Tsunami modeling

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TSUNAMI MODELING

1. **Tsunami characteristics**
2. **Mathematical models for earthquake generated tsunamis**
3. **Model results and case studies.**
4. **Mathematical models for landslide generated tsunamis**

What are tsunamis?

Tsunamis are water waves generated by a geophysical event:

Earthquake

Landslide

Volcano eruption/collapse

1896 Meiji tusnami

津波

tsu-nami

harbor waves

海嘯

hai-shou

sea outcry

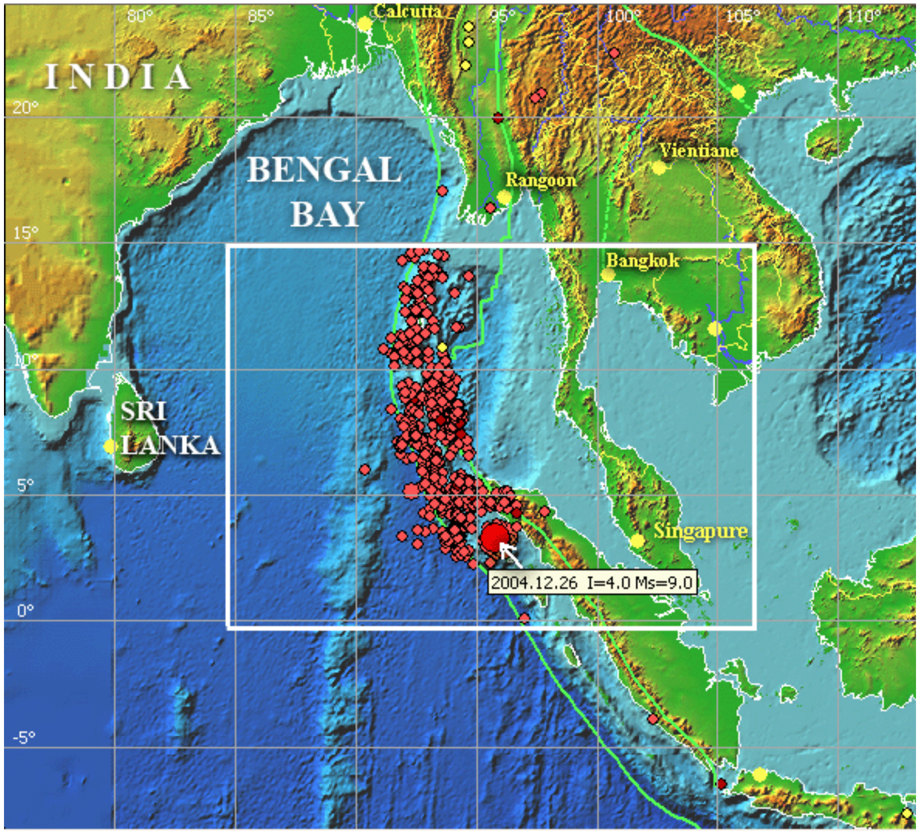
Motivations for tsunami research

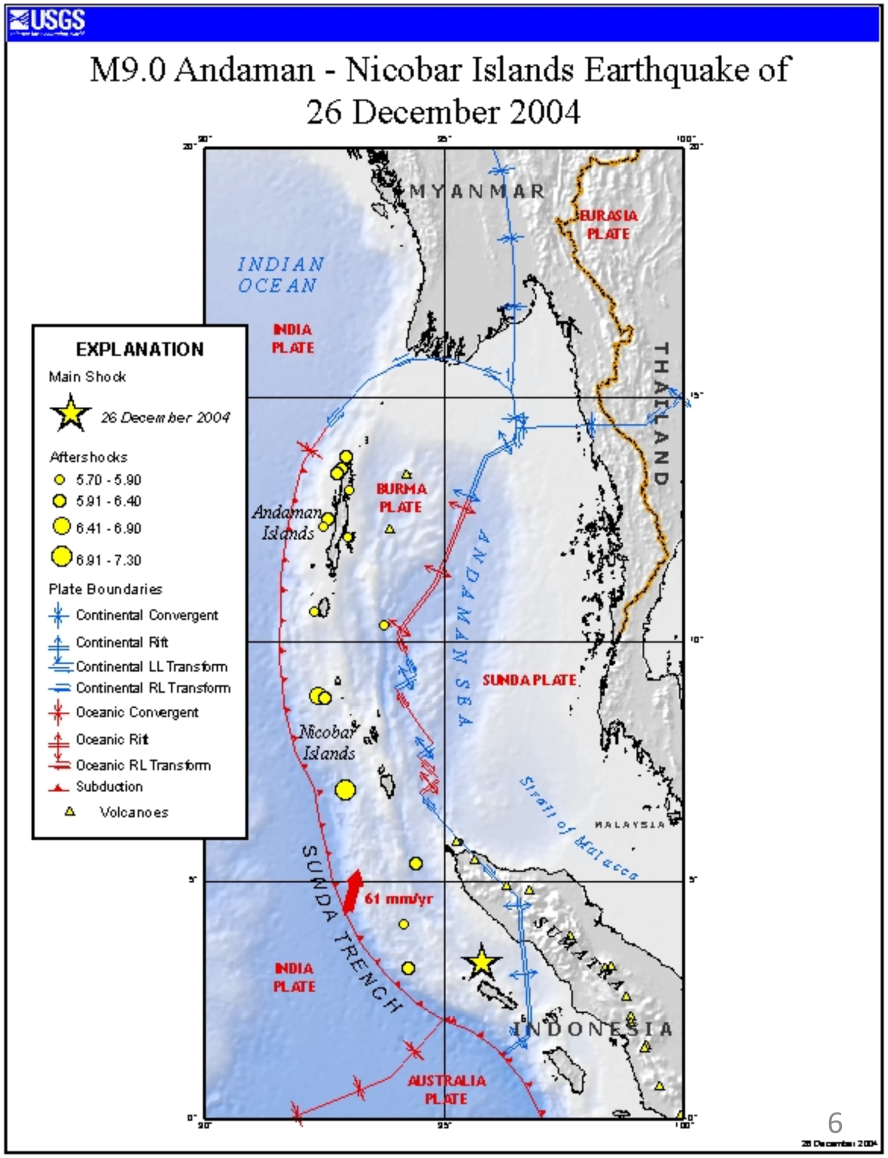
Two recent tsunamis generated by mega earthquakes

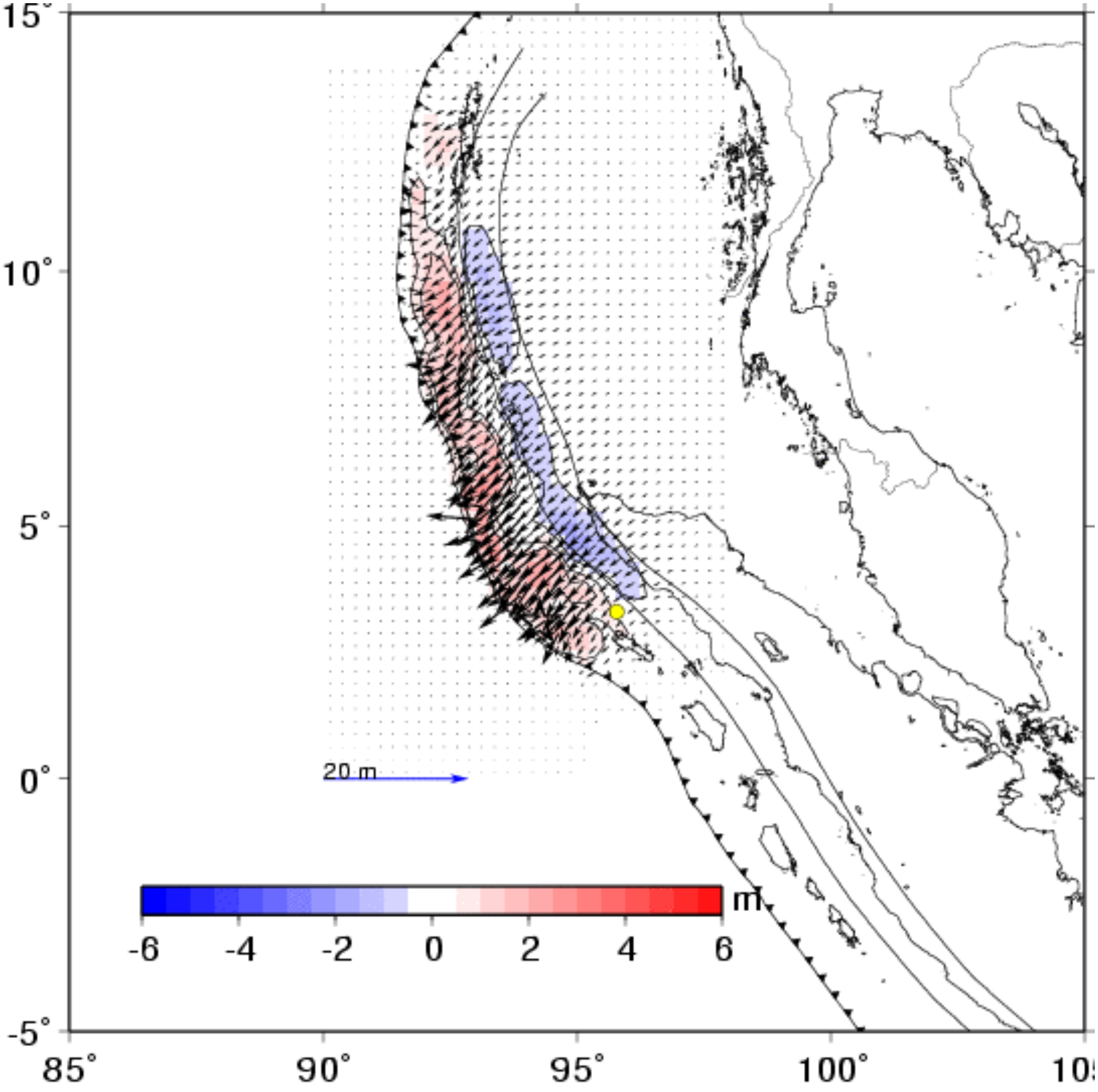
- **December 26, 2004 Indian Ocean Tsunamis**
- **March 11, 2011 Japan-Tohoku Tsunamis**

December 26 2004 Indian Ocean Tsunamis

Epicenter and after shocks







2004 Indian Ocean E/T

Rupture speed:
2 ~ 3 km/s

Rupture duration:
10 mins

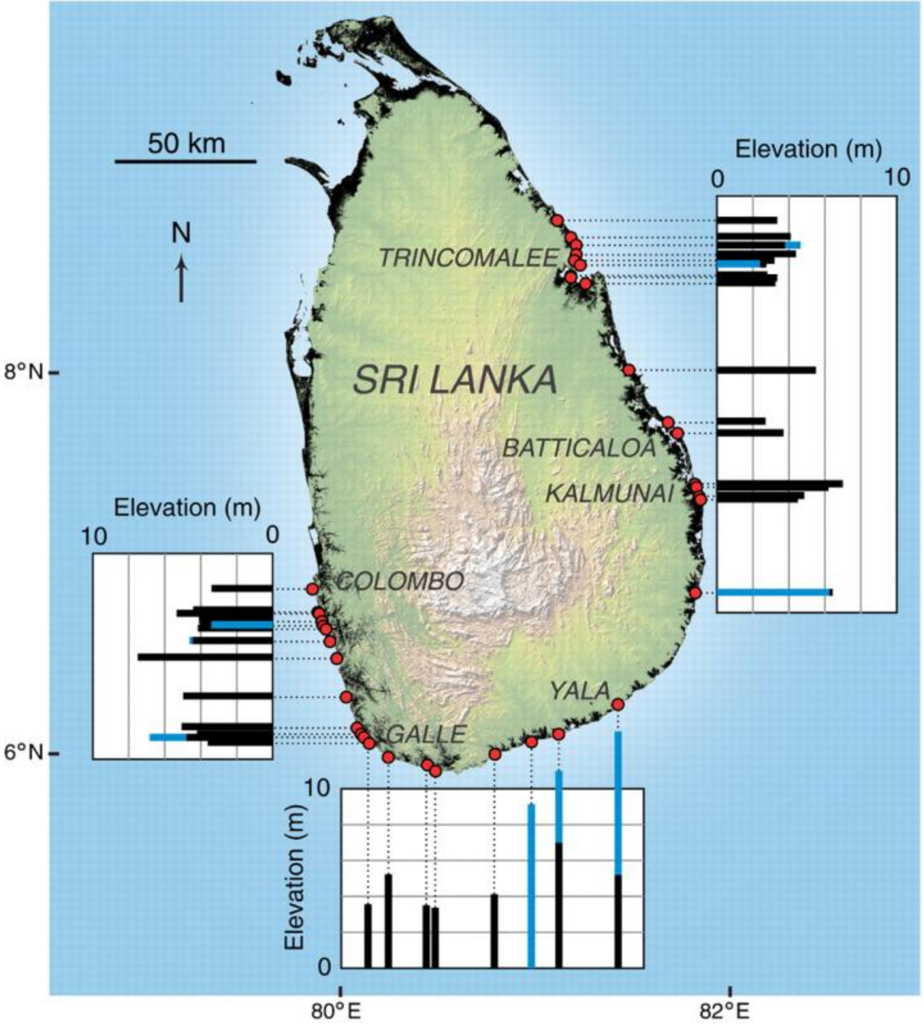
Fault Plane Width:
150 ~ 300 km

Maximum horizontal displacement: 20 m

Color scales the vertical displacement

2004 Indian Ocean Tsunami

Observations by the International Tsunami Survey Team in Sri Lanka (Liu et al., Science 2005)



Measured tsunami runups (blue) and maximum tsunami heights (black). Red dots show sites of elevation measurement; areas shaded in black are less than 10 m above sea level.

Sumatra earthquake and tsunami caused nearly 230,000 deaths and $10 billion in damage.

Measured in lives lost, this is one of the ten worst earthquakes in recorded history, as well as the single worst tsunami in history.

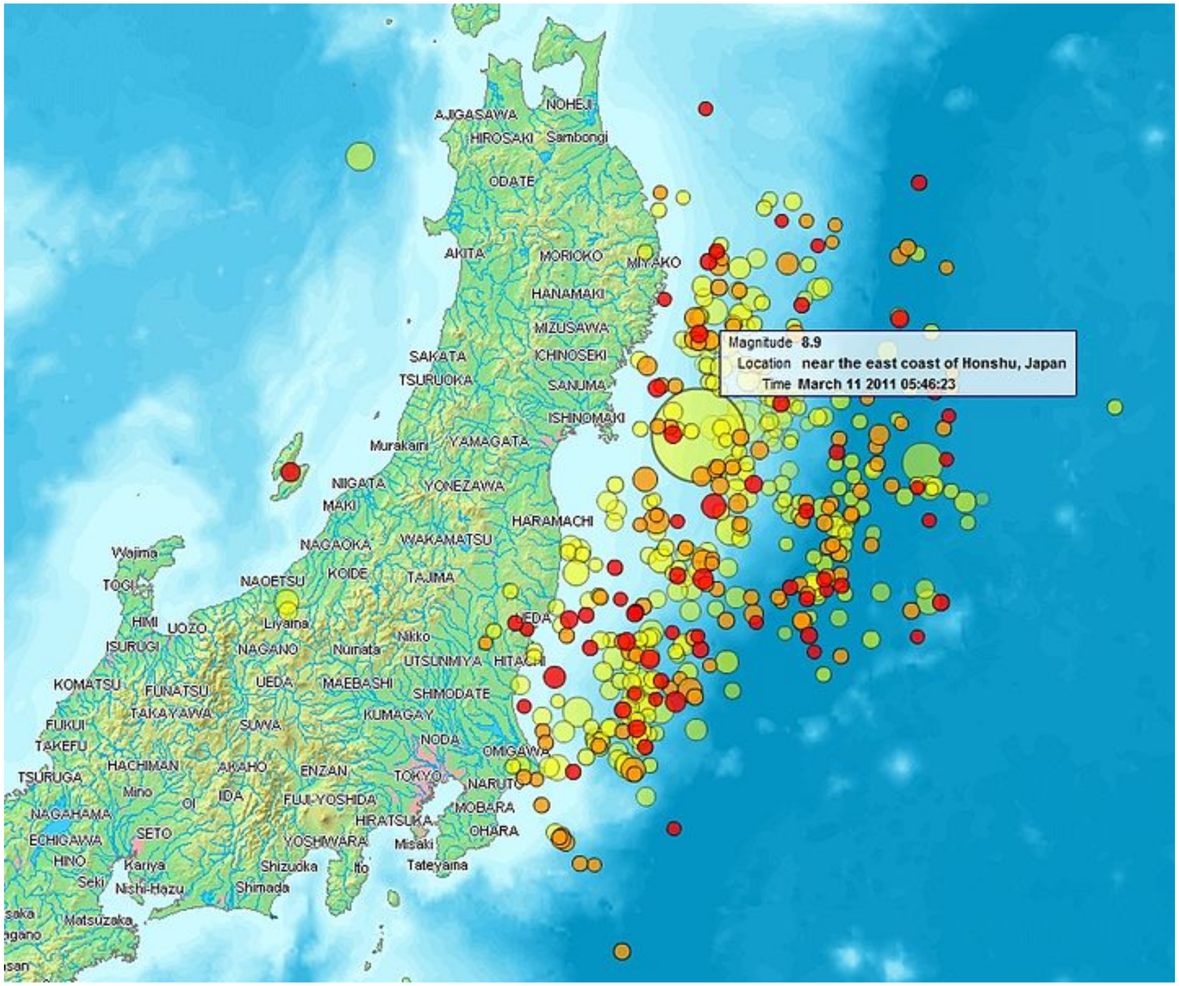
Indonesia was the worst affected area, with most death toll estimates at around 170,000.

The death toll is Sri Lanka and India is more than 50,000.   Tsunami caused serious damage and deaths as far as the east coast of Africa, with the farthest recorded death due to the tsunami occurring at Rooi Els in South Africa, 8,000 km (5,000 mi) away from the epicenter. In total, eight people in South Africa died due to abnormally high sea levels and waves.

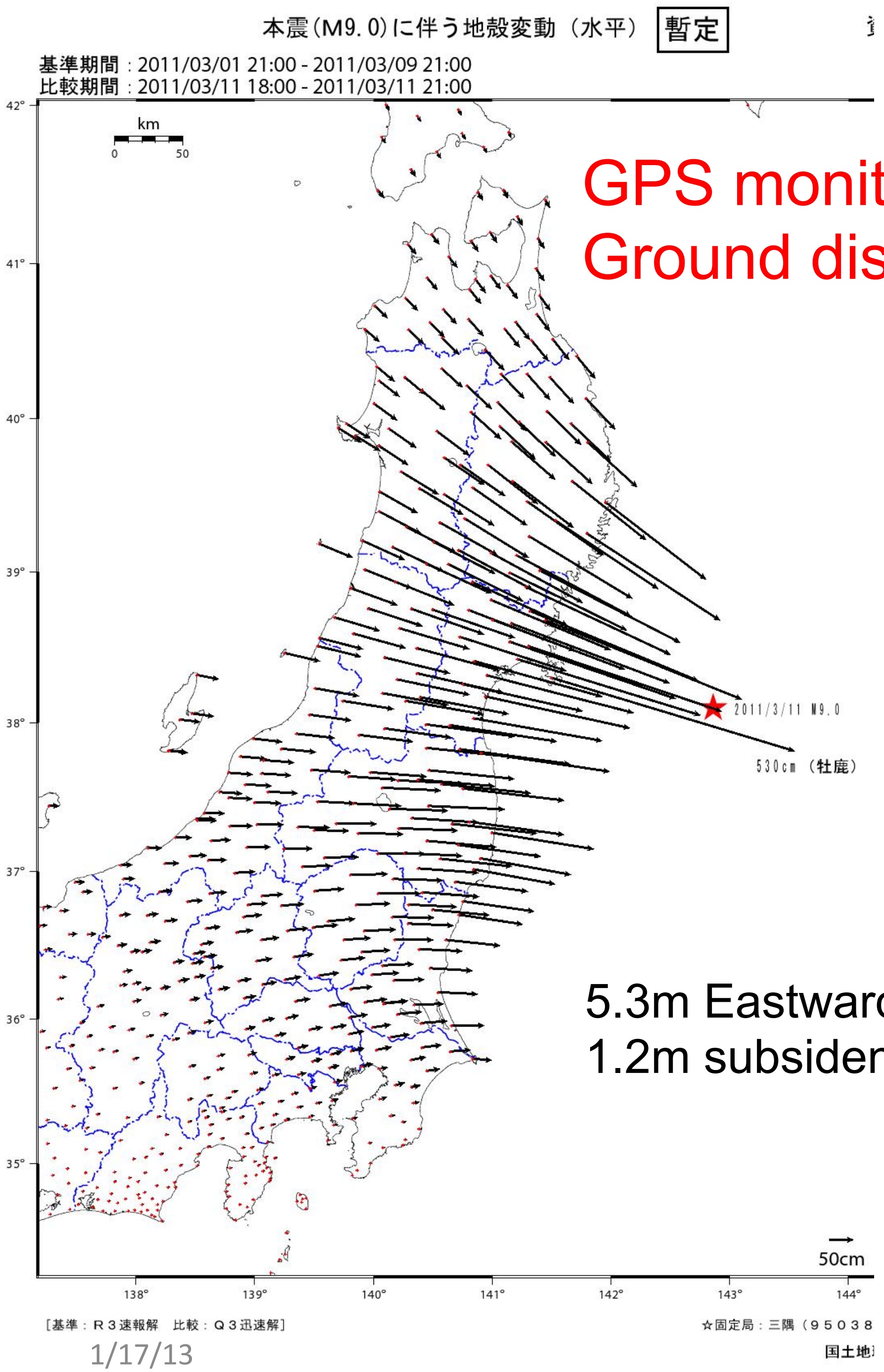
2011 Japan-Tohoku Earthquake/Tsunami

The **2011 earthquake off the Pacific coast of Tōhoku** was a magnitude 9.0 (M_w) undersea mega-thrust earthquake off the coast of Japan that occurred at 14:46 JST (05:46 UTC) on March 11, 2011,with the epicenter approximately 70 kilometers (43 mi) east of the Oshika Peninsula of Tōhoku and the hypocenter at an underwater depth of approximately 32 km (20 mi).

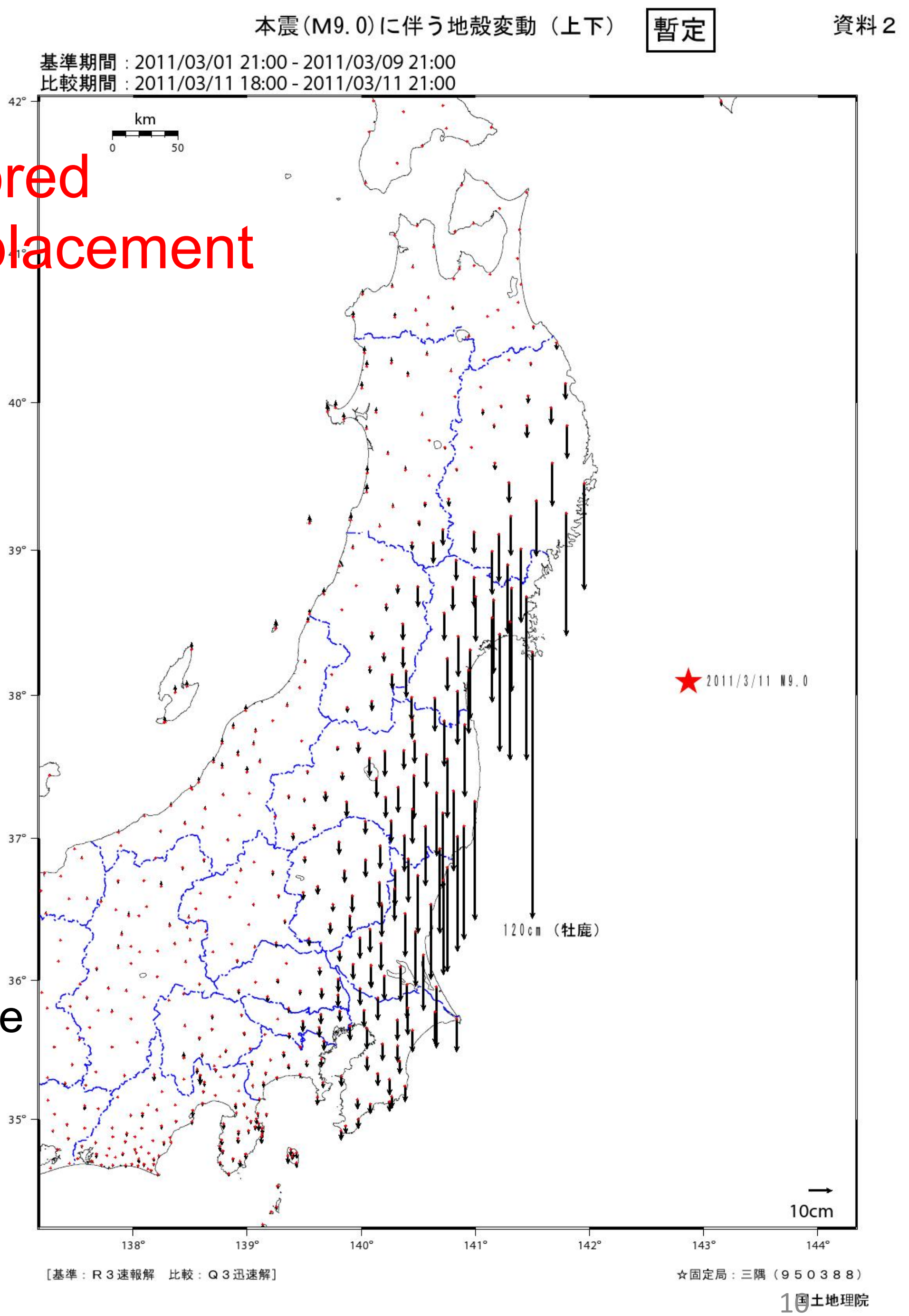
Aftershocks compiled by JMA



GPS monitored Ground displacement

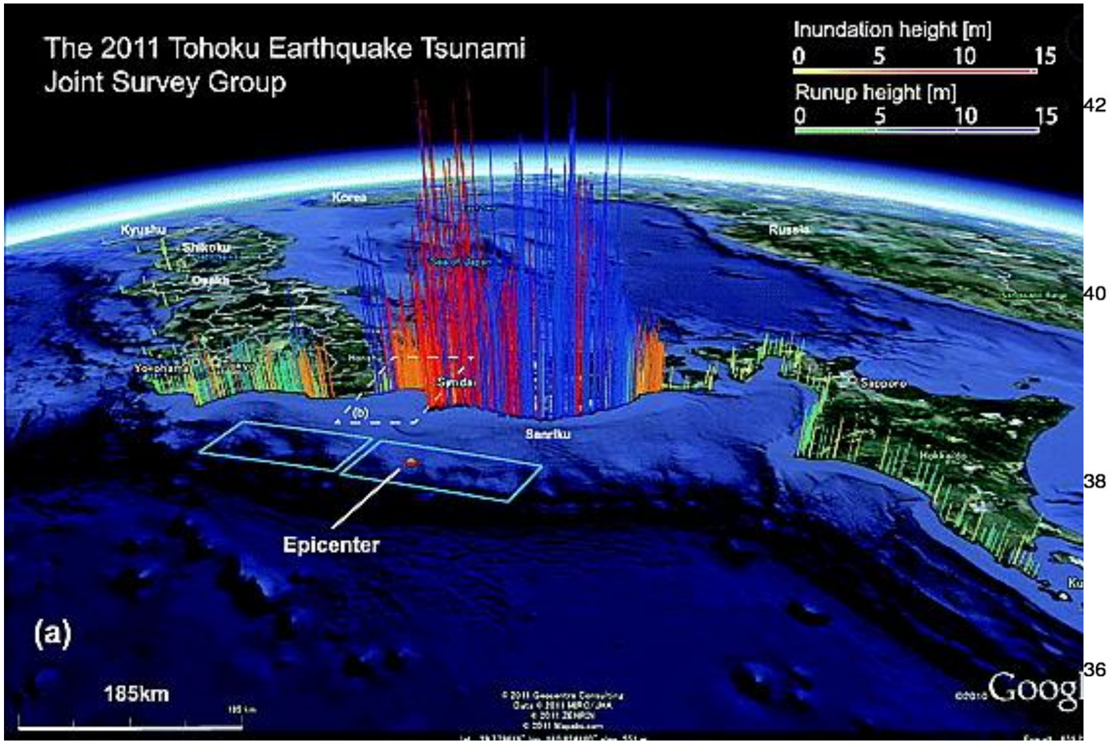


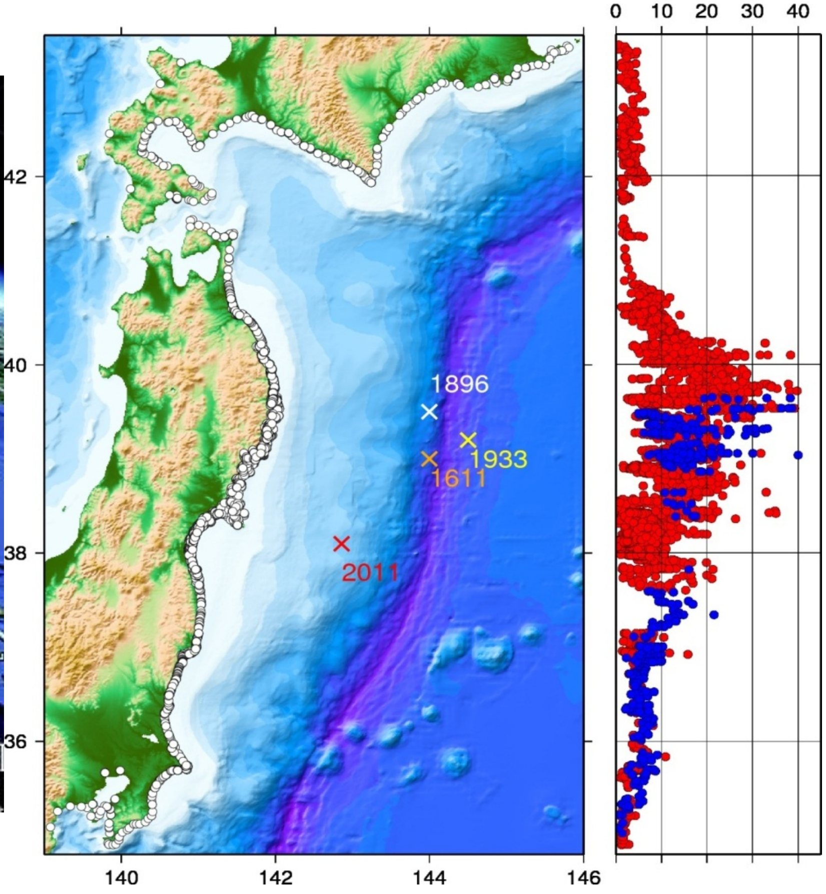
5.3m Eastward
1.2m subsidence



2011 Japan-Tohoku Earthquake/Tsunami

The earthquake triggered powerful tsunami waves that reached heights of up to 40.5 meters (133 ft) in Miyako in Tōhoku's Iwate Prefecture, and which, in the Sendai area, travelled up to 10 km (6 mi) inland.





2011 Japan-Tohoku Earthquake/Tsunami

- On 12 March 2012, a Japanese National Police Agency report confirmed 15,861 deaths, 6,107 injured, and 3,018 people missing across twenty prefectures, as well as 129,225 buildings totally collapsed, with a further 254,204 buildings 'half collapsed', and another 691,766 buildings partially damaged. The earthquake and tsunami also caused extensive and severe structural damage in north-eastern Japan, including heavy damage to roads and railways as well as fires in many areas, and a dam collapse.

- The tsunami caused a number of nuclear accidents, primarily the ongoing level 7 meltdowns at three reactors in the Fukushima Daiichi Nuclear Power Plant complex, and the associated evacuation zones affecting hundreds of thousands of residents.

- Early estimates placed insured losses from the earthquake alone at US$14.5 to $34.6 billion. The Bank of Japan offered ¥15 trillion (US$183 billion) to the banking system on 14 March in an effort to normalize market conditions. The World Bank's estimated economic cost was US$235 billion, making it the most expensive natural disaster in world history.

Objectives of tsunami research

- Forecasting tsunami: arrival time and wave amplitude
- Tsunami hazard mitigation: inundation/hazard maps, coastal management/planning, coastal structure design. Education/ communication

Developing research/engineering tools

- Appropriate mathematical models describing tsunami physics
- Accurate and efficient numerical models solving the mathematical model.

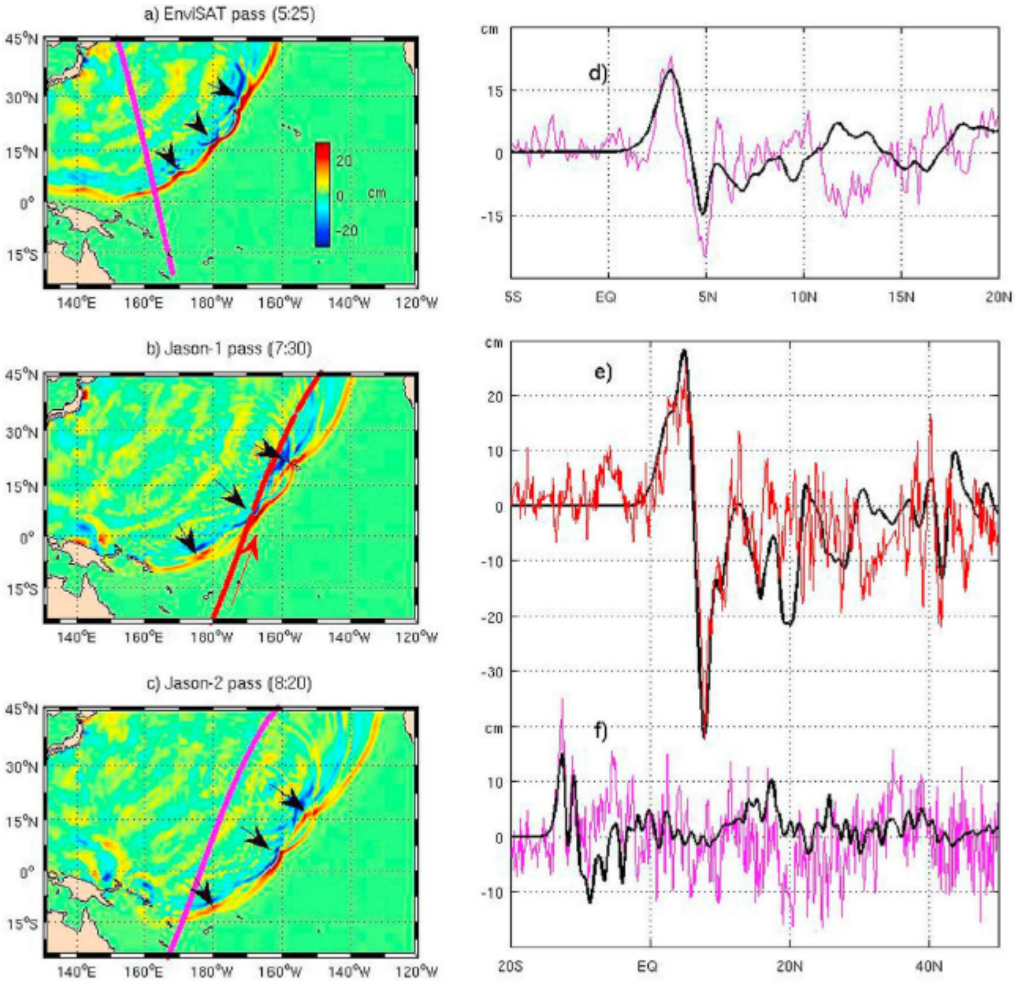
Characteristics of tsunamis in ocean basin

What can we learn from the field data?

2011 Tohoku Tsunamis
Satellite data
(Song et al. GRL, 39, 2012)

Amplitude < 0.3 m
Wavelength > 300 km

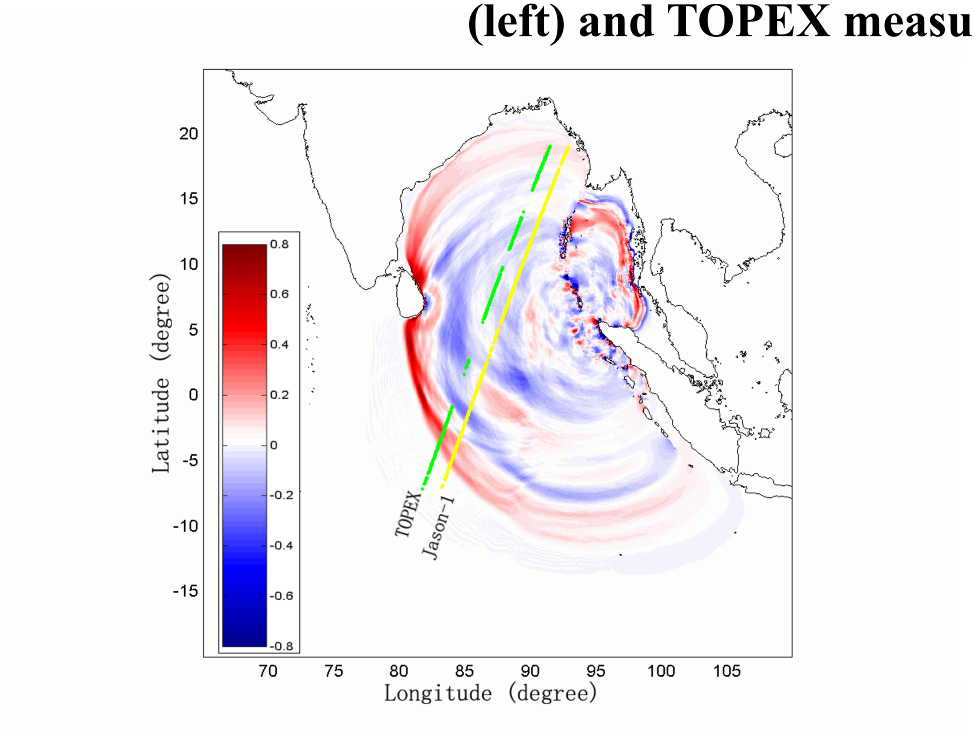
1 min = 1.8 km
1 degree = 111 km

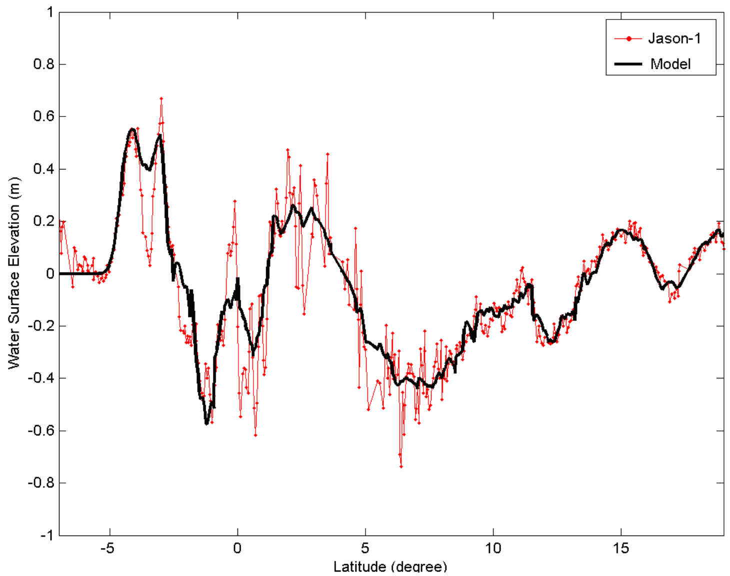


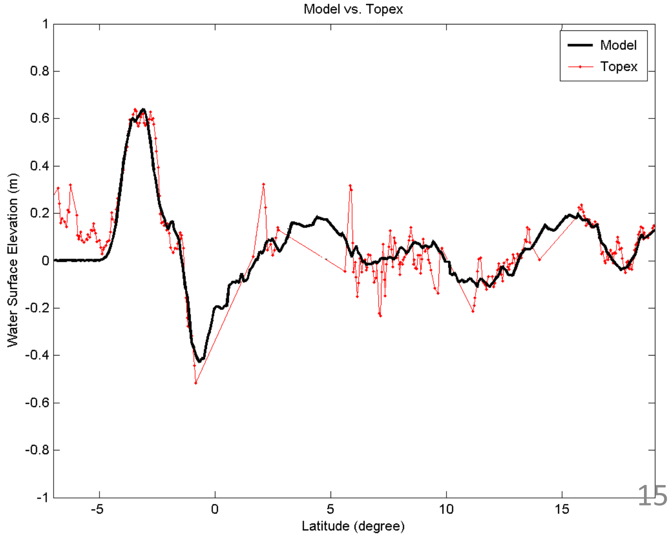
2004 Indian Ocean Tsunamis
Comparisons between model results and Jason-1 measurements

(Wang and Liu, JHR 44, 2006)

(left) and TOPEX measurements (right)



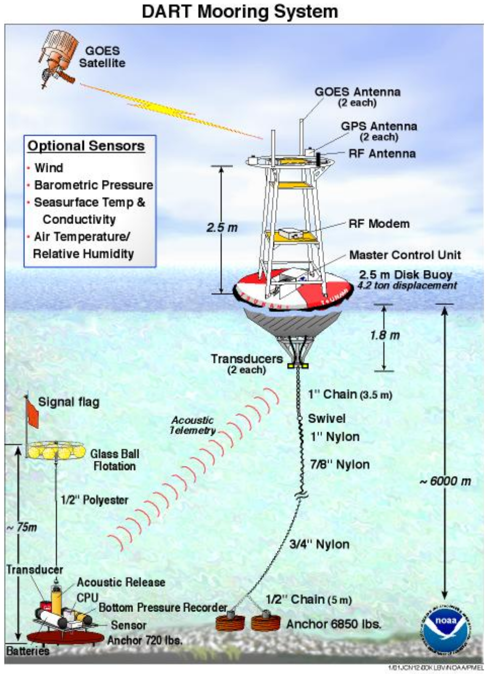


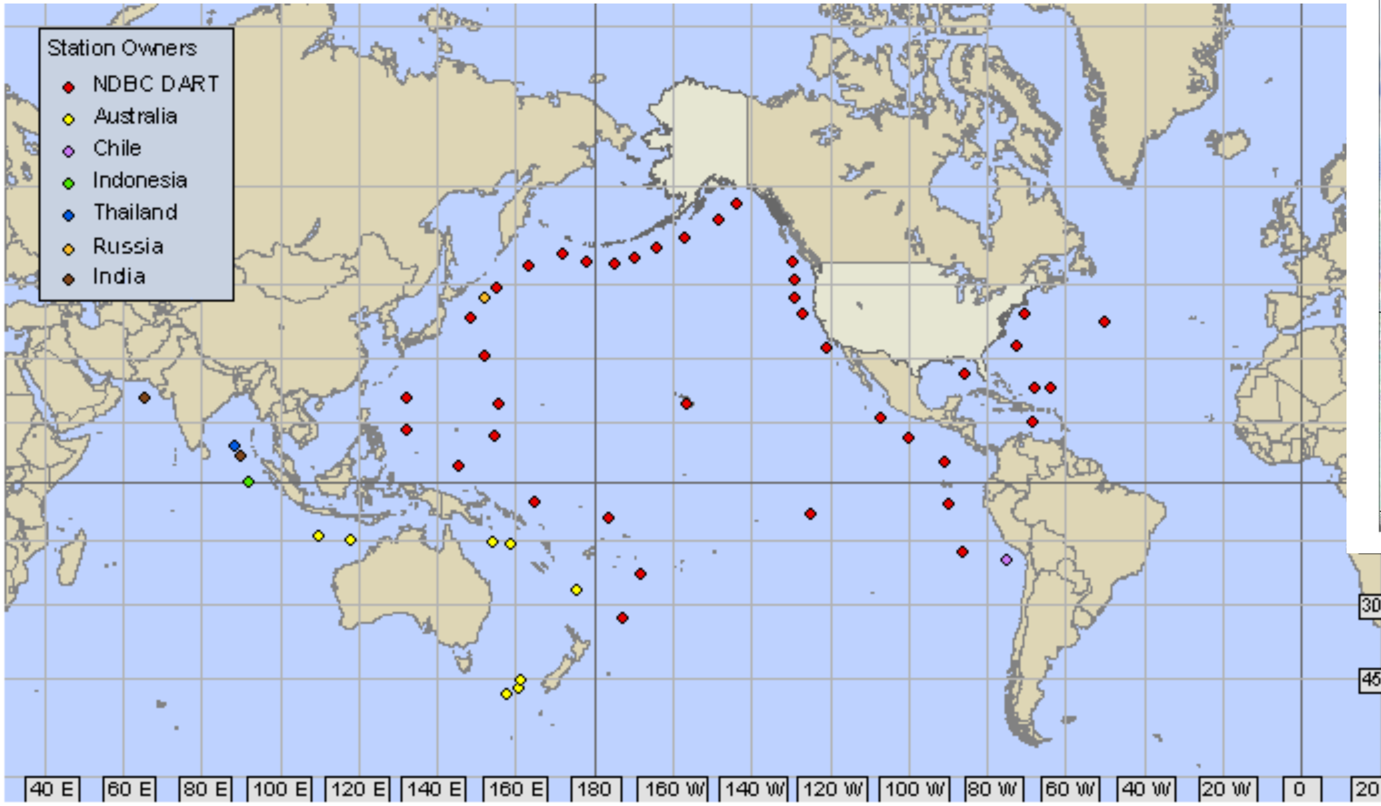


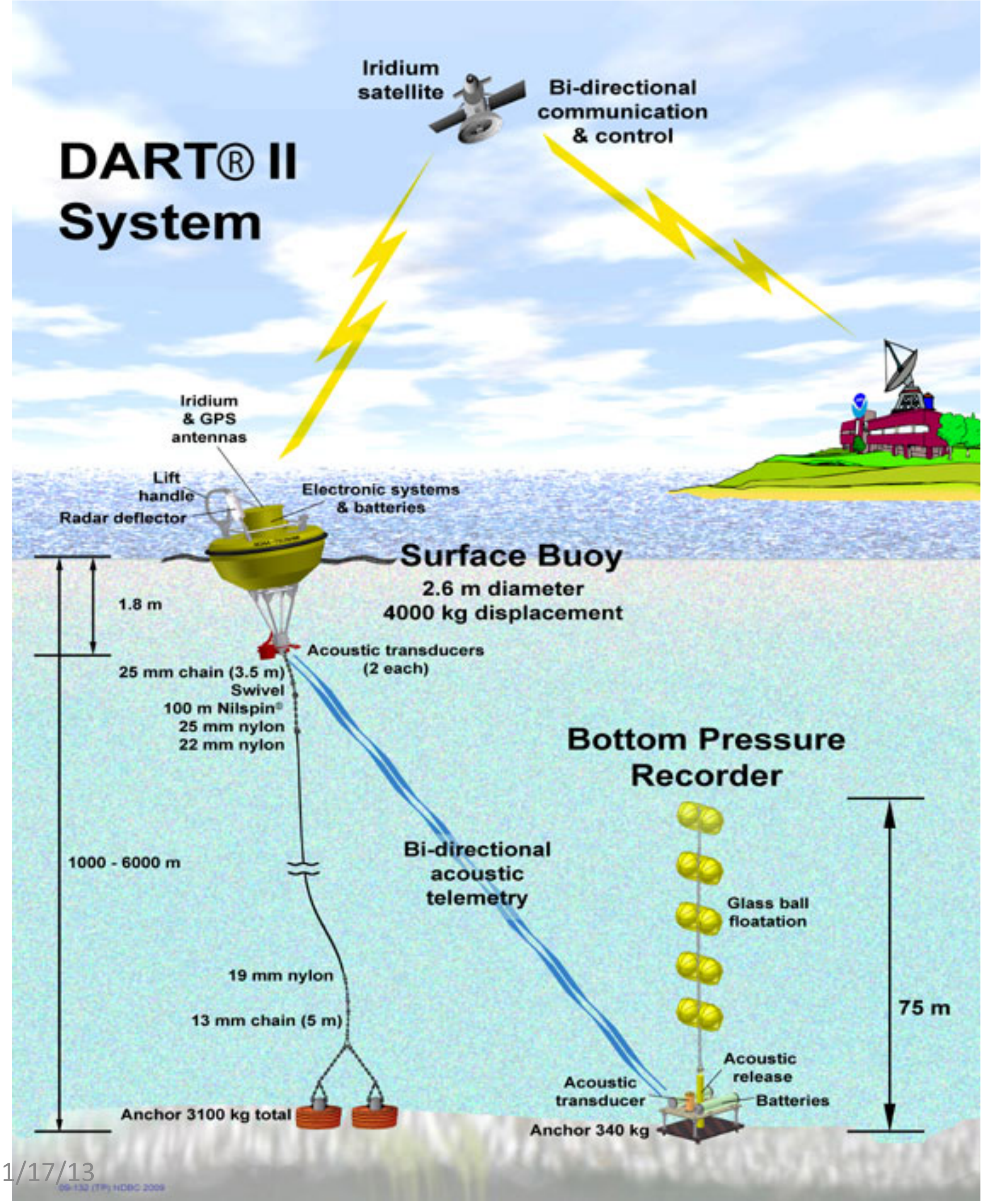
Wave amplitude <0.6 m
Wavelength > 300 km

Other field measurements

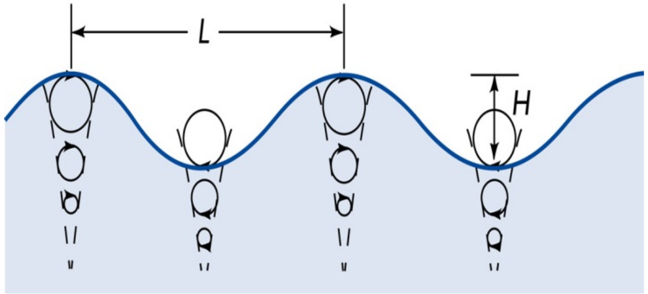
- DART: Deep-ocean Assessment and Reporting of Tsunamis
  - Real-time tsunami monitoring system
  - Tsunami warning system







Tsunami waves measured in deep ocean basin: DART BUOY SYSTEM



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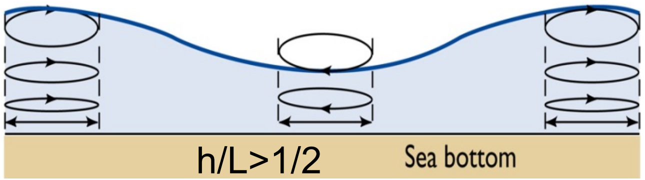
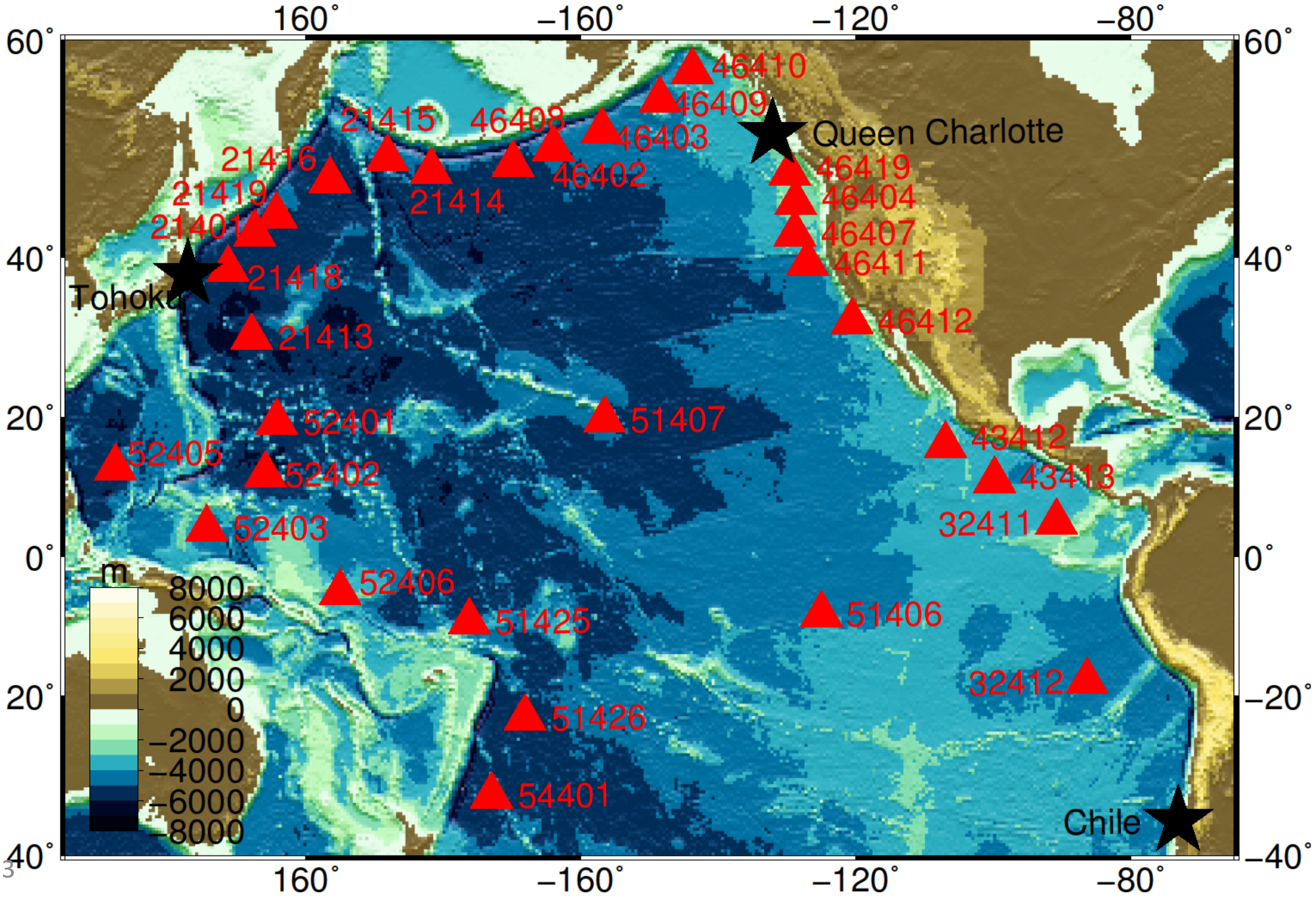


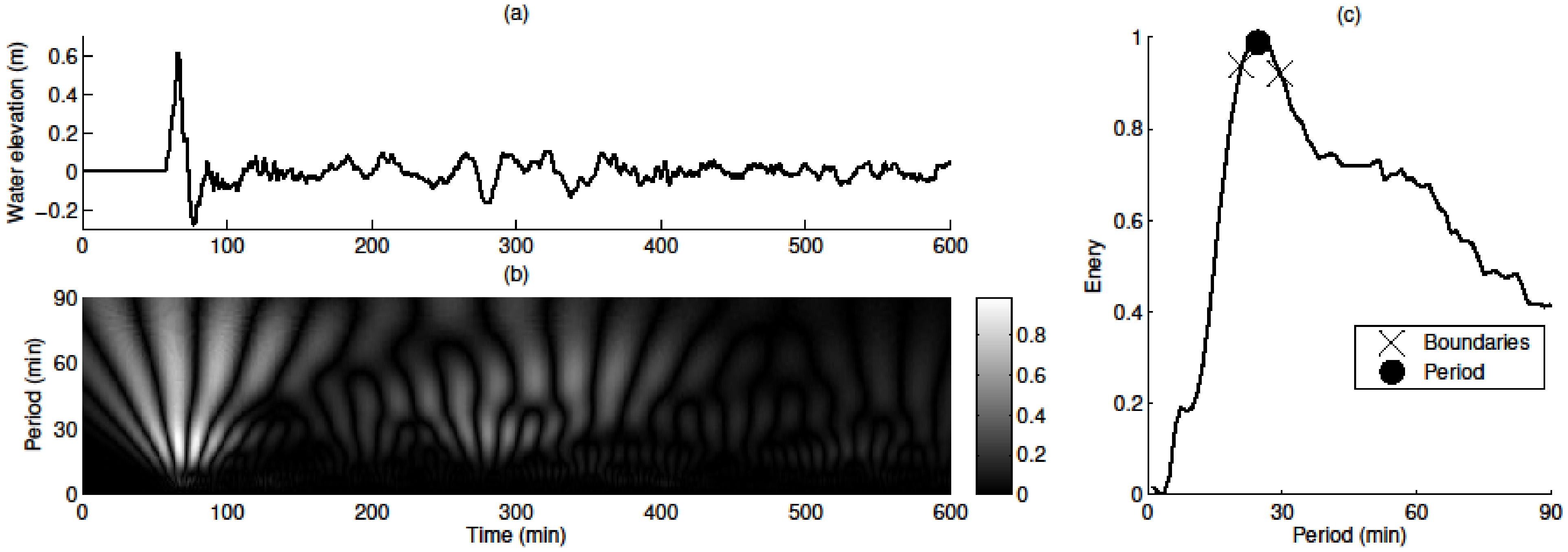
DART buoy data for three recent tsunamis:

9 DART stations for Queen-Charlotte tsunamis;

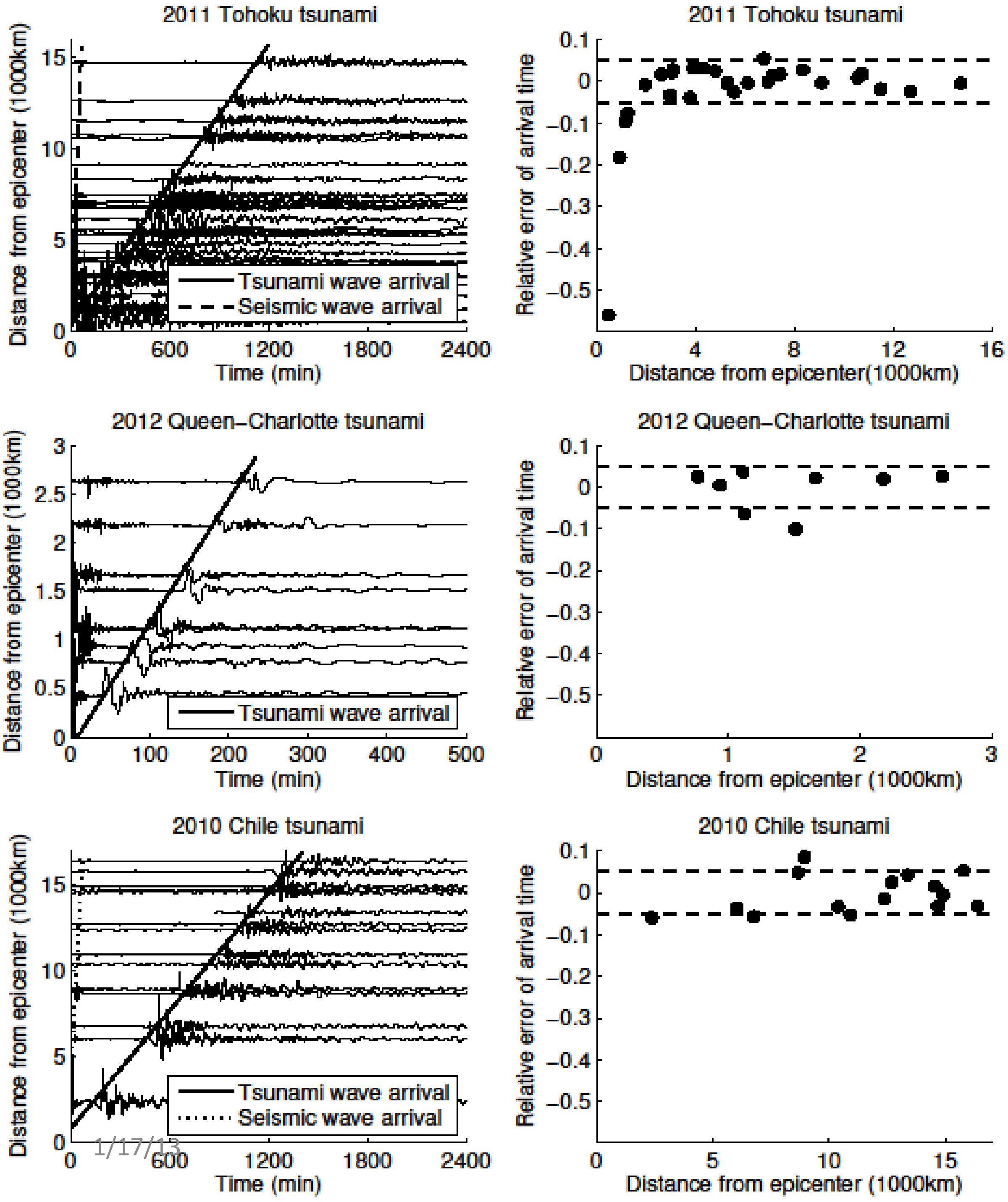
28 DART stations for 2011 Tohoku tsunamis;

15 DART stations for Chile tsunamis





Wavelet analysis of tsunami data at station 21401 for 2011 Tohoku event. (a): time series of wave elevation; (b): wavelet transformed result; (c): cross-section of wavelet transformed result at arrival of leading crest. From the cross-section, the wave period of the leading tsunami wave is defined as the wave period at the maximum value of the spectrum and is T = 24:62min. With 95% of the peak spectrum, the range of wave period is 20:92min < T < 29:53min.



C = Speed of the leading waves

$$h = C^2 / g = \text{mean depth}$$

C = 212.7 m/s and h = 4.61 km for Tohoku tsunamis

C = 211.9 m/s and h = 4.58 km for Queen-Charlotte tsunamis

C = 190.4 m/s and h = 3.70 km for Chile tsunami

The leading wave appears to behave like a linear shallow wave!

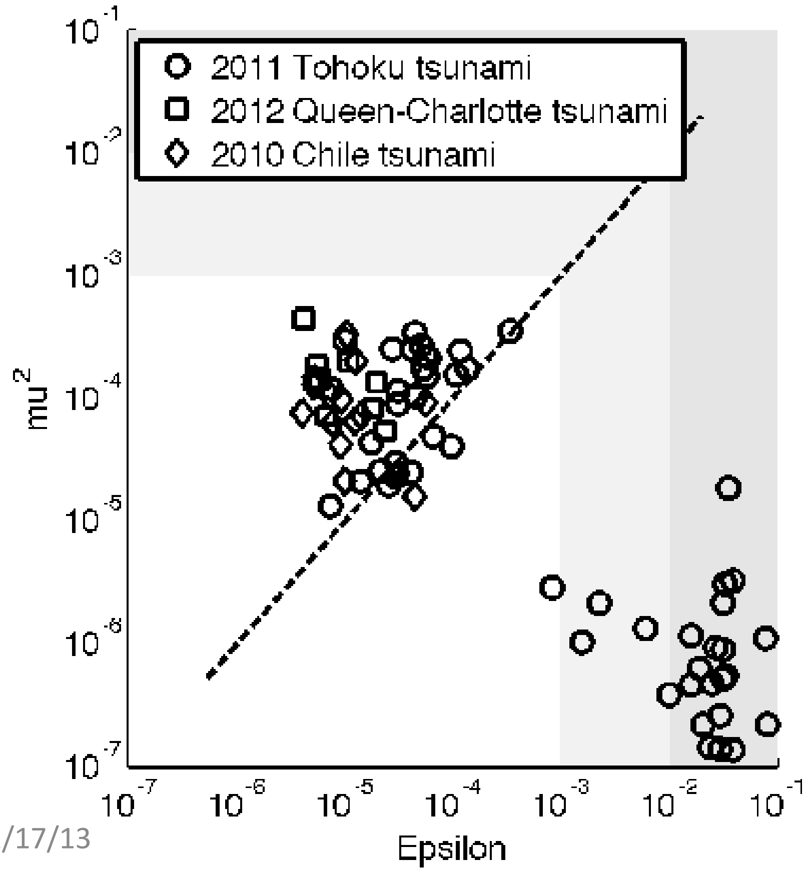
Note: 3.97km/s Rayleigh wave speed

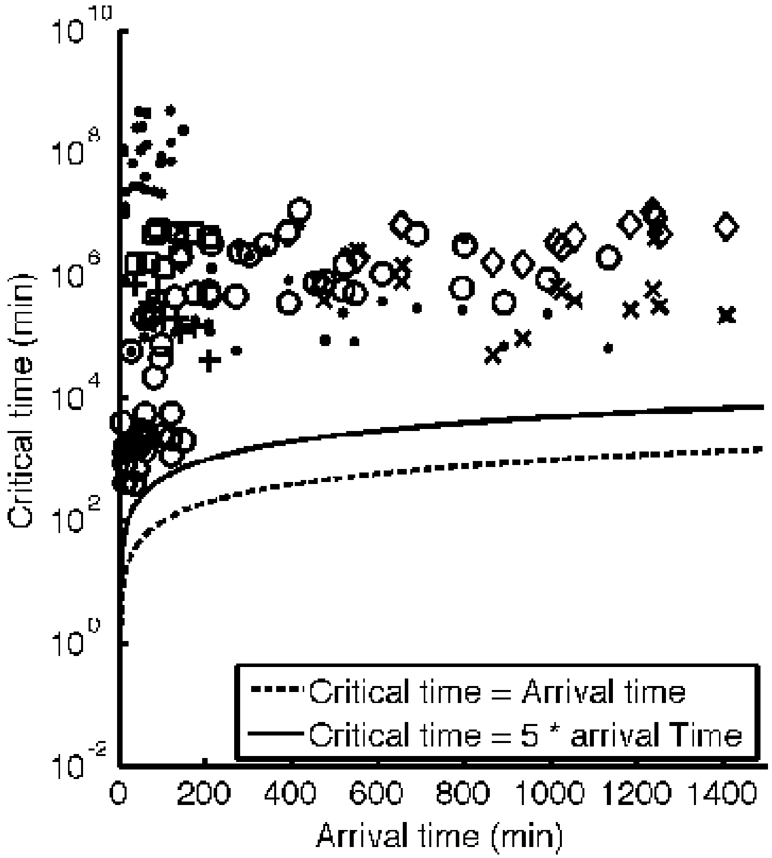
How do we evaluate the importance of “nonlinearity” and “frequency dispersion”?

Dispersion: $= h / \quad =$ water depth/wavelength

Nonlinearity: $= A / h =$ amplitude/water depth

For small amplitude wave in shallow water both and ^{2} must be small.





Effects of both nonlinearity and frequency dispersion, and ^{2} **are accumulative**.

Defining Ursell number as the ratio between

$$U_r = \frac{}{{}^{2}}$$

As waves propagate over a long distance for a long duration, these effects could become important.

The time scale required for these effects to become important is proportional to $T_{cr} = T/$ or $T/{}^{2}$, depending on if U_r ? 1 U_r = 1 or , where *T* is the wave period of the leading tsunami waves.

Boussinesq approximation: $U_r : O(1)$

What is the limitation of the linear and non-dispersive wave theory?

By examining the validity of the perturbation solution for the 1D *KdV* equation, we know that

if U_r ? $O(1)$, $(g/h)^{1/2}t = (\quad)^{1}$; $t/T = \quad^{1}$

if $U_r = O(1)$, $(g/h)^{1/2}t = \quad^{3}$; $t/T = \quad^{2}$

The maximum distance is $x/h \sim (g/h)^{1/2}t$

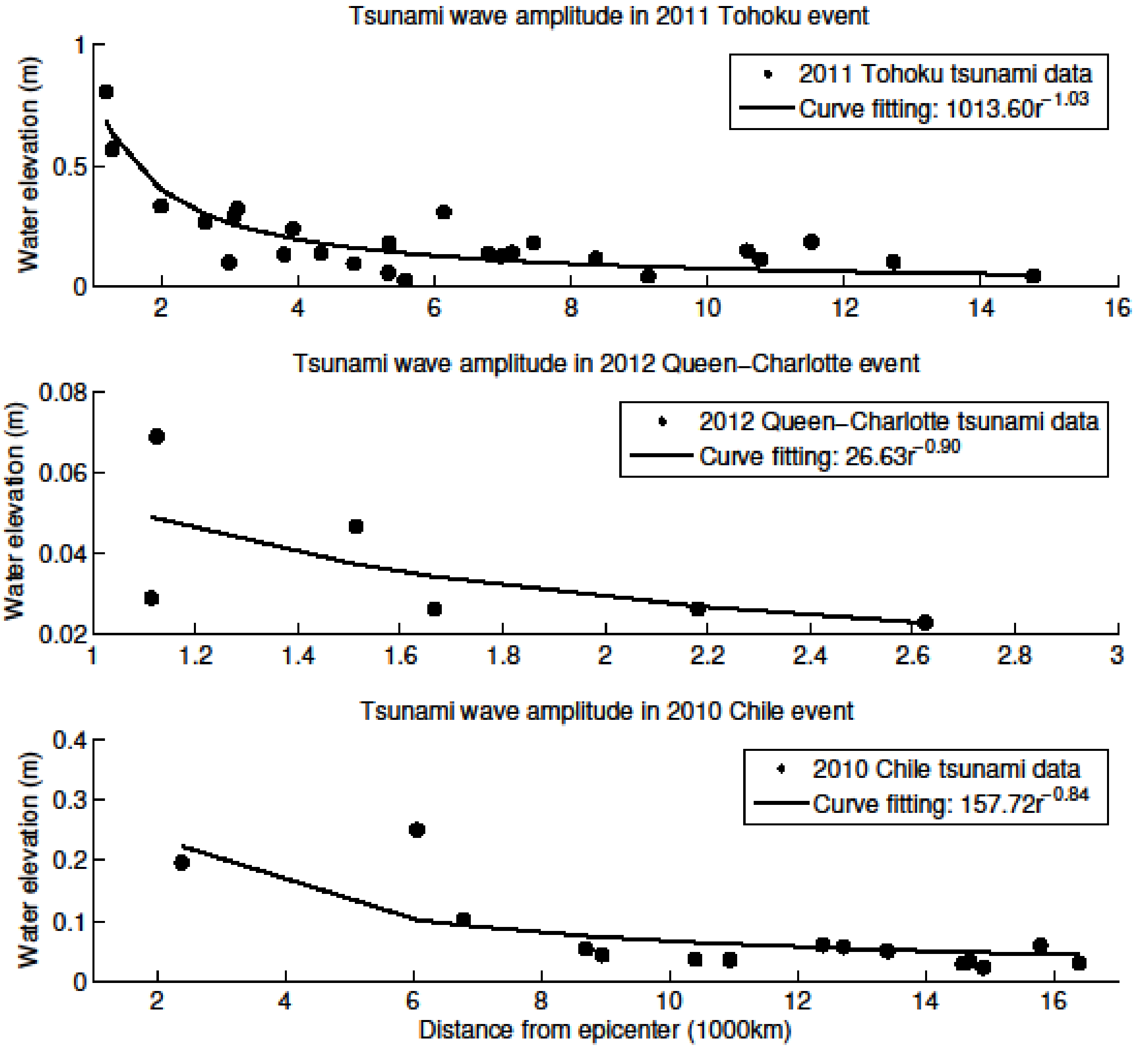
Choice of Approximate Theories for Modeling Tsunami Propagation

U_r	$= t\sqrt{g/h}$	t/T	**Approximate Equation**
? 1	$= (\quad)^{-1}$	$= \quad^{1}$	Linear nondispersive
	$\sim (\quad)^{-1}$	$: \quad^{1}$	Non-linear nondispersive
$= 1$	$= \quad^{-3}$	$= \quad^{-2}$	Linear nondispersive
	$: \quad^{-3}$	$: \quad^{-2}$	Linear dispersive
$O(1)$			
	$= \quad^{-3}$	$= \quad^{-2}$	Linear nondispersive
$(\quad)^{-3}$?	? $\quad^{3}$ $\quad^{-3}$	$\quad^{2}$? t/T ? $\quad^{2}$	Linear dispersive
		$: \quad^{-3}$ $\quad^{2}$	Nonlinear dispersive

Note that the importance of the nonlinearity could be over-estimated
Because of the 1D analyses.

2D spreading for linear shallow water wave in constant depth

$$A : \frac{1}{r} \text{ for large } r$$



Length- and time-scales for other recent earthquake generated tsunamis

Fault area (Width x Length)

- 1960 Chilean earthquake: 200km x 800km
- 1964 Alaskan earthquake: 100km x 700km
- 2003 Algerian earthquake: 20km x 36km
- 2004 Sumatra earthquake: 200km x 1200km

Maximum fault displacement (dislocation)

- 1960 Chilean earthquake: 24 m
- 1964 Alaska earthquake: 2 ~11 m
- 2003 Algerian earthquake: 1m
- 2004 Sumatra earthquake: 6 m

Resulting in an initial surface profile mimicking the seafloor deformation with a typical wavelength *L* ~ O(10 – 100 km) and an amplitude *A* ~ O(1 – 10 m). For a typical water depth *h* ~ 1 – 4 km,

$$= A / h \sim O(10^{2} \quad 2.5 \quad 10^{4}), \qquad\qquad {}^{2} = \left(h / L\right)^{2} \sim O(10^{1} \quad 10^{4})$$

$$U_{r} = \quad / \quad {}^{2} \sim O(10^{2} : \; 2.5 \quad 10^{3})$$

Example 1: For Chilean tsunami $h = 4$ km, $A = 6$ m, $L = 200$ km,

$$= 1.5 \quad 10^{\ 3}, \quad {}^{2} = 4 \quad 10^{\ 4}, U_r = 3.75.$$

Thus, $t = \ 0.66 \quad 10^{6}$ sec $= 183$hr and $x = \ 1.32 \quad 10^{5}$km.

Example 2: For Algerian tsunami $h = 2$ km, $A = 1$ m, $L = 20$ km,

$$= 2 \quad 10^{-3}, \quad {}^{2} = 10^{\ 2}, U_r = 0.2.$$

Thus, $t = \ \sqrt{2} \quad 10^{4}$ sec $= 3.93$hr and $x = \ 2 \quad 10^{3}$km.

Example 3: For 2012 Tohoku tsunami $h = 4$ km, $A = 5$ m, $L = 150$ km,

$$= 1.25 \quad 10^{-3}, \quad {}^{2} = 7.1 \quad 10^{\ 4}, U_r = 1.76.$$

Thus, $t = \ 1.14 \quad 10^{6}$ sec $= 316.7$hr and $x = \ 2.28 \quad 10^{5}$km.

The linear, non-dispersive wave theory is suitable to describe the propagation of seismically generated tsunami in ocean basin.

On the continental shelf the wave amplitude and the wavelength must be rescaled **according to water depth:**

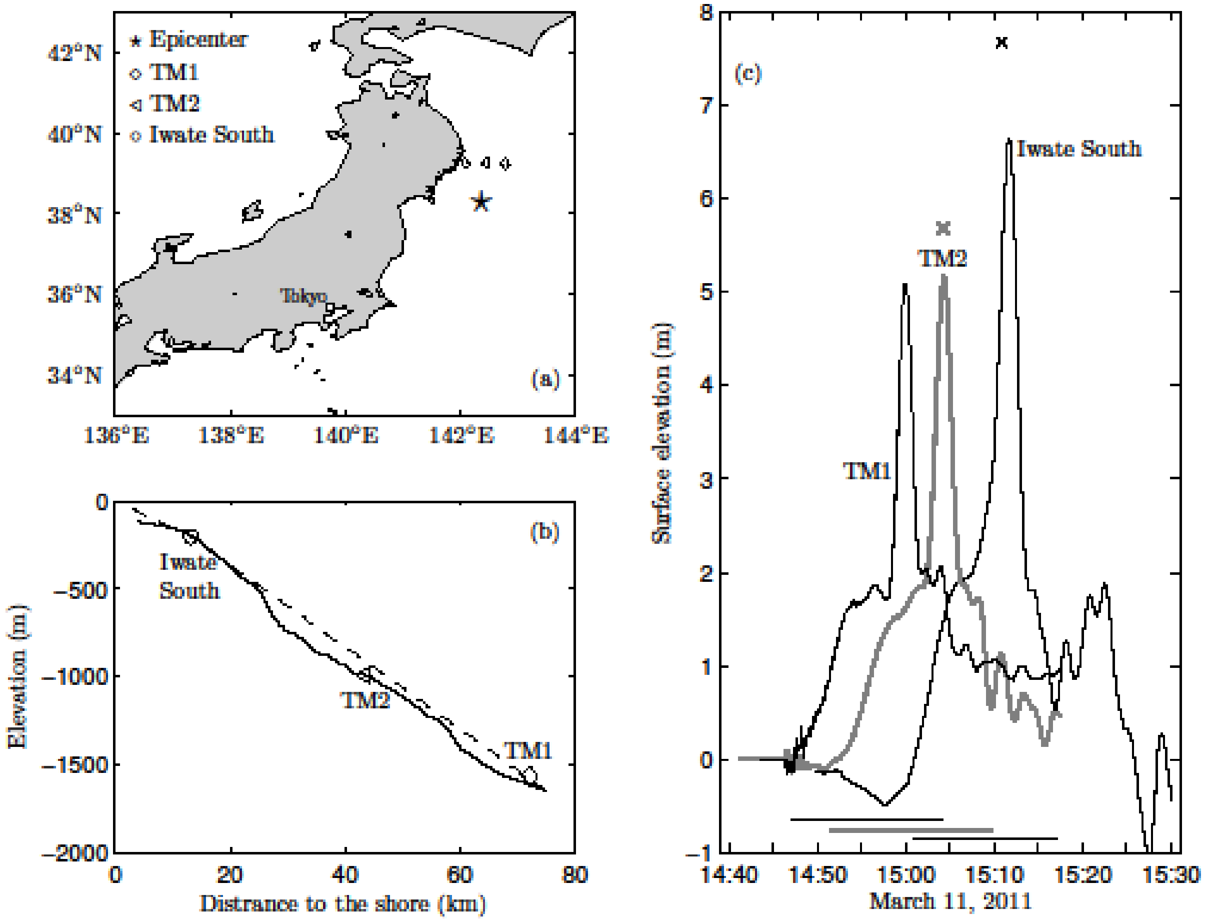
$$A \quad h^{1/4}, L \quad h^{1/2}, \quad = A / h \quad h^{5/4}, \quad = h / L \quad h^{1/2}$$

$$U_r = \quad / \quad ^{2} \quad h^{9/4}$$

This suggests that as the tsunami propagates into the coastal region, the importance of the nonlinearity will increase and that of the frequency dispersion will decrease with the decreasing water depth. Hence, in certain coastal region the Boussinesq wave theory might be necessary. However, in a very shallow depth, the nonlinearity dominates and the nonlinear shallow water wave theory becomes more adequate.

Physical parameters for leading waves of 2011 Tohoku tsunamis on the continental shelf

	h (km)	A (m)	L (km)	$= A/h$	$= h/L$
TM1	1.6	5.1	130	0.003	0.012
TM2	1.0	5.2	100	0.005	0.010
Iwate	0.2	6.7	45	0.032	0.014



$= 1, \quad {}^2 = 1$

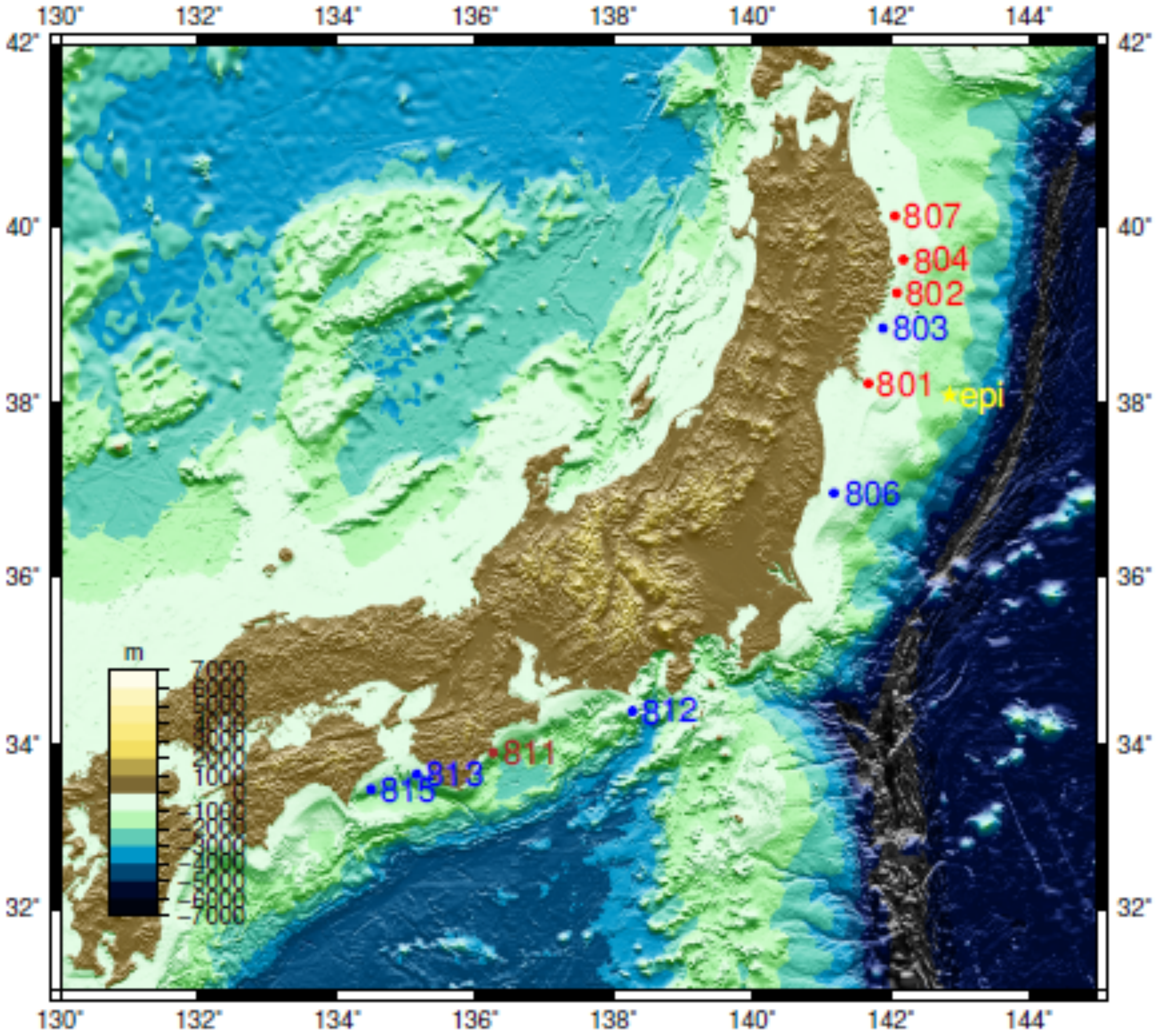
$> \; {}^2$

xxx

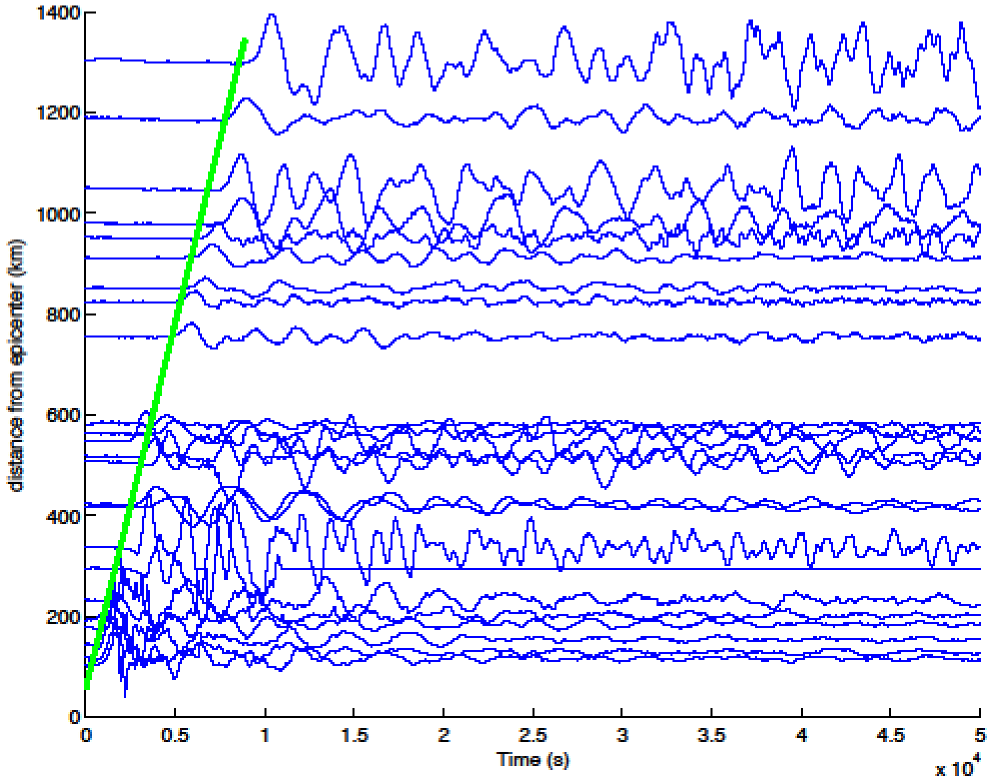
Green's Law

$A \quad h^{1/4}$

GPS, bottom mounted pressure gage and wave gage stations around Japan coasts



More nearshore tsunami data

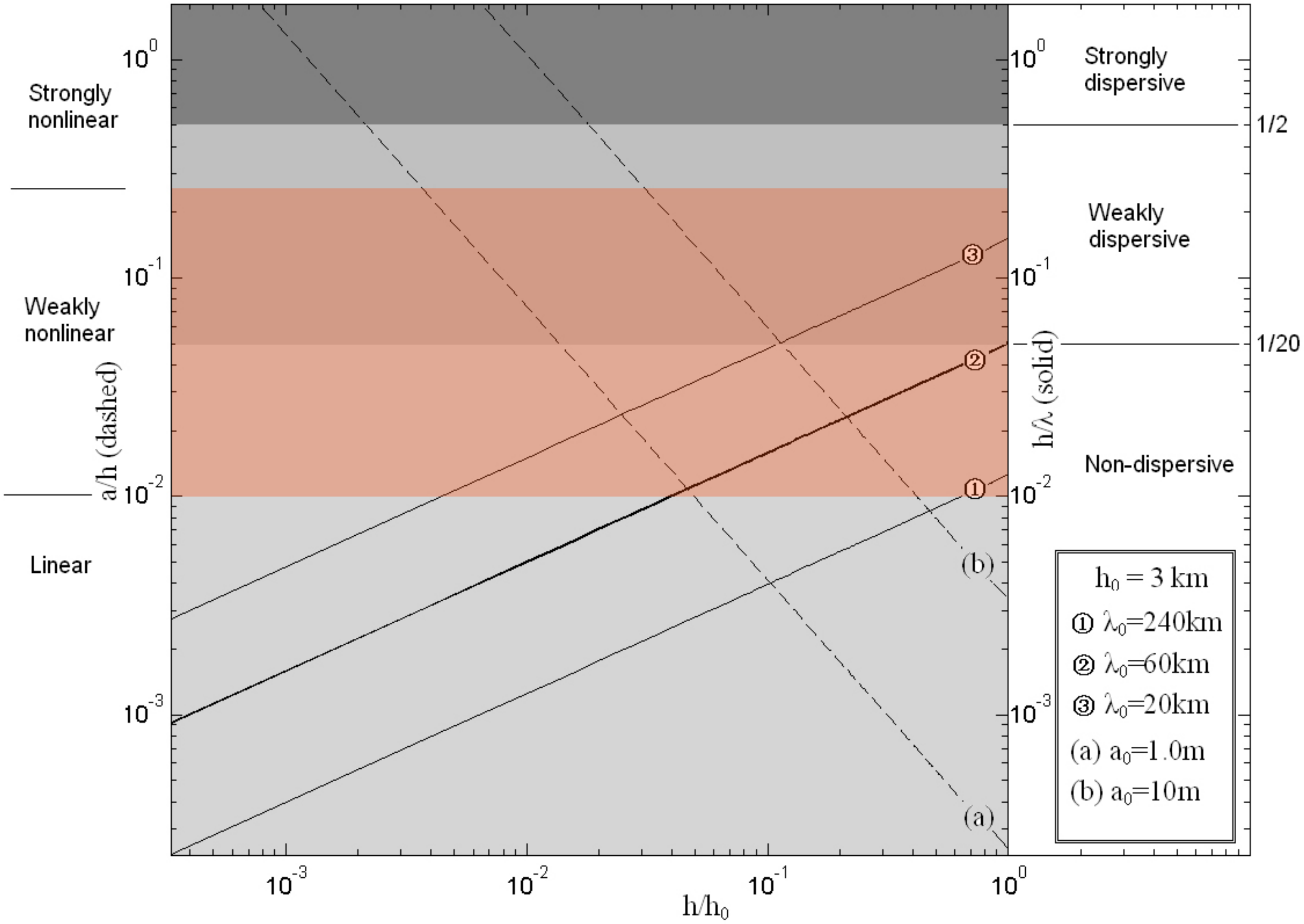


The averaged phase speed: 150 m/s

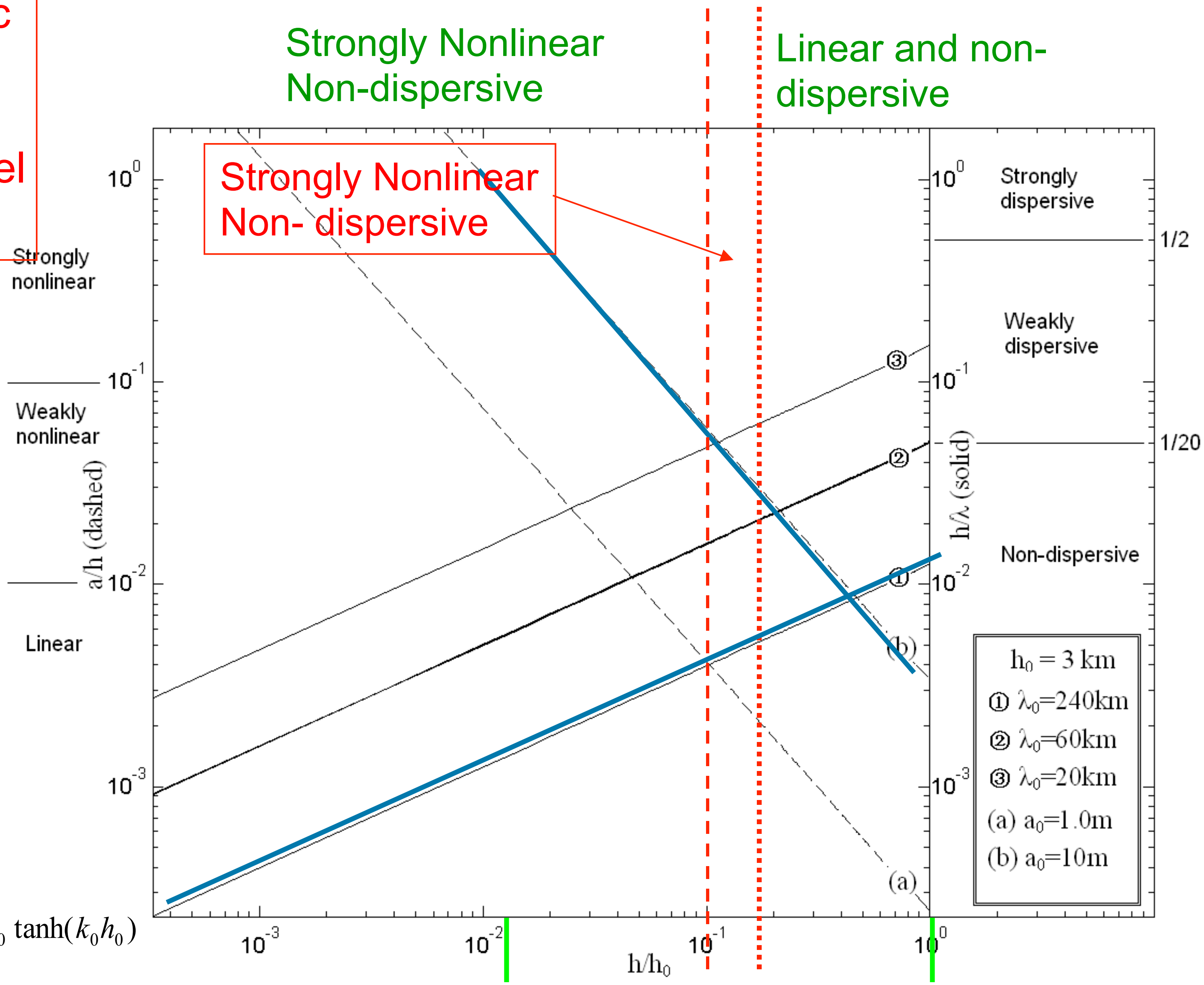
$$U_r ? 1$$

Station No.	Wave Amplitude, $A(m)$	Water Depth(m)	Wave Length $L(km)$	$\epsilon = A/h$	$\mu^2 = (h/L)^2$	$t = T/\epsilon$ (min)	$t = T/\mu^2$ (min)	Arrival Time (min)
801	5.687	144	78.8	3.95e-02	3.34e-06	8.85e+02	1.05e+07	12.95
803	5.663	160	36.8	3.54e-02	1.89e-05	4.38e+02	8.19e+05	14.62
802	6.674	204	115.8	3.27e-02	3.10e-06	1.32e+03	1.39e+07	14.7
804	6.3	200	136.5	3.15e-02	2.15e-06	1.63e+03	2.39e+07	16.53
806	2.605	137	171.6	1.90e-02	6.37e-07	4.10e+03	1.22e+08	6.87
807	3.929	125	131.6	3.14e-02	9.02e-07	1.99e+03	6.95e+07	30.87

Choice of models based on the dispersion relationship



Transoceanic model
+
Coastal Model
???



Strongly Nonlinear
Non-dispersive

Linear and non-dispersive

Strongly Nonlinear
Non- dispersive

Coastal Region

Continental Shelf

Ocean Basin

Nonlinearity

$$\frac{a}{a_0} = \frac{h_0}{h}^{1/4}$$

Dispersion

$${}^2 = gk \tanh(kh) = gk_0 \tanh(k_0 h_0)$$

Tsunami waves behave very differently in the coastal zone

塚原(Tsukahara)

14:57:50

富澤貞嗣氏撮影

塚原(Tsukahara)

15:11:32

富澤貞嗣氏撮影

塚原(Tsukahara)

15:32:14

富澤貞嗣氏撮影

塚原(Tsukahara)

15:34:24

富澤貞嗣氏撮影

塚原(Tsukahara) 15:39:00

富澤貞嗣氏撮影

塚原(Tsukahara)

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塚原(Tsukahara)

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富澤貞嗣氏撮影

塚原(Tsukahara) 15:39:39

富澤貞嗣氏撮影

塚原(Tsukahara)

15:39:40

富澤貞嗣氏撮影

塚原(Tsukahara)

15:39:41

富澤貞嗣氏撮影

塚原(Tsukahara)

15:39:45

富澤貞嗣氏撮影

塚原(Tsukahara)

15:53

富澤貞嗣氏撮影

Miyako Seawall

February 17, 2012

seawall: March 11, 2011

Miyako seawall

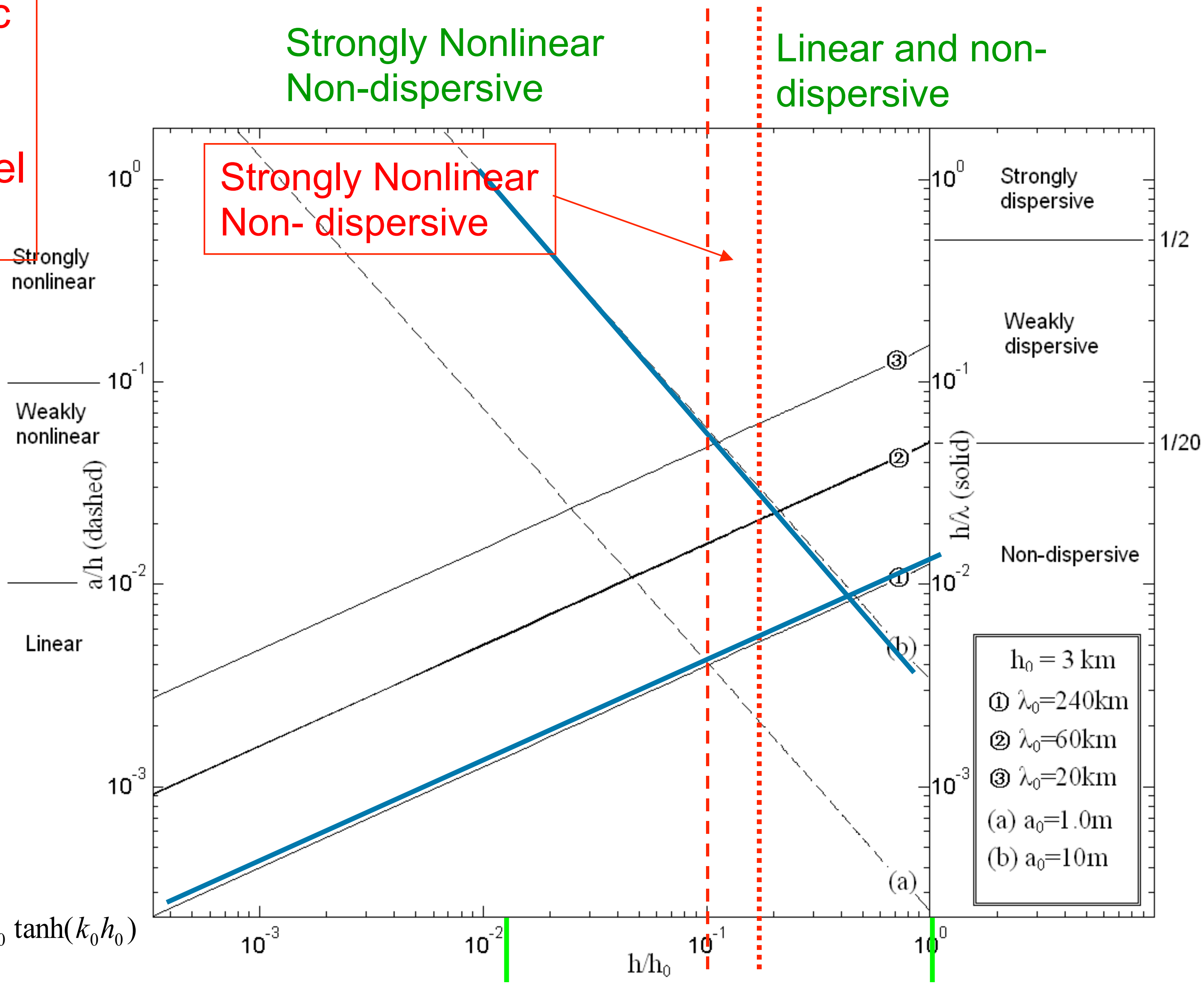
March 11, 2011

February 17, 2012

Transoceanic model
+
Coastal Model
???

Strongly Nonlinear
Non-dispersive

Linear and non-dispersive



Coastal Region

Continental Shelf

Ocean Basin

Nonlinearity

$$\frac{a}{a_0} = \frac{h_0}{h}^{1/4}$$

Dispersion

$${}^2 = gk\tanh(kh) = gk_0\tanh(k_0 h_0)$$

Summary
Characteristics of earthquake-generated tsunamis

- **Ocean basin and continental shelf**: Small amplitude waves (wave amplitude is much smaller than wavelength/water depth); Long waves (wavelength is much larger than water depth); Dissipation (wave breaking and bottom friction) insignificant.
- **Coastal region:** Strongly nonlinear transient flows; significant wave breaking induced and bottom boundary layer turbulence; sediment (debris) laden flows; 3D flows strongly affected by bathymetry, topography, and surface conditions.

Modeling earthquake-generated oceanic tsunamis

- Phenomenal
    - Generation and evolution of tsunami waves in the neighborhood of the source region
    - Wave propagation in the deep ocean and into the shallow waters
    - Terminal (Coastal) effects: wave runup/drawdown and inundation
- Methodology
    - Analytical solutions
    - Numerical simulations
    - Laboratory experiments

Purposes of tsunami modeling

- Forecasting and Warning
  - Accurate wave heights
  - Accurate arrival times
  - Accurate inundation
  - Accurate currents
- Hazard Mitigation/Coastal Zone Planning
  - Accurate inundation areas and water levels
  - Impact potential (forces, erosion, interaction with structures and structural response, etc.)

How to model tsunami generation?

- Earthquake causes seafloor motions
- Seafloor displacement generate surface gravity waves (tsunamis)

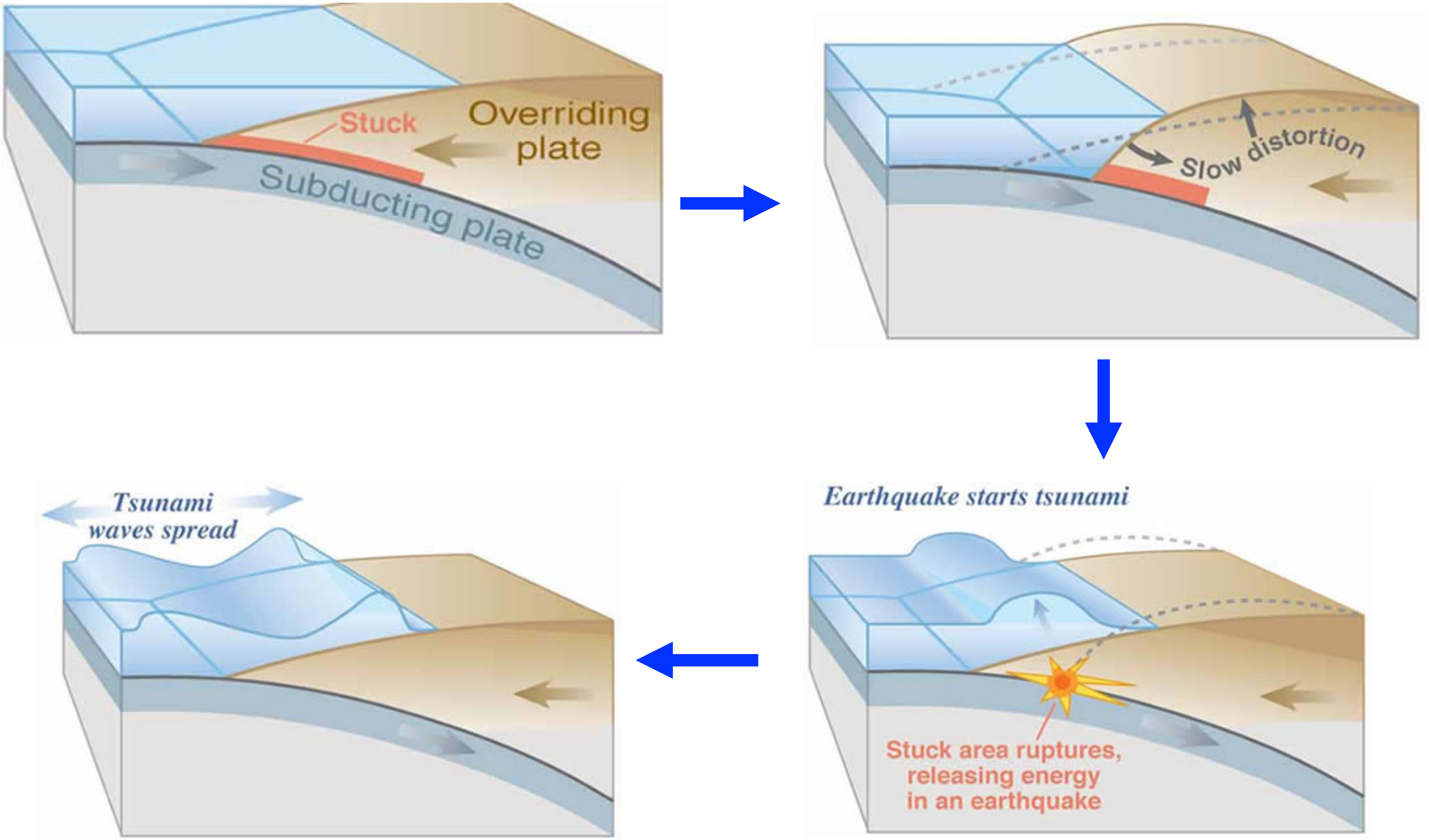
Difficulties:

- Earth is a complex and heterogeneous media
- Different time scales for seismic waves and tsunamis (Rayleigh wave speed 4km/s; tsunami wave speed 0.2 km/s)

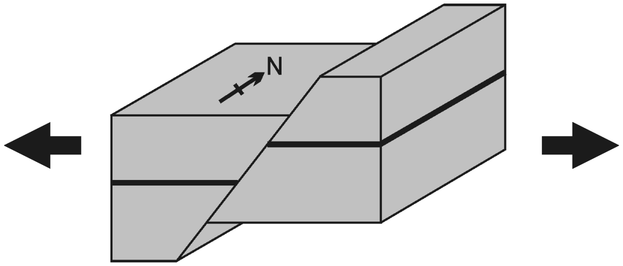
Simplified Models:

Linear elastic dislocation model (Okada, *Bull. Seism. Soc. Am*.1985)

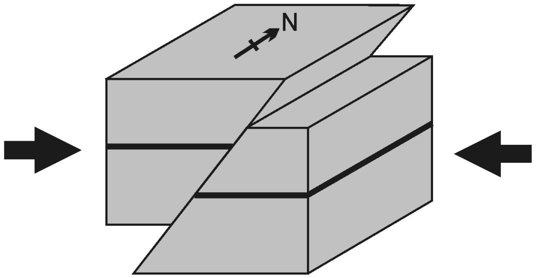
Example of earthquake-generated tsunamis



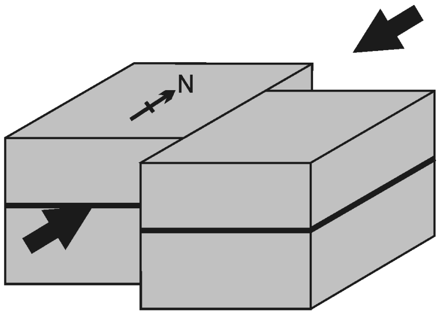
Not all seafloor earthquakes cause tsunamis



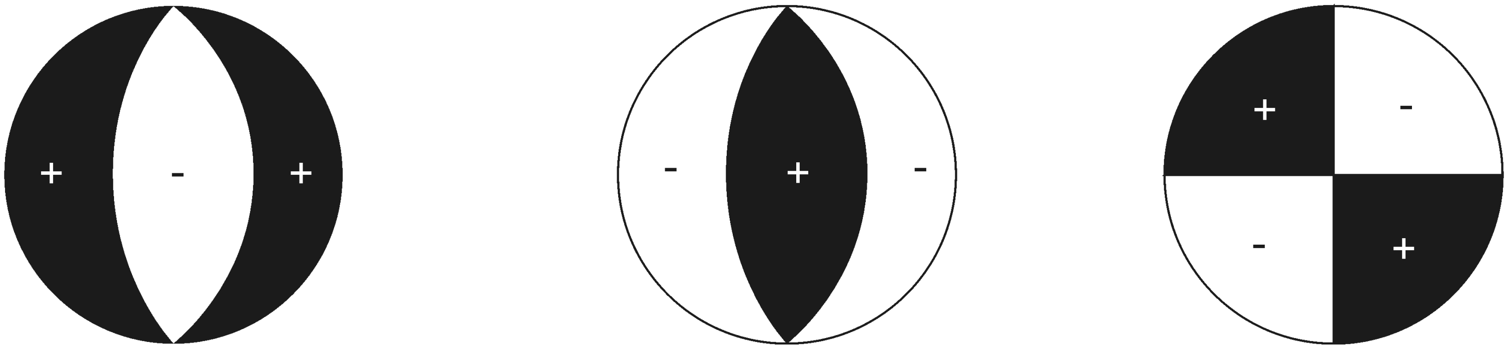
Normal Fault



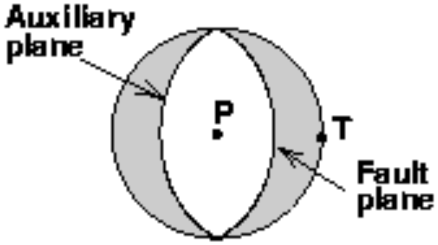
Reverse Fault



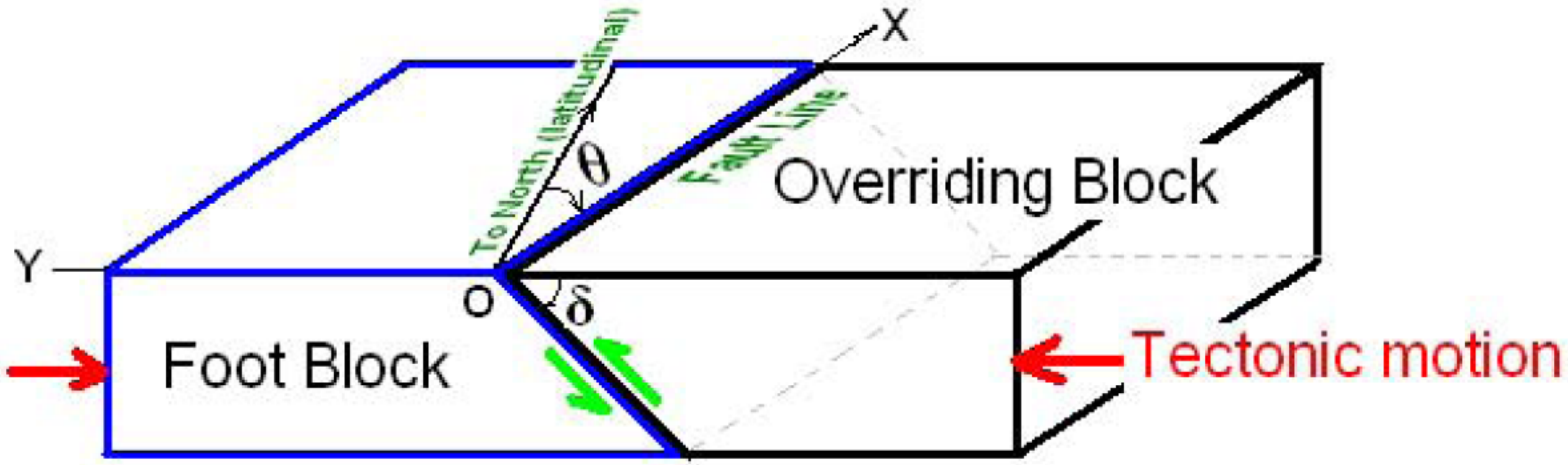
Strike-Slip Fault

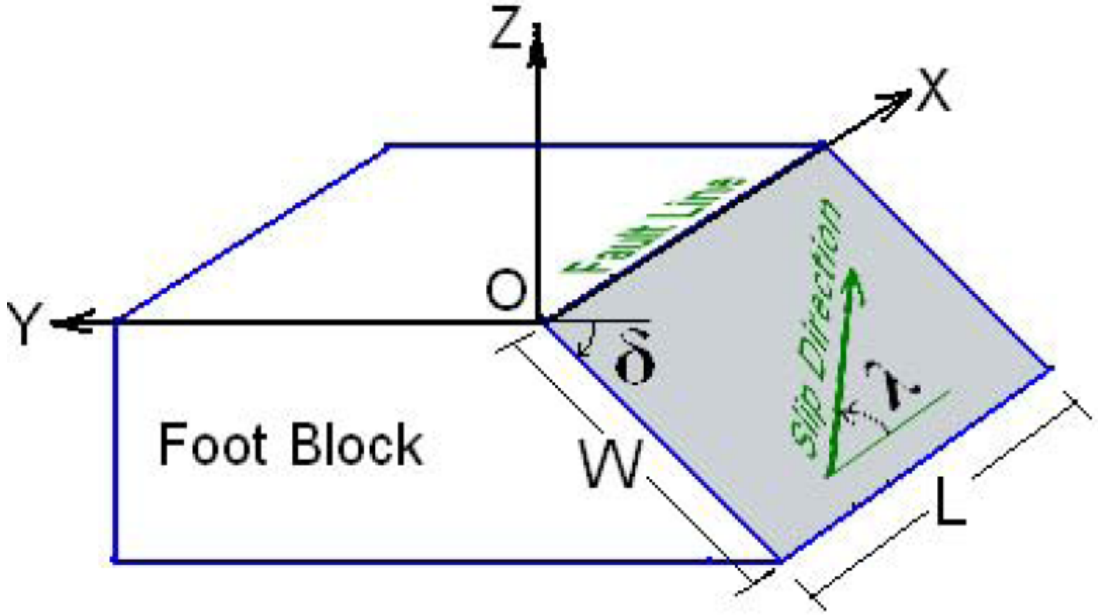


View from above



"Beach ball"



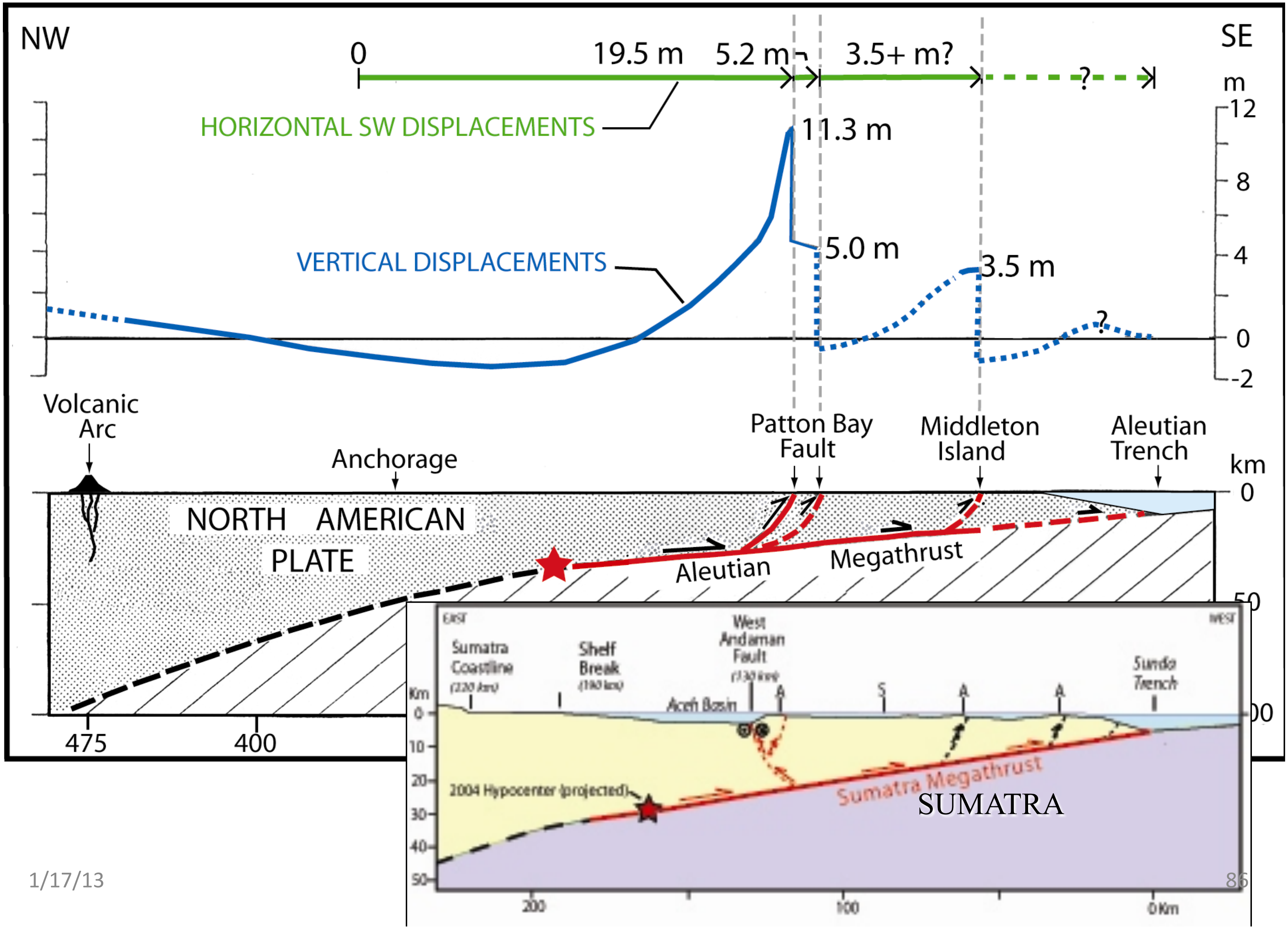


Symbols

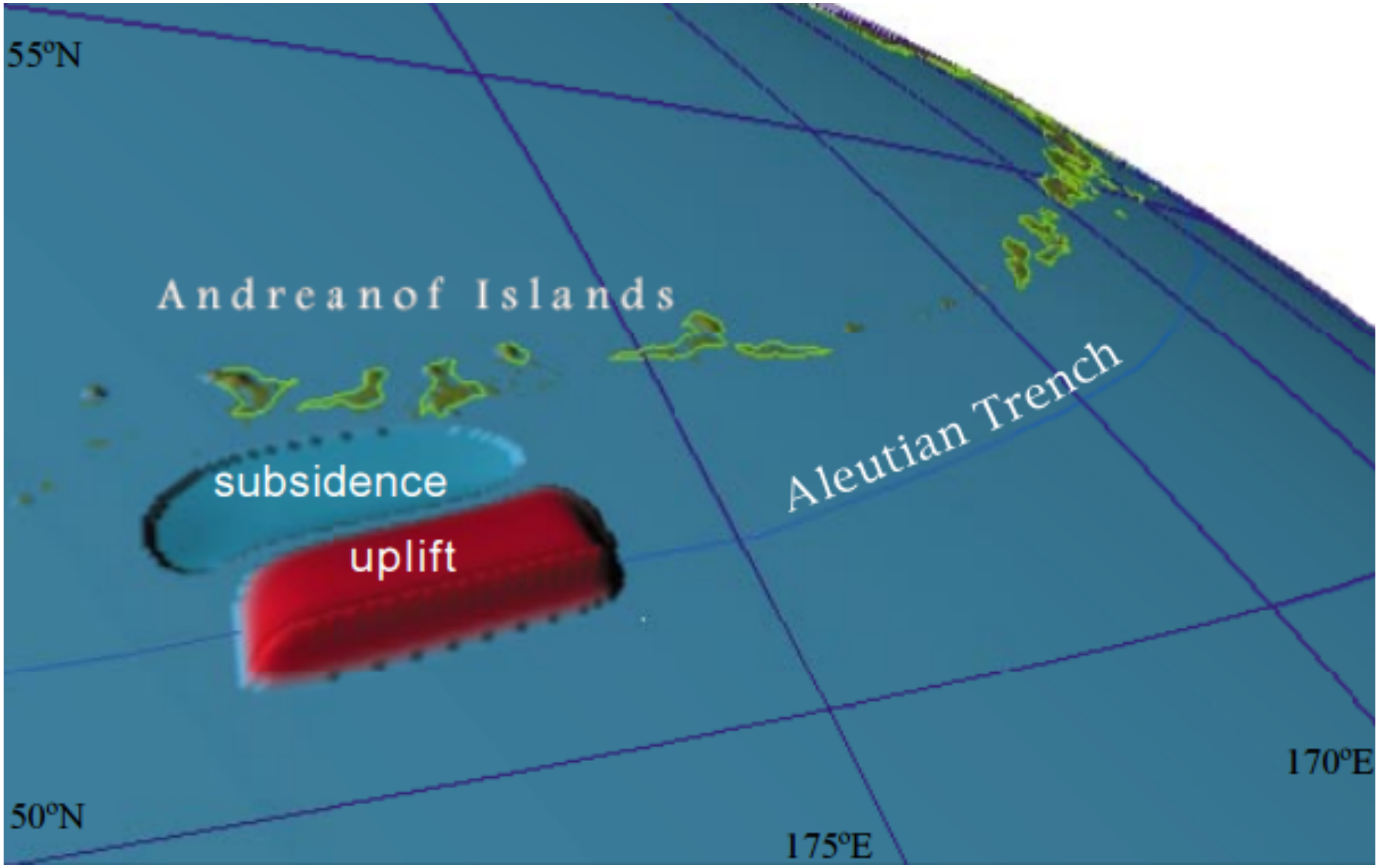
L	Length of Fault Plane
W	Width of Fault Plane
θ	Strike angle
δ	Dip angle
λ	Slip angle

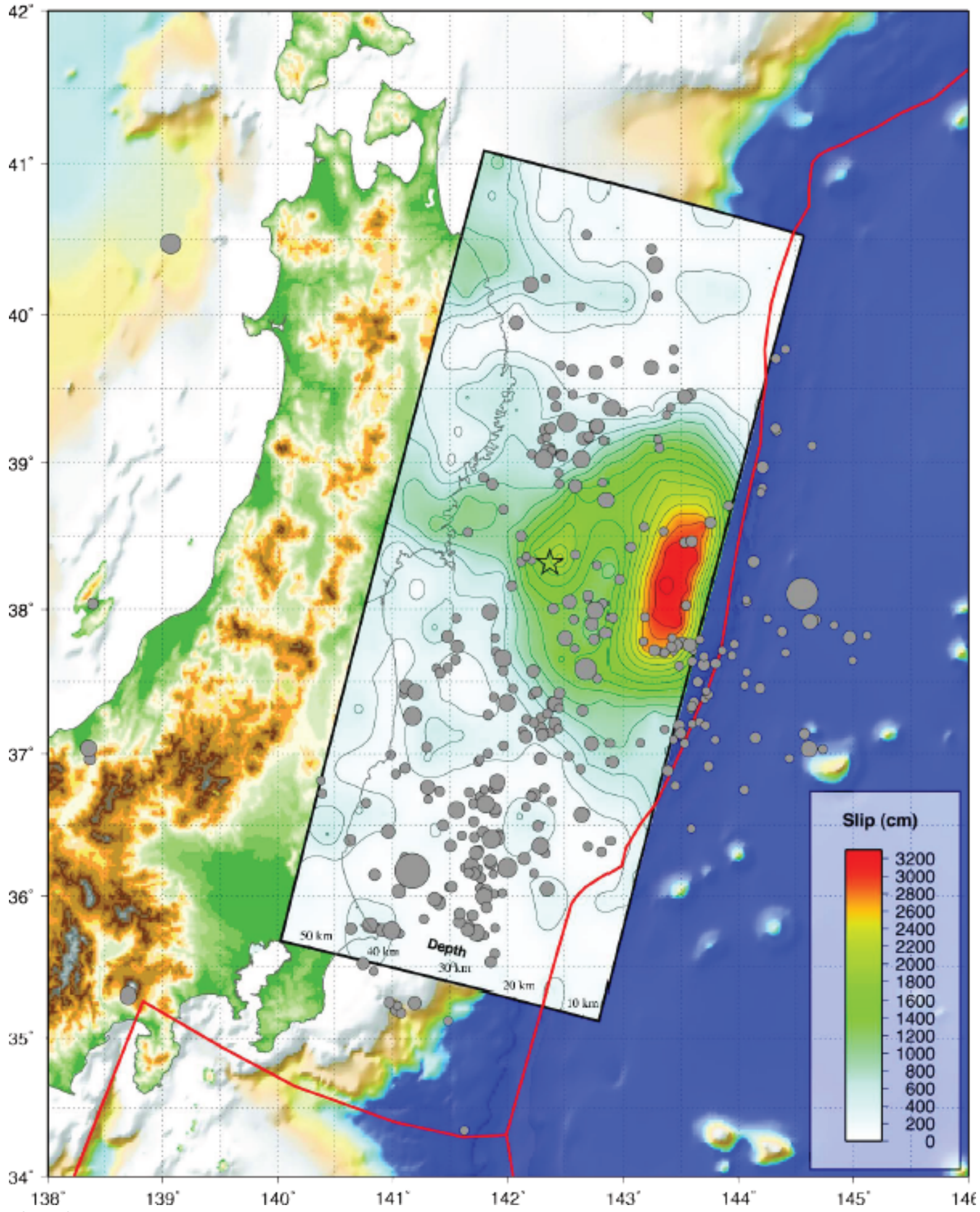
XOY parallel to the horizontal earth surface; OZ pointing upward; θ is the azimuth of OX measuring clockwise from the latitudinal.

Comparison of 1964 Alaska & 2004 Sumatra Structures



Okada's dislocation theory provides the final seafloor displacement.





2011 Tohoku E/T

Dip angle = 10 degree;
Strike angle = 200 degree;
Slip angle = 90 degree;

The slip varies 30m to 50m;

W = 200~300 km;
L = 500 ~ 800 km.