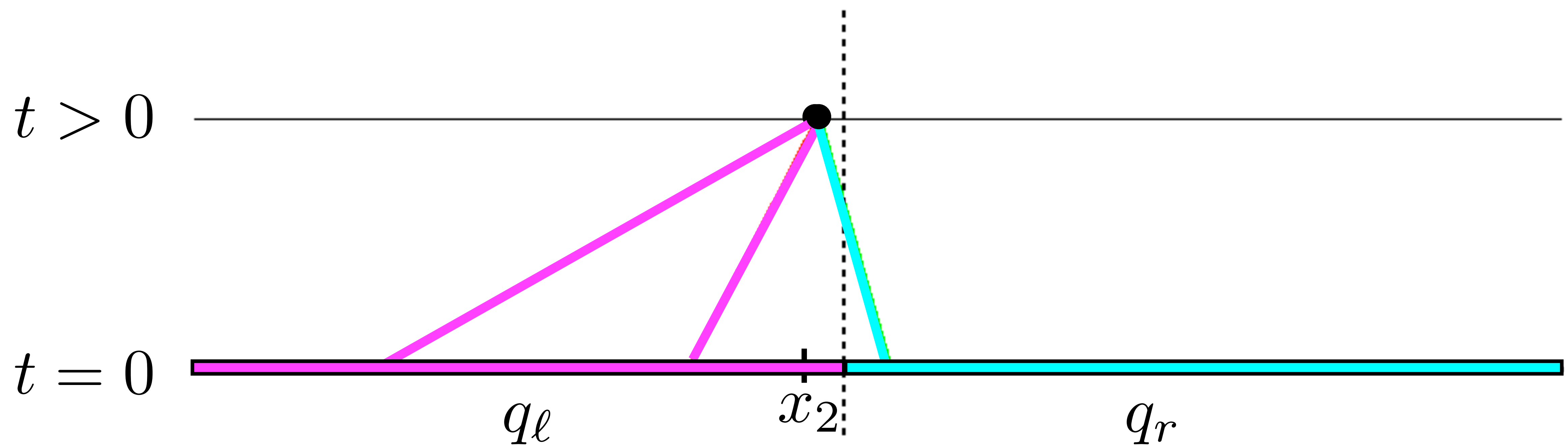
- $q(x_1, t) = \omega_\ell^1\, r^1 + \omega_\ell^2\, r^2 + \omega_\ell^3\, r^3$

$= q_\ell$

$$q_\ell = \sum_{p=1}^{m} \omega_\ell^p\, r^p \qquad q_r = \sum_{p=1}^{m} \omega_r^p\, r^p$$

Riemann problem for systems

$$q_t + A\, q_x = 0$$

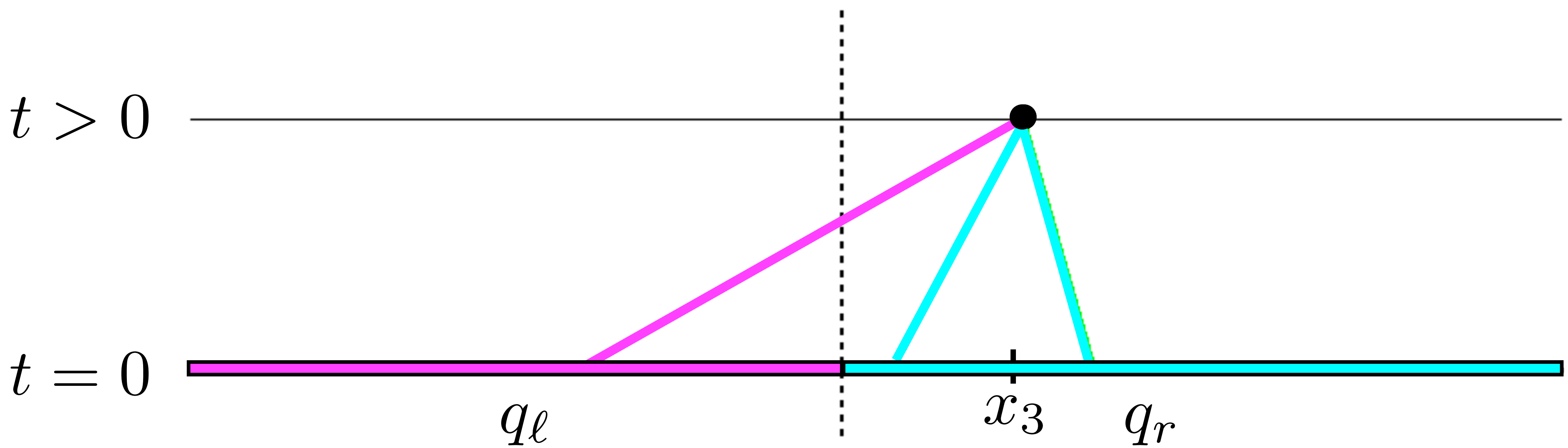


- $q(x_2, t) = \omega_r^1\, r^1 + \omega_\ell^2\, r^2 + \omega_\ell^3\, r^3$

$$q_\ell = \sum_{p=1}^{m} \omega_\ell^p\, r^p \qquad q_r = \sum_{p=1}^{m} \omega_r^p\, r^p$$

Riemann problem for systems

$$q_t + A\,q_x = 0$$

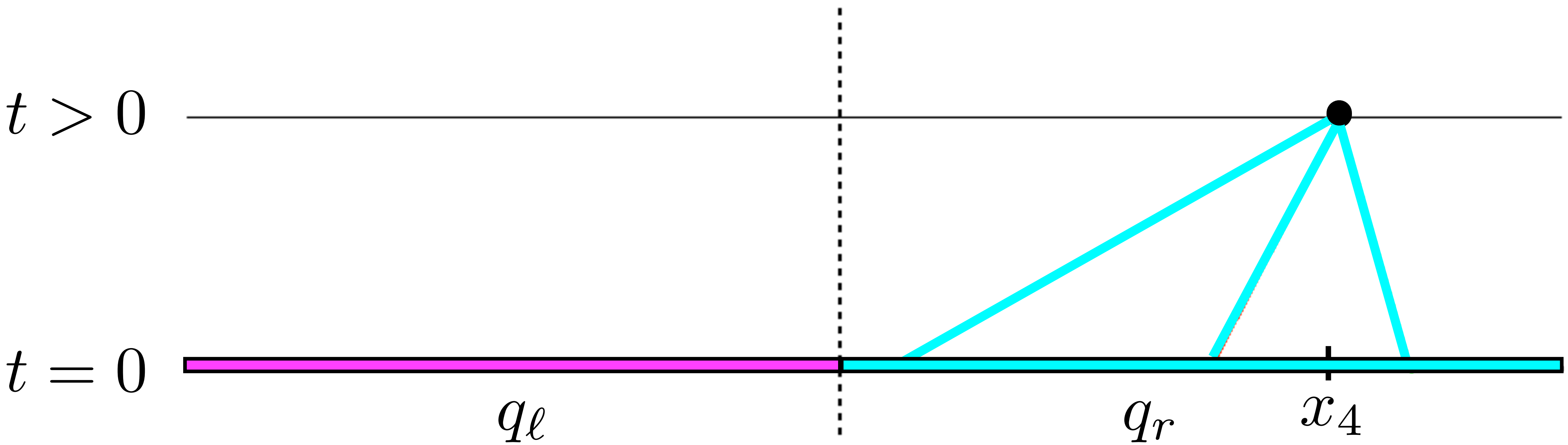


- $q(x_3, t) = \omega_r^1\, r^1 + \omega_r^2\, r^2 + \omega_\ell^3\, r^3$

$$q_\ell = \sum_{p=1}^{m} \omega_\ell^p\, r^p \qquad q_r = \sum_{p=1}^{m} \omega_r^p\, r^p$$

Riemann problem for systems

$$q_t + A\,q_x = 0$$



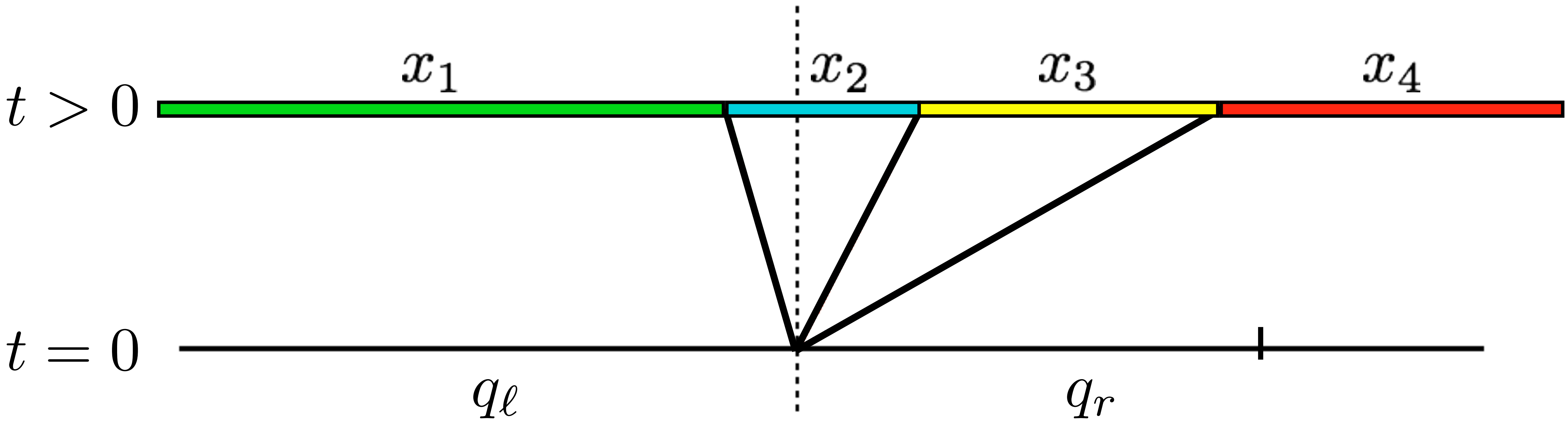
- $q(x_4, t) = \omega_r^1\, r^1 + \omega_r^2\, r^2 + \omega_r^3\, r^3$

$= q_r$

$$q_\ell = \sum_{p=1}^{m} \omega_\ell^p\, r^p \qquad q_r = \sum_{p=1}^{m} \omega_r^p\, r^p$$

Riemann problem for systems

$$q_t + A\,q_x = 0$$



$$q(x_1,t) = \omega_\ell^1\, r^1 + \omega_\ell^2\, r^2 + \omega_\ell^3\, r^3$$

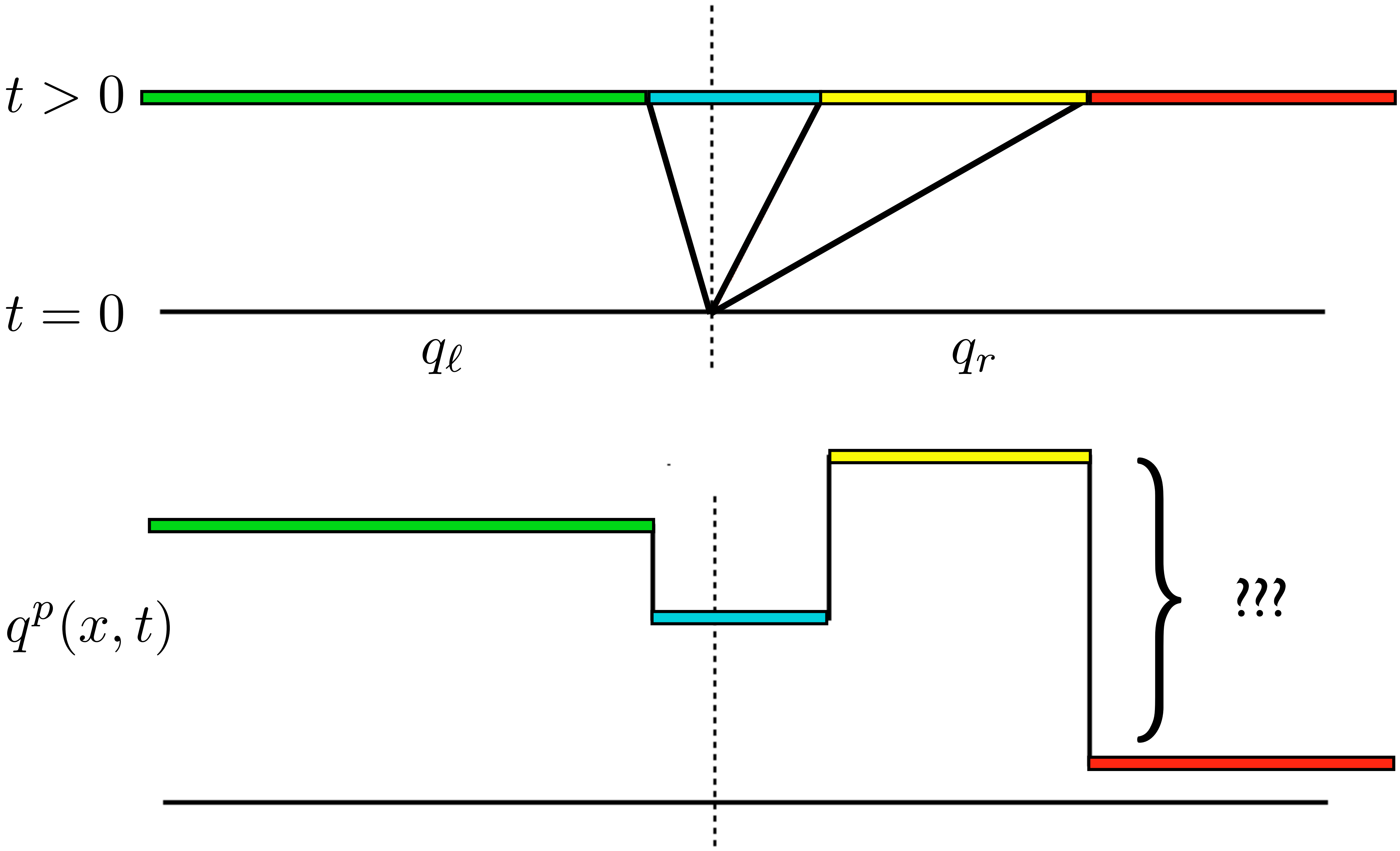
$$q(x_2,t) = \omega_r^1\, r^1 + \omega_\ell^2\, r^2 + \omega_\ell^3\, r^3$$

$$q(x_3,t) = \omega_r^1\, r^1 + \omega_r^2\, r^2 + \omega_\ell^3\, r^3$$

$$q(x_4,t) = \omega_r^1\, r^1 + \omega_r^2\, r^2 + \omega_r^3\, r^3$$

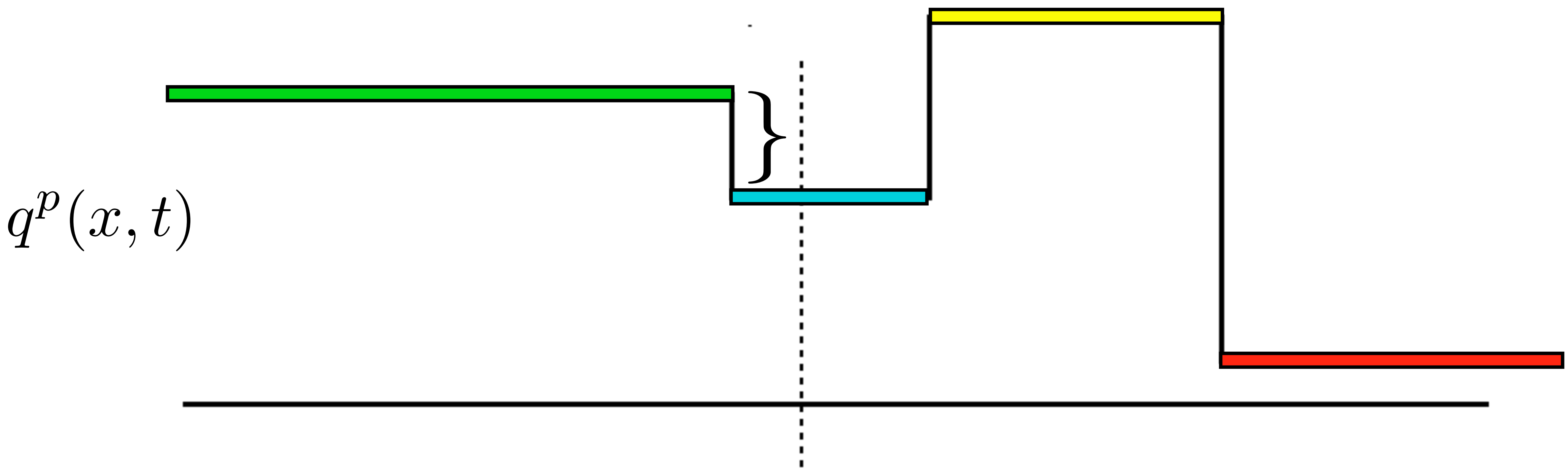
Riemann problem for systems

$$q_t + A\,q_x = 0$$



Riemann problem for systems

$$q_t + A\,q_x = 0$$



$$q(x_1,t) = \omega_\ell^1\,r^1 + \omega_\ell^2\,r^2 + \omega_\ell^3\,r^3$$

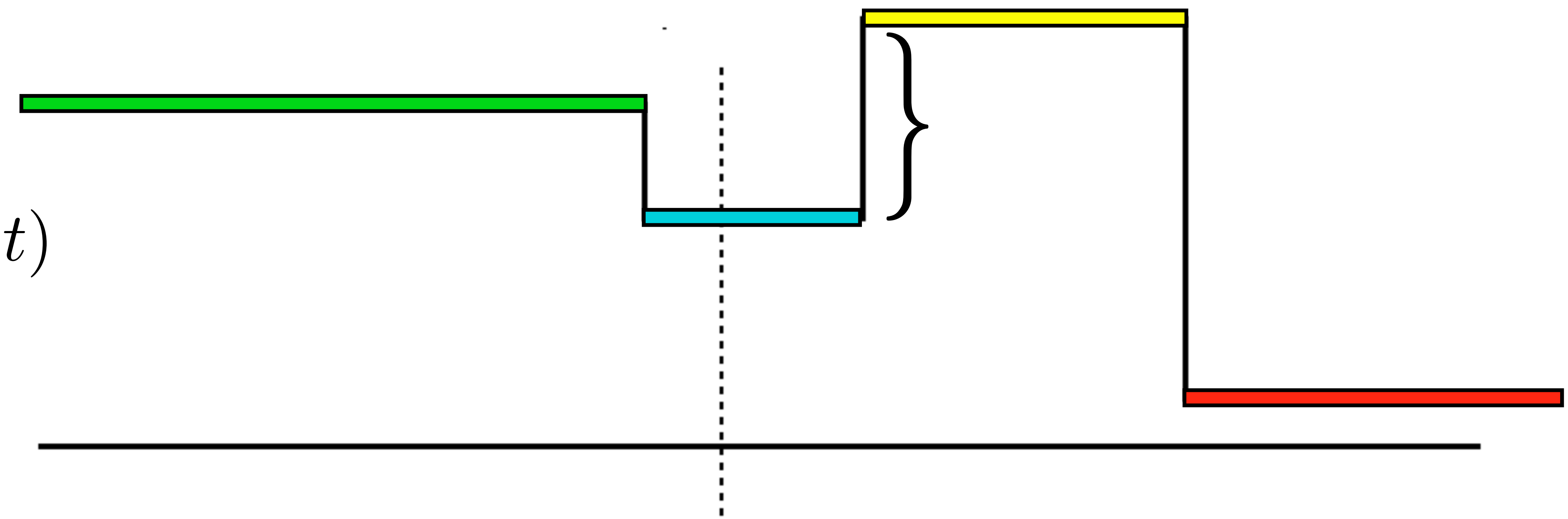
$$q(x_2,t) = \omega_r^1\,r^1 + \omega_\ell^2\,r^2 + \omega_\ell^3\,r^3$$

$$\boxed{q(x_2,t) - q(x_1,t) = (\omega_r^1 - \omega_\ell^1)\,r^1}$$

Riemann problem for systems

$$q_t + A\,q_x = 0$$

$q^p(x,t)$



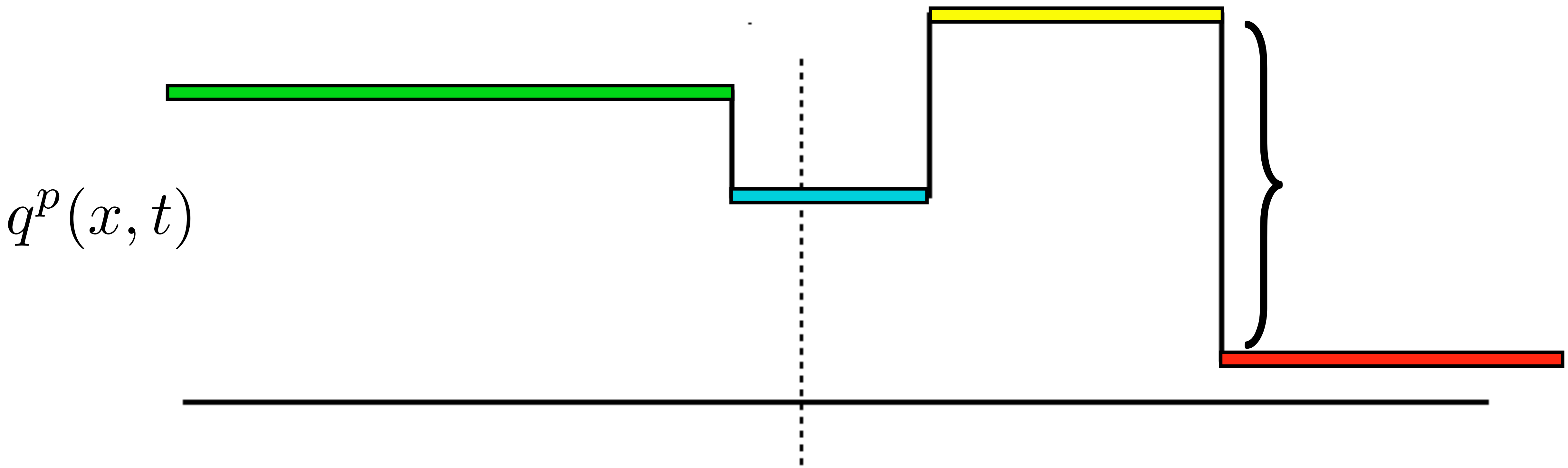
$$q(x_2,t) = \omega_r^1\,r^1 + \omega_\ell^2\,r^2 + \omega_\ell^3\,r^3$$

$$q(x_3,t) = \omega_r^1\,r^1 + \omega_r^2\,r^2 + \omega_\ell^3\,r^3$$

$$\boxed{q(x_3,t) - q(x_2,t) = (\omega_r^2 - \omega_\ell^2)\,r^2}$$

Riemann problem for systems

$$q_t + A\,q_x = 0$$



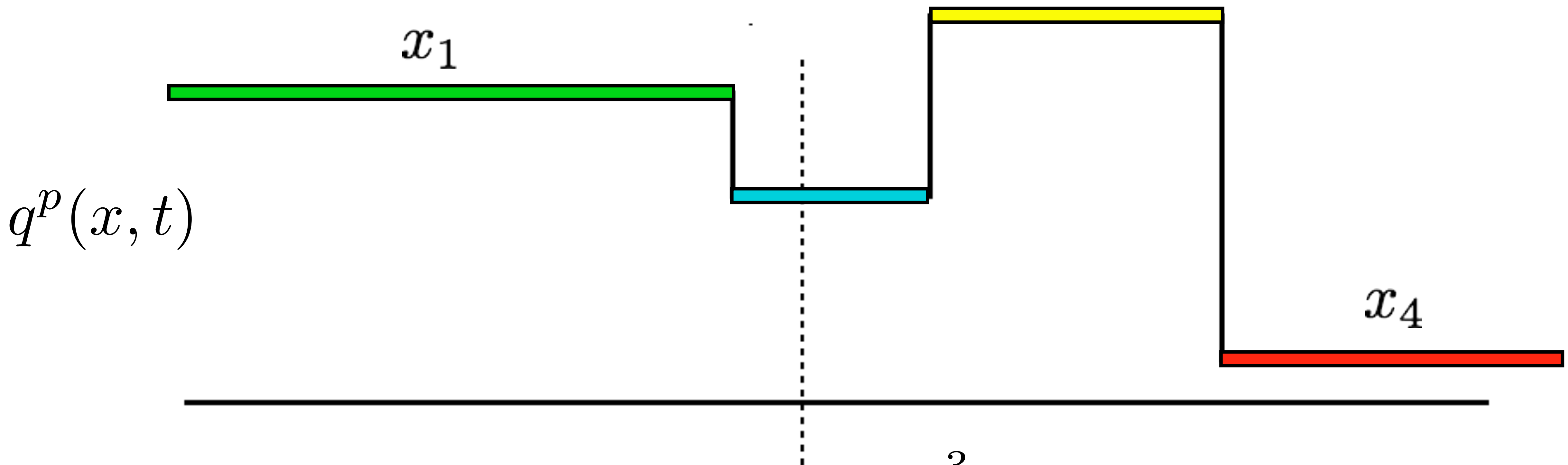
$$q(x_3,t) = \omega_r^1\,r^1 + \omega_r^2\,r^2 + \omega_\ell^3\,r^3$$

$$q(x_4,t) = \omega_r^1\,r^1 + \omega_r^2\,r^2 + \omega_r^3\,r^3$$

$$\boxed{q(x_4,t) - q(x_3,t) = (\omega_r^3 - \omega_\ell^3)\,r^3}$$

Riemann problem for systems

$$q_t + A\, q_x = 0$$



$$\underbrace{q(x_4,t) - q(x_1,t)}_{q_r - q_\ell} = \sum_{p=1}^{3} \underbrace{(\omega_r^p - \omega_\ell^p)}\, r^p$$

$$\equiv \sum_{p=1}^{3} \alpha^p\, r^p$$

$$R\,\alpha = q_r - q_\ell$$

Riemann problem for systems

Solving the Riemann problem for linear problem

$$q_t + A\, q_x = 0$$

(1) Compute eigenvalues and eigenvectors of matrix A

(2) Compute characteristic variables by solving

$$R\,\alpha = q_r - q_\ell$$

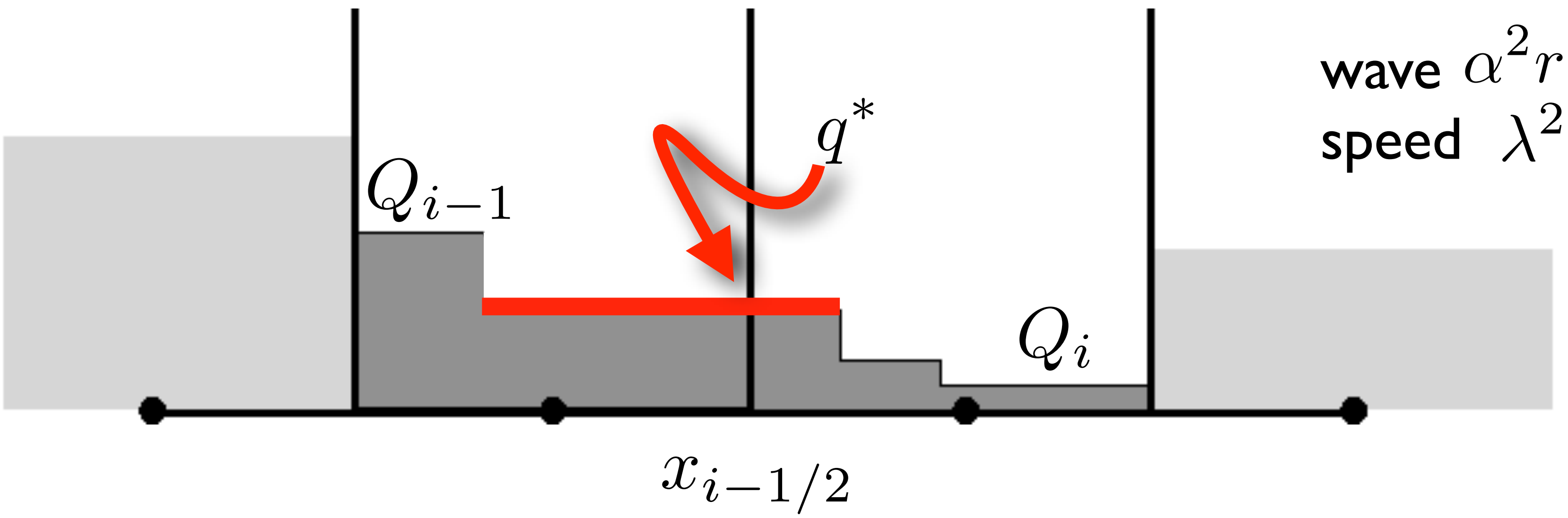
(3) Use eigenvalues or "speeds" to determine piecewise constant solution

$$\begin{aligned} q(x,t) &= q_\ell + \sum_{p\,:\,\lambda^p < x/t} \alpha^p\, r^p \\ &= q_r - \sum_{p\,:\,\lambda^p > x/t} \alpha^p\, r^p \end{aligned}$$

Numerical solution

$$q_t + f(q)_x = 0, \qquad f(q) = Aq$$

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2}^n - F_{i-1/2}^n\right]$$



wave $\alpha^2 r^2$ travels at speed $\lambda^2 > 0$

Decompose jump in Q at the interface into waves :

$$q^* = Q_{i-1} + \alpha^1 r^1 = Q_i - \alpha^3 r^3 - \alpha^2 r^2$$

$$F_{i-1/2} \approx \frac{1}{\Delta t}\int_t^{t+\Delta t} f(q(x_{i-1/2}, t))dt = Aq^*$$

Example : Linearized shallow water

$$q_t + A\,q_x = 0, \qquad A = \begin{pmatrix} U & H \\ g & U \end{pmatrix}, \qquad q = \begin{pmatrix} h \\ u \end{pmatrix}$$

Characteristic information :

Eigenvalues : $\lambda^1 = U - \sqrt{gH}, \qquad \lambda^2 = U + \sqrt{gH}$

Eigenvectors : $r^1 = \begin{pmatrix} -\sqrt{gH} \\ g \end{pmatrix}, \qquad r^2 = \begin{pmatrix} \sqrt{gH} \\ g \end{pmatrix}$

Example : Linearized shallow water

Characteristic variables : $R\alpha = q_r - q_\ell$

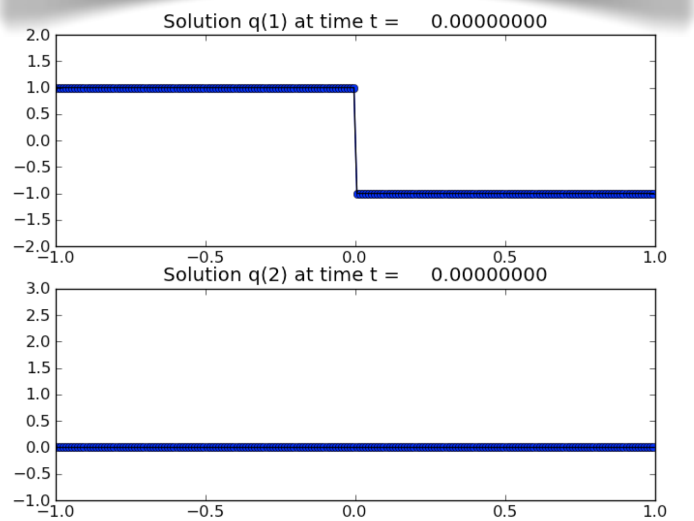
Define : $\delta = q_r - q_\ell \quad \rightarrow \quad \begin{array}{rl} \delta^1 = & h_r - h_\ell \\ \delta^2 = & u_r - u_\ell \end{array}$

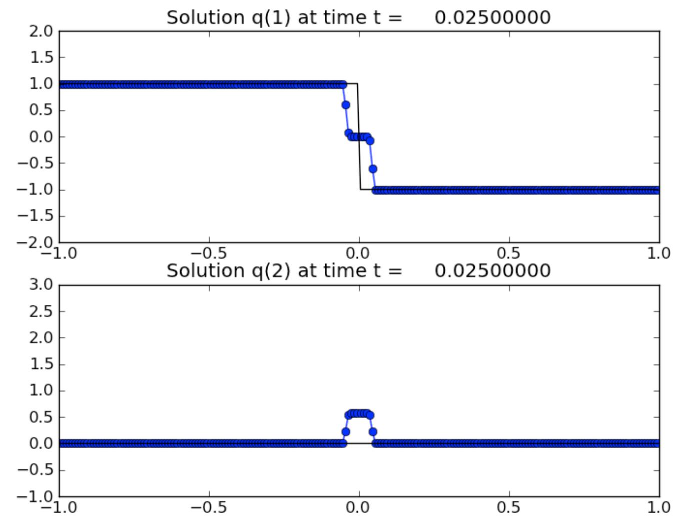
$$\left(\begin{array}{c} \alpha_1 \\ \alpha_2 \end{array}\right) = \frac{1}{2gH}\left(\begin{array}{cc} -\sqrt{gH} & H \\ \sqrt{gH} & H \end{array}\right)\left(\begin{array}{c} \delta_1 \\ \delta_2 \end{array}\right)$$

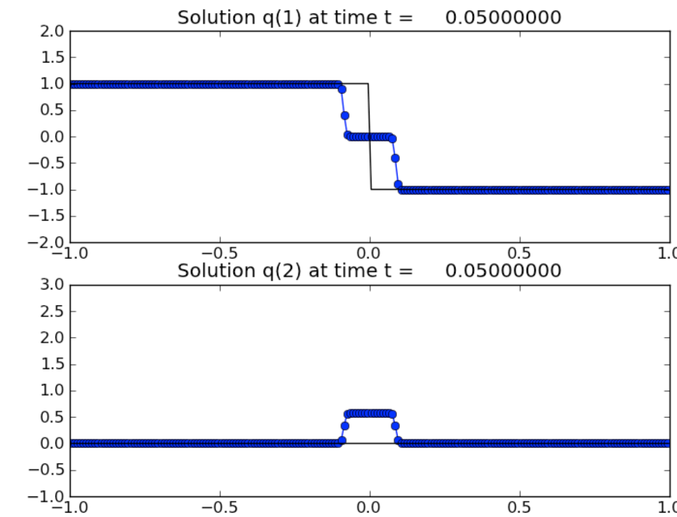
$$q(x,t) = \begin{cases} q_\ell = \left(\begin{array}{c} h_\ell \\ u_\ell \end{array}\right) & x/t < U - \sqrt{gH} \\ q_\ell + \alpha^1 r^1 & U - \sqrt{gH} < x/t < U + \sqrt{gH} \\ q_r = \left(\begin{array}{c} h_r \\ u_\ell \end{array}\right) & x/t > U + \sqrt{gH} \end{cases}$$

Linear shallow water wave equations

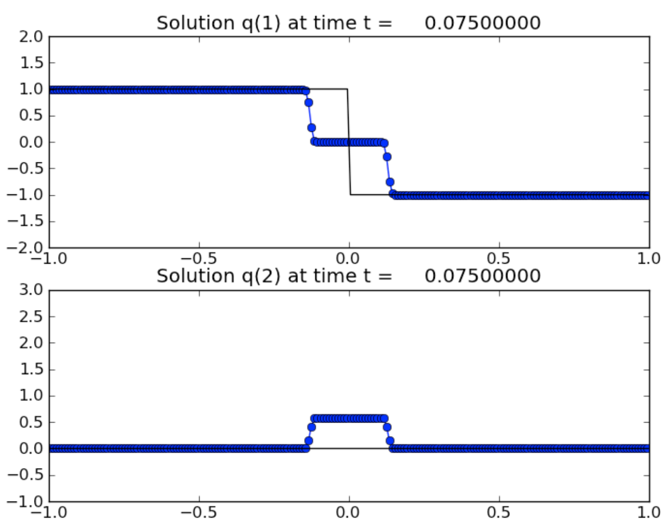
Initial height and velocity

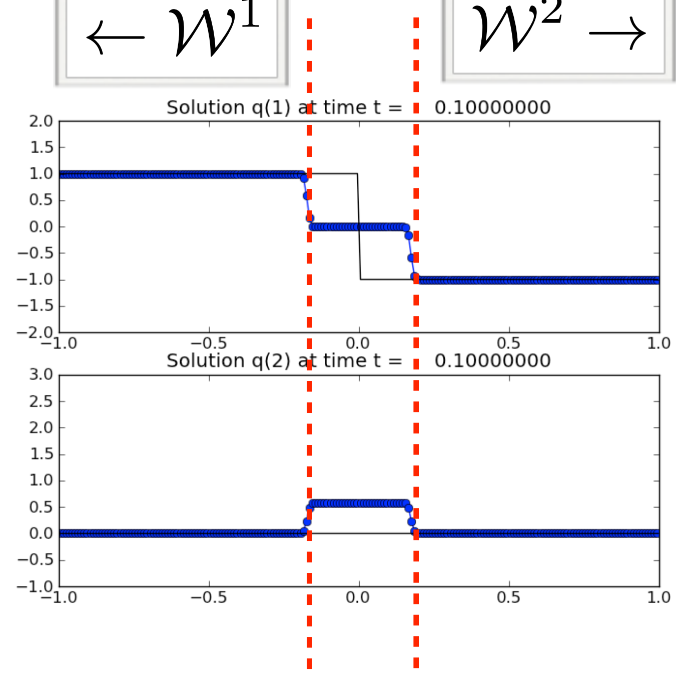


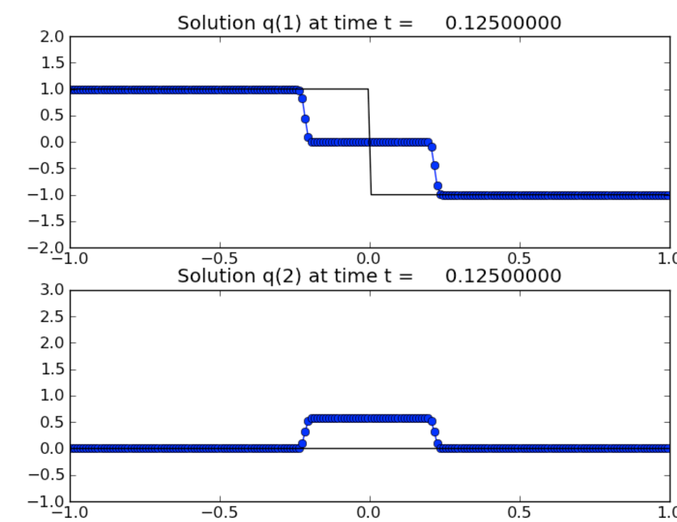




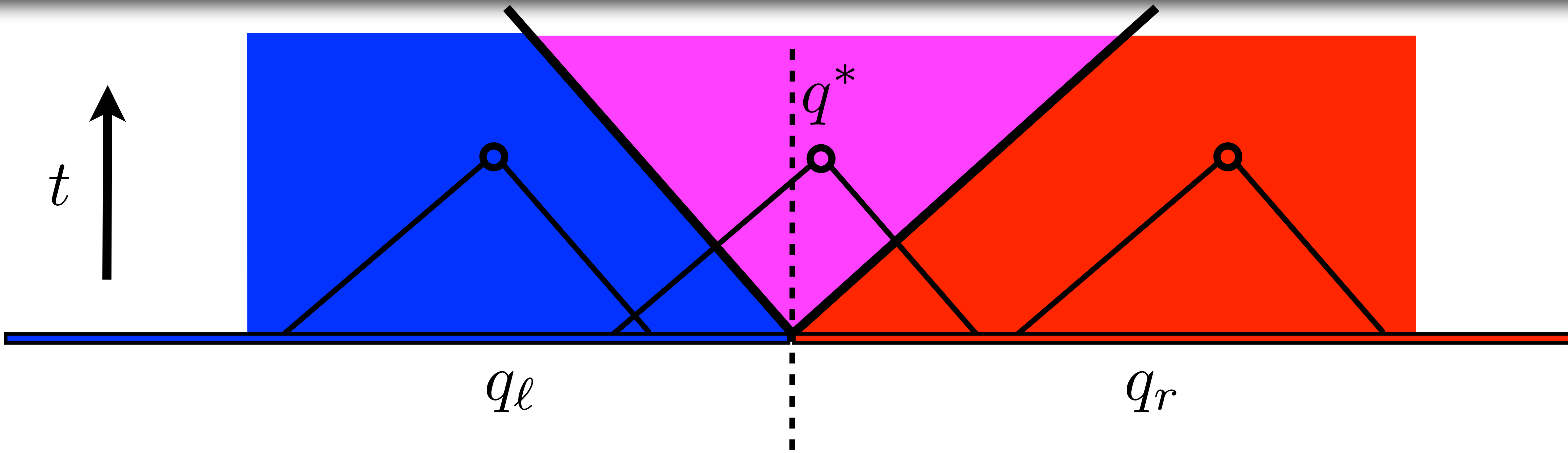
$\leftarrow \mathcal{W}^1$ $\mathcal{W}^2 \rightarrow$







Extending to nonlinear systems



$$q^* - q_\ell = \alpha^1 r^1$$

$$A(q^* - q_\ell) = \alpha^1 A r^1$$

$$A(q^* - q_\ell) = \lambda^1 (\alpha^1 r^)$$

$$A(q^* - q_\ell) = \lambda^1 (q^* - q_\ell)$$

Rankine-Hugoniot condition for the constant coefficent linear system

$$f(q) = Aq \quad \rightarrow \quad f(q^*) - f(q^\ell) = \lambda^1 (q^* - q_\ell)$$

Rankine-Hugoniot conditions

For a 2x2 linear system, we have

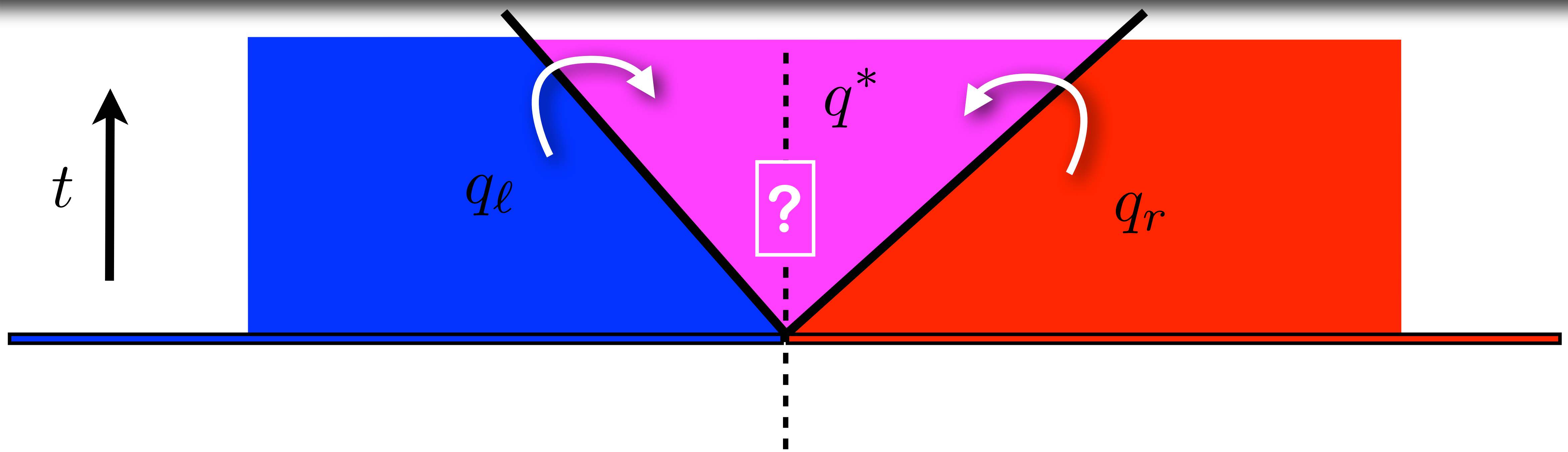
$$A(q^* - q_\ell) = \lambda^1 (q^* - q_\ell)$$
$$A(q_r - q^*) = \lambda^2 (q_r - q^*)$$

For $f(q) = Aq$, we can write this as :

$$f(q^*) - f(q_\ell) = \lambda^1 (q^* - q^\ell)$$
$$f(q_r) - f(q^*) = \lambda^2 (q_r - q^*)$$

The left and right states q_ℓ and q_r as "connected" by an intermediate state q^*.

Constant coefficient Riemann problem

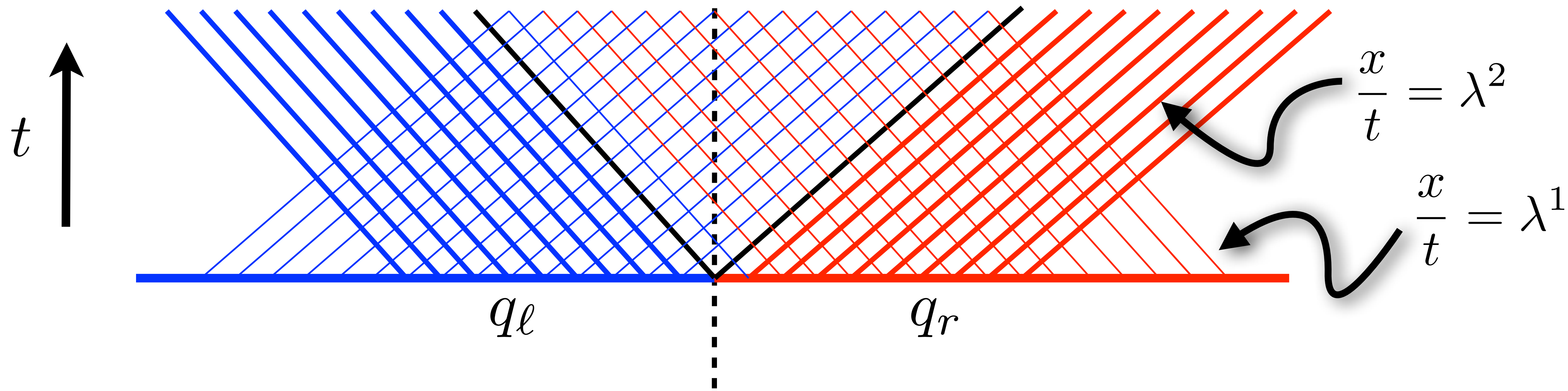


We could have asked "Find an intermediate state q^* such that

$$f(q^*) - f(q_\ell) = \lambda^1(q^* - q^\ell)$$
$$f(q_r) - f(q^*) = \lambda^2(q_r - q^*) \quad \text{"}$$

For $f(q) = Aq$, this leads to the eigenvalue problem that we solved.

Extending to nonlinear systems



Reminder : Solutions to the constant coefficient linear system travel along characteristic curves $(X(t), t)$:

$$\frac{d}{dt} q(X(t), t) = q_t + X'(t) q_x = 0$$

$$X'(t) q_x = A q_x$$

$X'(t)$ must be an eigenvalue of A, i.e. $X'(t) = \lambda^1, \lambda^2$

Solution remains constant along straight lines

Nonlinear shallow water wave equations

$$q_t + f(q)_x = 0$$

where

$$q = \begin{pmatrix} h \\ hu \end{pmatrix}, \qquad f(q) = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix}$$

for smooth solutions, this can also be written as

$$q_t + f'(q)q_x = 0$$

where

$$f'(q) = \begin{pmatrix} 0 & 1 \\ -u^2 + gh & 2u \end{pmatrix}$$

Show!

is the *flux Jacobian matrix.*

What changes in the nonlinear case?

$$q_t + f(q)_x = 0$$

We can still ask "Are there characteristic curves on which the solution remains constant?

$$\frac{d}{dt} q(X(t), t) = q_t + X'(t) q_x = 0$$

For smooth solutions, we have

$$q_t + f'(q) q_x = 0$$

where $f'(q) \in \mathcal{R}^{m \times m}$ is the flux Jacobian matrix.

$$\longrightarrow \qquad f'(q) q_x = X'(t) q_x$$

Characteristics are governed by eigenvalues of the flux Jacobian

Shallow water wave equations

$$q = \begin{pmatrix} h \\ hu \end{pmatrix}, \qquad f(q) = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix}$$

$$f'(q) = \begin{pmatrix} 0 & 1 \\ -u^2 + gh & 2u \end{pmatrix}$$

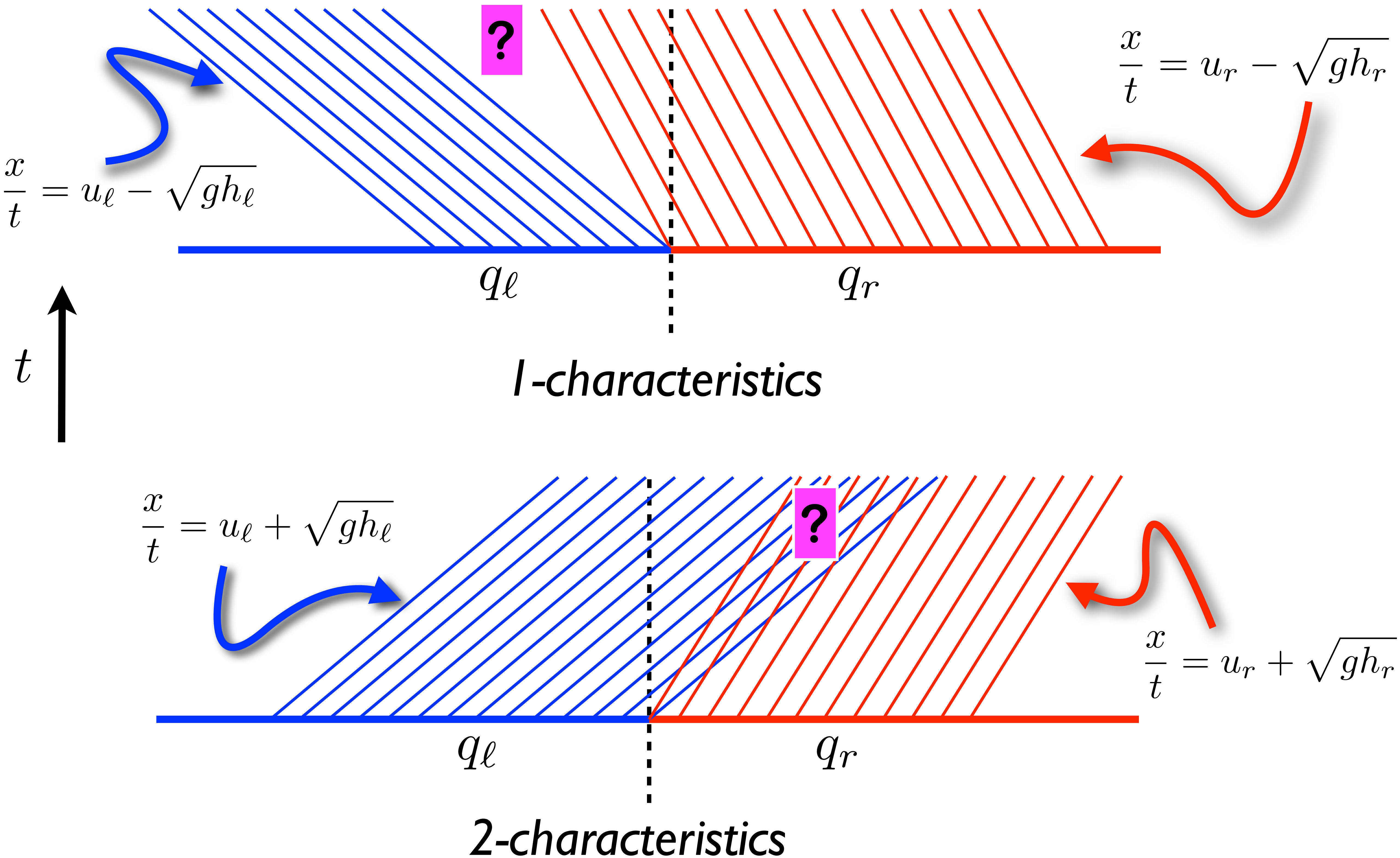
Eigenvalues and eigenvectors of the flux Jacobian $f'(q)$:

$$\lambda^1 = u - \sqrt{gh}, \qquad \lambda^2 = u + \sqrt{gh}$$

$$r^1 = \begin{pmatrix} 1 \\ u - \sqrt{gh} \end{pmatrix}, \qquad r^2 = \begin{pmatrix} 1 \\ u + \sqrt{gh} \end{pmatrix}$$

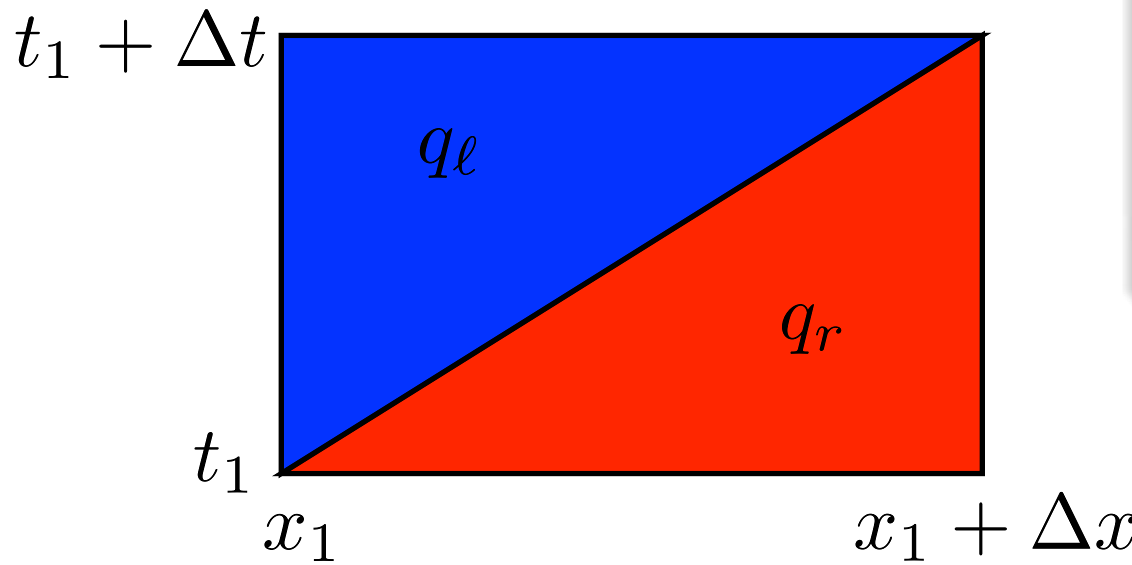
Eigenvalues and eigenvectors depend on q!

What can happen?



Riemann solution for the SWE

Consider the case where $\lambda^p(q_\ell) > \lambda^p(q_r)$:



Assume that left and right states are constant in this infinitesimal box

Using the conservation law, we can write

$$\frac{d}{dt}\int_{x_1}^{x_1+\Delta x} q(x,t)dx \quad + \quad \int_{x_1}^{x_1+\Delta x} f(q(x,t))_x \, dx = 0$$

Show!

$$\frac{q_\ell - q_r}{\Delta t} + \Delta x(f(q_r) - f(q_\ell)) = 0$$

Riemann problem for SWE

This leads to

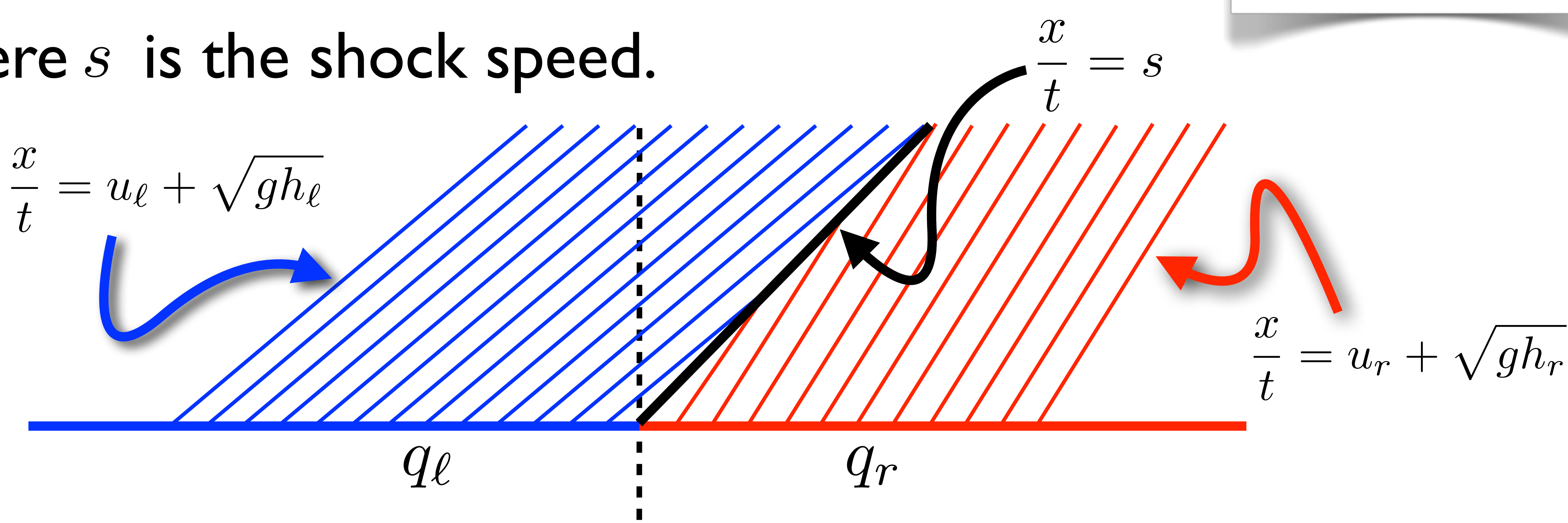
$$f(q_r) - f(q_\ell) = \frac{\Delta x}{\Delta t}(q_r - q_\ell)$$

This is the required jump condition across shocks. More generally we can write this condition as

$$f(q_r) - f(q_\ell) = s(q_r - q_\ell)$$

Rankine Hugoniot jump condition

where s is the shock speed.

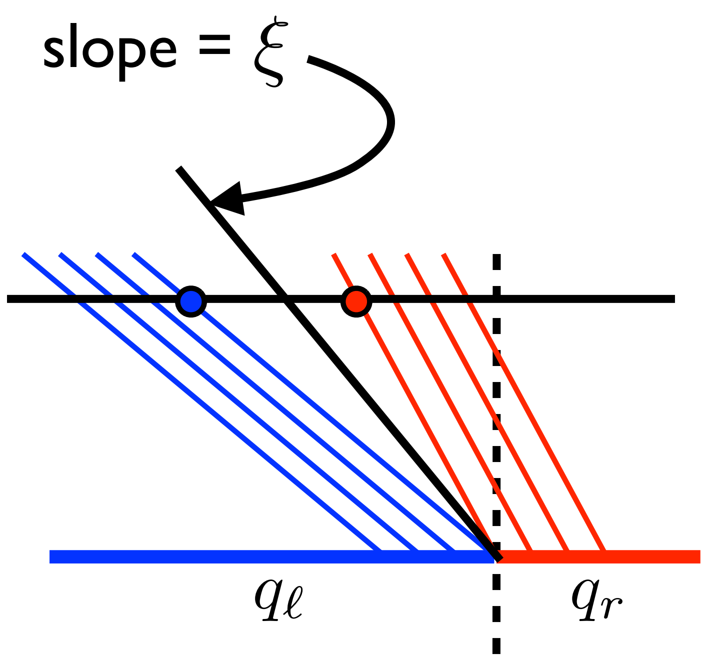


Rarefaction waves

Consider the case where $\lambda^p(q_\ell) < \lambda^p(q_r)$:

Let $\xi = \dfrac{x}{t}$ be the slope of the characteristic. We need to find $q(\xi)$ for $\lambda^1(q_\ell) < \xi < \lambda^1(q_r)$. Recall that $f'(q(\xi))q'(\xi) = \xi q'(\xi)$. Then

$$q'(\xi) = \alpha(\xi) r^1(q(\xi))$$



$$\xi = \lambda^1(q(\xi))$$

$$\rightarrow 1 = \nabla\lambda^1(q(\xi)) \cdot q'(\xi)$$

$$= \alpha(\xi)\nabla\lambda^1(q(\xi)) \cdot r^1(q(\xi))$$

$$\rightarrow q'(\xi) = \frac{r^1(q(\xi))}{\nabla\lambda^1(q(\xi)) \cdot r^1(q(\xi))}$$

The denominator is never 0!

Solve resulting ODE to get $q(\xi)$ in the centered rarefaction.

Centered rarefaction

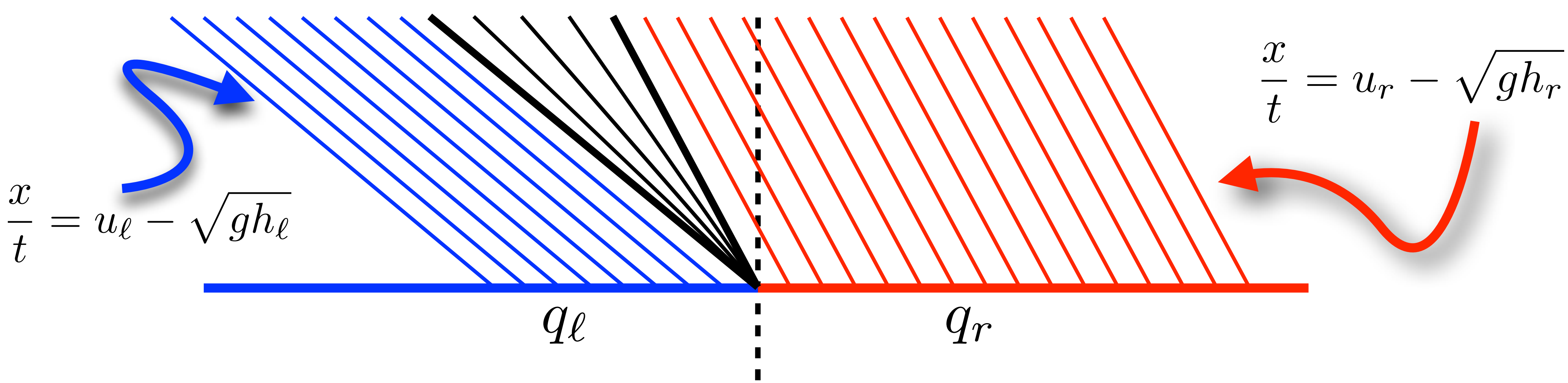
Solve the system of two ODEs (for SWE) :

$$q'(\xi) = \frac{r^1(q(\xi))}{\nabla\lambda^1(q(\xi)) \cdot r^1(q(\xi))}$$

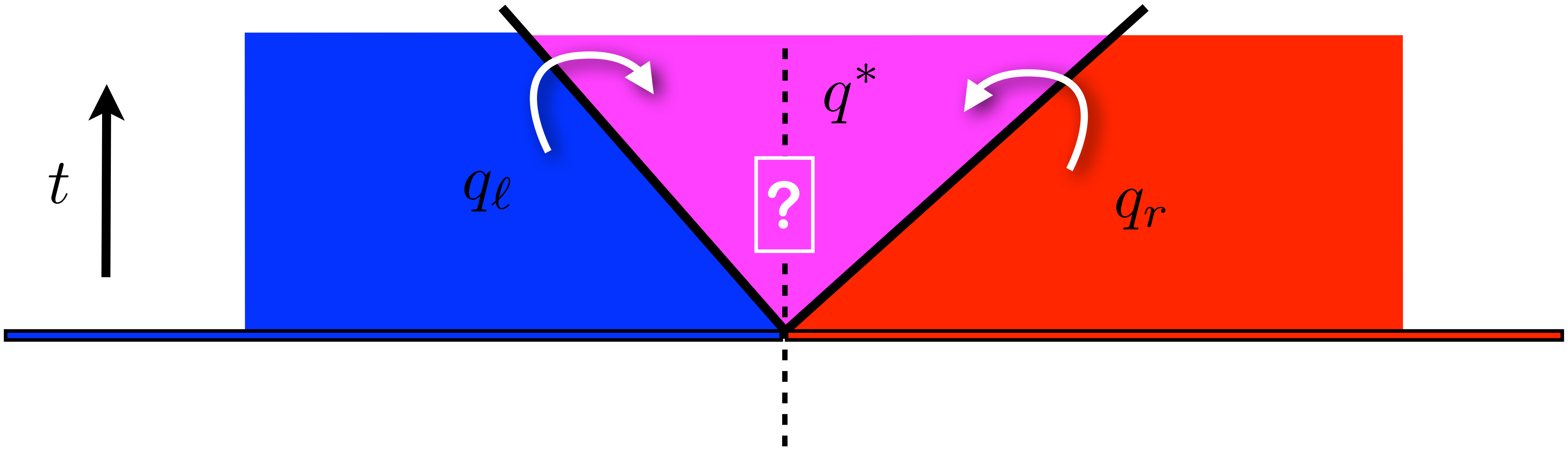
subject to

$$\xi_1 = \lambda^1(q_\ell), \qquad q(\xi_1) = q_\ell$$
$$\xi_2 = \lambda^1(q_r), \qquad q(\xi_2) = q_r$$

Use *Riemann invariants* to solve for unknown constants.



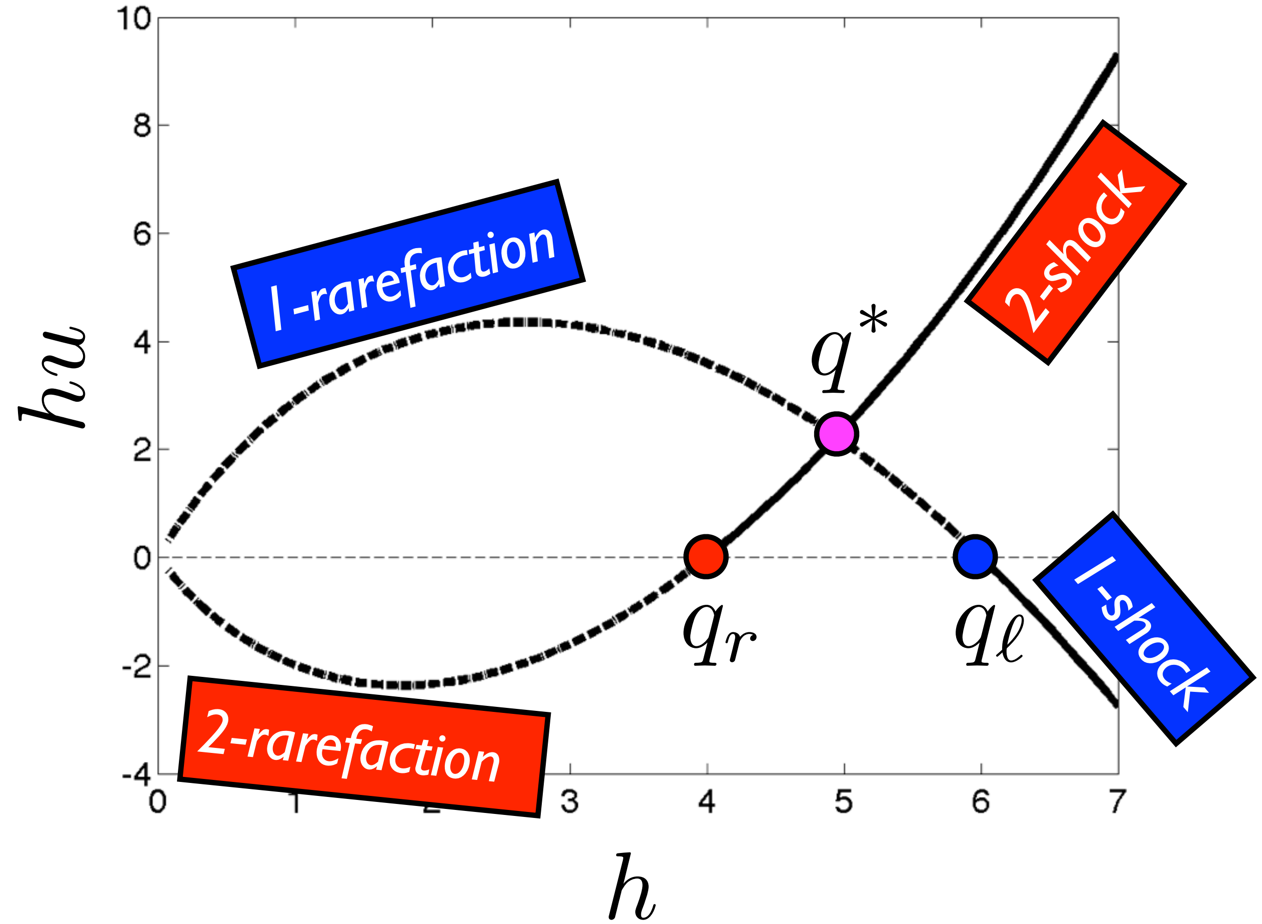
Solving the Riemann problem



Find a state q^* such that q_ℓ is connected to q^* by a physically correct 1-shock wave or 1-rarefaction, *and* q_r is connected to q^* by a physically correct 2-shock or 2-rarefaction.

The need to find a state q^* that *simultaneously* satisfies both conditions above means we have to solve something...

Solving the Riemann problem

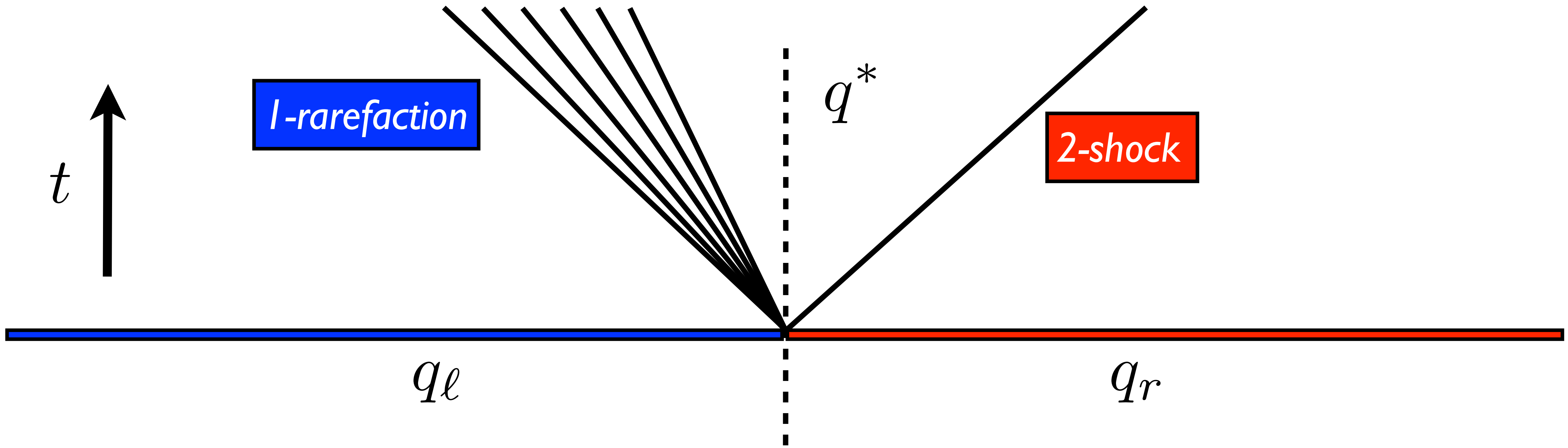


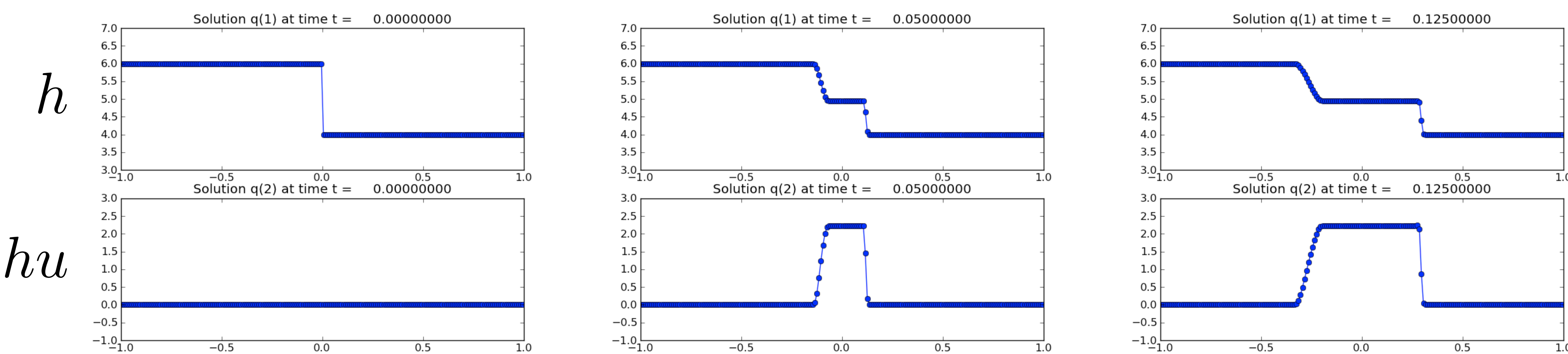
Formulas for these curves are relatively simple.

Curves represent states that can be connected to q_r or q_ℓ by a shock or a rarefaction. Use a nonlinear root-finder to find the middle state q^*.

- Determine the structure of the rarefaction (if there is one).

Riemann solution





The structure of the Riemann solution depends on the initial conditions.

Next :

- Can we avoid the nonlinear solve?
- How does this extend to the two dimensional shallow water equations?
- How does GeoClaw use Riemann solvers?

For details, see *Finite Volume Methods for Hyperbolic Problems, R. J. LeVeque (Cambridge University Press, 2002).*

Lab sessions :

- Experiment with linear SWE using ClawPack.
- Experiment with nonlinear SWE using ClawPack.

Goals :

- Learn about how Riemann solvers are used in Clawpack and GeoClaw
- Include bathymetry to see the effects of well-balancing.
- Learn about various plotting parameters in Clawpack
- Leave Chile with a simple 1d solver for the linear and nonlinear shallow water wave equations.

See PASI website on Piazza for website describing project.