The Importance of Tides

Important for commerce and science for thousands of years

- Tidal heights are necessary for navigation.
- Tides affect mixing, stratification and, as a result biological activity.
- Tides produce strong currents, up to 5m/s in coastal waters
- Tidal currents generate internal waves over various topographies.
- The Earth's crust “bends” under tidal forces.
- Tides influence the orbits of satellites.
- Tidal forces are important in solar and galactic dynamics.

The Nature of Tides

"The truth is, the word "tide" as used by sailors at sea means horizontal motion of the water; but when used by landsmen or sailors in port, it means vertical motion of the water."

"One of the most interesting points of tidal theory is the determination of the currents by which the rise and fall is produced, and so far the sailor's idea of what is most noteworthy as to tidal motion is correct: because before there can be a rise and fall of the water anywhere it must come from some other place, and the water cannot pass from place to place without moving horizontally, or nearly horizontally, through a great distance. Thus the primary phenomenon of the tides is after all the tidal current; …"

The Tides, Sir William Thomson (Lord Kelvin) – 1882, Evening Lecture To The British Association

TIDAL HYDRODYNAMICS AND MODELING

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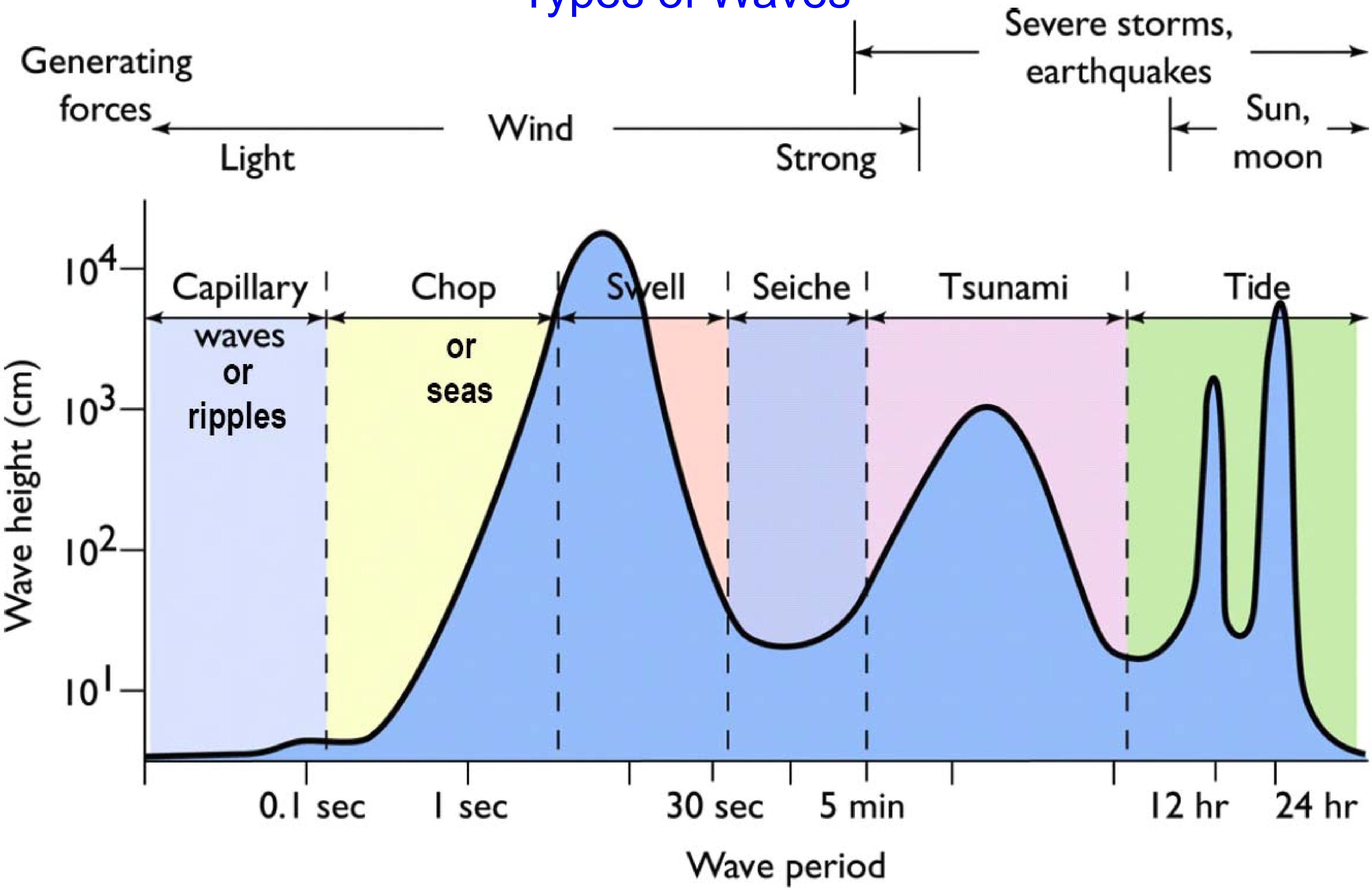
TIDAL HYDRODYNAMICS AND MODELING

Outline

1) Tides within the Wave Spectrum
2) Tide-Generating Forces
3) Equilibrium Theory of Tides – Diurnal, Semi-diurnal, Mixed
4) Tidal Propagation in Ocean Basins - Rotation, Resonance
5) Dynamical Theory of Tides - Laplace's Equations
6) Tidal Prediction – Tidal Potential, Harmonic Analysis
7) Non-linear Tides
8) Tidal Currents

Types of Waves

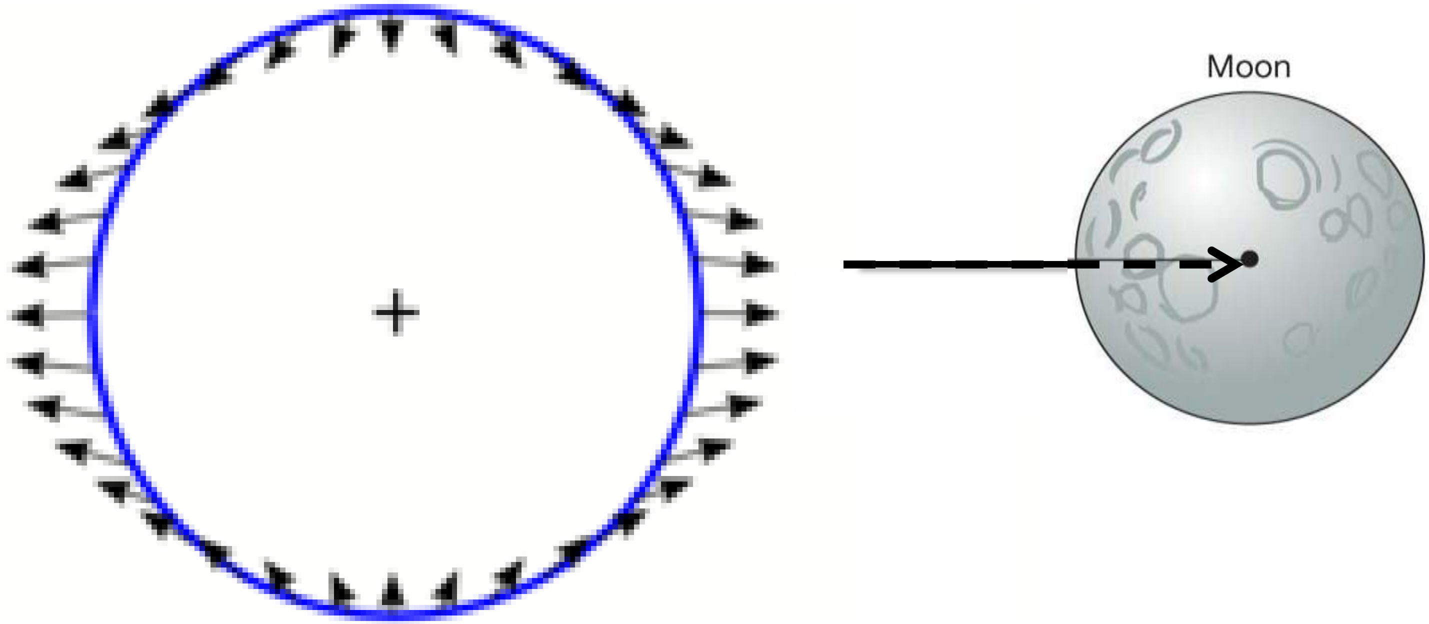


Types of Waves

Name	Typical Periods	Wave lengths	Forcing Mechanism
Ripples	< 0.2 s	10^{-2} m	wind on sea surface
Sea	0.2 - 9 s	10s of m	"
Swell	9 - 30 s	100s of m	"
Internal	min to several hrs	1 - 300 m	current shear on stratification
Planetary and Topographic	hours to days	100-1000s km	bathymetry/atmospheric pressure
Tsunamis	15 min to 1 hr	few 100s km	seismic, landslide, meteorite impact
Tides	several hrs	100s -1000s km	gravitational (moon and sun mainly)

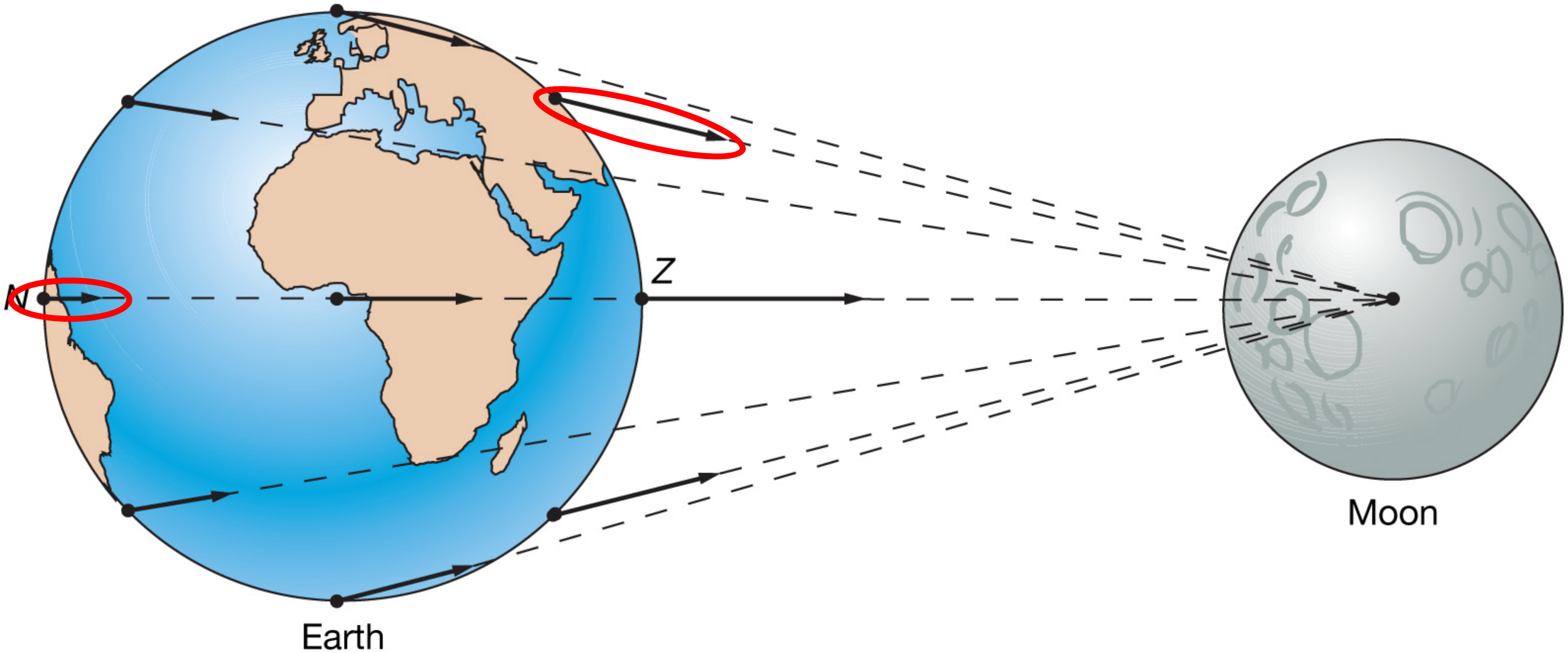
Tide-Generating Forces

Ocean tides are a result of the combined action of differential gravitational attractions and the centrifugal forces within the Earth-Moon-Sun system.



Gravitational Forces

- Gravitational Force between two objects (F) – every particle attracts every other particle

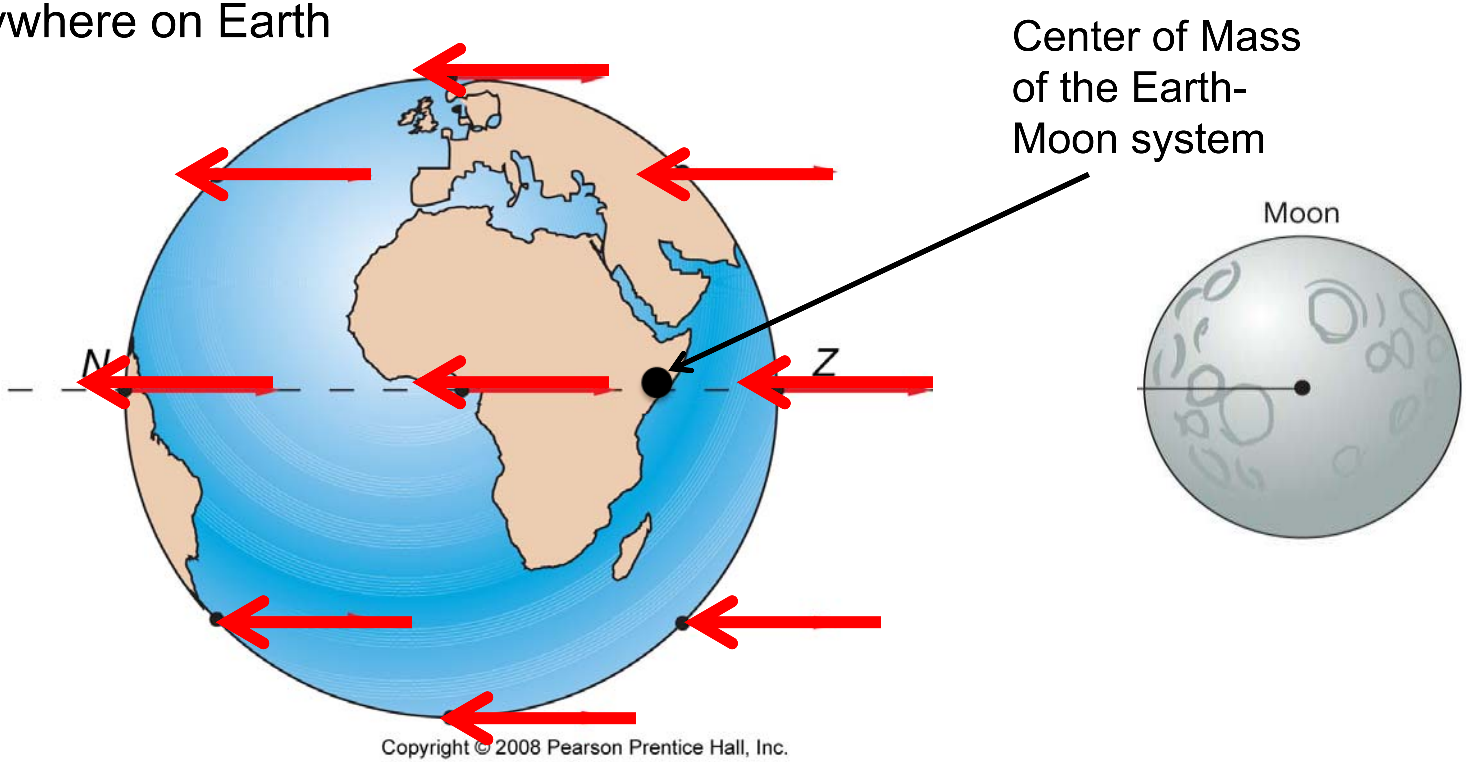


$$F_m = G\frac{M_m M_E}{r^2}$$

G = 6.6 × 10^{-11} Newton m^2/kg^2
M_m = mass of the Moon
M_E = mass of Earth
r = distance between Moon and Earth

Centrifugal Forces

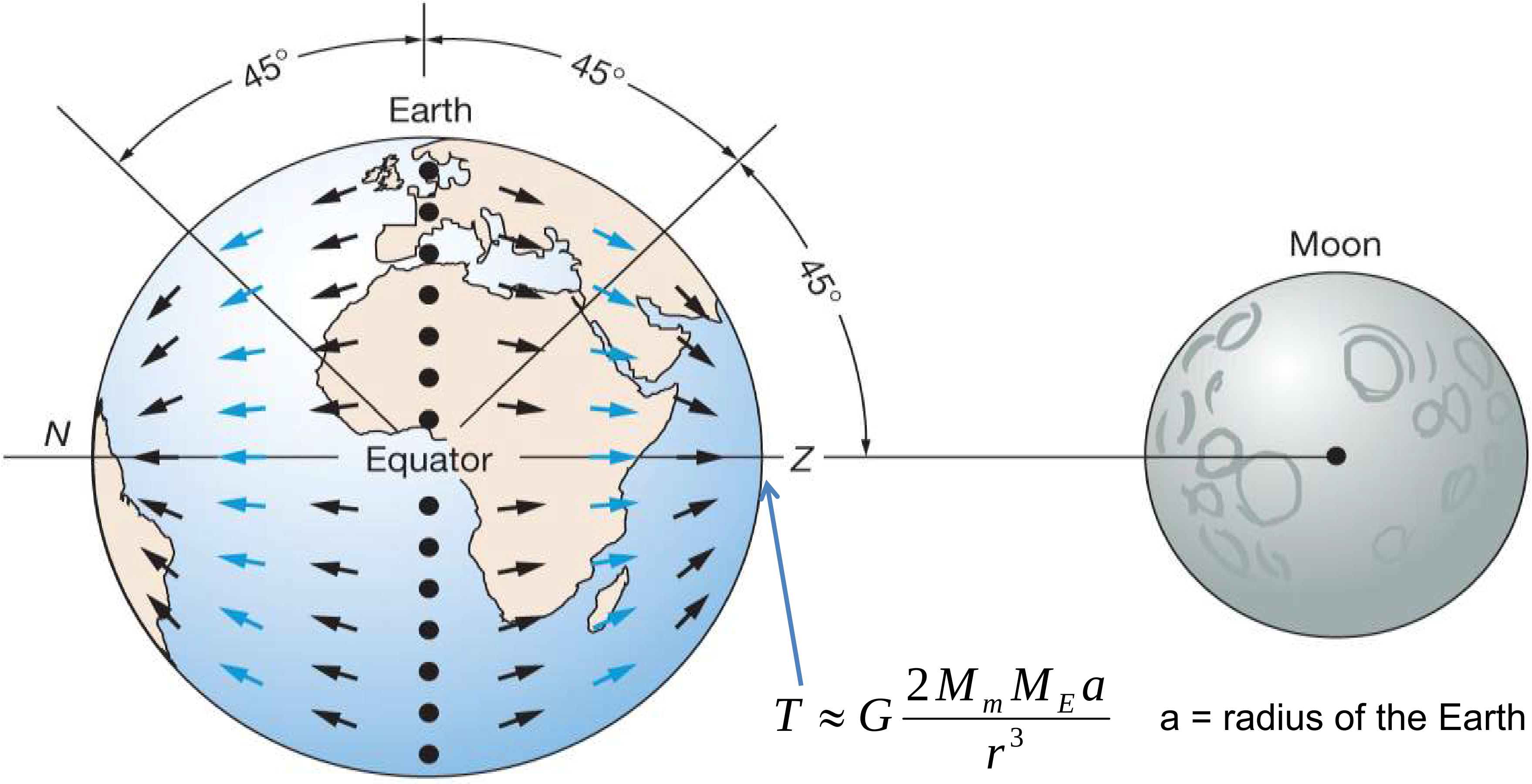
- Center-seeking force arising from the revolution of the Earth and the Moon about their common center of mass
- Uniform everywhere on Earth



$$F_c = \frac{M_{E/M} v^2}{r}$$

v = speed of the Earth or Moon

Resultant Tide-Generating Forces, T

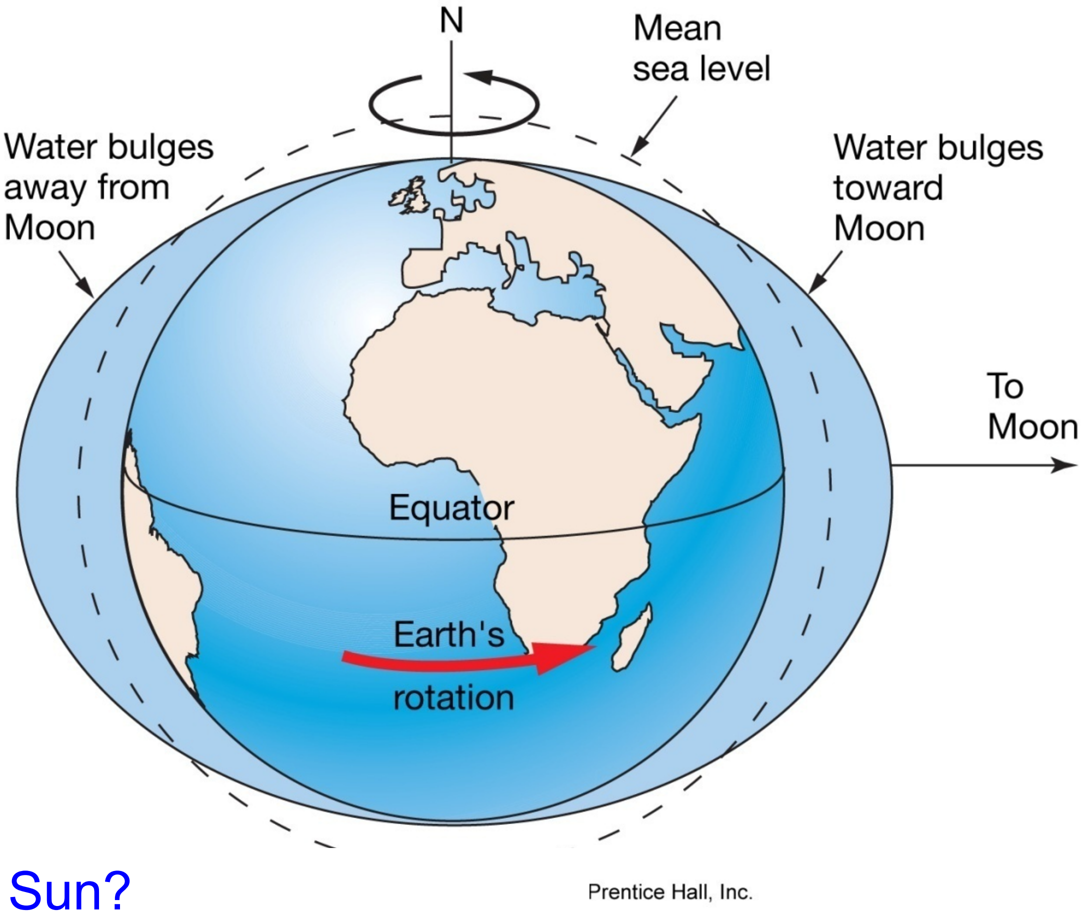


$$T \approx G\frac{2M_m M_E a}{r^3}$$

a = radius of the Earth

- Resultant force has significant horizontal component
- Resultant force is inversely proportional to the cube of the distance to the Moon.
- Pushes water into two simultaneous bulges, one toward and one away from Moon

Lunar Effect – Tidal Bulges



What about the Sun?

Sun is 27 million times more massive than moon, but 390 times farther away.

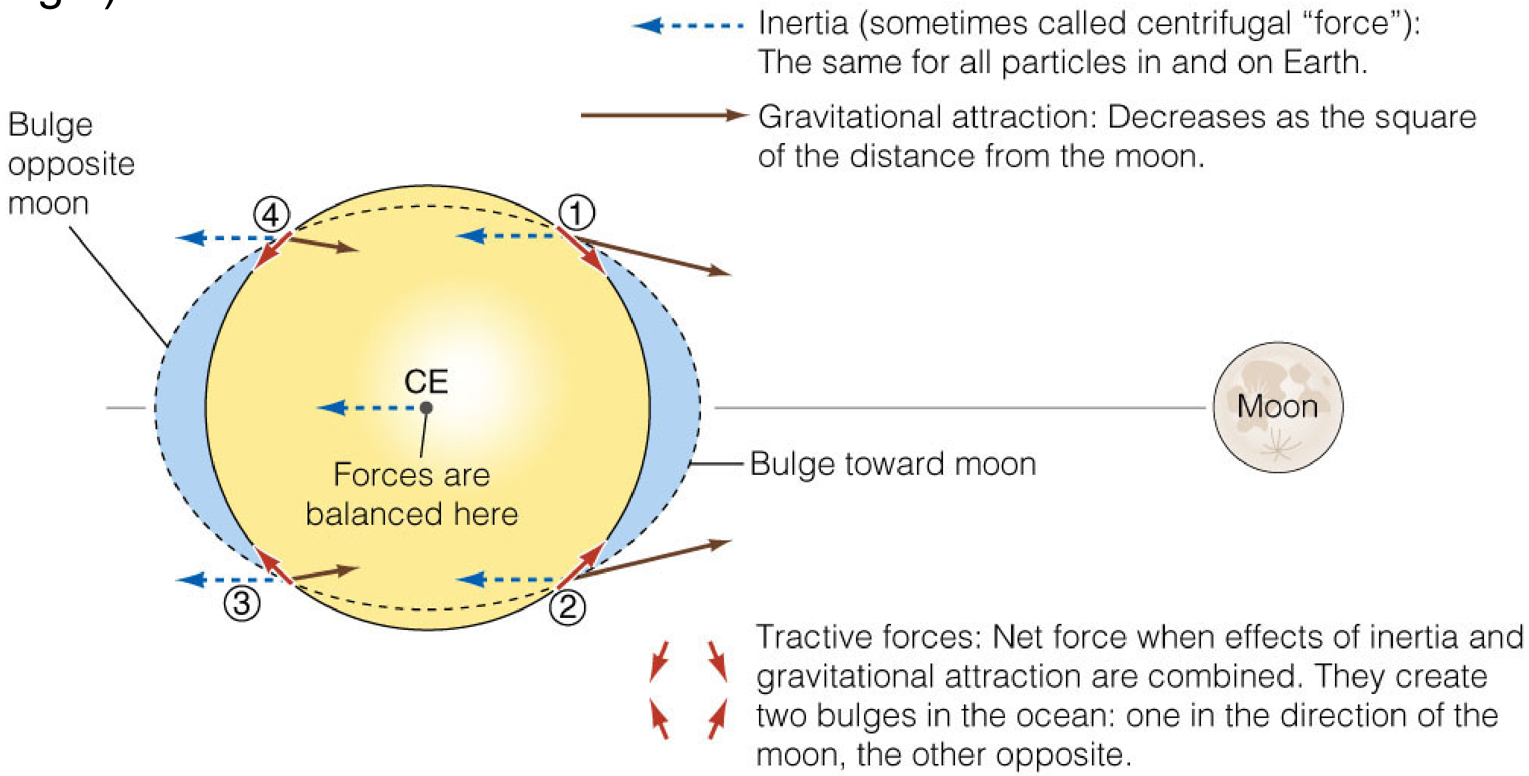
$$\frac{T_m}{T_s} = \frac{M_m}{M_S}\frac{r_s^3}{r_m^3} = \frac{1}{2.7\times 10^7}\times 390^3 = 2.2$$

Moon more than twice as important for creating tides.

Equilibrium Theory of Tides

Assumptions:

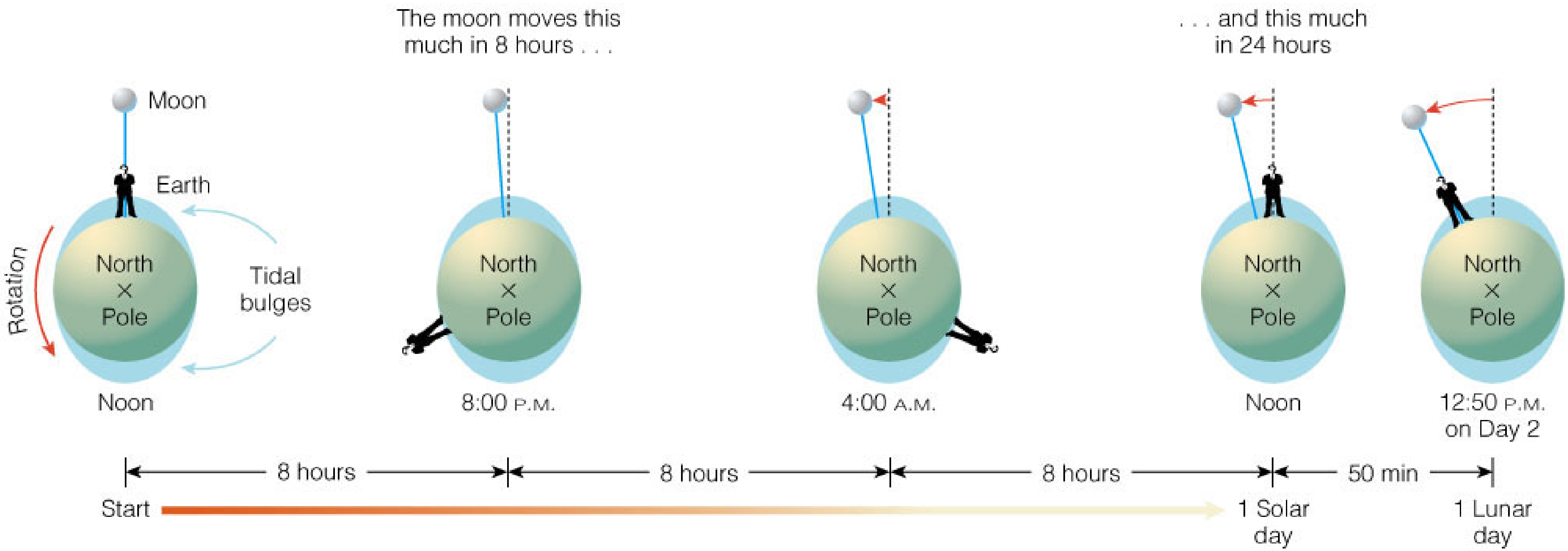
- Earth's surface completely covered by water of infinite depth
- Wave is progressive, moves significant distances relative to Earth's surface
- Tidal wave in equilibrium with tide generating forces (gravitational and centrifugal)



The two forces that can move the ocean—inertia and gravitational attraction—are precisely equal in strength but opposite in direction, and thus balanced, only at the center of Earth (point **CE**).

Lunar Day is 24 hours 50 minutes

This gives two high (flood) and two low (ebb) tides, each 12.42 hours apart.
So tidal period is 12.42 hours (the tide is a very long wave).



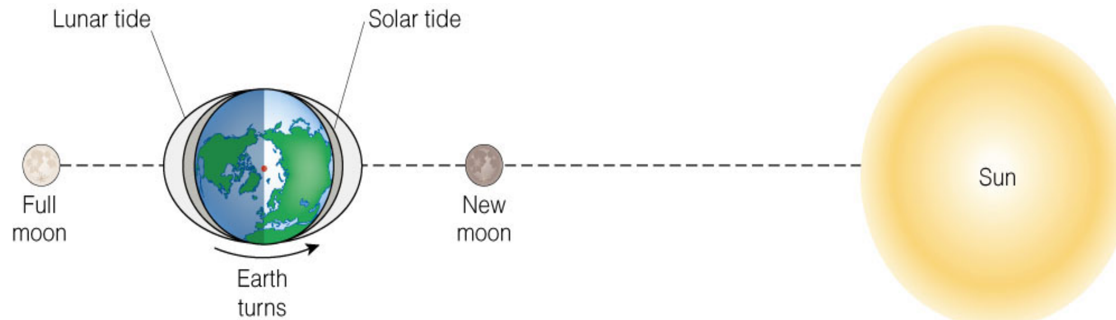
A lunar day is longer than a solar day. A lunar day is the time that elapses between the time the moon is highest in the sky and the next time it is highest in the sky. In a 24-hour solar day, the moon moves eastward about 12.2. Earth must rotate another 12.2° - 50 minutes to again place the moon at the highest position overhead. A lunar day is therefore 24 hours 50 minutes long. Because Earth must turn an additional 50 minutes for the same tidal alignment, lunar tides usually arrive 50 minutes later each day.

Both Sun and Moon Create Tides

Remember, moon's tide generating force is 2.2 times bigger than sun's.

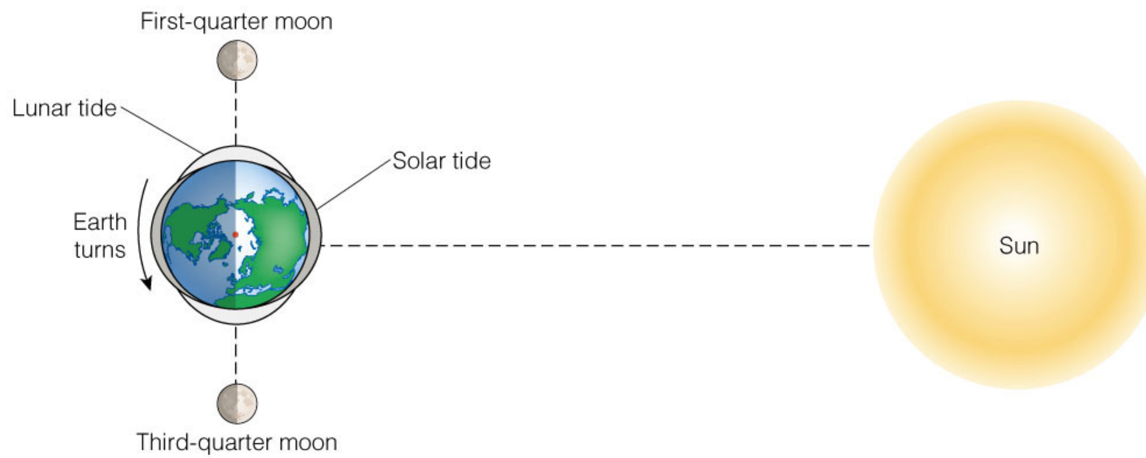
Spring Tide

- Largest tidal range
- Full, new moons
- Lunar and solar tides align
  - constructive interference
- Approximately 2-weeks between spring tides



Neap Tide

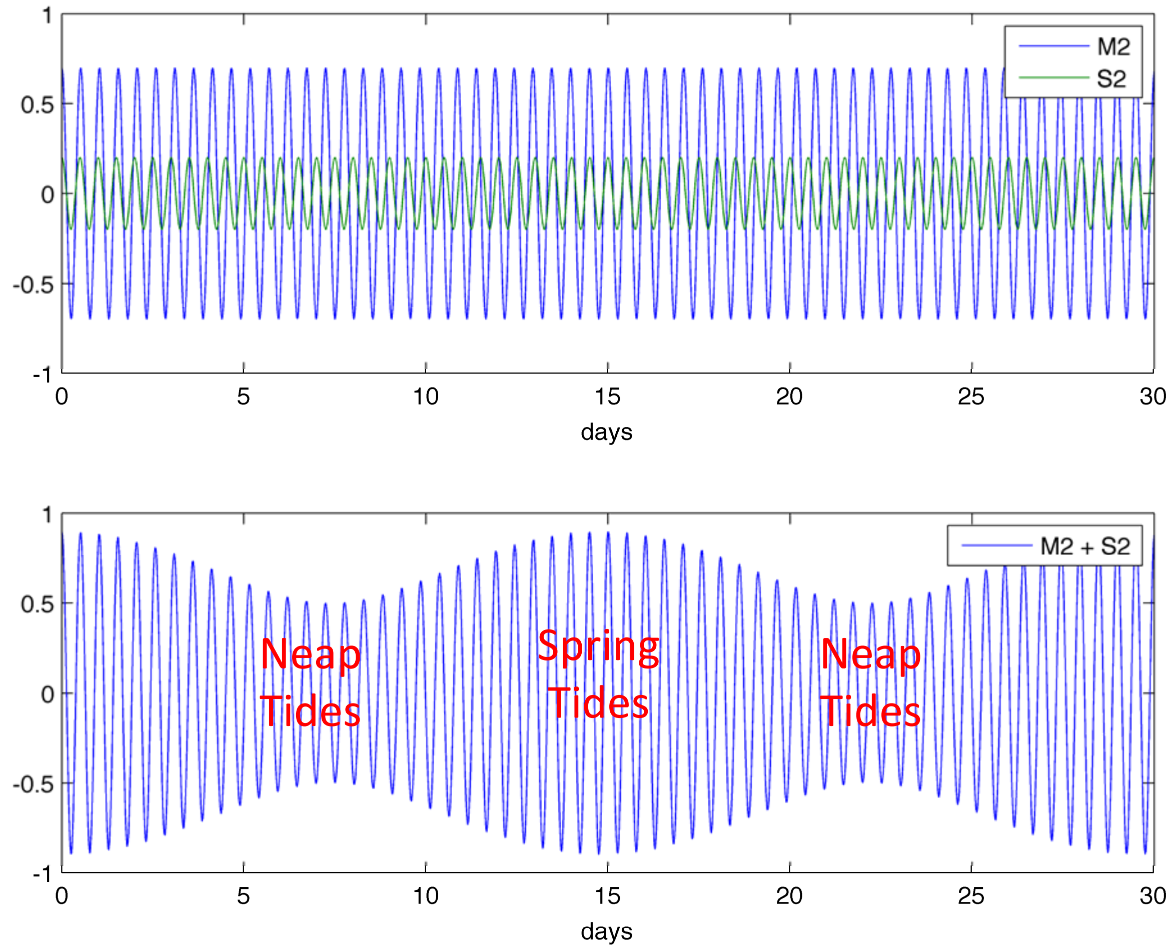
- Least tidal range
- Quarter moons
- Lunar and solar tides at 90^{O}
  - destructive interference



Tidal waves can constructively and destructively interfere with each other.

Wave period due to moon = 12.42 hours. This called the M2 tide.
Wave period due to sun = 12.00 hours. This called the S2 tide.

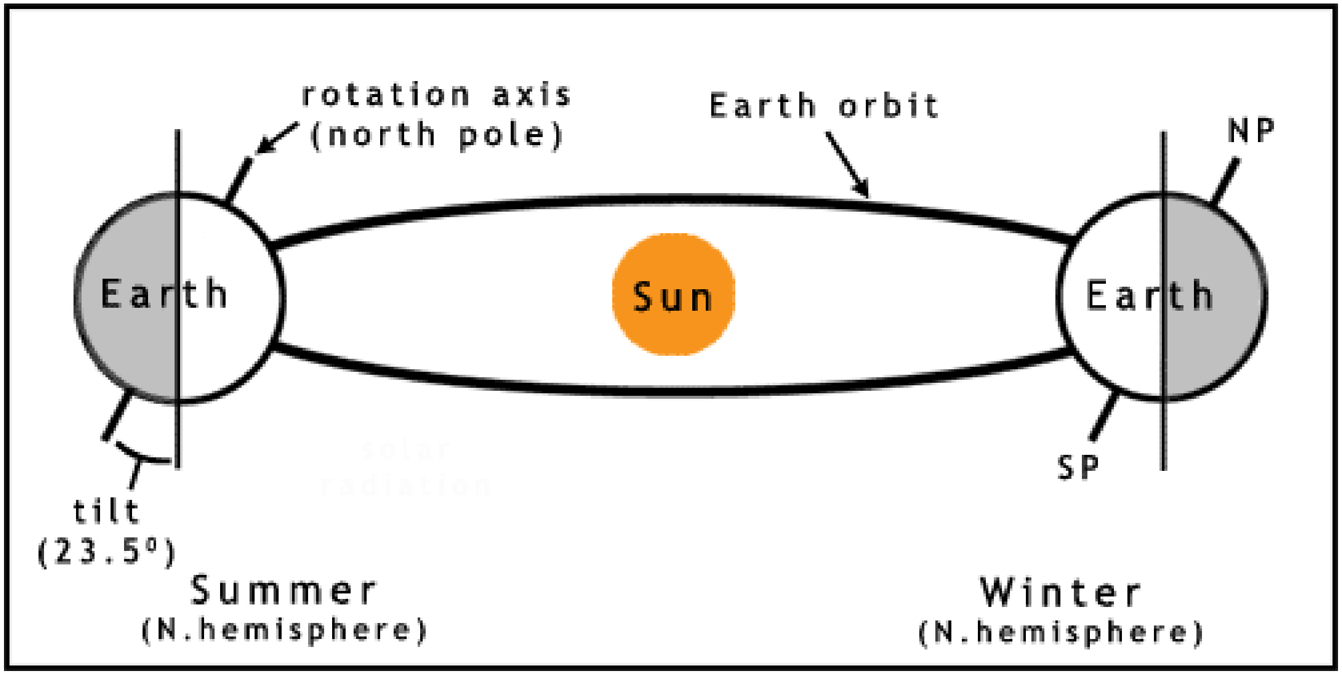
Tides that occur twice a day (2 highs and 2 lows) are called semi-diurnal tides.



Dominant Diurnal and Semidiurnal Tidal Constituents

Symbol	Period (hours)	Description
M_2	12.42	Main Lunar Semidiurnal constituent
S_2	12.00	Main Solar Semidiurnal constituent
N_2	12.66	Lunar constituent due to monthly variation in the Moon's distance
K_2	11.97	Solar-lunar constituent due to changes in declination of the sun and the moon
K_1	23.93	Solar-lunar constituent
O_1	25.82	Main lunar diurnal constituent
P_1	24.07	Main solar diurnal constituent

Other complicating factors: Declination

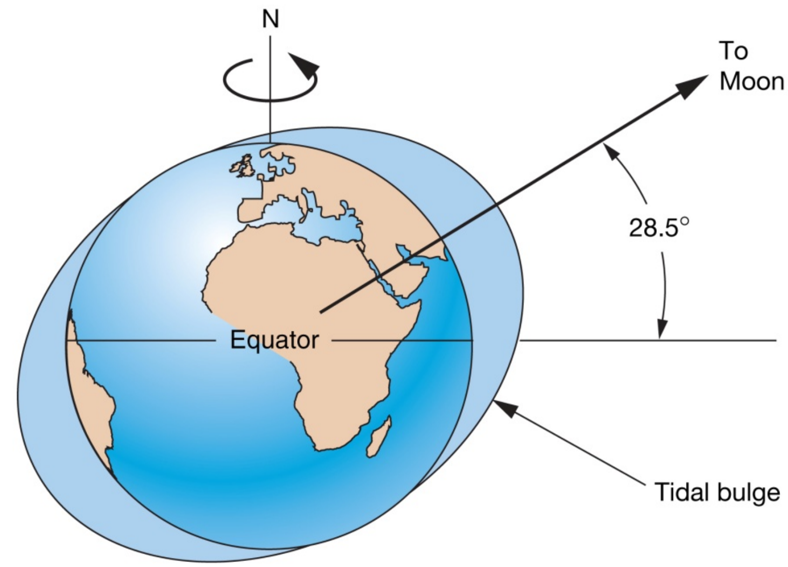


Earth's axis is tilted 23.5 degrees relative to its rotation about the Sun

Other complicating factors: Declination

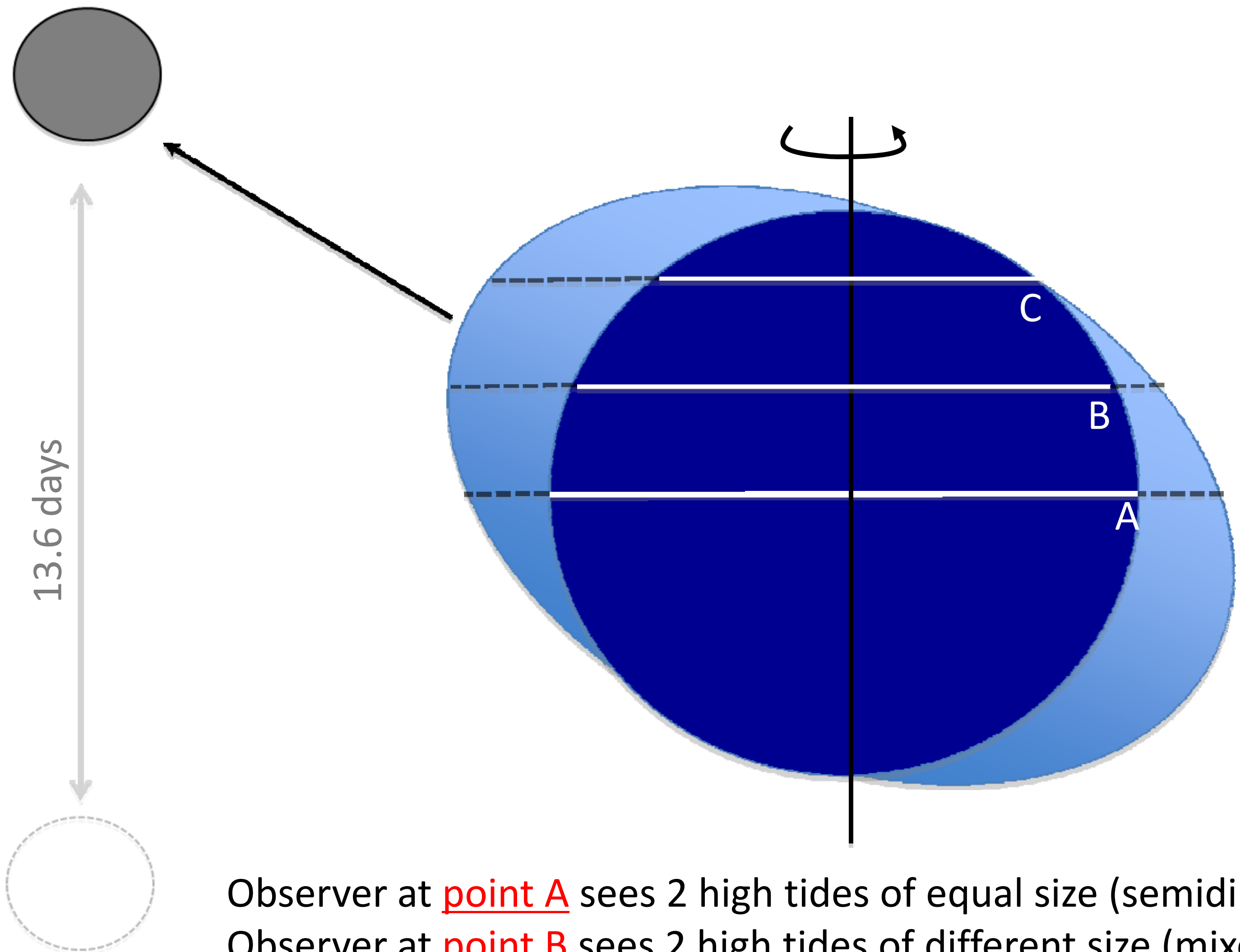
- Angular distance of the Moon or Sun above or below Earth's equator
  - Sun to Earth: 23.5° N or S of equator
  - Moon to Earth: 28.5° N or S of equator

- Shifts lunar and solar bulges from equator resulting in unequal tides at a given latitude



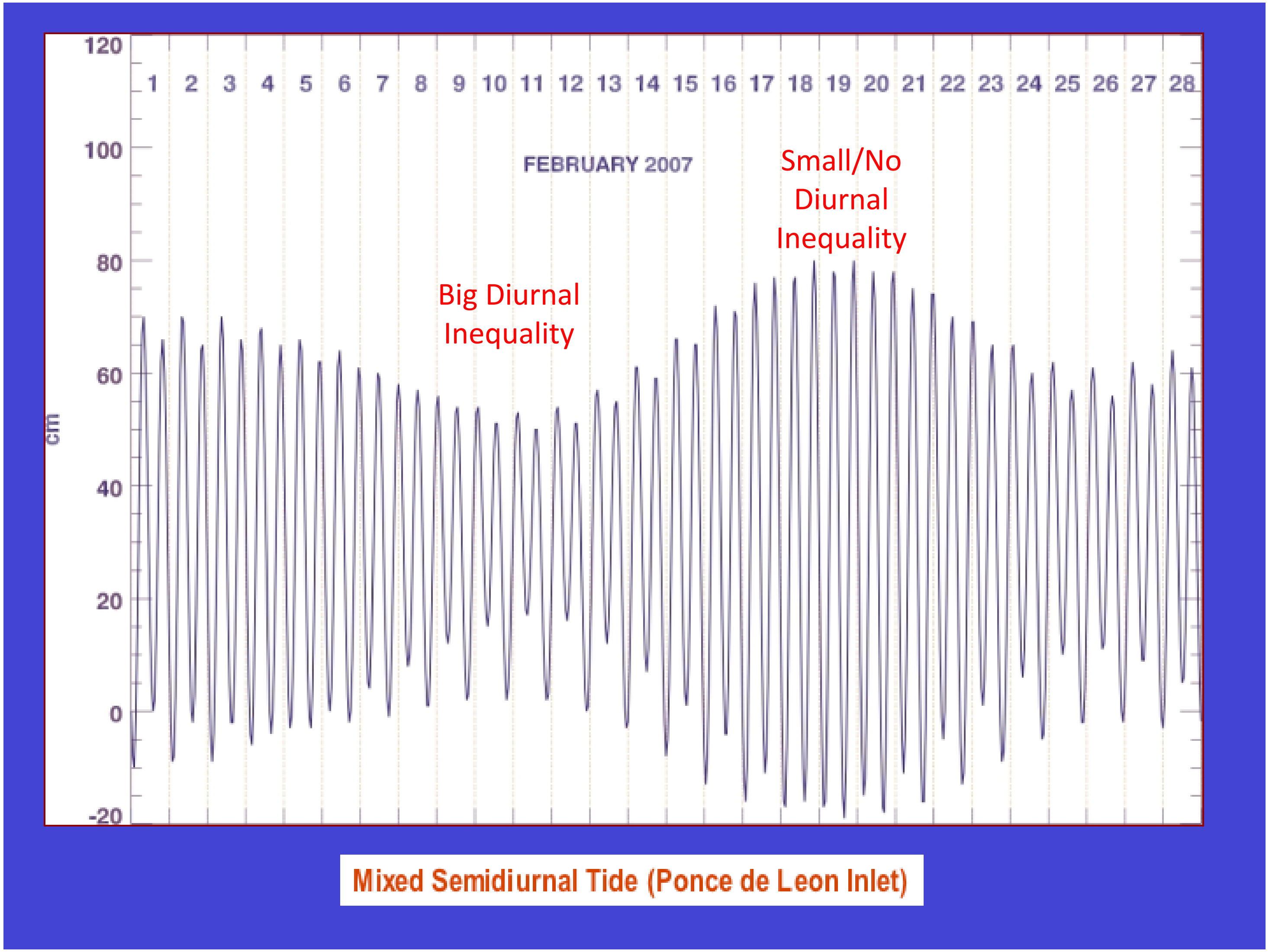
Moon orbits Earth every 27.3 days

Declination of Earth's Axis Leads to Diurnal and Mixed Tides



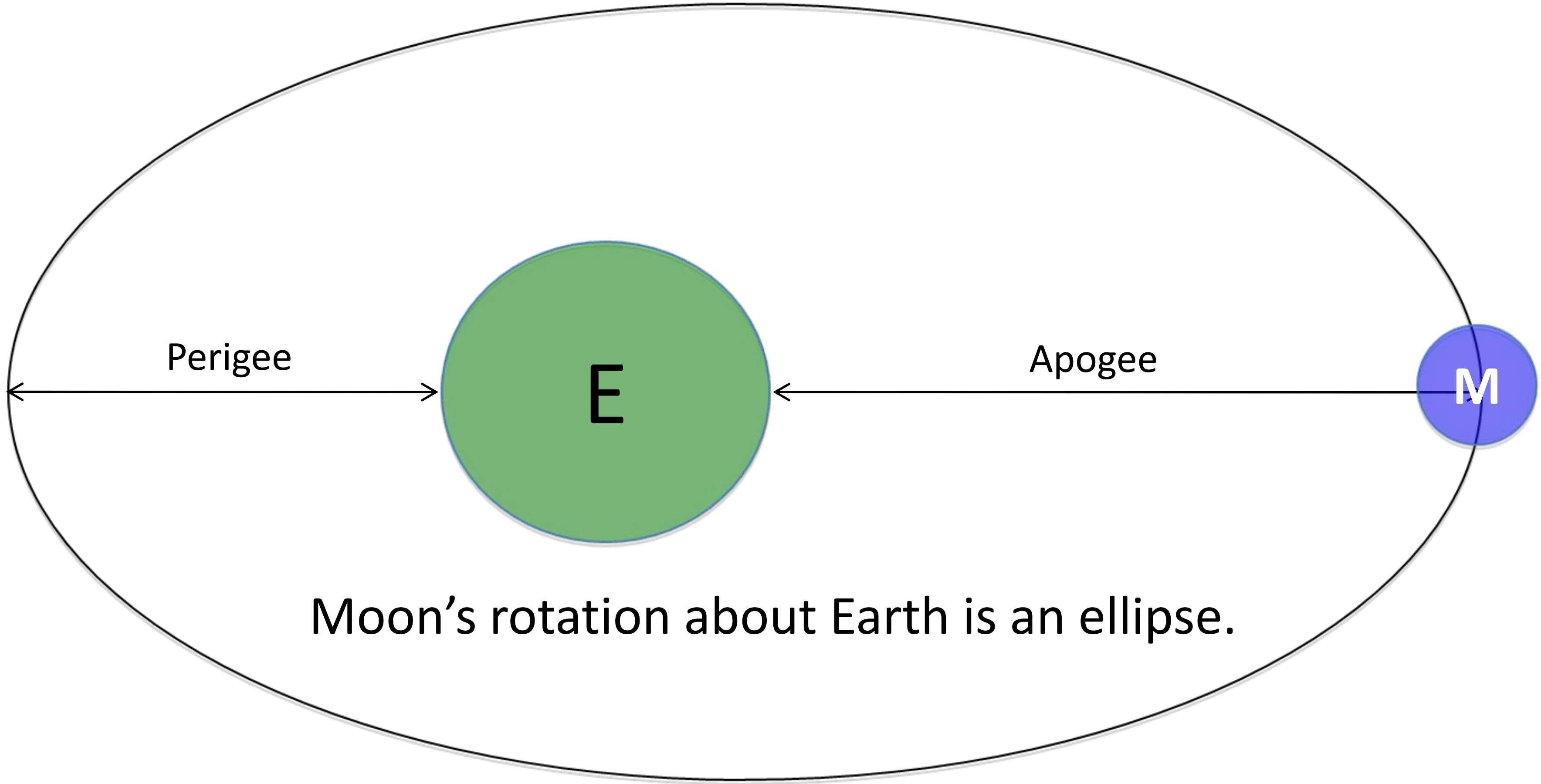
Observer at point A sees 2 high tides of equal size (semidiurnal).
Observer at point B sees 2 high tides of different size (mixed tides).
Observer at point C see only 1 high tide (diurnal tides).

Mixed Tides have 2 unequal highs and lows during the day.



Diurnal Inequality – When two tides have different amplitudes.

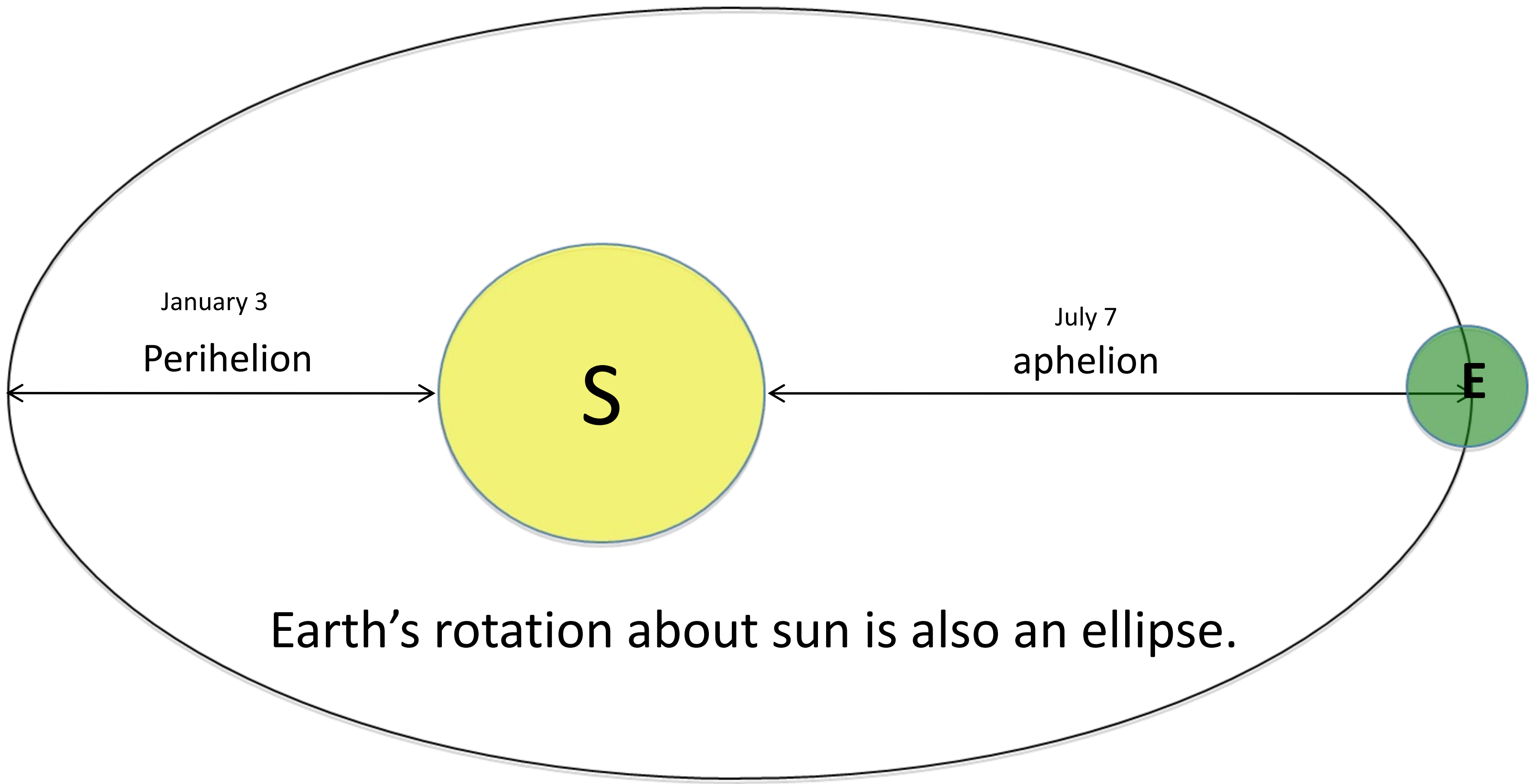
Other complicating factors: Elliptical orbits



Moon's orbit around Earth is elliptical not circular so the size of the bulge will vary with the moon's distance from earth. This has period of 27.55 days. So there are also small variations of the tides over monthly time scales.

*The *Highest Astronomical Tide* is a perigean spring tide when both the sun and moon are closest to Earth.

Other complicating factors: Elliptical orbits



Perihelion to aphelion occurs in one anomalistic year (365.2596 days).

Tide generating forces are greater in the northern hemisphere winter.

Equilibrium Theory Explains:

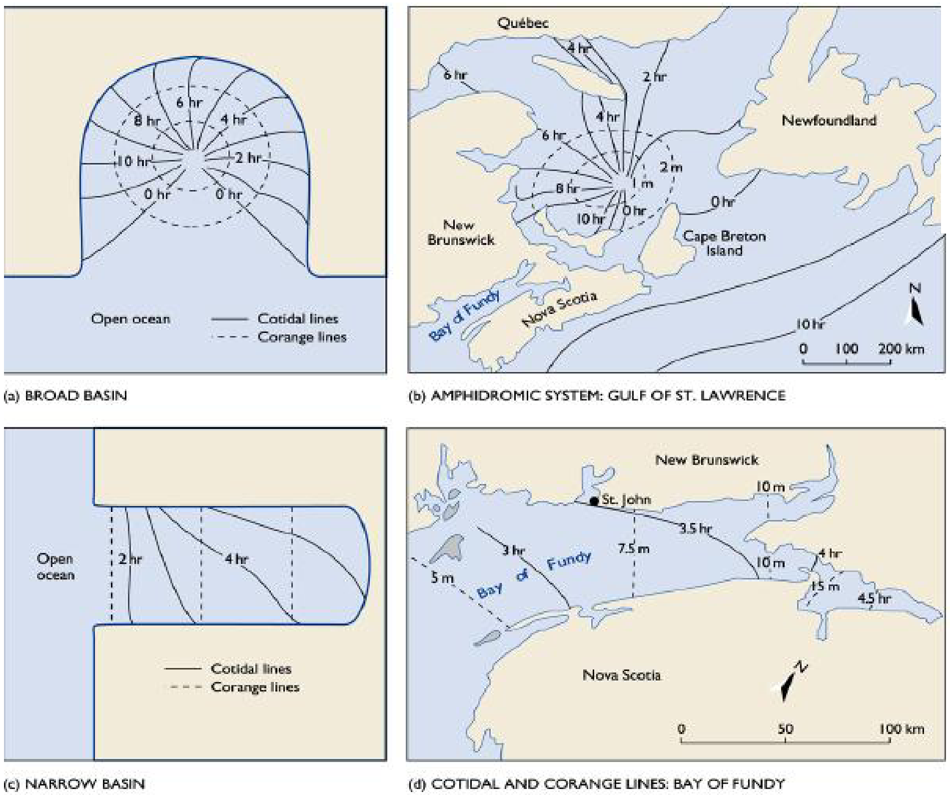
• Geographic variation of tidal periods (diurnal, semi-diurnal, mixed)
- Sun and Moon's declination causes latitudinal variation in sizes of "bulges" seen at a a point on the rotating Earth

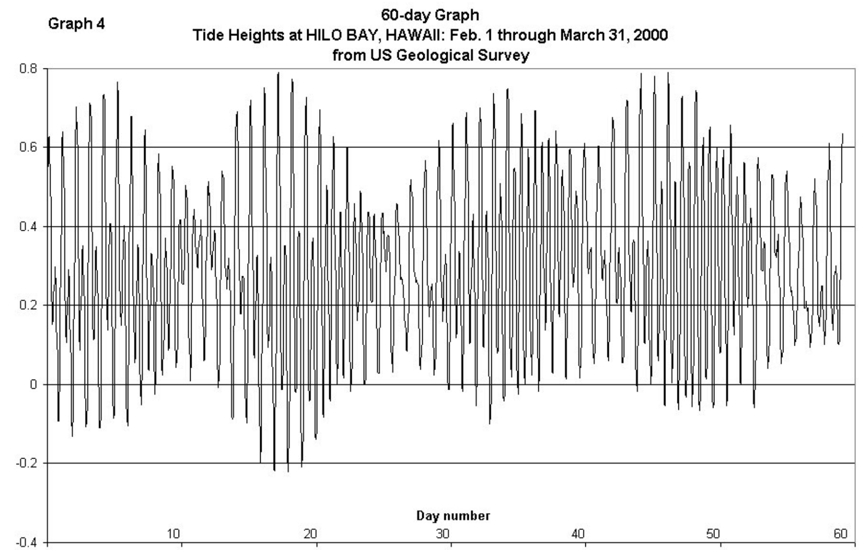
• Temporal variations of tidal periods (spring, neap, perihelion, aphelion)
Moon's phases and alignment of the sun and moon orbits

But, the Earth is not totally covered in water, which makes tides a little more complicated.

Real Tides Affected by Many Factors

- Tides are shallow water waves with speed determined by depth of water
- Continents and friction with seafloor modify tidal bulges

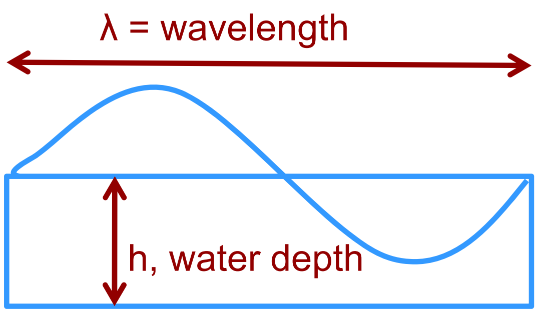




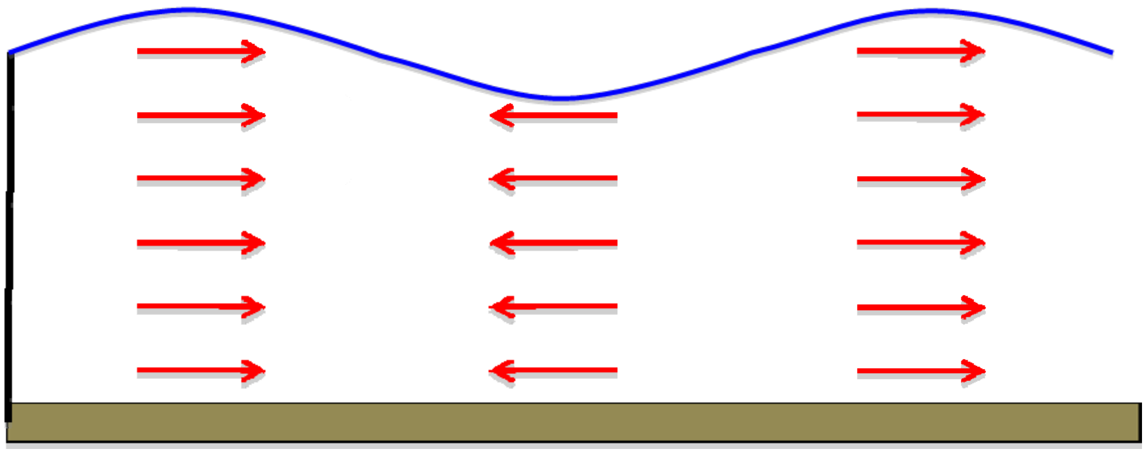
- Tidal movement blocked by land masses
- Tides deflected by Earth's rotation (Coriolis effect)

Tides as Shallow Water Waves

Considered shallow water waves since $\lambda >> 20h$

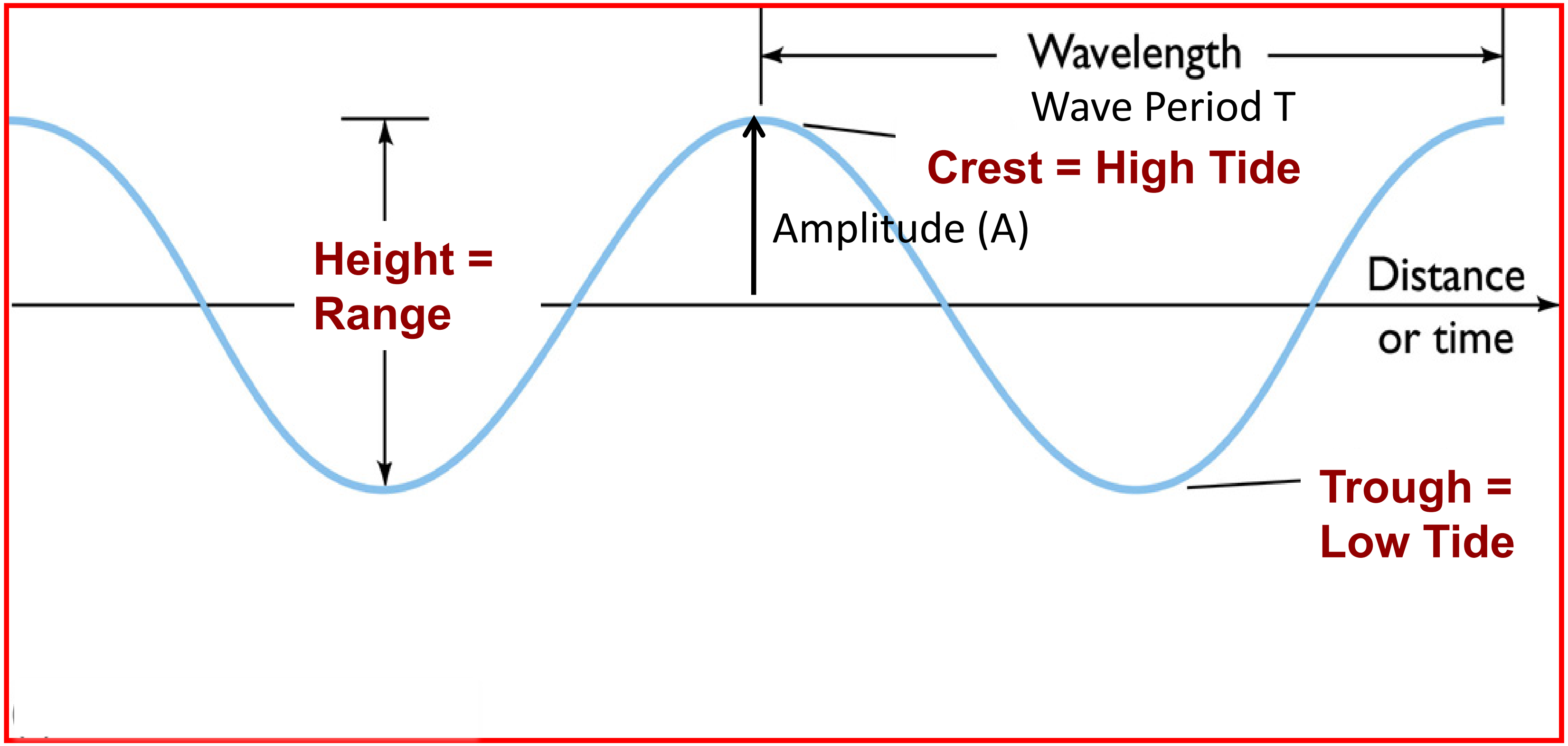


- wavelength is similar to water depth
- wave speed depends only on water depth, $c = \sqrt{gh}$



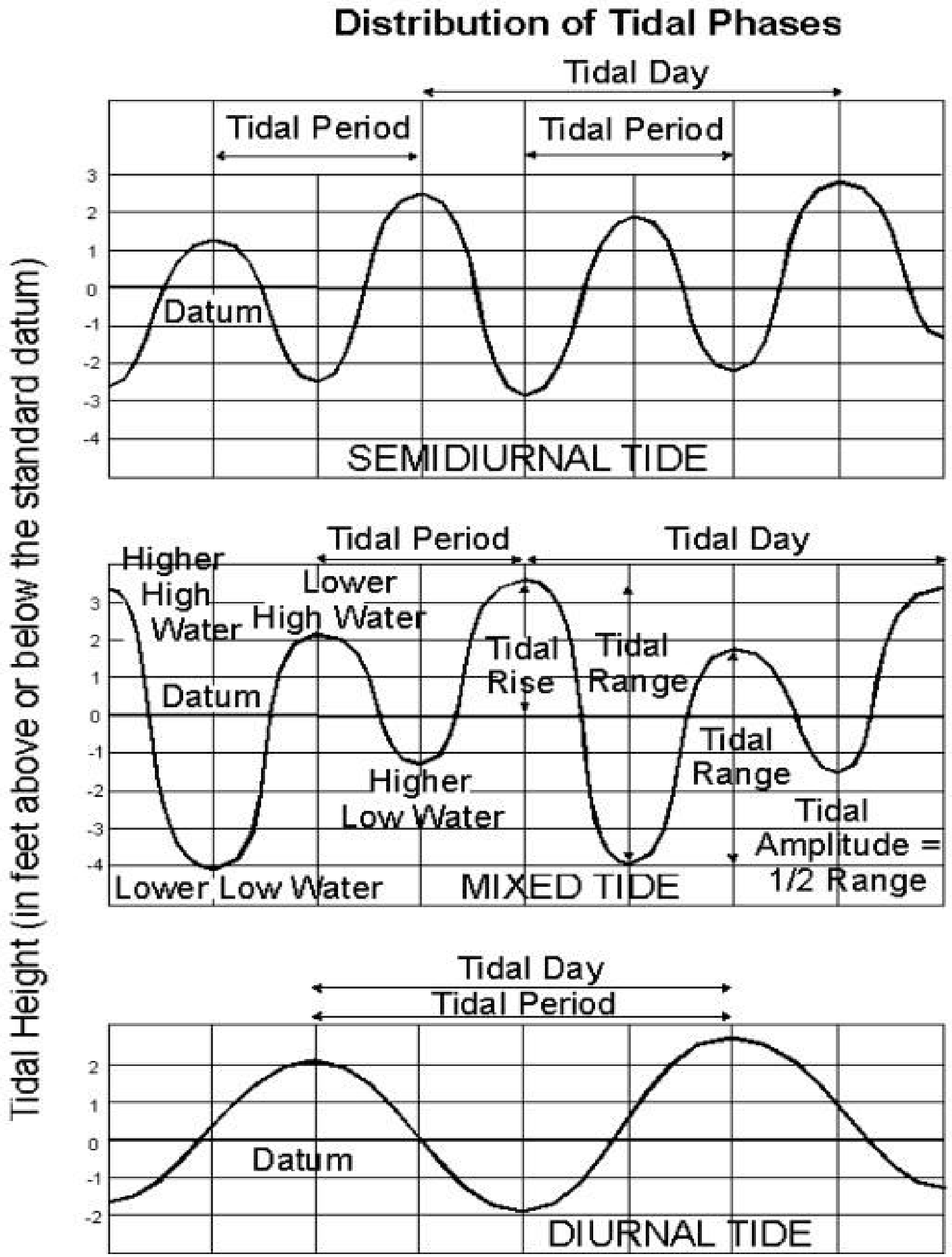
For very long waves like tides, orbital velocities become flat.

Tidal Wave Form for Coastal Regions

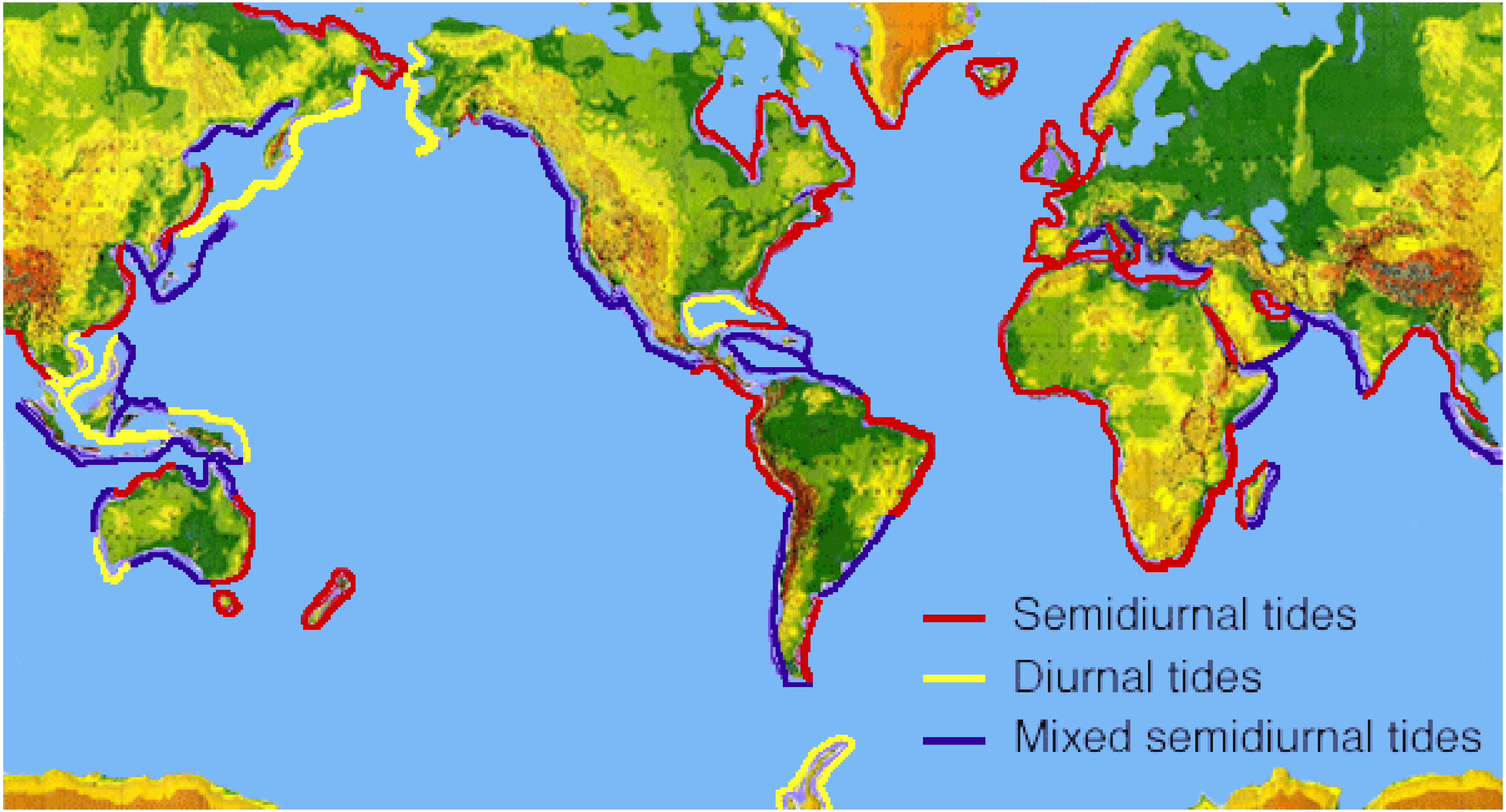


Tidal Patterns

- Semidiurnal
  - Two high tides/two low tides per day
  - Tidal range about same
- Diurnal
  - One high tide/one low tide per day
  - Rarest
- Mixed
  - Two high tides/two low tides per day
  - Tidal range different
  - Most common



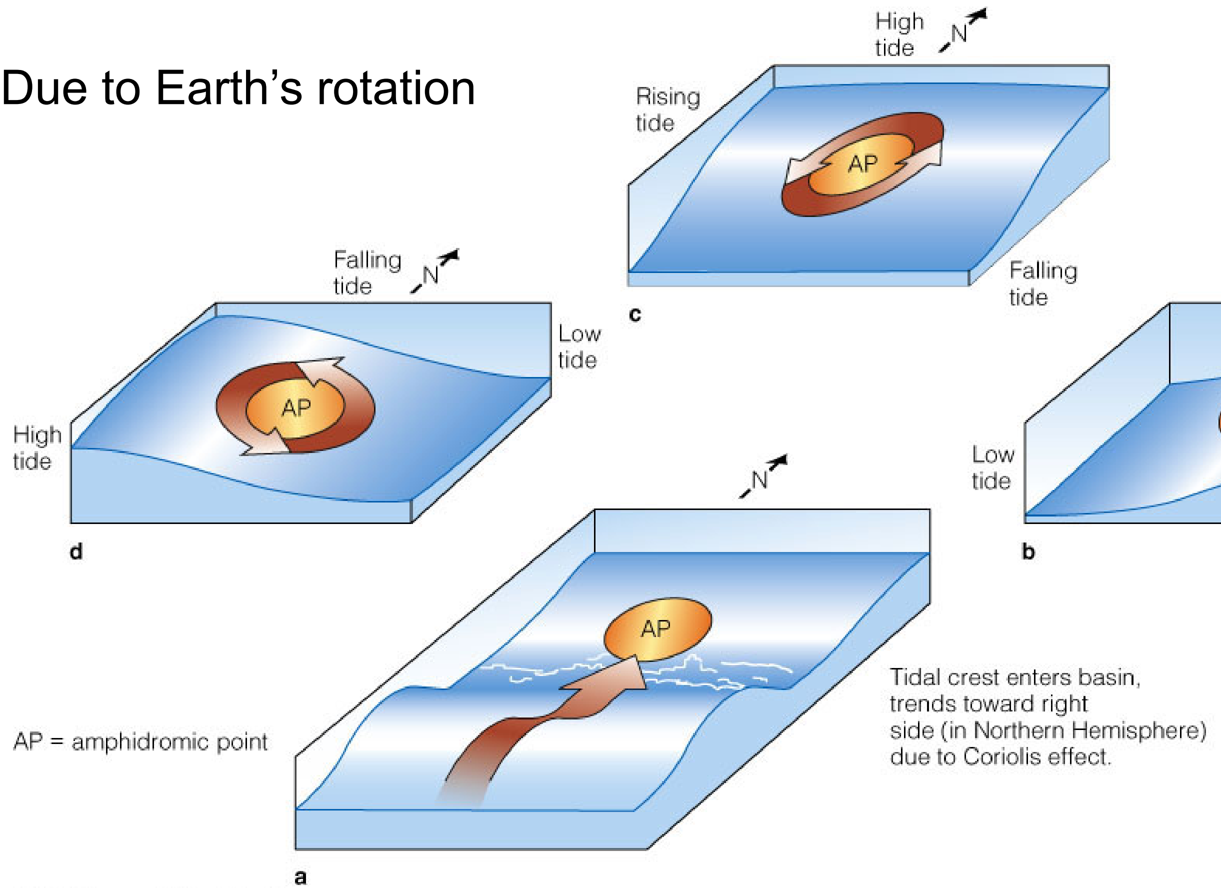
Global Distribution of Tides



www.nos.noaa.gov/.../ tides/media/

Tide Propagation around Ocean Basins

Due to Earth's rotation



Tidal Bulge moves like a wave, that is turned by rotation of the Earth. Wave turns right in the Northern Hemisphere and left in the Southern Hemisphere.

When it encounters the coastline, it becomes a coastally-trapped Kelvin Wave.

Kelvin Wave Solution (for *u = 0*)

$$\eta = \eta_o e^{-x/R} \cos(ky + \omega t) \qquad v = -c\frac{\eta_o}{h} e^{-x/R} \cos(ky + \omega t)$$

Where R is the Rossby radius of deformation.

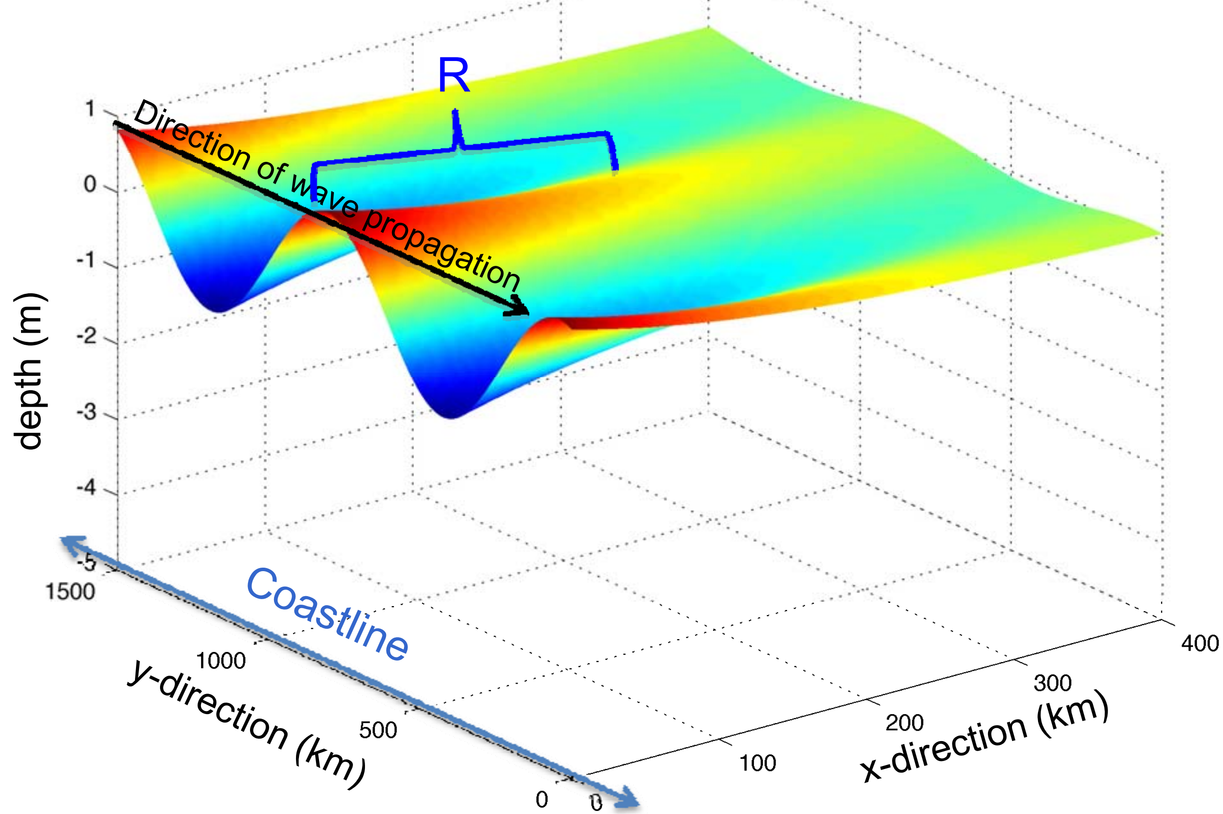
$$R = \frac{c}{f} = \frac{\sqrt{gh}}{f}$$

Ratio of shallow water wave speed to Coriolis parameter

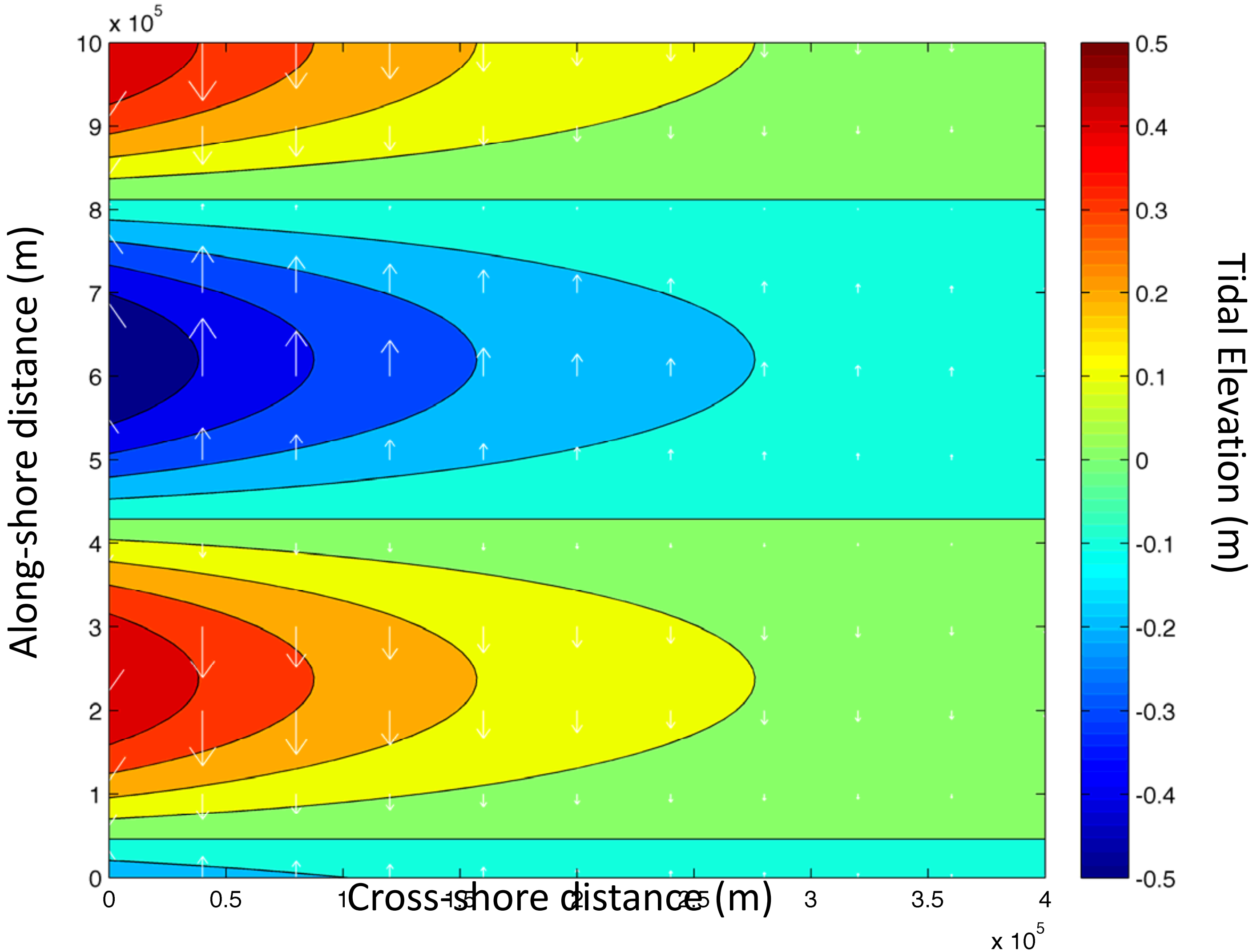
The length scale over which rotational effects become important

Large R - rotation important only at large scales

Small R – tight radius of curvature, rotation important at smaller length scales



Kelvin Wave Solution

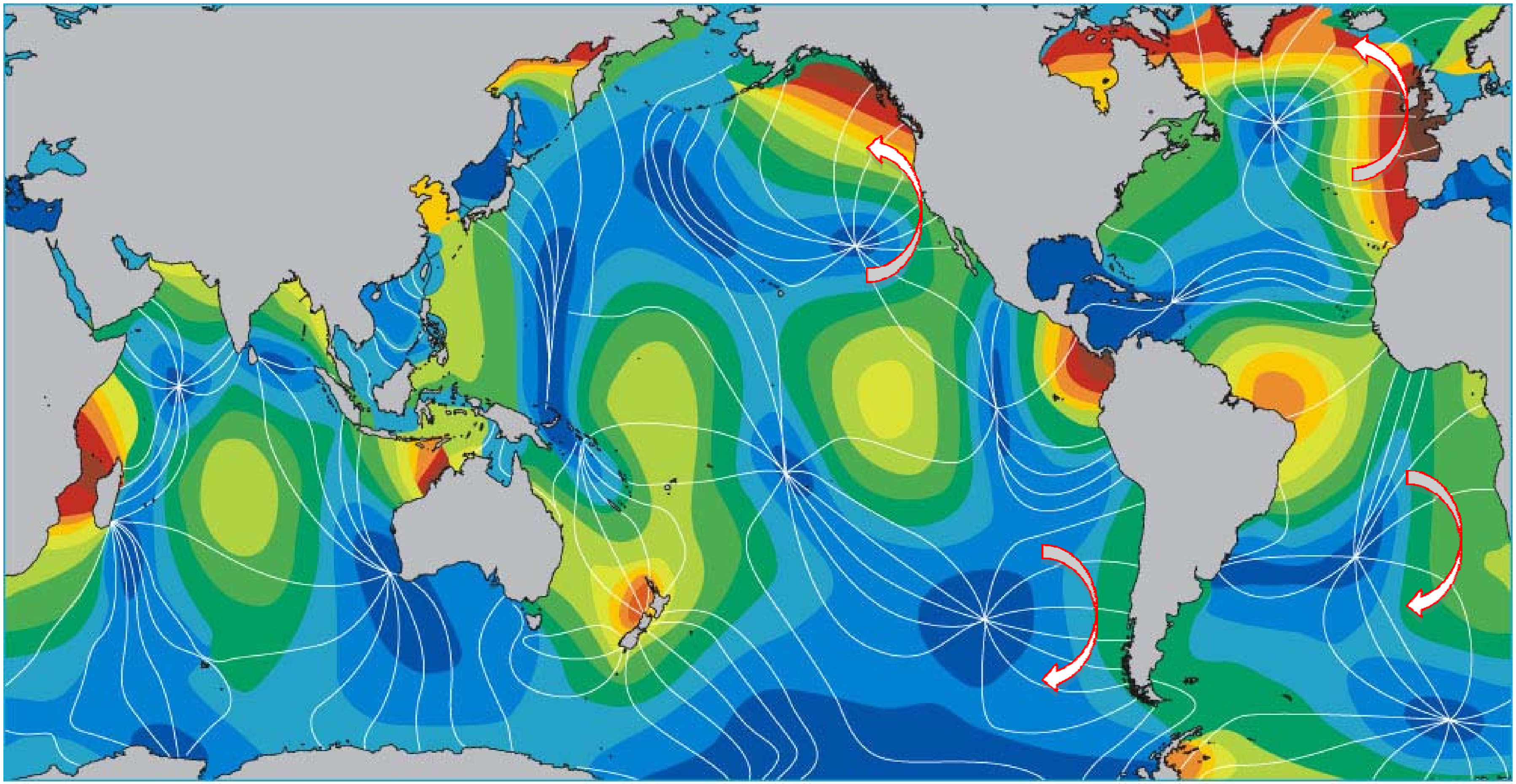


White arrows show wave orbital velocity. Kelvin wave is a progressive wave, max velocity at high tide!!!

Tidal Propagation Around Ocean Basins

Amphidromic System (rotary standing wave):

- Balance of Coriolis and pressure gradient
- Rotation with coastline <u>to the right/left in the Northern/Southern Hemisphere</u>



Rotary motion of the tides: CCW in northern hemisphere; CW in southern hemisphere

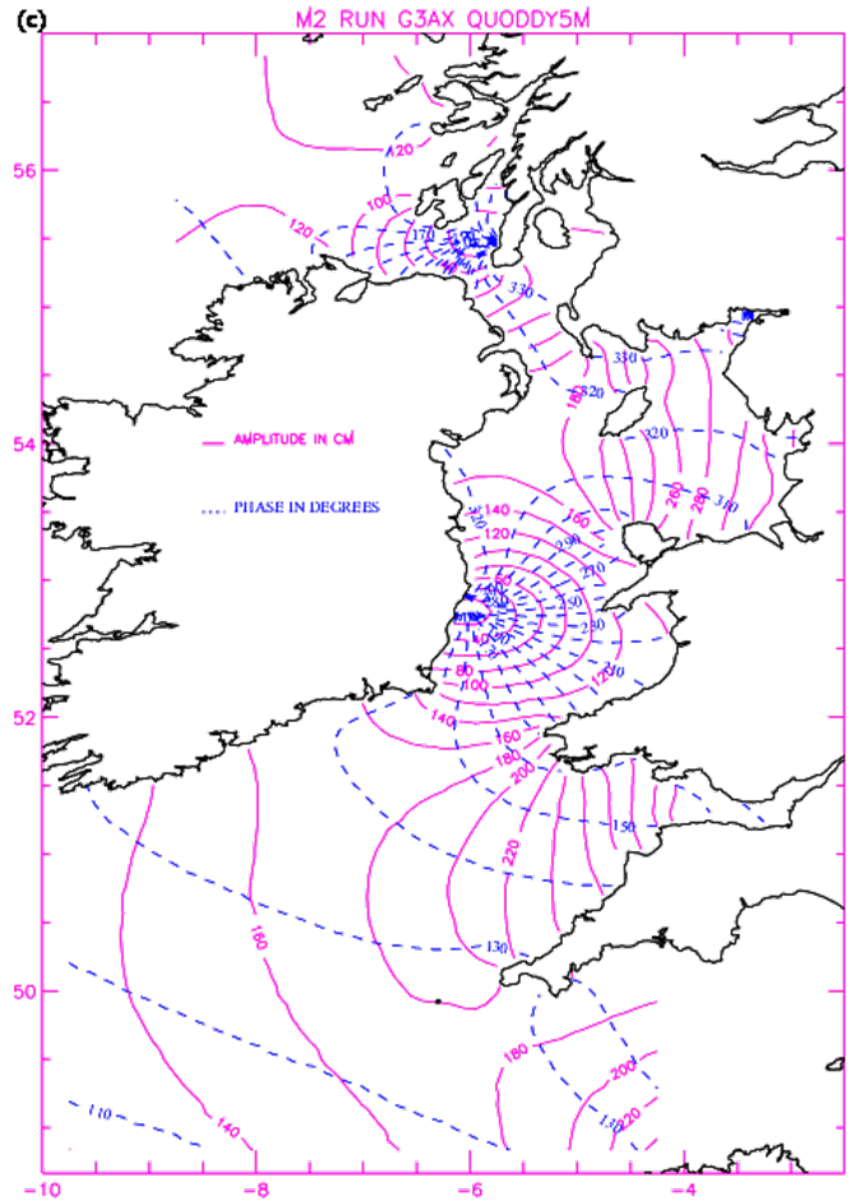
Tides as Rotary Standing Waves

Amphidromic System (rotary standing wave):

- Tidal wave progress about a node (no vertical displacement)
- Antinode (Maximum vertical displacement) rotates about the basin edges

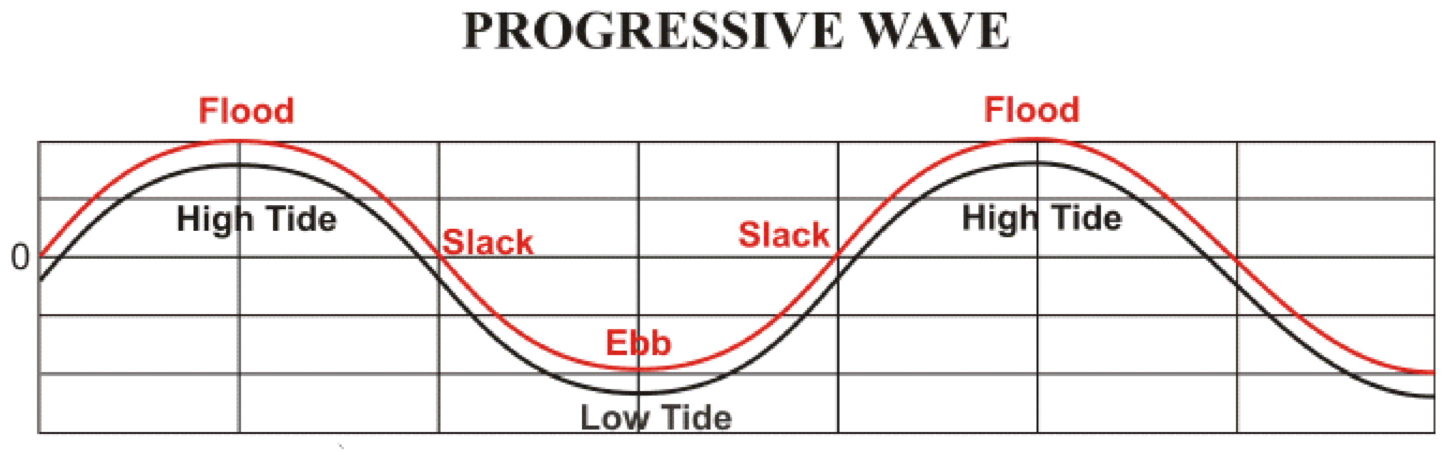
Co-tidal lines connect points that experience high tide at the same time

Co-range circles connect points of the same tidal range



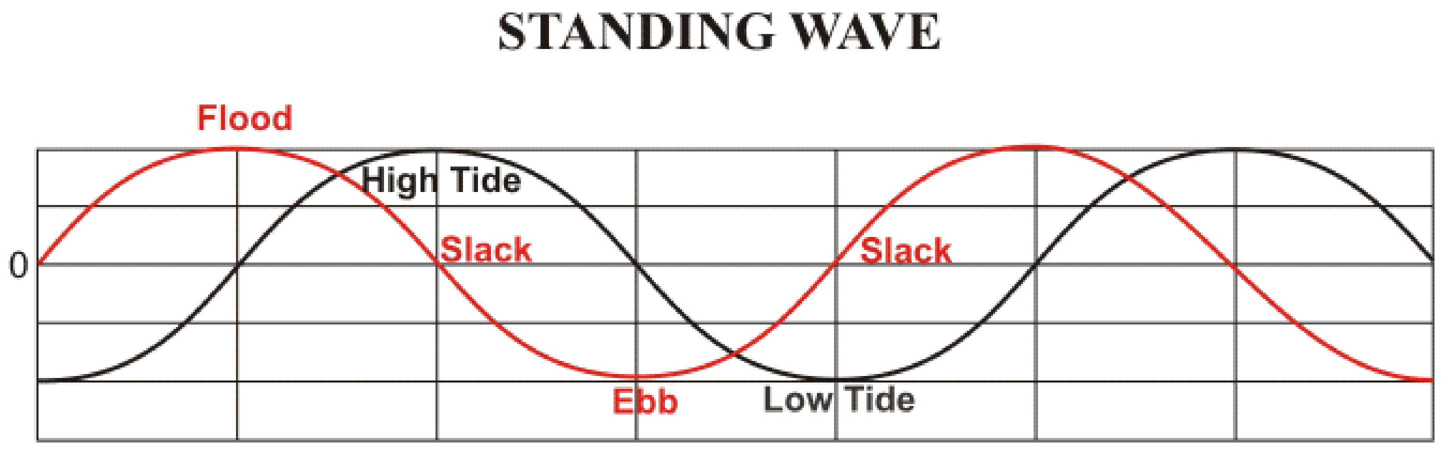
Along open coast lines, tides are usually "progressive waves".

Maximum velocity magnitude occurs at high and low water.

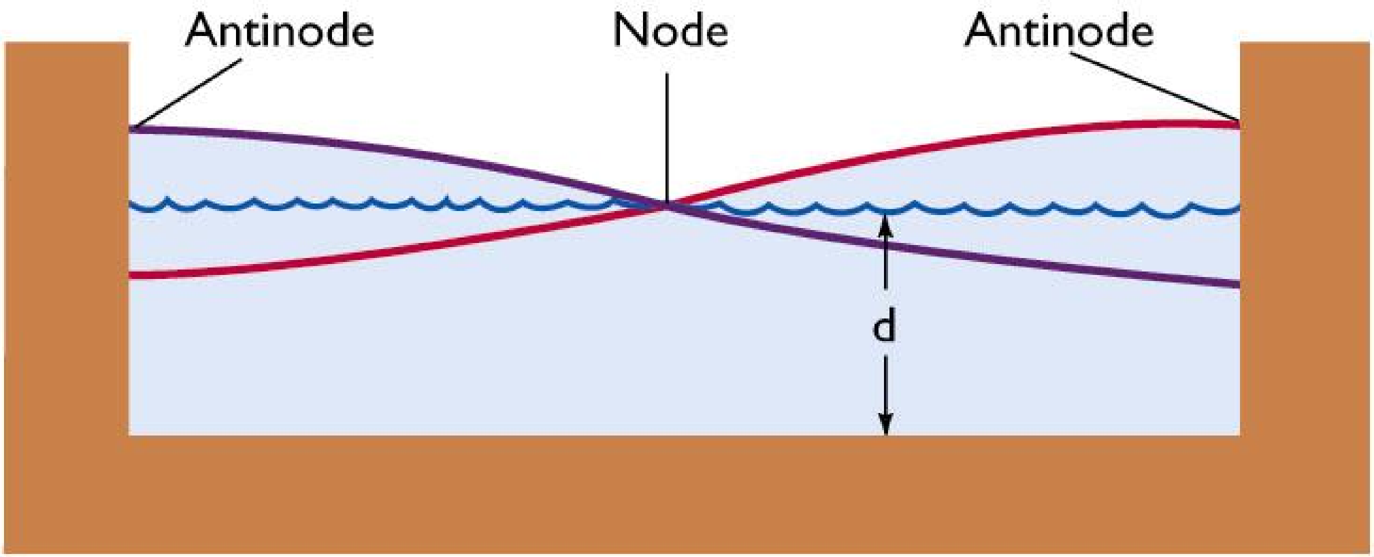


In some embayments or estuaries, the tide is a "standing wave".

Velocity is zero at high and low water.



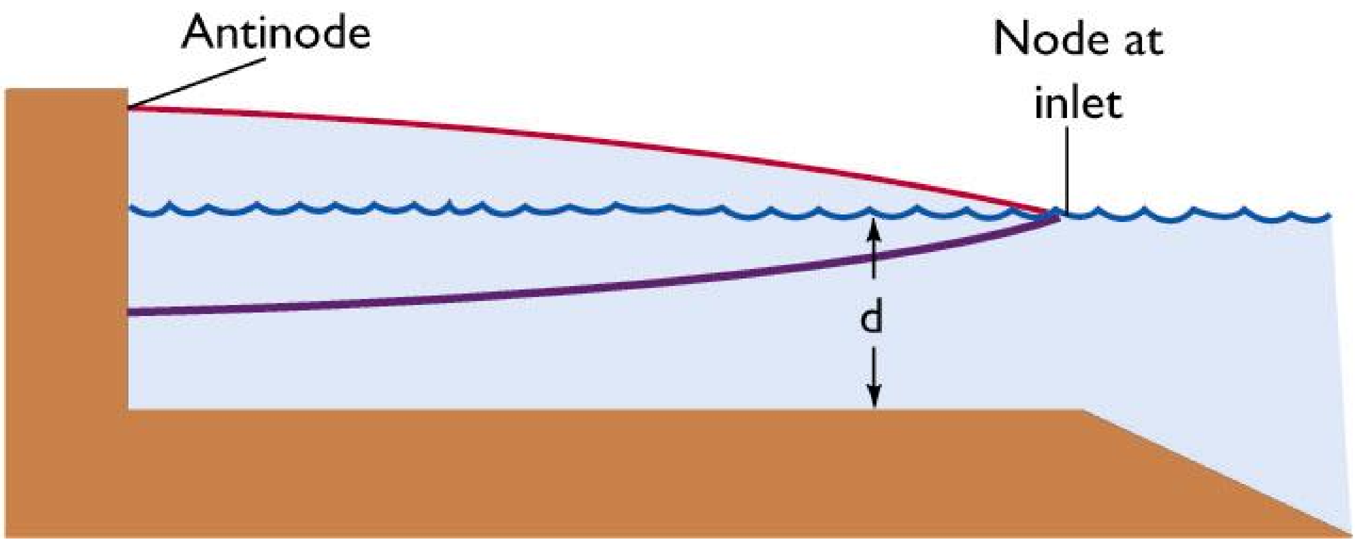
Natural Period of Standing Waves



Closed basins

- teacup
- lake
- ocean basin

$$T = \frac{2l}{\sqrt{gd}} \text{ sec}$$



Open basins

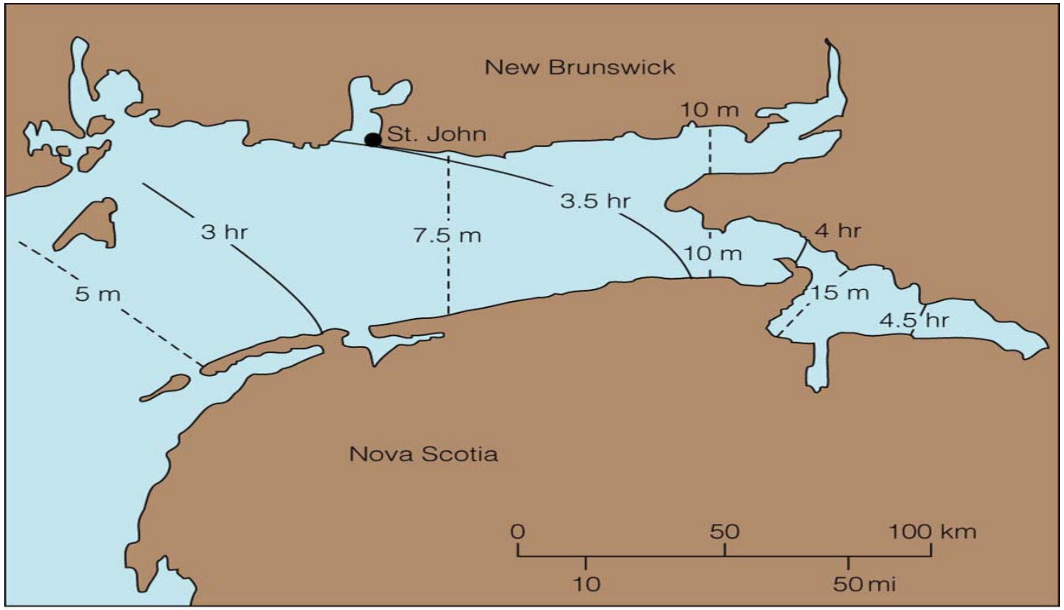
- estuary
- harbor

$$T = \frac{4l}{\sqrt{gd}} \text{ sec}$$

Coastal Standing Waves

- Tide waves reflected by coast
- Amplification of tidal range (resonance)

Resonance is the tendency of a system to oscillate at maximum amplitude at certain frequencies. At these frequencies, even small periodic driving forces can produce large amplitude vibrations, because the system stores vibrational energy.



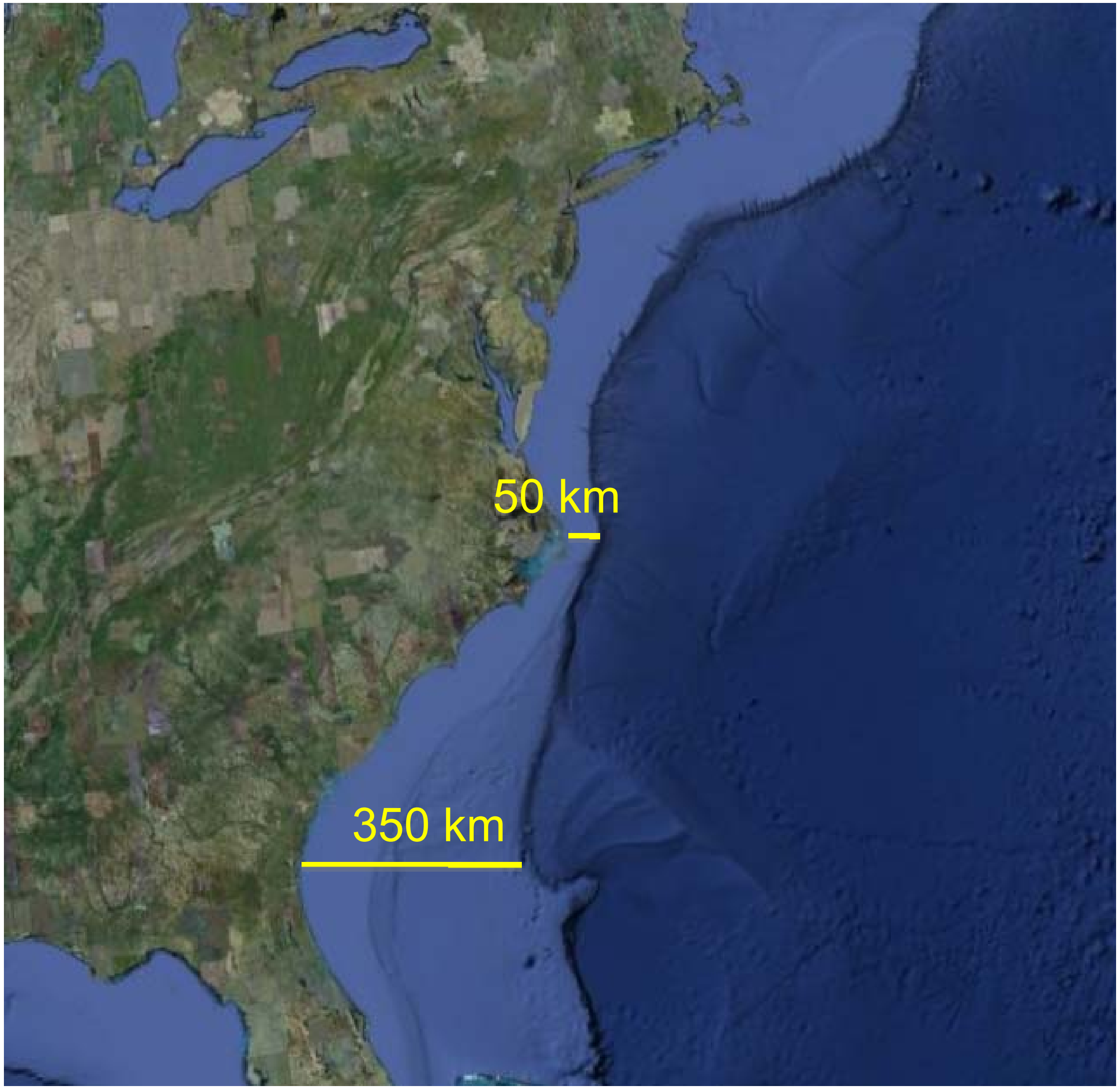
Bay of Fundy has 15 meter tides (over 50 feet)

Tides can produce resonance, when the basin length is ¼ the tidal wave length:

Natural period of the basin:

$$T = \frac{2L}{\sqrt{gH}}$$

Interaction of tide and continental shelf



Assume depth over continental shelf is 100m.

Resonance?

Is width of shelf ¼ of the tidal wavelength?

Tidal range at Cape Hatteras is less than 3 m.

Tidal range along Georgia coast is more than 6 m.

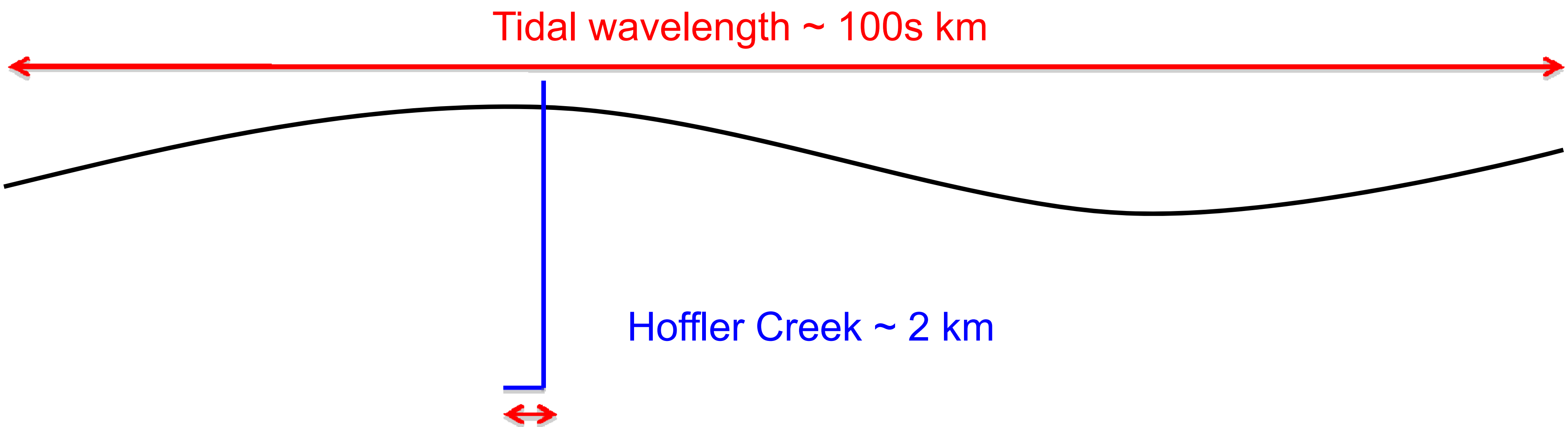
Tides can also increase due to convergence

Remember the continuity equation

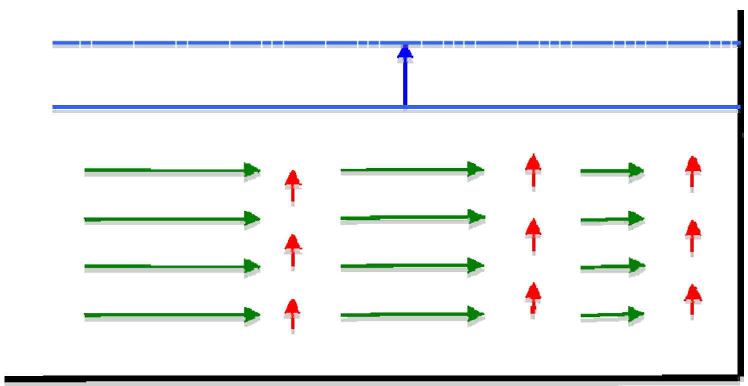
Converging channel can increase the water elevation and tides get bigger.

Tides in Short Embayments and Creeks

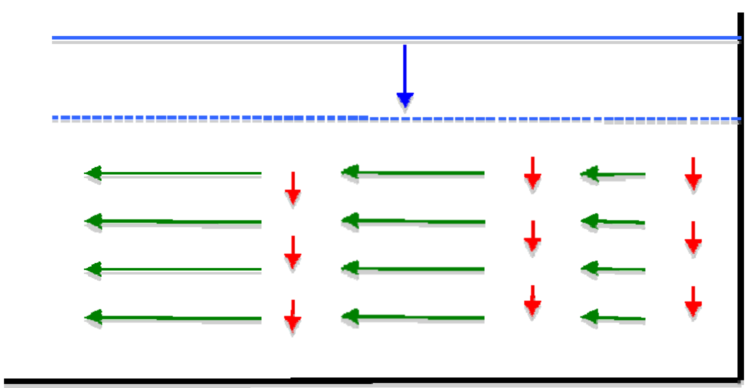
What happens in basins that are short compared to the tidal wavelength?
No rotation, just direction reversal between high and low tide



Because of continuity relationship:



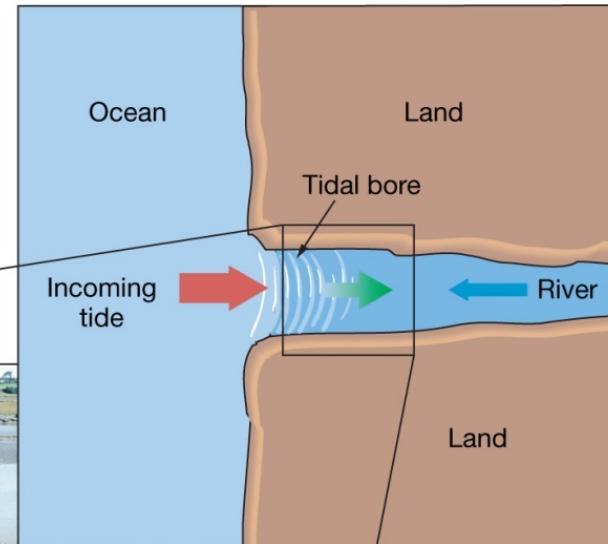
Water surface goes up during flood.



Water surface goes down during ebb.

Tidal Bores

- High tidal range
- Rapid convergence



Tidal bore in low-gradient rivers

Dynamical Theory of Tides

- Equilibrium theory ignored the inertia of the water
- Laplace formulated the basic form of the governing shallow water equations in 1775

- Tides considered hydrostatically as waves that respond to periodic forcing and thus have the same frequencies as the forcing
- Included depth of the ocean, configuration of the ocean basins, frictional forces, and Coriolis (rotation) effects
- Result are the conservation of momentum and vertically integrated continuity equation (mass conservation) equations

Momentum

$$\frac{\partial \mathbf{U}}{\partial t} \mp 2\omega \mathbf{U}\cos\gamma = -\frac{g}{a}\begin{bmatrix}\csc\gamma \\ 1\end{bmatrix}\partial\frac{\partial}{\partial \boldsymbol{\mu}}\left(\eta_i - \frac{\Phi}{g}\right) + dissipation$$

Continuity

$$\frac{\partial \eta_i}{\partial t} + \frac{\csc\gamma}{a}\left[\frac{\partial}{\partial \mu}\cdot\begin{bmatrix}Hv\sin\gamma \\ Hu\end{bmatrix}\right] = 0$$

Tidal Potential

Tidal Potential

• Tides are a measure of changes in gravity, caused by the attraction of the moon and sun.

• Tidal potential is the gravitational potential that varies with the position of the Moon and Sun relative to the Earth

• Components of the tidal potential
  – Deformation of the solid earth due to gravitational potential (***solid earth tide)***
  – Movement of ocean water due to changing potential (***ocean tides)***
  – Deformation of the solid earth due to the changing load of ocean tides (***ocean tidal loading)***

Tides as Harmonic Constituents

- Tidal variations can be described as a sum of harmonic terms
  - any periodic motion can be resolved into a sum of a series of single harmonic motions

Harmonic of the tidal potential

$$\Omega = A\cos(\omega t + V)$$

Dynamic theory predicts the component for constituent, *i:*

$$\eta(t) = \sum_i A_i \cos\left(\omega_i t + \phi_i + V_i\right)$$

Location (latitude) dependence

Amplitude

Frequency

Phase angle of equilibrium tide (phase lag relative to equilibrium tide)

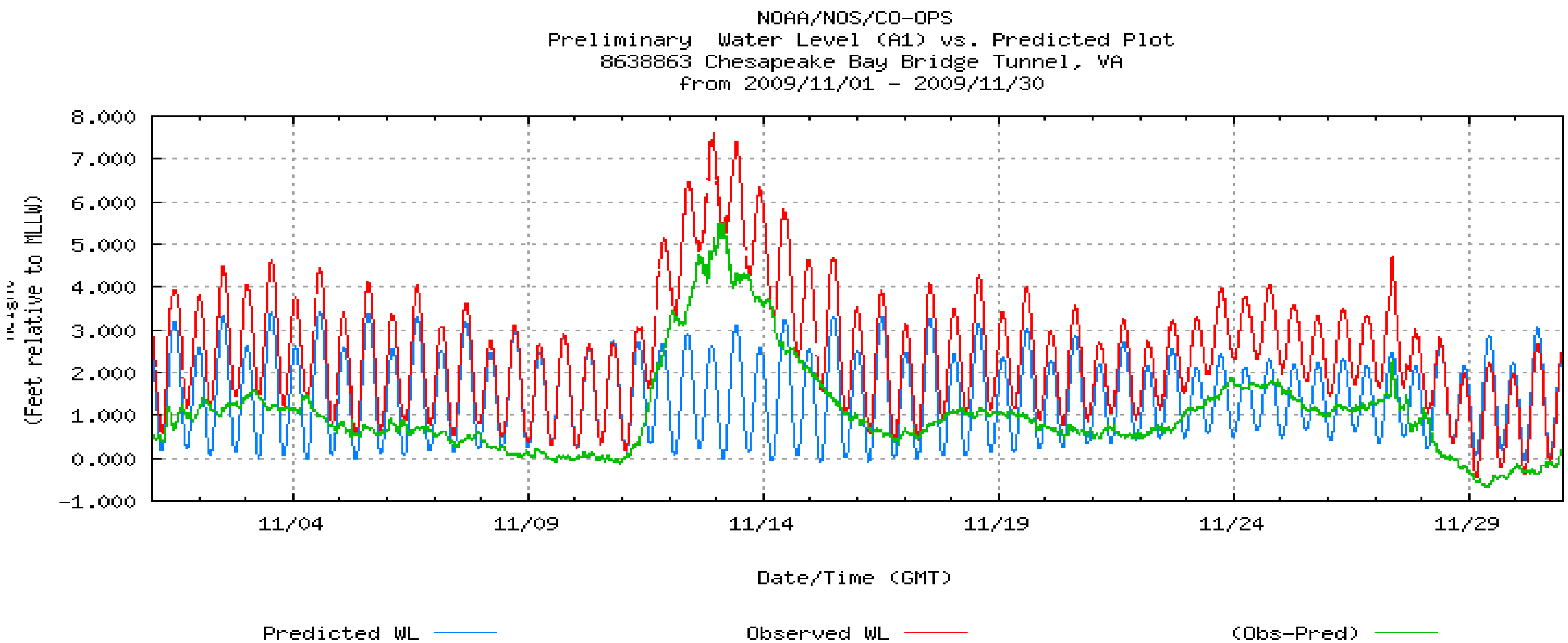
Local phase of constituent

Tides as Harmonic Constituents

Since harmonic decomposition of tides is done on records that are not sufficiently long to separate the 8.85 and 18.6 year tides, add corrections:

$$\eta(t) = \eta_o + \sum_i f_i A_i \cos\left[\omega_i t - \phi_i + \left(V_i + u_i\right)\right]$$

Mean sea level   Nodal factor   Nodal angle



Harmonic Analysis

• Orbital paths are very nearly circular, so sinusoidal variations are suitable for tides.

• The tidal patterns are decomposed into many sinusoids having many fundamental frequencies, corresponding to many different combinations of the motions of the Earth, the moon, and the angles that define the shape and location of their orbits.

• For tides, *harmonic analysis* is not limited to harmonics of a single frequency. Harmonics are multiples of many fundamental frequencies. Their representation as a Fourier series having only one fundamental frequency and its (integer) multiples would require many terms, and would be severely limited in the time-range for which it would be valid.

The study of tide height by harmonic analysis was begun by Laplace, William Thomson (Lord Kelvin), and George Darwin. A.T. Doodson extended their work, introducing the *Doodson Number* notation to organize the hundreds of resulting terms.

Doodson's Frequencies

	Frequency (°/hour)		Period	Source
f_1	14.49205211	1	lunar day	Local mean lunar time
f_2	0.54901653	1	month	Moon's mean longitude
f_3	0.04106864	1	year	Sun's mean longitude
f_4	>0.00464184	8.847	years	Longitude of Moon's perigee
f_5	-0.00220641	18.613	years	Longitude of Moon's ascending node
f_6	0.00000196	20,940	years	Longitude of sun's perigee

Doodson (1922) - Fourier Series Expansion using 6 frequencies

$$f = n_1 f_1 + n_2 f_2 + n_3 f_3 + n_4 f_4 + n_5 f_5 + n_6 f_6$$

The Tidal Constituents

Tidal Species	Name	n_1	n_2	n_3	n_4	n_5	Equilibrium Amplitude* (m)	Period (hr)
Semidiurnal	$n_1 = 2$							
Principal lunar	M_2	2	0	0	0	0	0.242334	12.4206
Principal solar	S_2	2	2	-2	0	0	0.112841	12.0000
Lunar elliptic	N_2	2	-1	0	1	0	0.046398	12.6584
Lunisolar	K_2	2	2	0	0	0	0.030704	11.9673
Diurnal	$n_1 = 1$							
Lunisolar	K_1	1	1	0	0	0	0.141565	23.9344
Principal lunar	O_1	1	-1	0	0	0	0.100514	25.8194
Principal solar	P_1	1	1	-2	0	0	0.046843	24.0659
Elliptic lunar	$>Q_1$	1	-2	0	1	0	0.019256	26.8684
Long Period	$n_1 = 0$							
Fortnightly	M_f	0	2	0	0	0	0.041742	327.85
Monthly	M_m	0	1	0	-1	0	0.022026	661.31
Semiannual	S_{sa}	0	0	2	0	0	0.019446	4383.05

Nonlinear Tides

If a forced frequency gives rise to motions at the forced frequency as well as non-forced frequencies, the tidal response is non-linear.

- Nonlinear tides arise from the advective terms resulting in sums and differences between frequencies

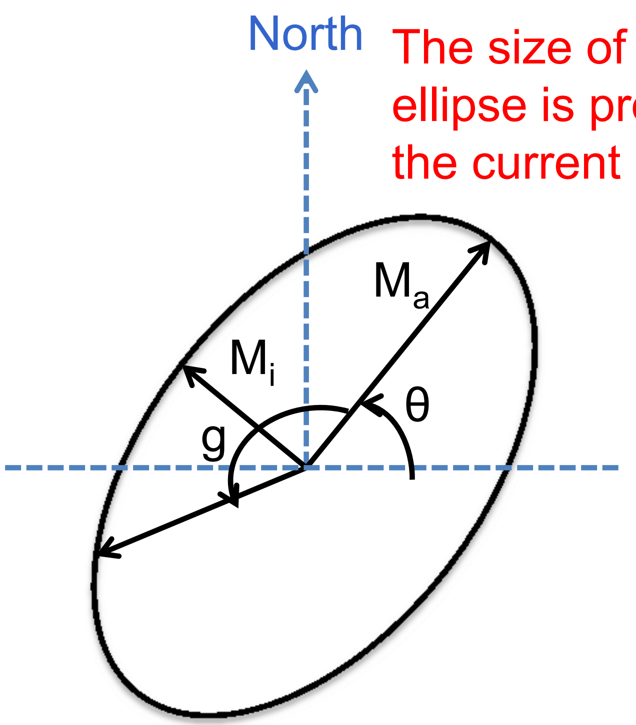
- Overtides – interactions of a tidal frequency with itself
  - M_2 interacting with itself gives rise to M_4, M_6, M_8, etc.
- Compound Tides – interactions between tidal frequencies
  - M_2 interacting with M_2 gives rise to MS_4, MSf, etc.

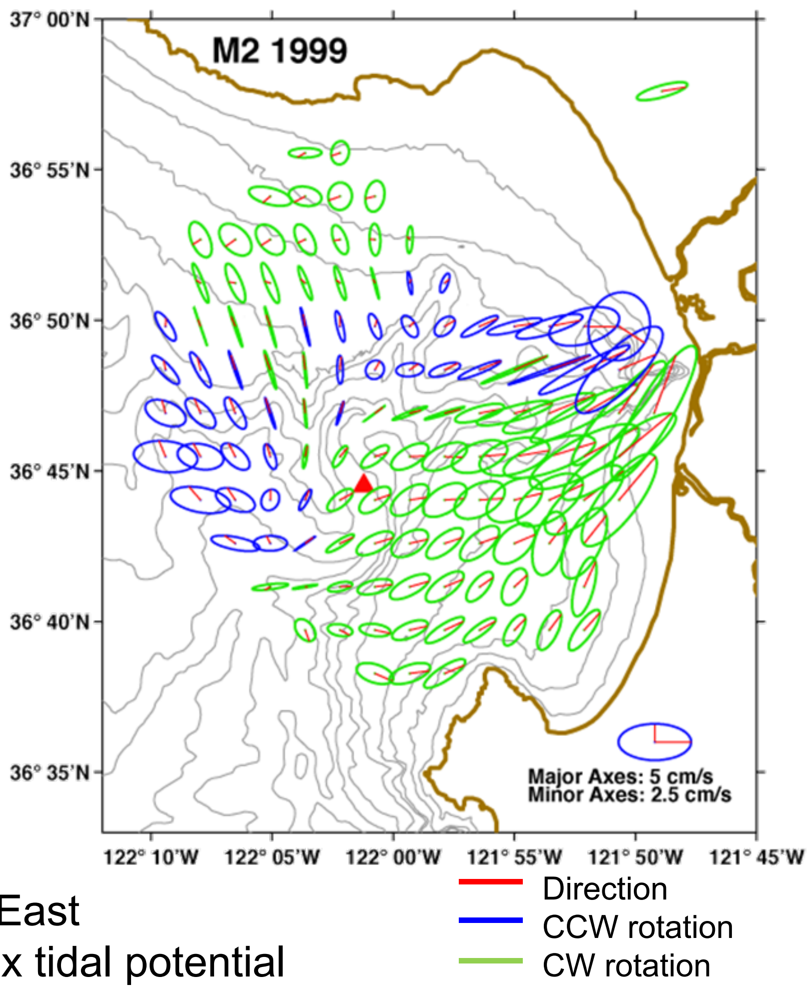
  These tides are know as shallow water tides and are generally smaller than their primary frequencies

- Residual Tide – non-oscillatory component of the tide
  - important for transport, interacts with wind, density, other processes; hard to separate tidal contribution

Tidal Current Ellipses

- Horizontal currents are two dimensional
- Harmonic analysis performed on component vectors
- Results for each constituent are combined and reported using ellipse parameters

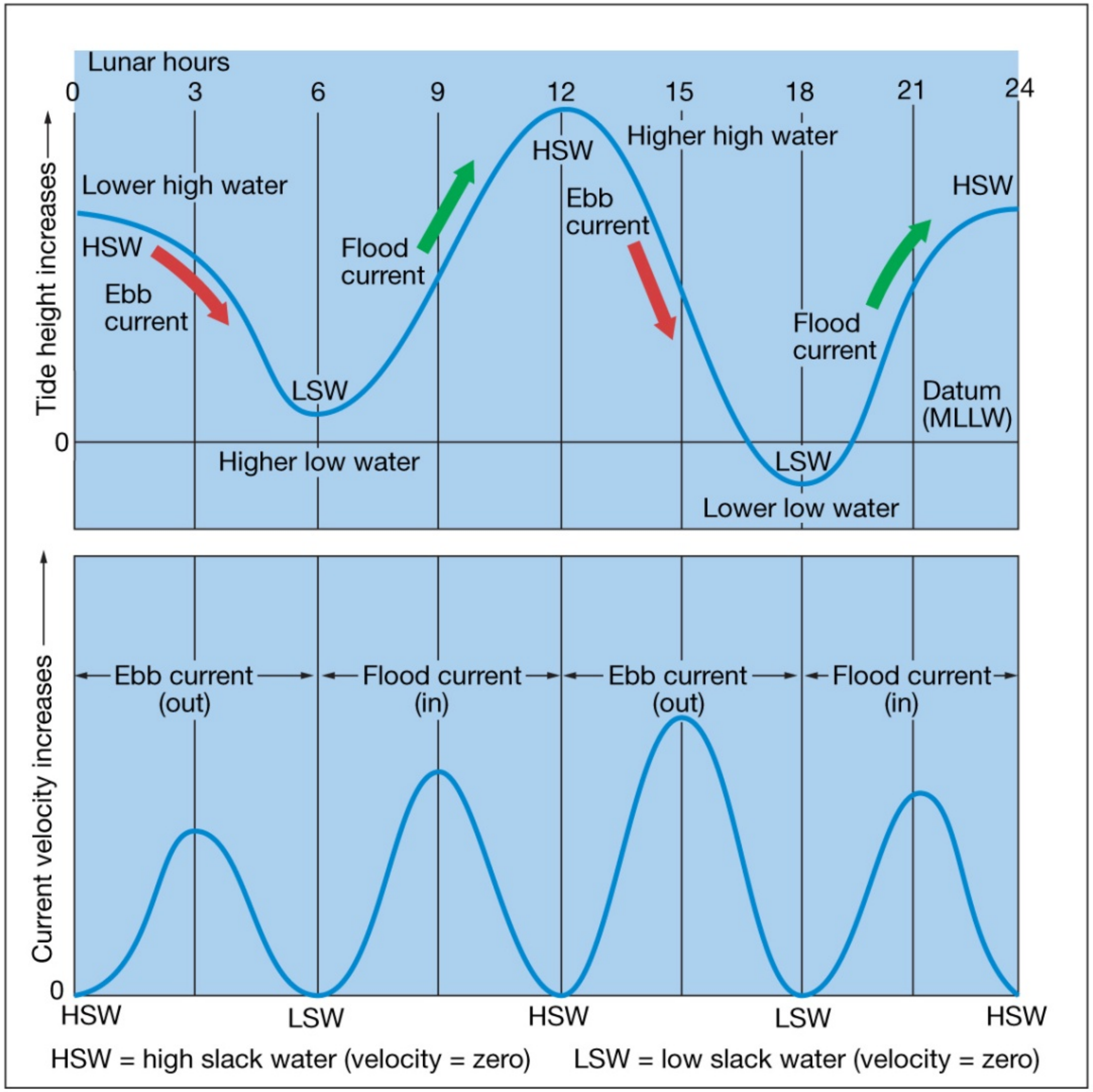




M_a = major axis – max. current velocity
M_i = minor axis
θ = inclination – angle semi-major axis makes to the East
g = phase lag – time of maximum velocity behind max tidal potential

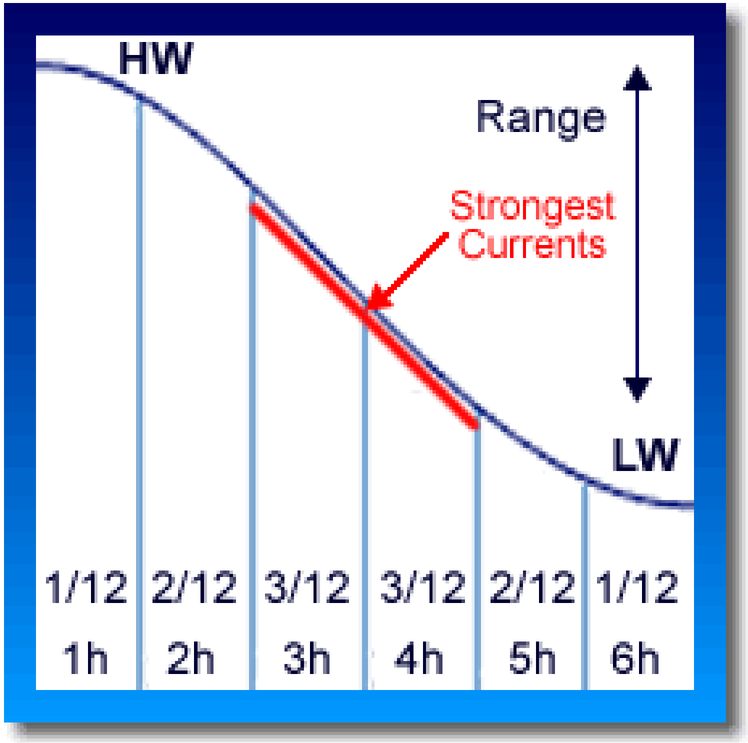
Coastal Tidal Currents

- Reversing current
  - Flood current or tide – approaching high tide (crest moves into harbor)
  - Ebb current or tide – approaching low tide (trough moves into harbor)
  - Slack tide – period of little current at high or low tide
  - High velocity flow in restricted channels



Coastal Tidal Currents

- Peak flood and ebb currents occur in the middle of the tidal cycle
- ~1/2 of the tidal flow moves within the middle two hours of the tidal cycle



For semi-diurnal tide - “rule of 12ths”

Hr 1 – 1/12 of volume - slack tide
Hr 2 – 2/12 of volume
Hr 3 – 3/12 of volume - fastest current
Hr 4 – 3/12 of volume - fastest current
Hr 5 – 2/12 of volume
Hr 6 – 1/12 of volume - slack tide

Tidal range is not a good predictor of tidal currents

Currents are modified by:

- Shape and volume of basin
- Restriction of flow at basin mouth
- Winds
- Friction at the basin boundaries

Considerations for Tidal Modeling and Prediction

- Allow applied tidal forcing to be applied gradually using a ramp function for a period of 12-15 days to eliminate the excitation of numerical artifacts in a modeled solution

- Account for both the tidal potential (body force) and tides entering at the open ocean boundary.

- Maintain consistency between model forcing and harmonic analysis
  - same constituents, same nodal and phase corrections

- Select constituents for the analysis that are appropriate to the applied forcing and subsequent nonlinear interactions

- Ensure the time series length for harmonic analysis is sufficient to separate the desired constituents

- Comparisons between modeled and observed tides should be done by constituent, amp vs. amp (magnitude), phase vs. phase (timing) or by using a complex-valued amplitude

Primary Considerations for Accurate Tidal Modeling

- **Bathymetric variations**
- **Representation of the coastal boundary geometry**
- **Specification of tidal constituents at the open ocean boundary**
- **Frictional characteristics of the region**
- **Model resolution appropriate for scales associated with the nonlinear tidal interactions**

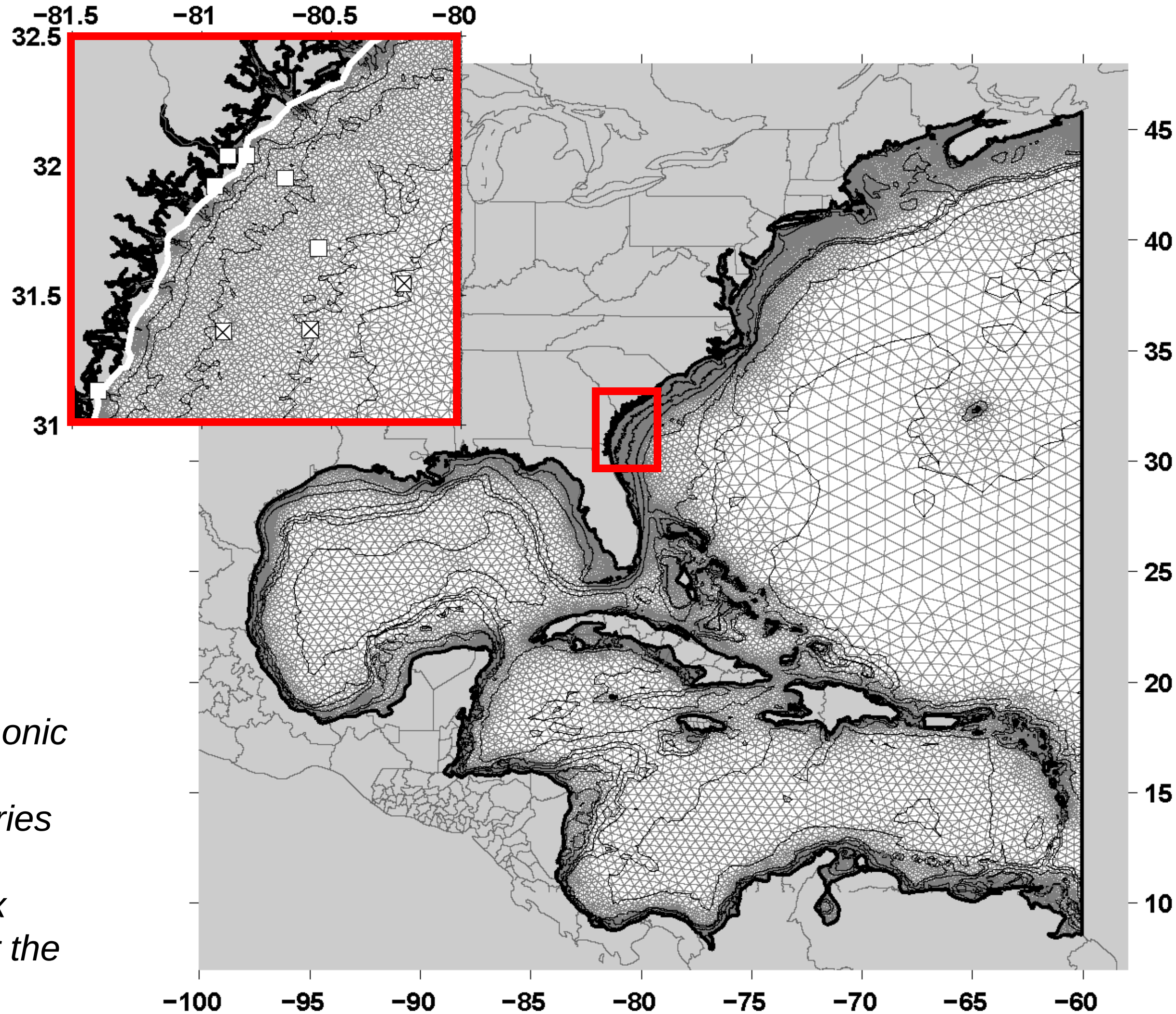
Project: Modeling Tides in the South Atlantic Bight

Finite element model domain and mesh for South Atlantic Bight

Follow Blanton et al. 2004, *J. Geophs. Res.*

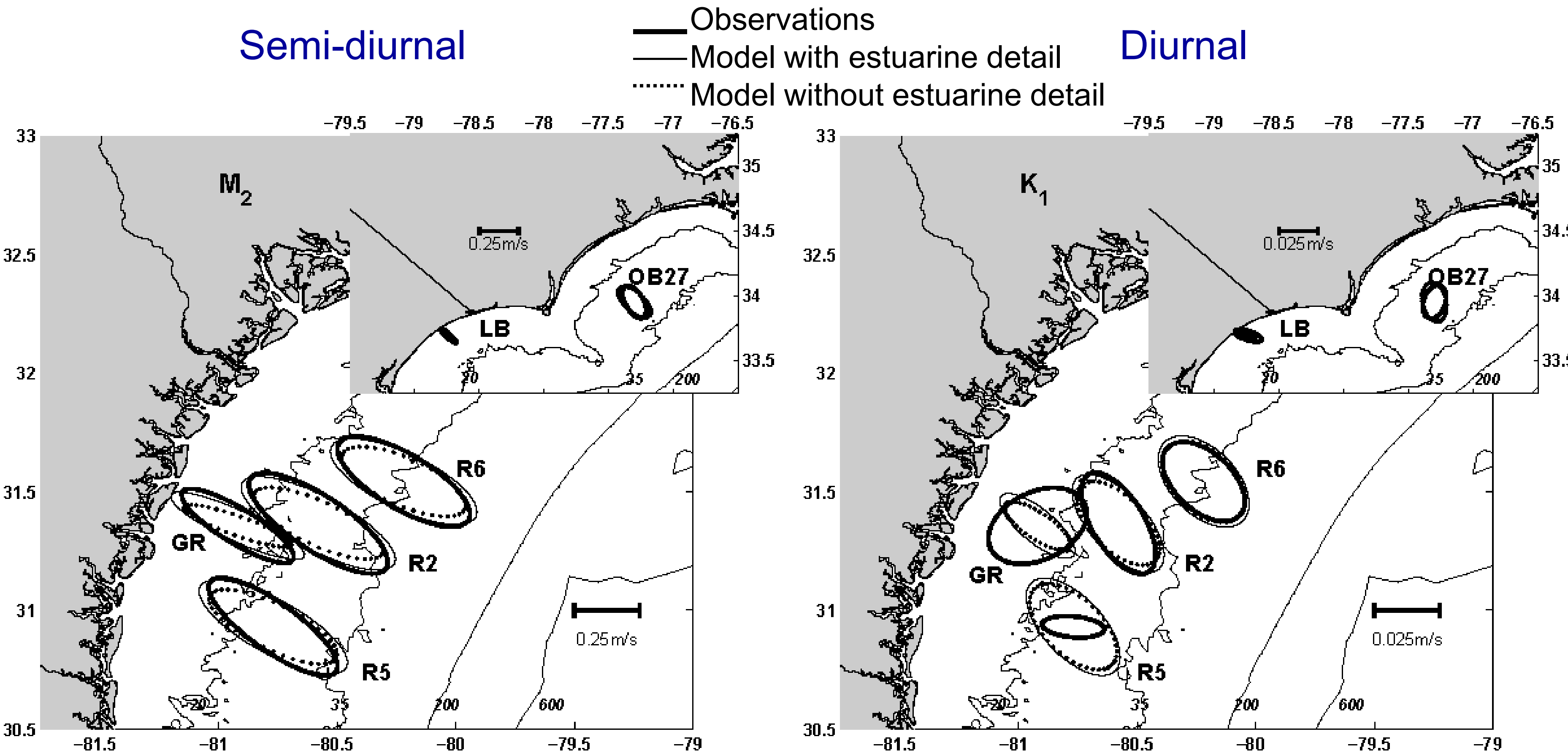
Provided:

- *Fort.14 – grids with and without estuarine detail*
- *Fort.15 – input file*
- *Fort.51, 52, 53, 54 – harmonic output files*
- *Fort.61, 62 – station time series for elevation and velocity*
- *maxele.63, maxvel.63 – max elevation and velocities over the simulation*



Blanton et al., 2005

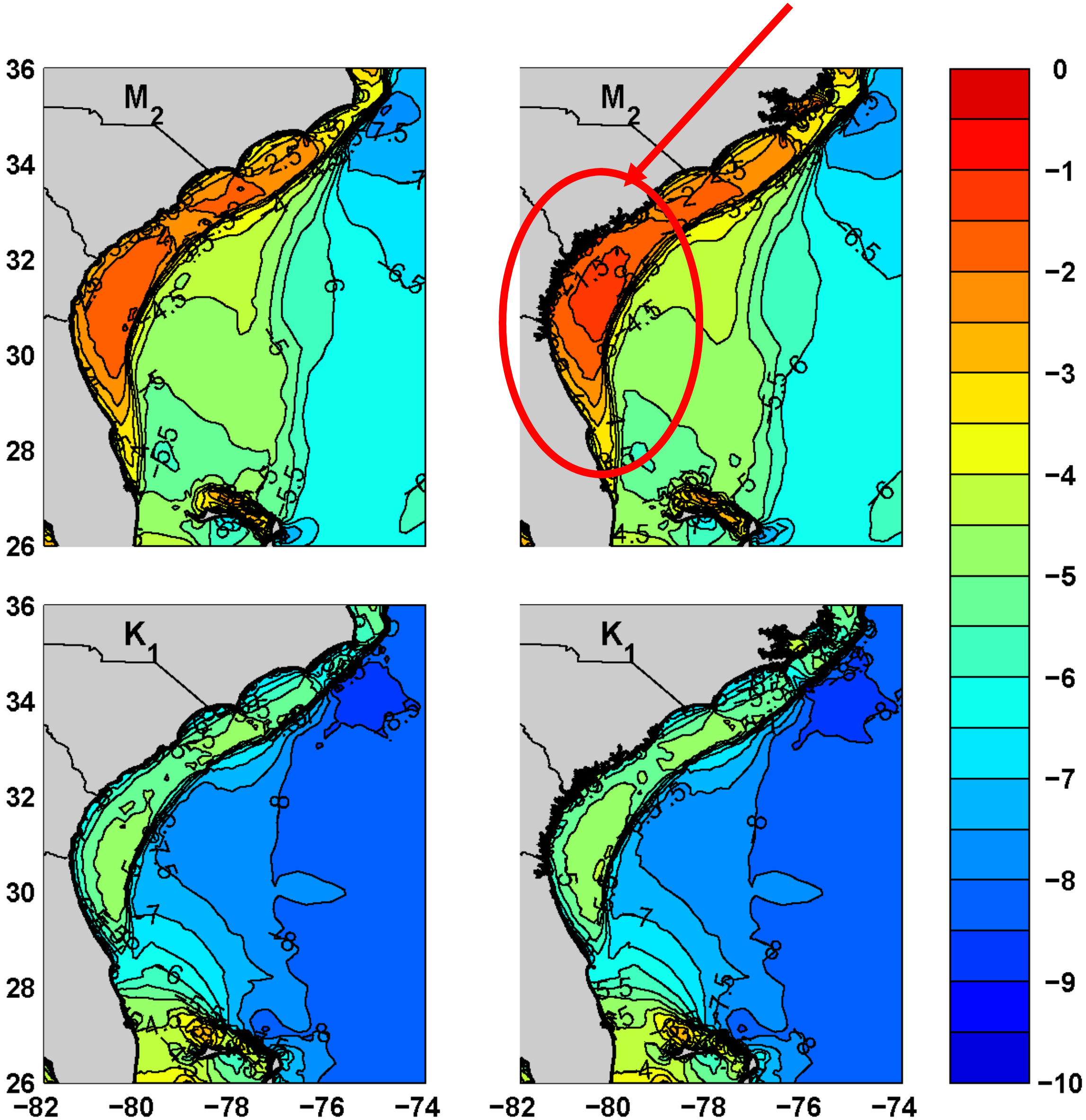
Importance of Including Estuarine/Tidal Inlet Morphology



Tidal current ellipse model-data comparisons
Dotted lines do not include estuary/inlet detail

Blanton et al., 2004

Enhanced Tidal Dissipation



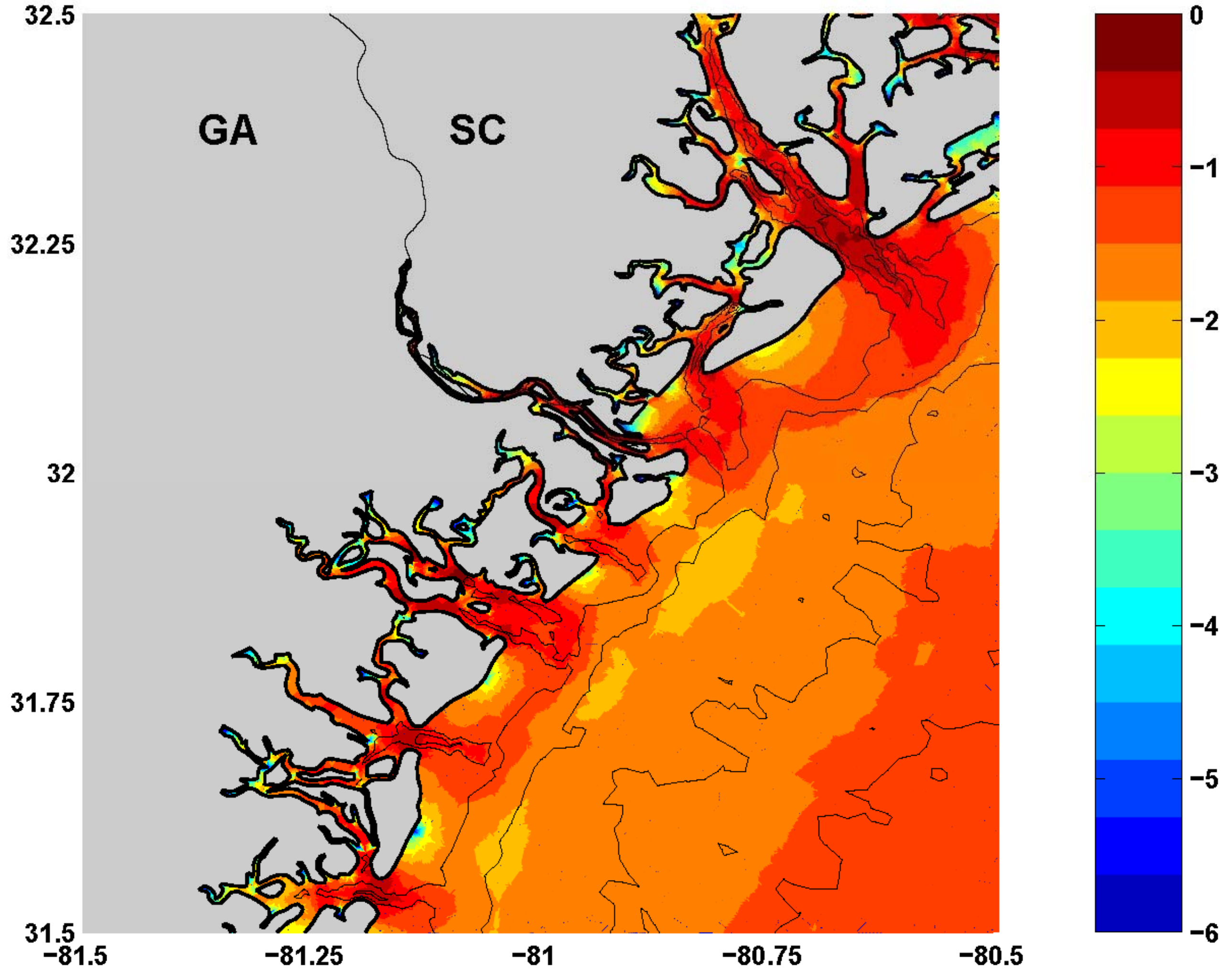
Semi-diurnal

Diurnal

No estuary/inlet detail

Blanton et al., 2005

Enhanced tidal dissipation in upper reaches of estuary/inlet complex



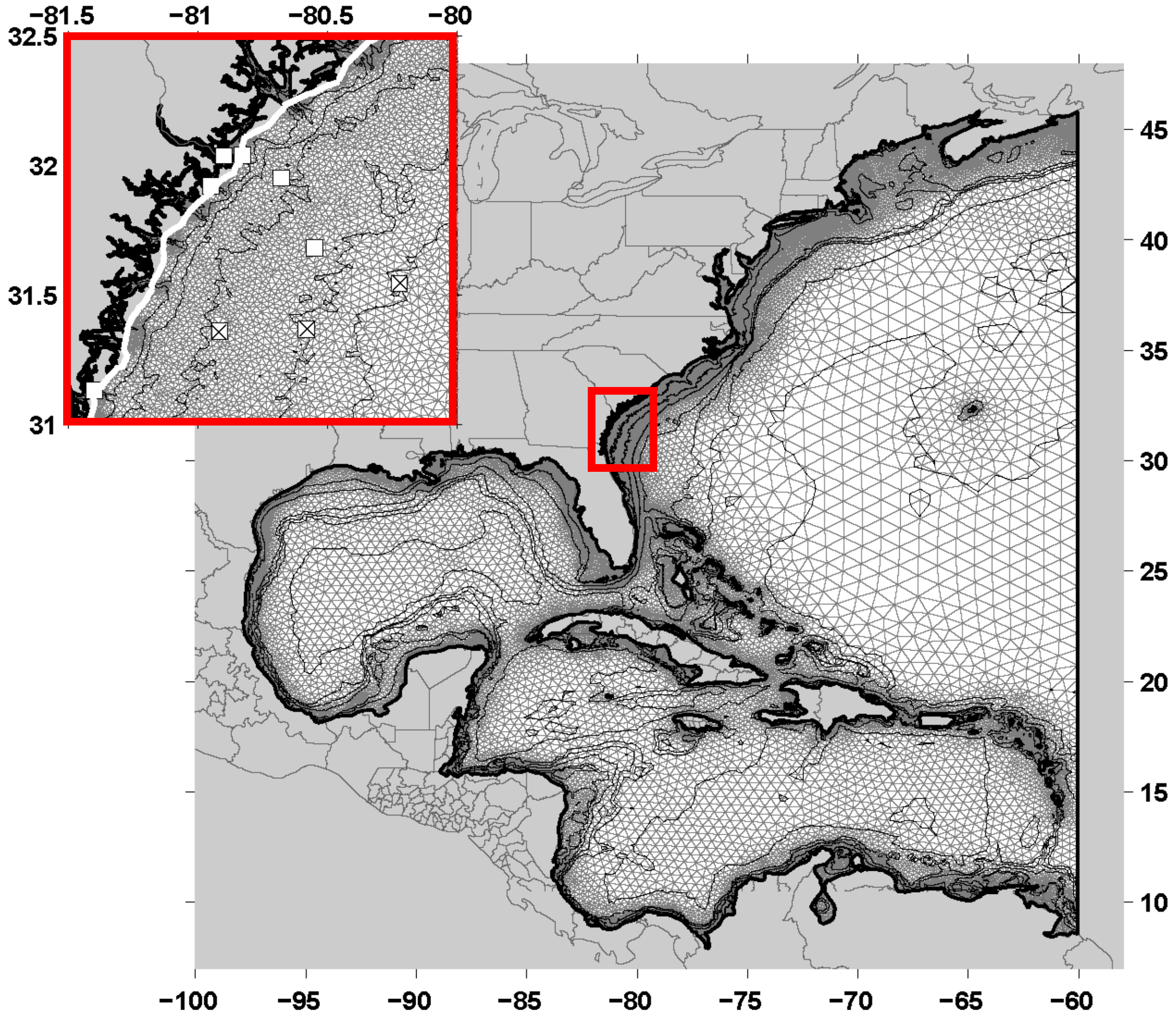
Tidal dissipation effects turbulent energy available for mixing and dispersion

Blanton et al., 2004

Project: Modeling Tides in the South Atlantic Bight

Suggested activities:

1) Run the ADCIRC model for SAB
2) Plot max tidal water level and velocity variations
3) Plot tidal time series at station locations
4) Apply harmonic analysis to time series data
5) Compare derived and model computed harmonic results
6) Convert tidal velocity amplitudes and phases into tidal ellipse parameters
7) Plot tidal ellipses on bathymetry contours
8) Compute energy flux and tidal dissipation
9) Compare solutions from both grids using quantitative errors



Blanton et al., 2005

Project: Modeling Tides in the Bight of Abaco

Finite element model domain and mesh for Bight of Abaco

See Westerink et al. 1989, *J. Phy. Oc.* for modeling

See Filloux and Snyder, 1979, *J. Phy. Oc.* for observations

Provided:

- *Fort.14 – grids with differing resolutions*
- *Fort.15 – input file*
- *Abaco.MEASLOC – file containing observation station locations*

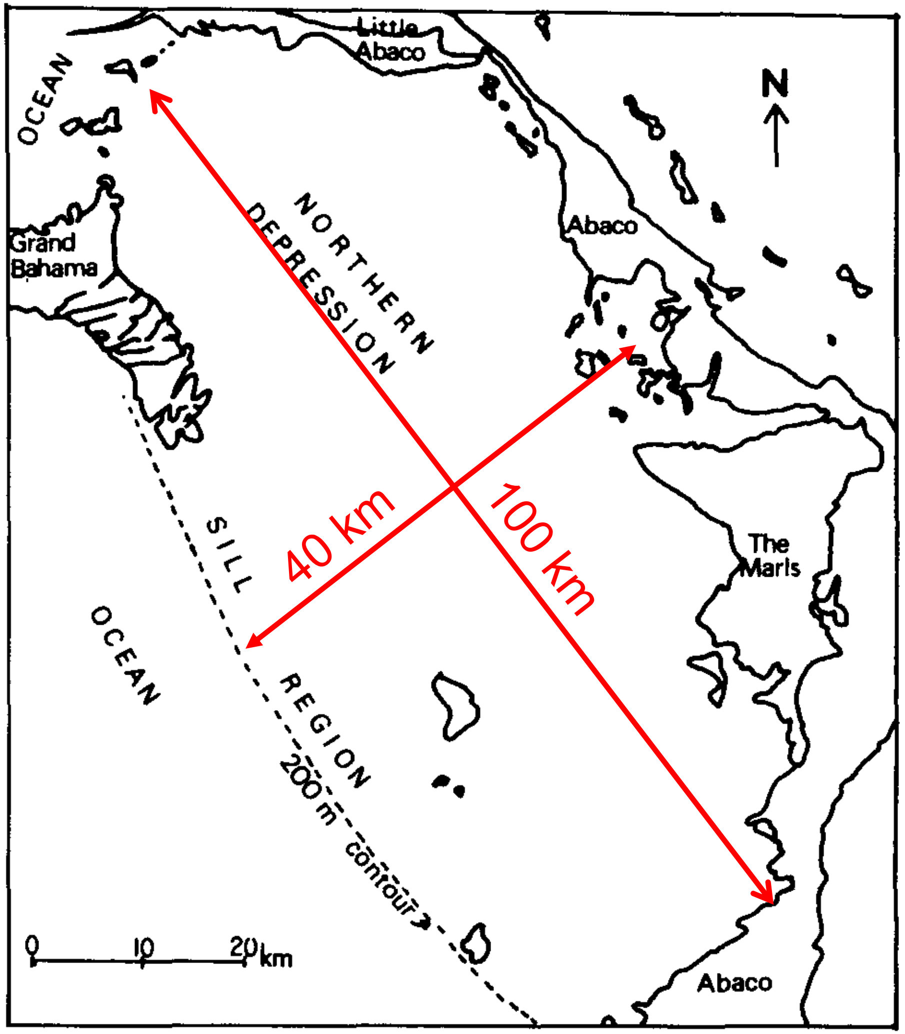


FIG. 1. Bight of Abaco, Bahamas.

Project: Modeling Tides in the Bight of Abaco

Suggested activities:

1) Run the ADCIRC model for the Bight of Abaco for two grid resolutions
2) Evaluate the effect of grid resolution – examine global time series, station time series, harmonically analyzed constituents
3) Examine the interaction between tidal constituents (Cases O-1 to O-6)
4) Examine effects of various nonlinear terms in the model (Cases O-7 to O-11, O-4)

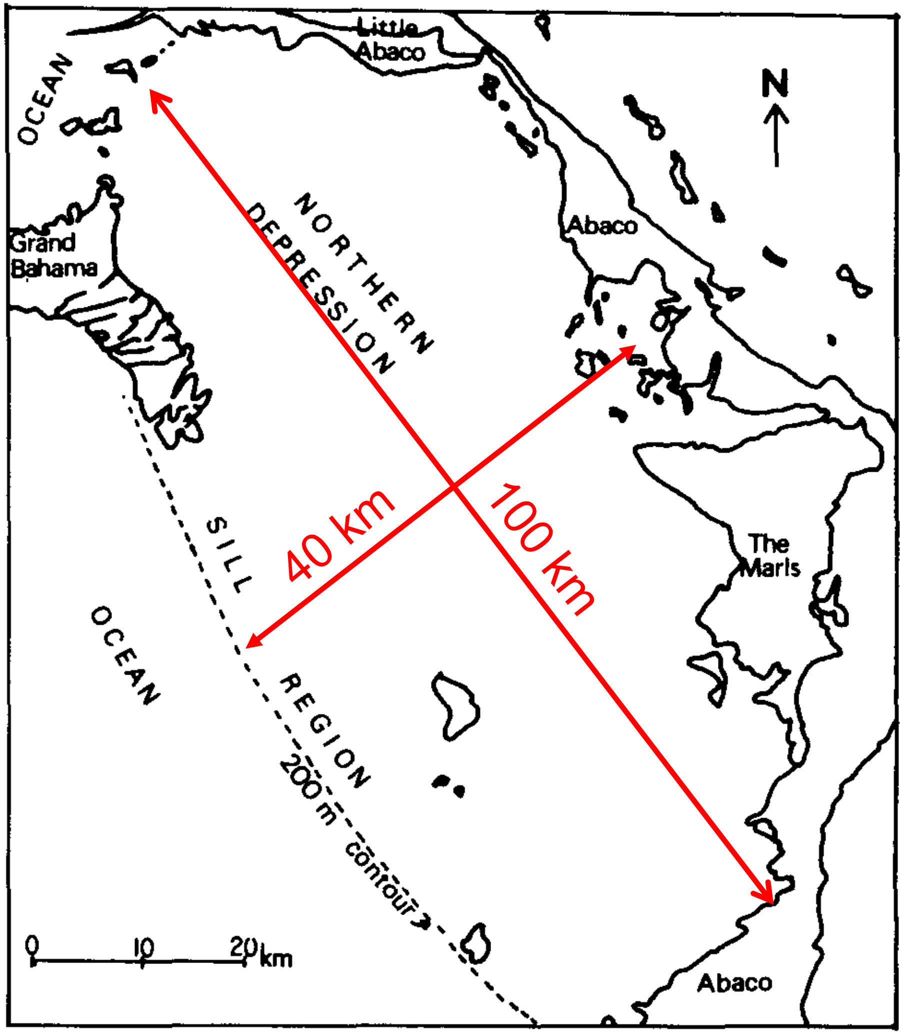


FIG. 1. Bight of Abaco, Bahamas.