Lecture-1:

FVCOM-An unstructured grid Finite-Volume Community Ocean Model

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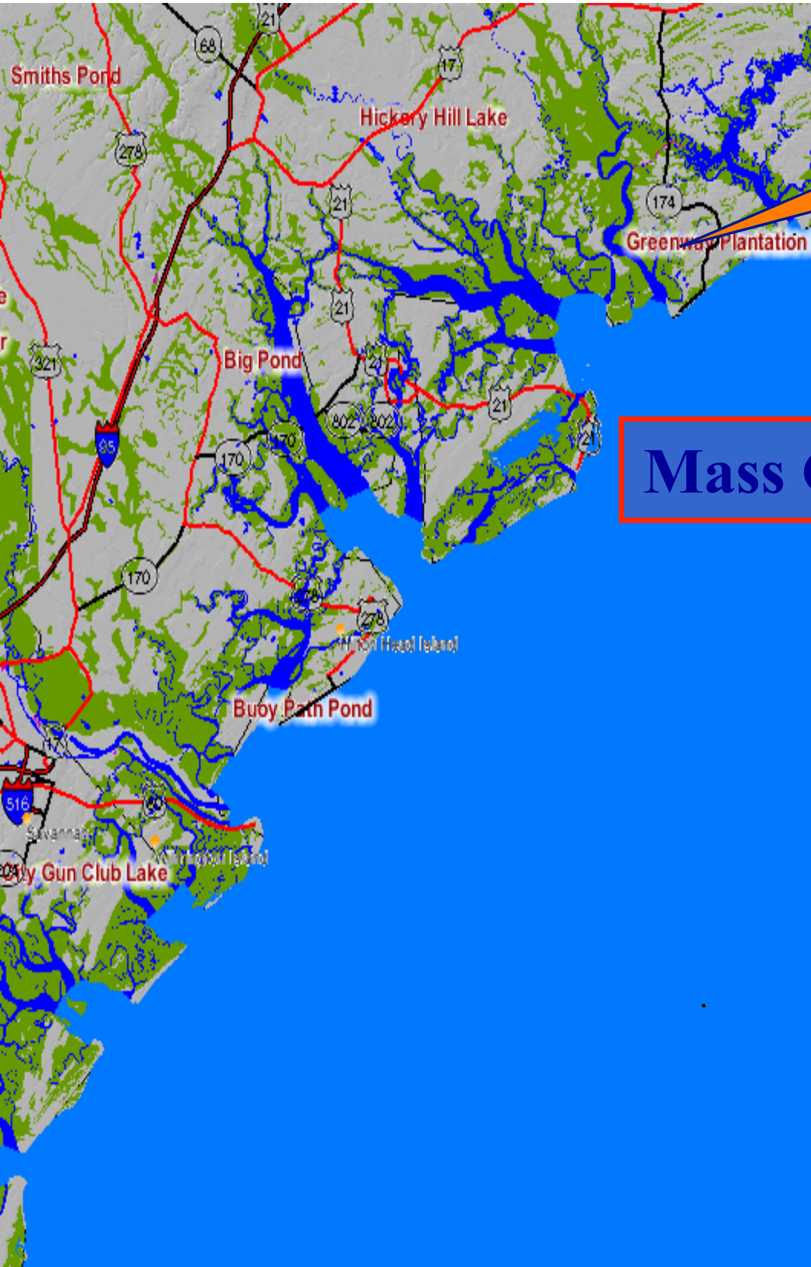
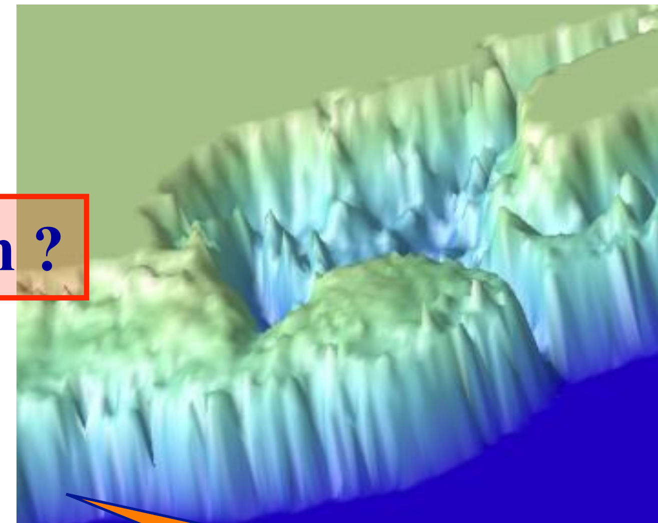
Critical Issues in Coastal Ocean Modeling



Irregular geometry

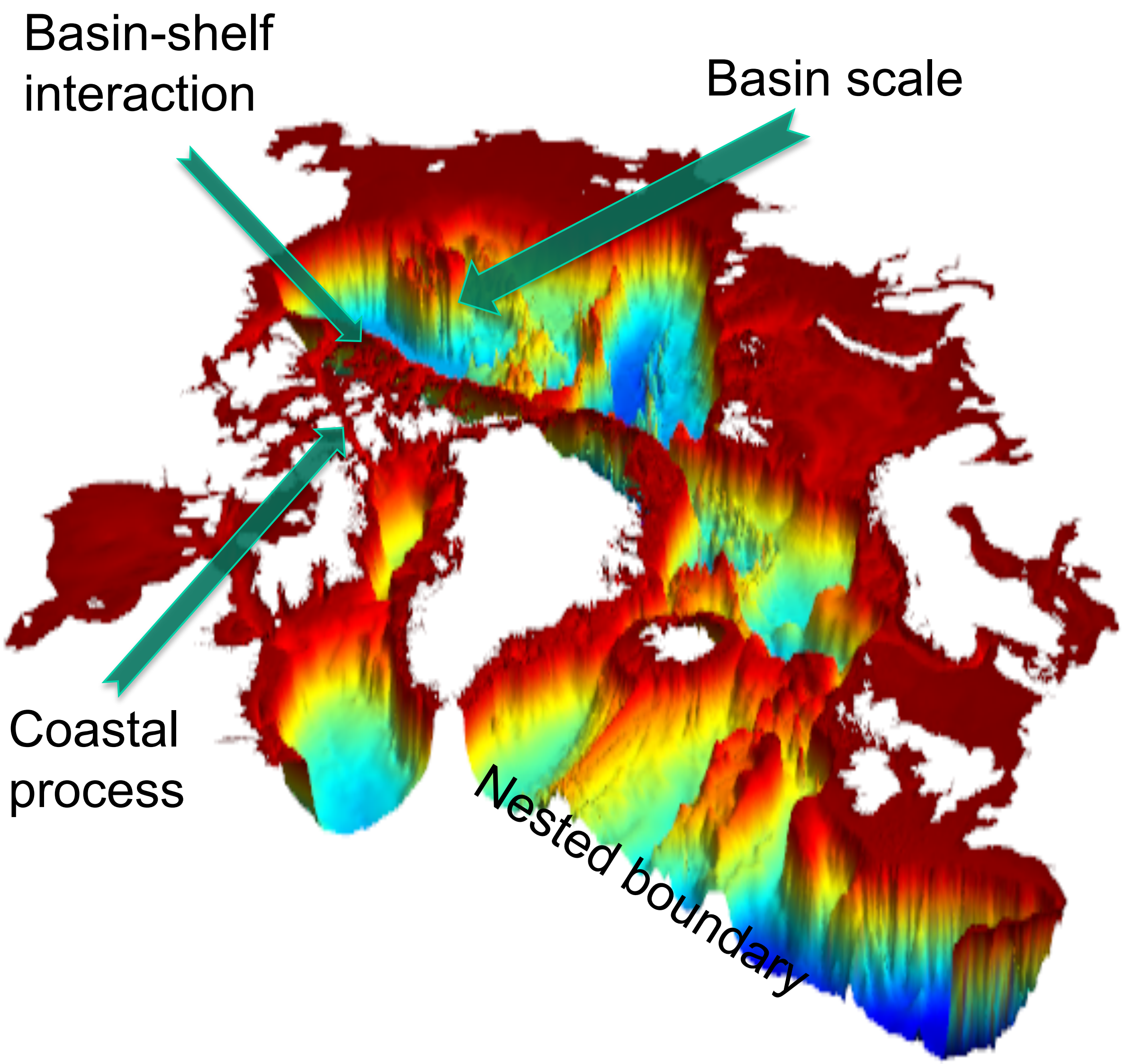
Mass Conservation ?



Steep topography

Intertidal wetlands

Critical Issues for Global Ocean:



Multi-scale dynamics: Basin-shelf interaction, convection via advection, etc.

Resolving irregular coastal geometries connected to the North Atlantic Ocean and Pacific Ocean

Possible solutions:

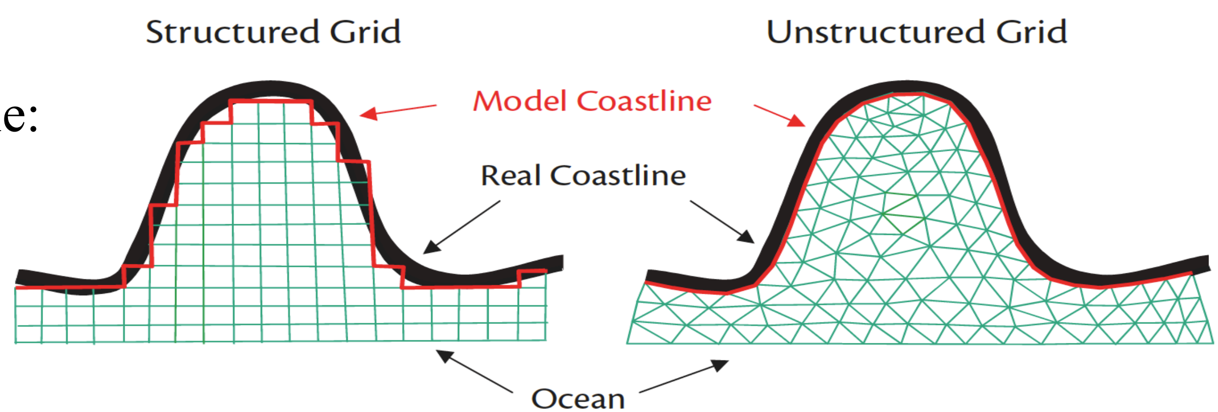
1. Finite-volume algorithms: Configuring the computational domains with individual control volumes and calculating the variables by a net flux through control volumes

An example:

$$\iint \frac{\partial \zeta}{\partial t} dxdy = -\iint [\frac{\partial(\bar{u}D)}{\partial x} + \frac{\partial(\bar{v}D)}{\partial y}] dxdy = -\oint_S \bar{v}_n D ds$$

2. Unstructured grids: Configuring the computational domains with unstructured grids to provide the accurate fitting of the irregular coastal geometry.

An example:



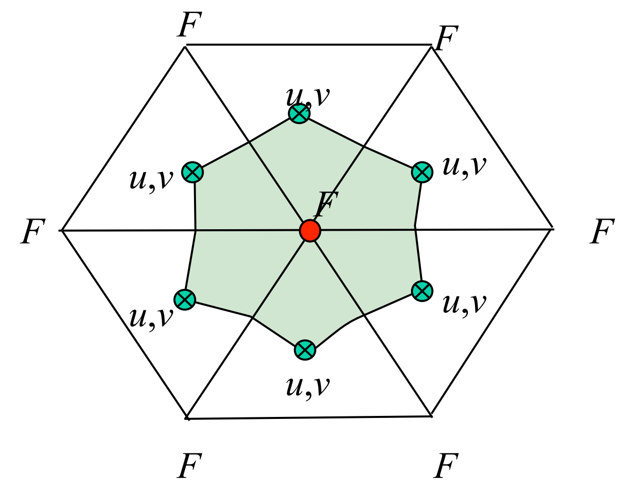
FVCOM: Unstructured-grid, Finite-Volume Coastal Ocean Model (Chen, C. R. H. Liu and R. C. Beardsley, JAOT, 2003)

- All variables are computed in the integral form of the equations, which provides a better representation of the conservative laws of mass, momentum and heat in the coastal region with complex geometry.

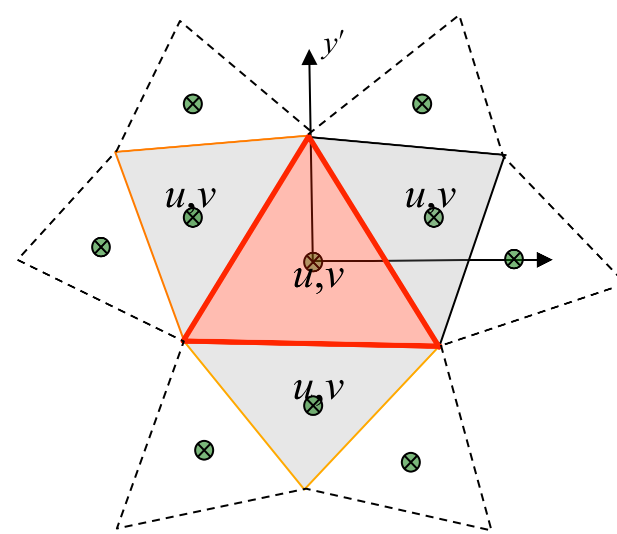
- The numerical computational domain consists of non-overlapping unstructured cells.

- Combines the best from the finite-element method for the geometric flexibility and finite difference method for the simplest discrete computation.

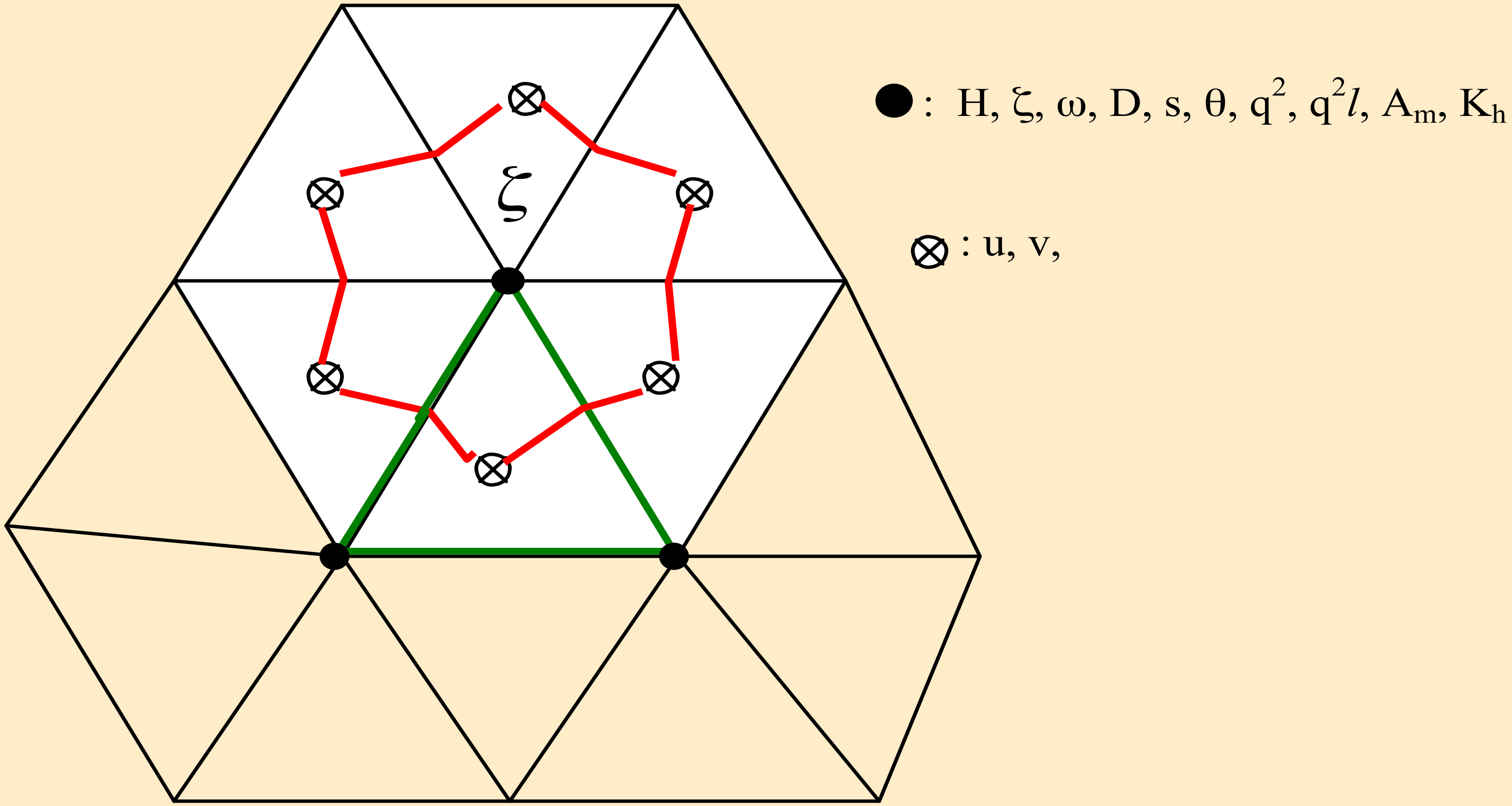
- Both current and tracer remain the second-order accuracy.



$F: \quad \zeta, T, S, \rho, H, K_m, K_h$, *etc*

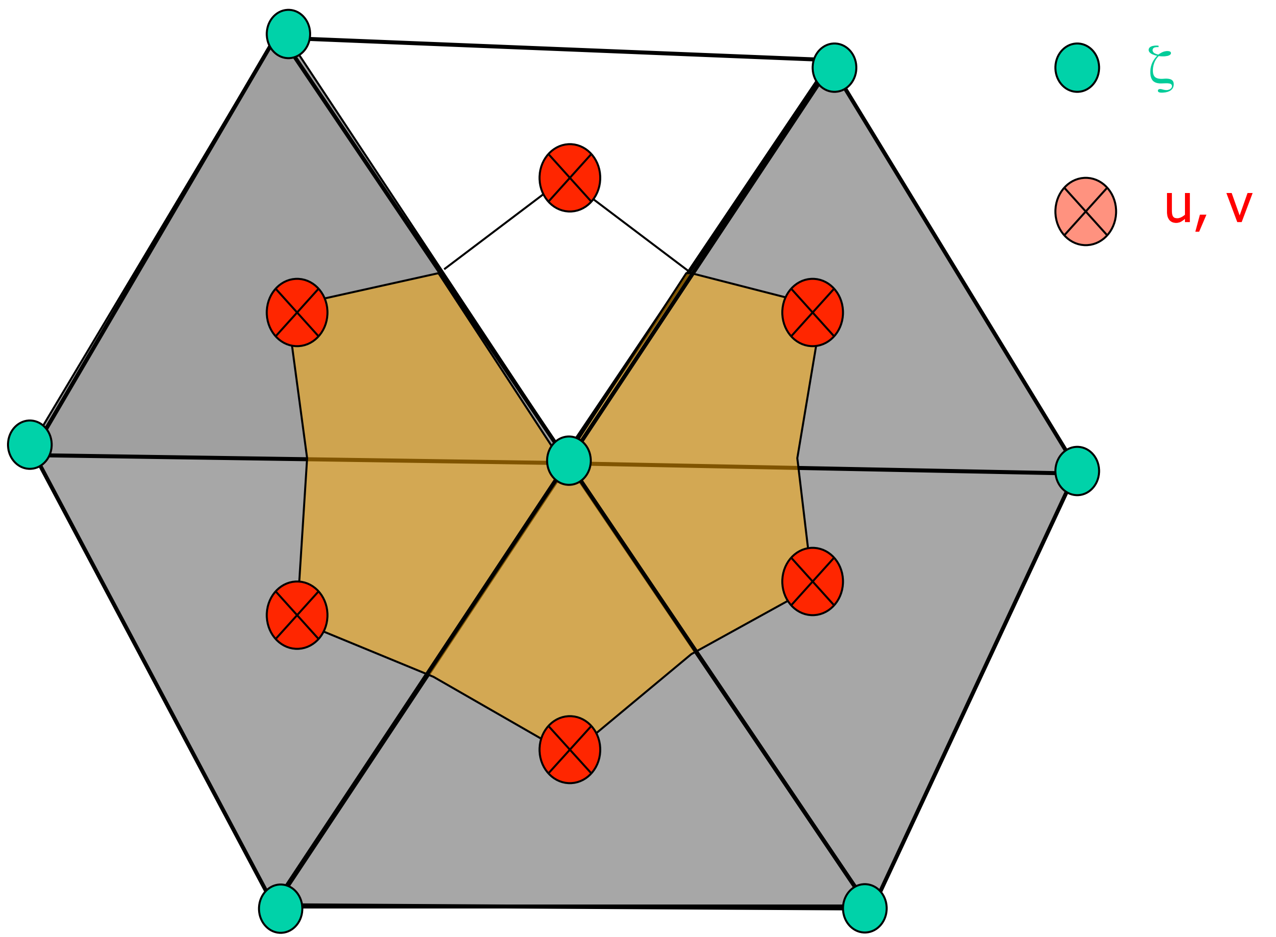


For example: The Continuity Equation:



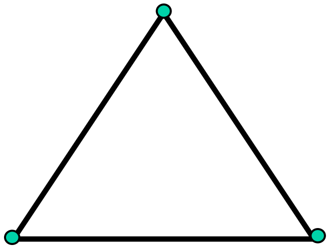
$$\iint \frac{\partial \zeta}{\partial t} dxdy = -\iint [\frac{\partial(\bar{u}D)}{\partial x} + \frac{\partial(\bar{v}D)}{\partial y}] dxdy = -\oint_S \bar{v}_n D ds$$

FVCOM Wet/Dry Treatment Technology-Coastal Inundation



Spherical Coordinate at the Arctic

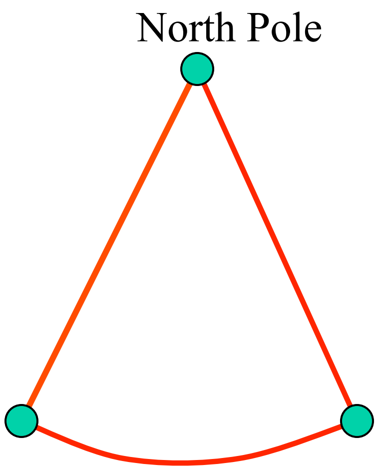
In the Cartesian coordinates,



$$\oint_s dx = 0; \quad \int_s dy = 0$$

A line integral is closed

In the spherical coordinates,

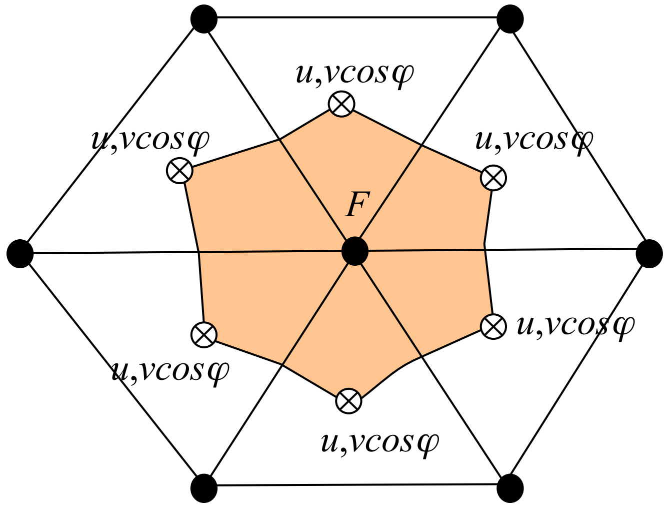


$$\oint_s r\cos\varphi d\lambda \neq 0$$

A line integral is not always closed

$$-\iint_\Omega \frac{\partial\zeta}{r\partial\varphi} r^2 \cos\varphi d\lambda d\varphi \xrightarrow{?} \int_s (\quad) d\lambda$$

Spherical Coordinate FVCOM



F: Scalar variables such as ζ, T, S, K_m, K_h...and vertical velocity ω.

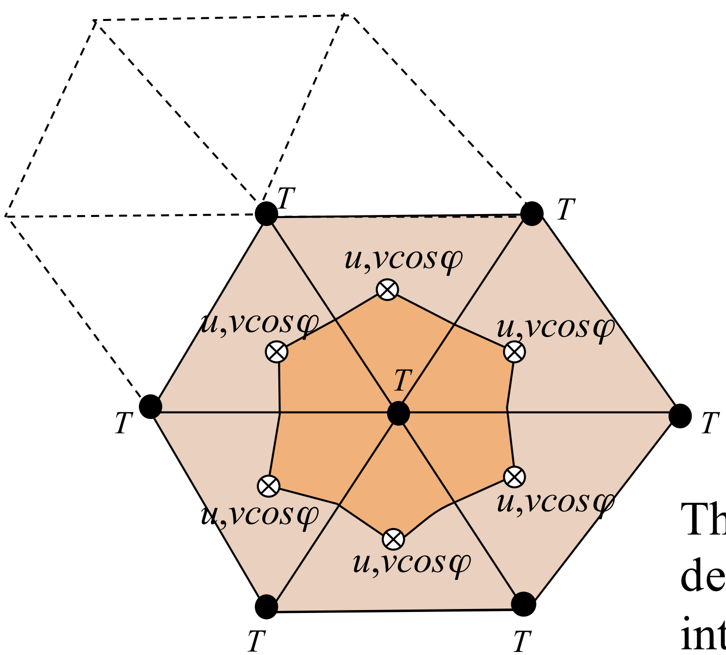
•: The node of triangles where scalar variable or vertical velocity is calculated

⊗: The centroid of a triangle where the horizontal velocity is calculated.

Example: Continuity equation

$$\frac{\partial \zeta}{\partial t} + \frac{1}{r\cos\varphi}\left(\frac{\partial \bar{u}D}{\partial \lambda} + \frac{\partial \bar{v}\cos\varphi D}{\partial \varphi}\right) = 0$$

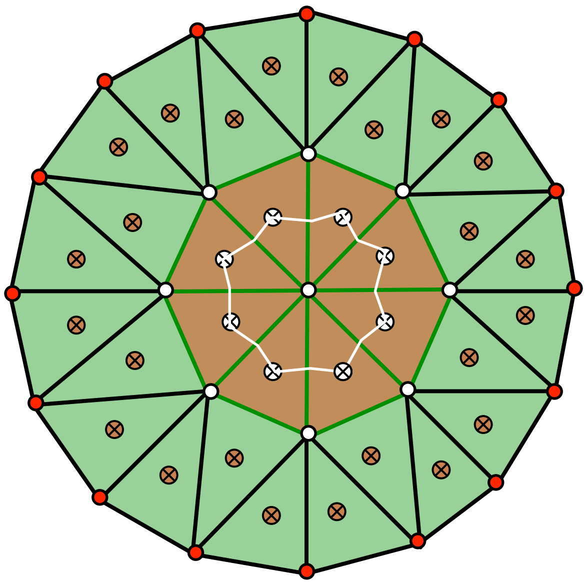
$$\iint_{\Omega} \frac{\partial \zeta}{\partial t} r^2 \cos\varphi d\lambda d\varphi + \iint_{\Omega} \frac{1}{r\cos\varphi}\left(\frac{\partial \bar{u}D}{\partial \lambda} + \frac{\partial \bar{v}\cos\varphi D}{\partial \varphi}\right) r^2 \cos\varphi d\lambda d\varphi = 0$$



$$\boxed{\frac{\partial \zeta}{\partial t} = -\frac{r}{\Omega}\left[\oint (D\bar{u})d\varphi - \oint (D\bar{v}\cos\varphi)d\lambda\right]}$$

The gradient of the water temperature (or salinity) is determined by the Green's function through the integration over the larger volume (with boundaries linked to nodes).

Treatment of North Pole



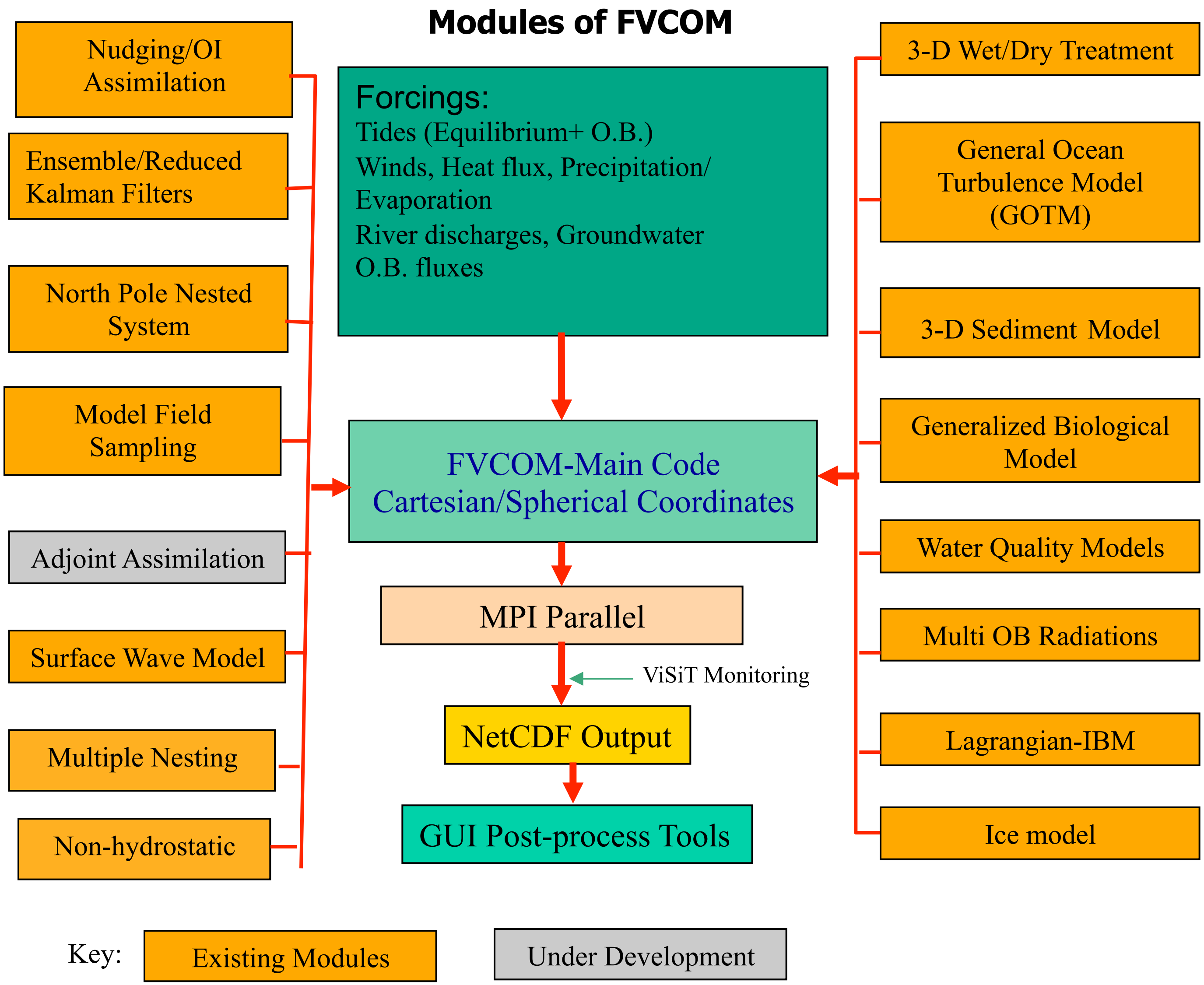
- ○ Node calculated using the polar stereographic projection coordinate.
- ⊗ Centroid calculated using the polar stereographic projection coordinate.
- ● Node calculated directly in the spherical coordinate system.
- ⊗ Centroid calculated directly in the spherical coordinate system.

Conversion formulae between (u_p, v_p) and (u_s, v_s):

$$\begin{pmatrix} u_p \\ v_p \end{pmatrix} = \begin{pmatrix} -\sin\lambda & -\cos\lambda \\ \cos\lambda & -\sin\lambda \end{pmatrix} \begin{pmatrix} u_s \\ v_s \end{pmatrix}$$

$$u_p = h_x \frac{dx}{dt} \quad h_x = \frac{1+\sin\varphi}{2}$$

$$v_p = h_y \frac{dy}{dt} \quad h_y = \frac{1+\sin\varphi}{2}$$



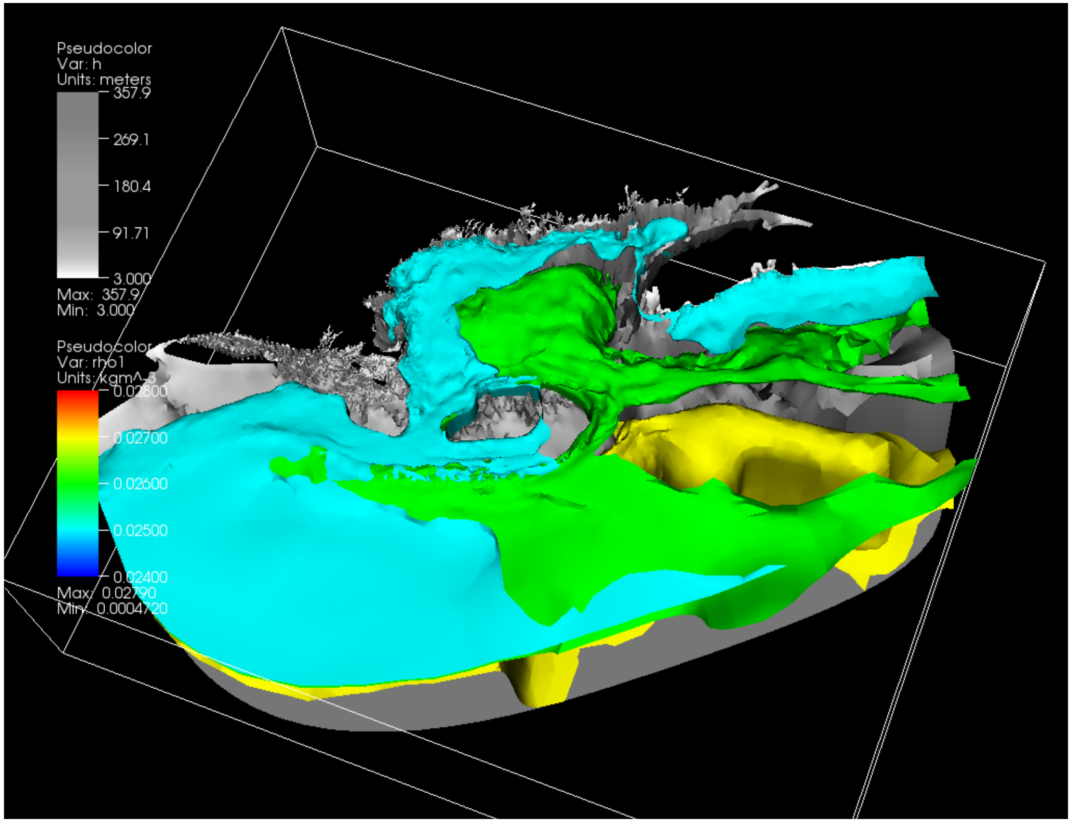
Solver: Mode-split or semi-implicit; 2-D and 3-D,
Types: Research version (FVCOM-v2.7) and Forecast version (FVCOM-v3.1).

ViSiT

Software developed by Lawrence Livermore National Laboratory

http://www.llnl.gov/visit/

- Open source; Parallel visualization; Interactive simulation support
- Multiple platform support (LINUX, UNIX, PC, MAC)



FVCOM Plug-in for VISIT

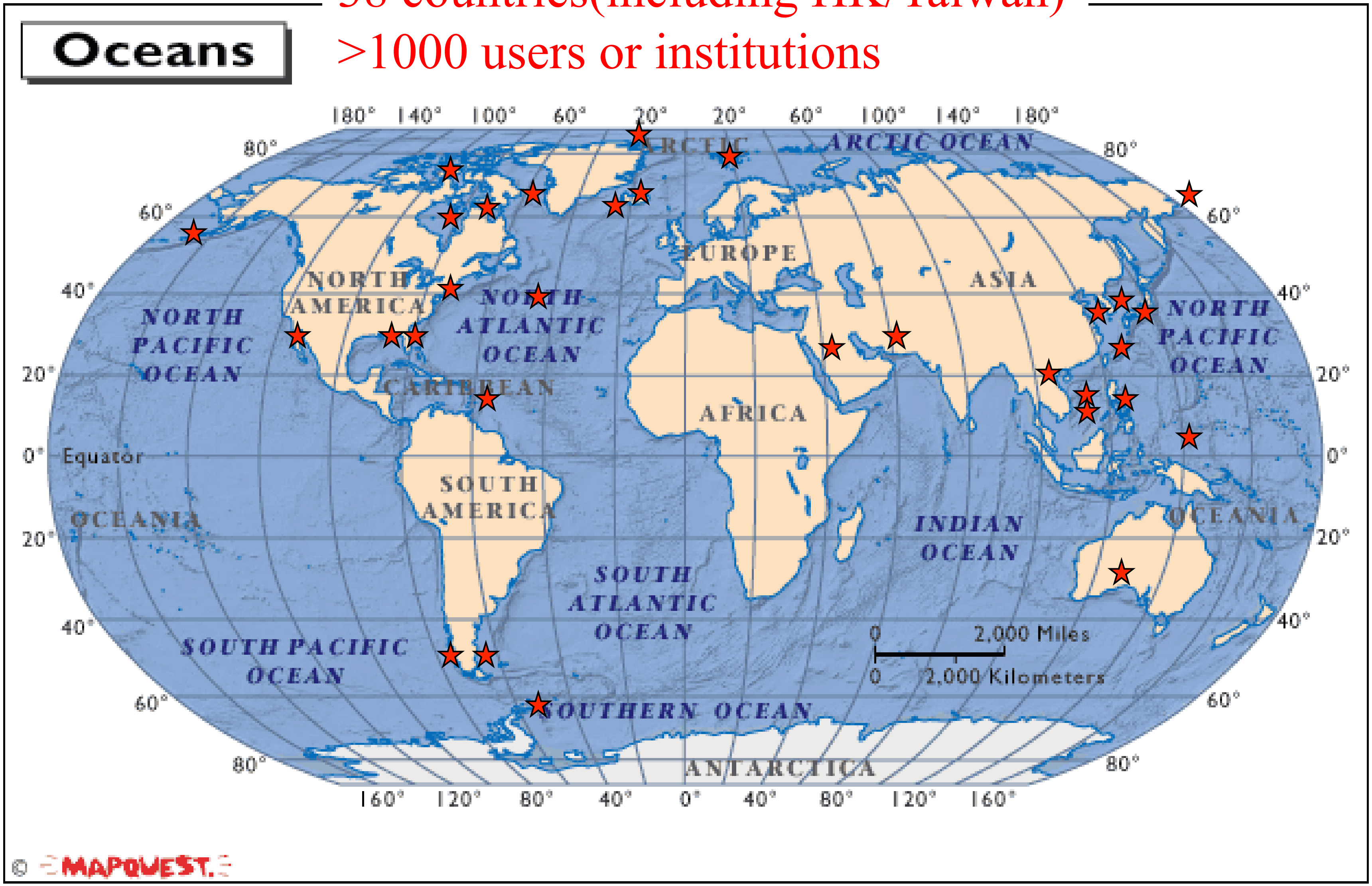
- FVCOM NETCDF files
- Visualization and animation of 3D vector and scalar fields
- Database linking to NETCDF formatted particle tracking output

Example of density isosurfaces from a high resolution FVCOM GOM model

FVCOM Users

38 countries(including HK/Taiwan)

>1000 users or institutions



Australia, Bangladesh, Brazil, Canada, Chile, China, Colombia, Denmark, Egypt, France, Germany, Hong Kong, Hungary India, Indonesia, Iran, Israel, Italy, Jamaica, Japan, Korea, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway Peru, Saudi Arabia, Singapore, Sweden, Taiwan, Thailand, Turkey, U.S., UK, Venezuela, Vietnam

Hydrostatic FVCOM Validation Experiments

1. Advection scheme;
2. Wind-induced oscillation (POM, ECOM-si);
3. Wind-induced waves over the slope bottom topography (POM, ECOM-si);
4. Tidal Resonances in a semi-enclosed channel and a sector (POM, ECOM-si);
5. Freshwater discharge plume (POM, ECOM-si);
6. Bottom boundary layer over a step bottom slope (POM; ECOM-si)
7. Equatorial Rossby soliton (ROMs)
8. Wind-induced lateral boundary (ROMs)
9. Super-critical current (ROMs)

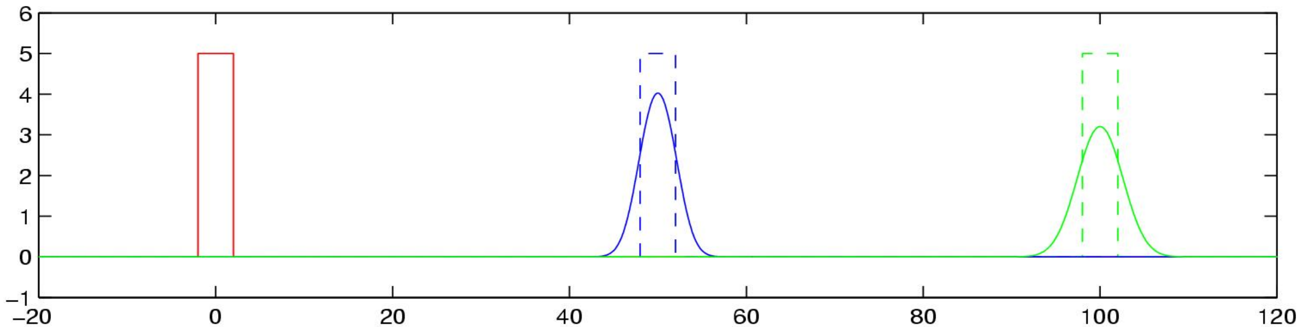
Non-Hydrostatic FVCOM Validation Experiments

1. Standing surface short gravity waves;
2. Lock exchanges;
3. Solitary waves in both homogeneous and layer fluids.

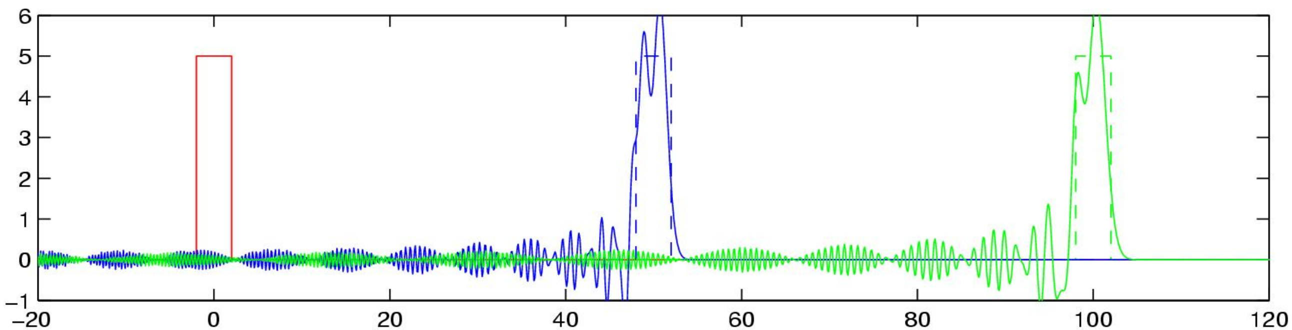
1. Advection Scheme

$$\frac{\partial F}{\partial t} + C\frac{\partial F}{\partial x} = 0 \qquad F(x,0) = \begin{cases} 5, & -2 \le x \le 2 \\ 0, & otherwise \end{cases} \qquad \text{and } \ C = 1$$

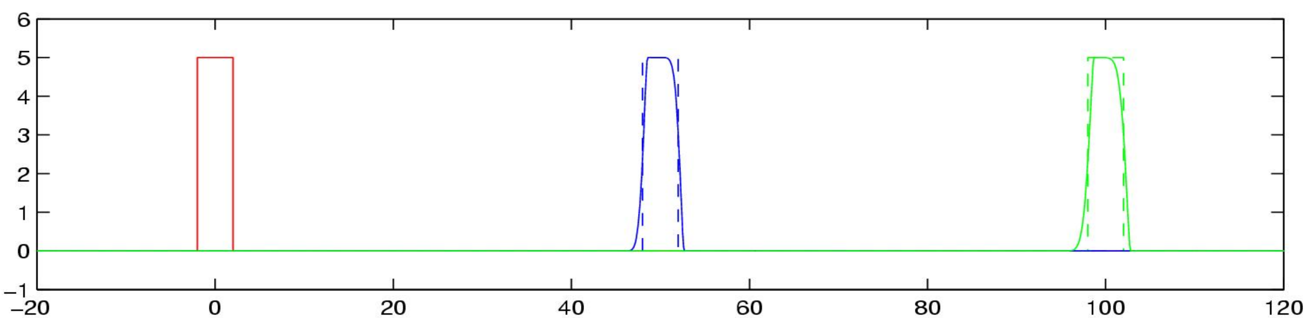
$\Delta t = 0.05, \ \Delta x = 0.1$



Upwind finite-difference stream

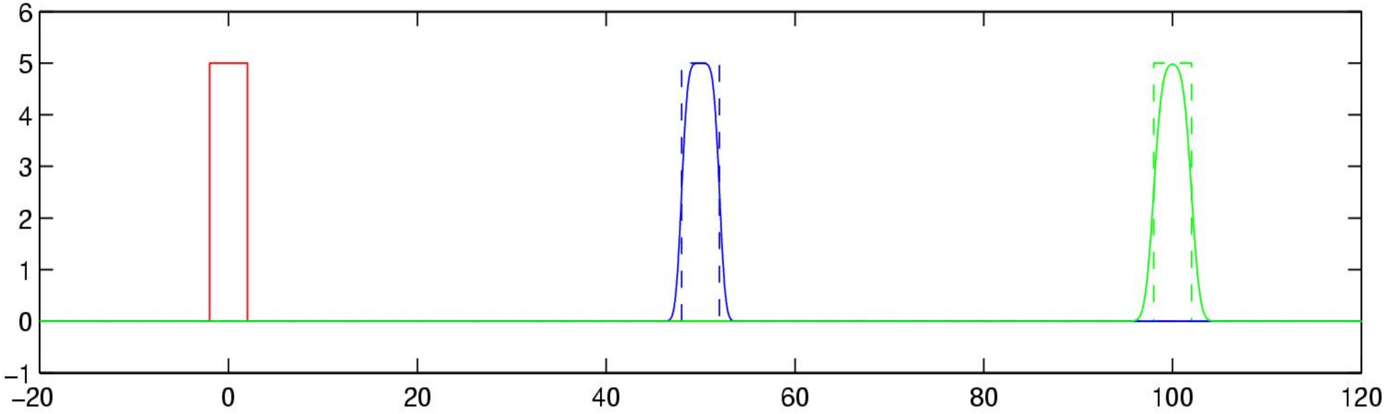


Central finite-difference stream

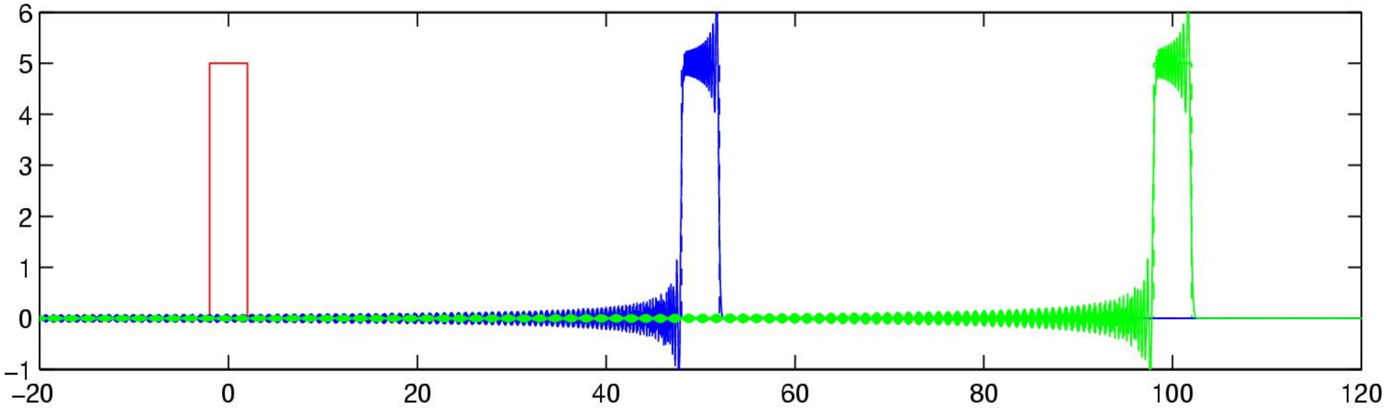


FVCOM finite-volume flux scheme

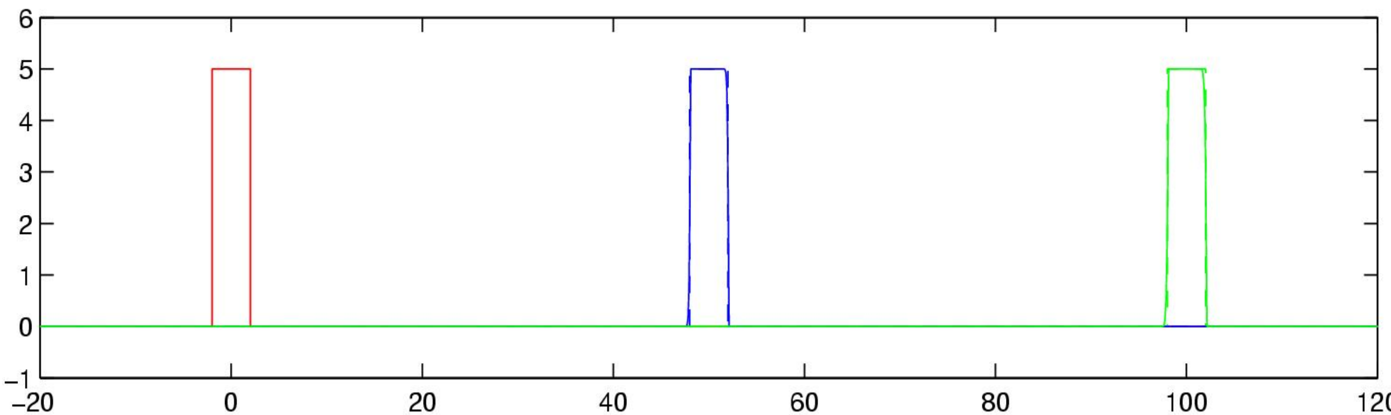
$\Delta t = 0.005$,
$\Delta x=0.01$



Upwind finite-difference stream



Central finite-difference stream



FVCOM finite-volume flux scheme

Wind-induced oscillation

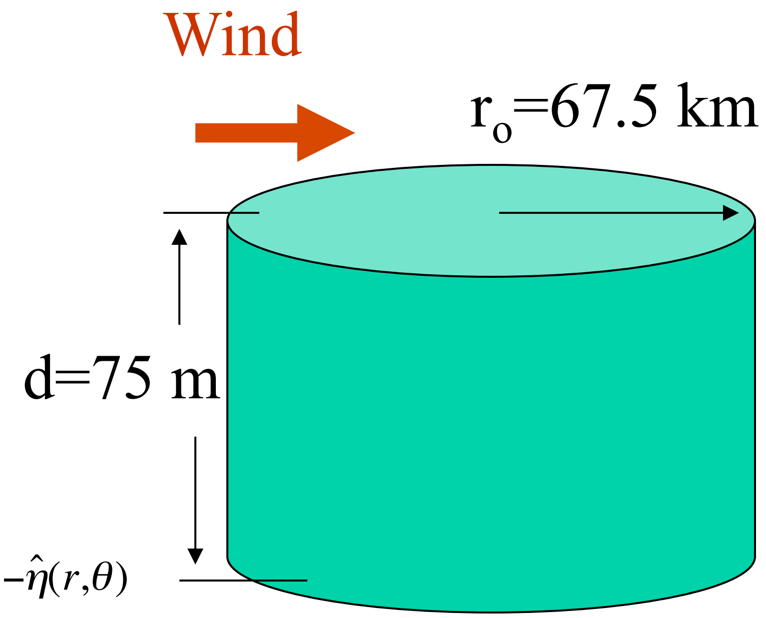
Wind is suddenly imposed at initial

Linear, non-dimensional equations:

$$\frac{\partial u}{\partial t} - v = -\lambda \frac{\partial \zeta}{\partial r}$$

$$\frac{\partial v}{\partial t} + u = -\lambda \frac{\partial \zeta}{r \partial \theta}$$

$$\frac{\partial \zeta}{\partial t} + \frac{\lambda}{r}[\frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial \theta}] = 0$$



where $\lambda = \frac{\sqrt{gd}}{r_o f}; \zeta = \eta - \hat{\eta}; \hat{\eta} = \frac{\tau_o r \cos\theta}{\lambda^4}; \tau_o = \frac{g\tau}{r_o^3 f^4}$

and $u|_{r=1} = 0; (u, v, \zeta)_{r=0} \rightarrow finite; u|_{t=0} = v|_{t=0} = 0; \zeta|_{t=0} = -\hat{\eta}(r,\theta)$

Solution:

$$\eta(r,\theta,t) = \frac{\tau_o}{\lambda^4}[A_o(r)\cos\theta + \sum_{k=1}^{\infty} a_k A_k(r)\cos(\theta - \sigma_k t)]$$

$$u(r,\theta,t) = \frac{\tau_o}{\lambda^3}[(\frac{A_o(r)}{r} - 1)\sin\theta - \sum_{k=1}^{\infty} b_k F_k(r)\sin(\theta - \sigma_k t)]$$

$$v(r,\theta,t) = \frac{\tau_o}{\lambda^3}[(\frac{dA_o(r)}{dr} - 1)\cos\theta - \sum_{k=1}^{\infty} b_k G_k(r)\cos(\theta - \sigma_k t)]$$

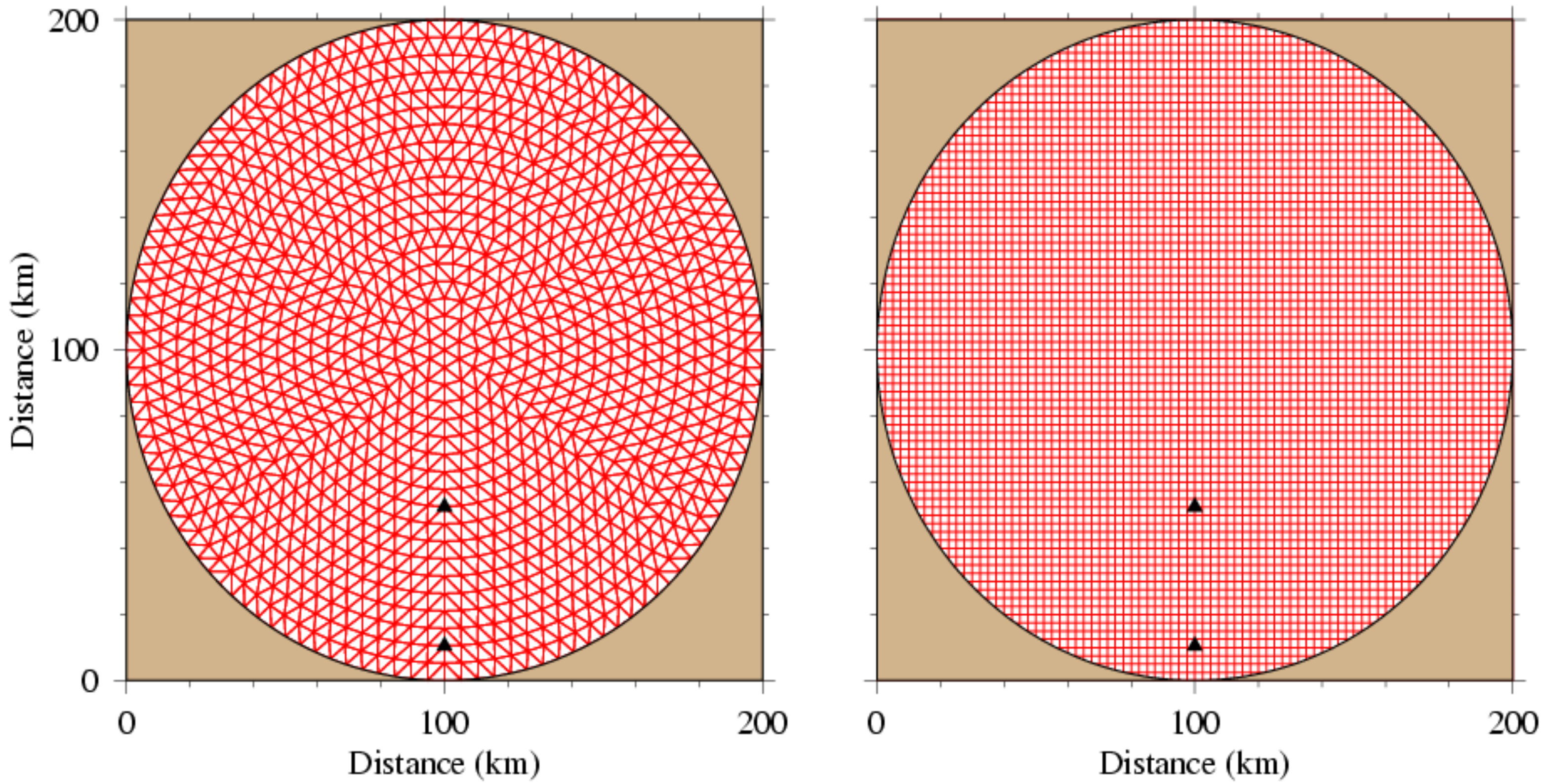
Reference:

Csanady (1968)

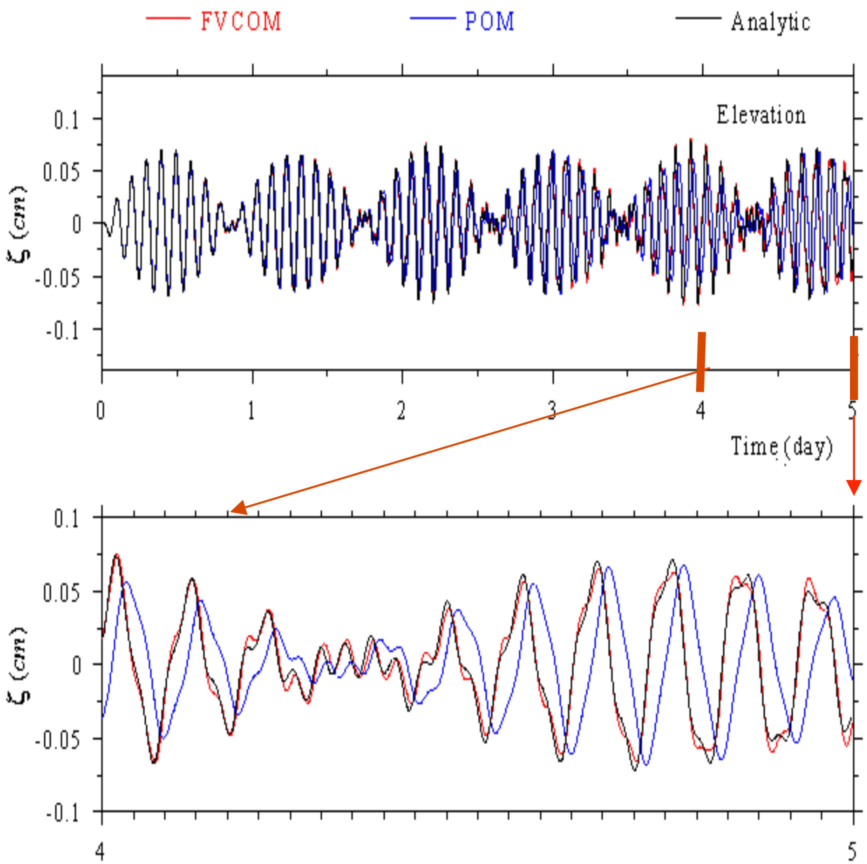
Birchfield (1969)

Unstructured (FVCOM)

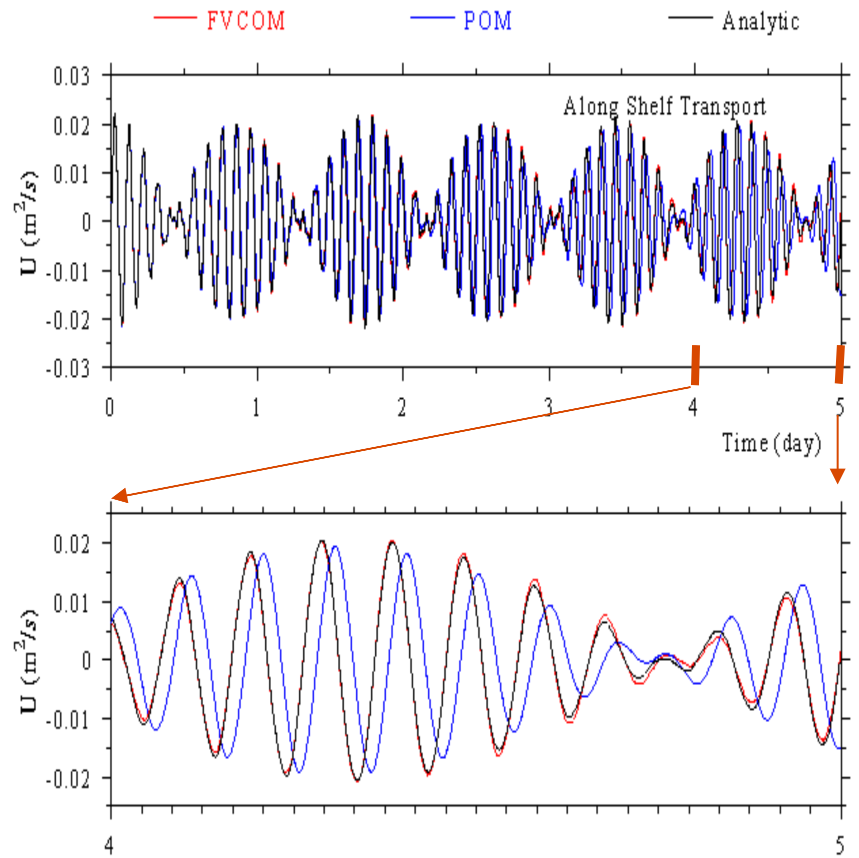
Structured (POM)

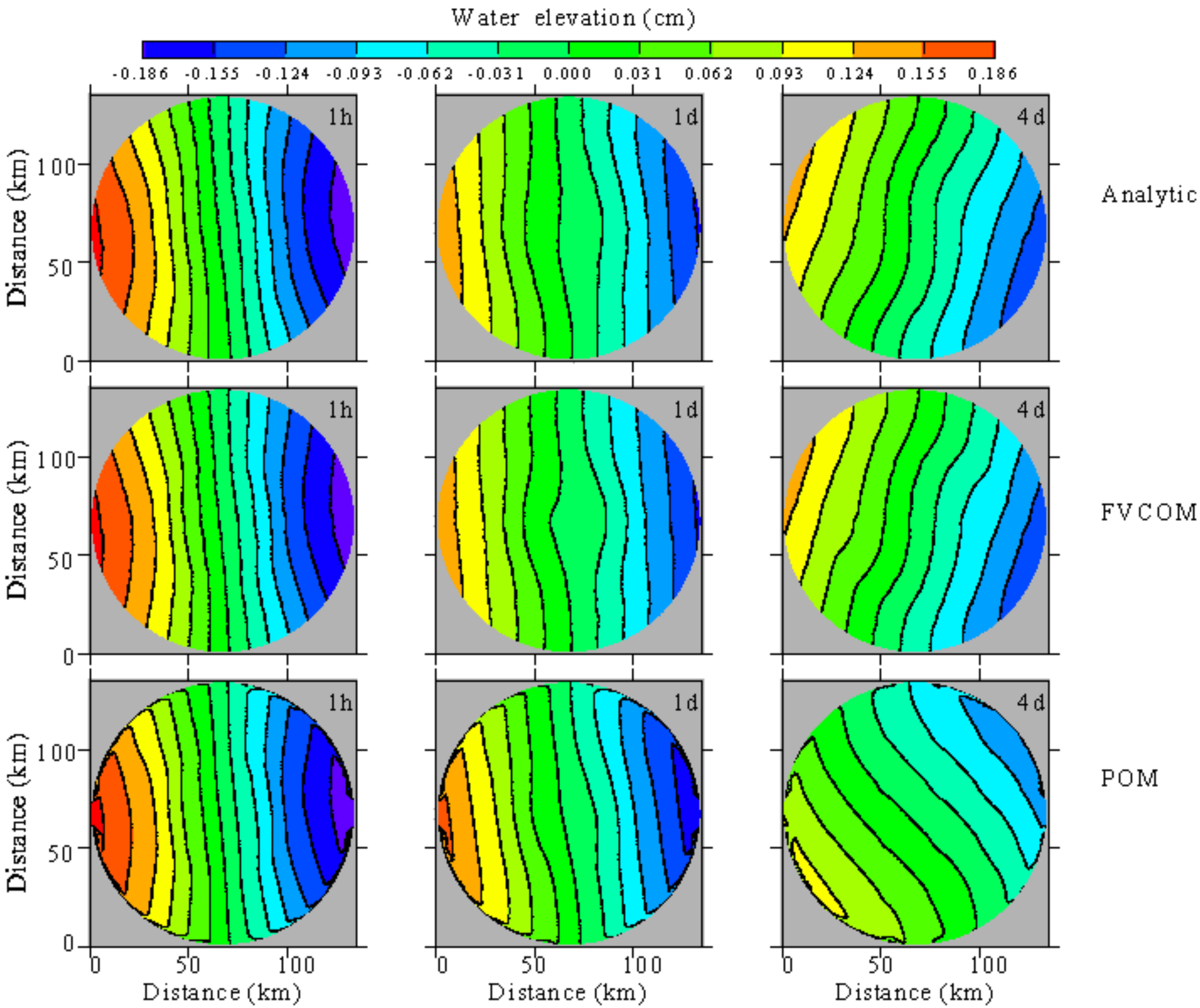


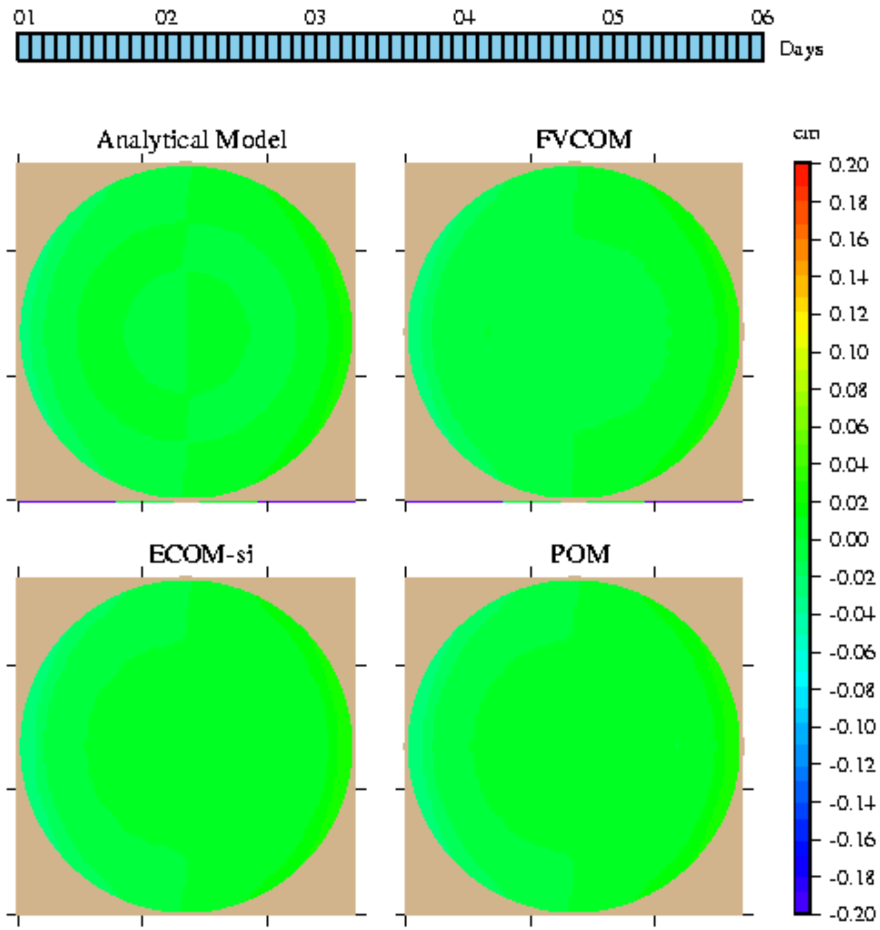
Water elevation



Alongshore transport

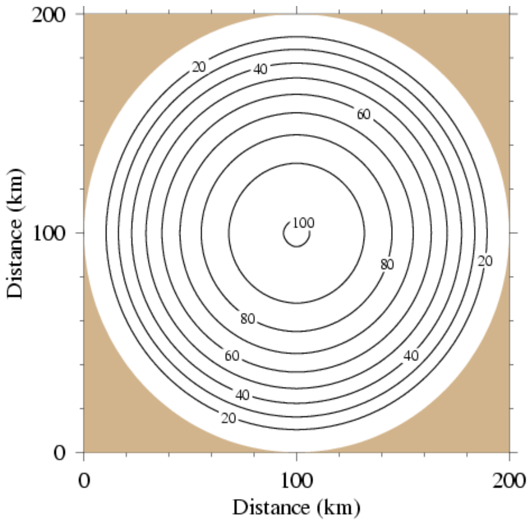


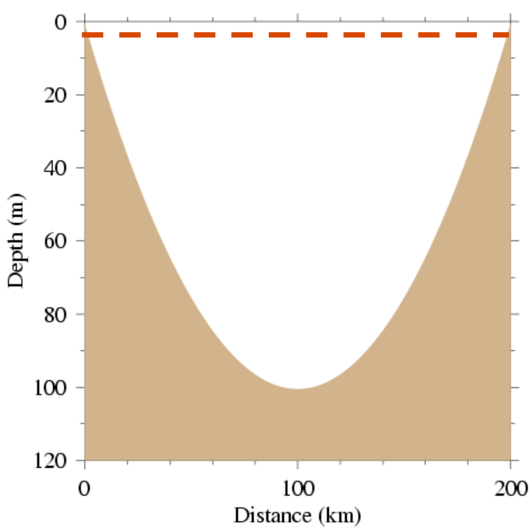

Water elevation (cm)
-0.186 -0.155 -0.124 -0.093 -0.062 -0.031 0.000 0.031 0.062 0.093 0.124 0.155 0.186
1h
1d
4d
Analytic
FVCOM
POM
Distance (km)
0
50
100


01
02
03
04
05
06
Days
Analytical Model
FVCOM
ECOM-si
POM
cm
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
-0.12
-0.14
-0.16
-0.18
-0.20

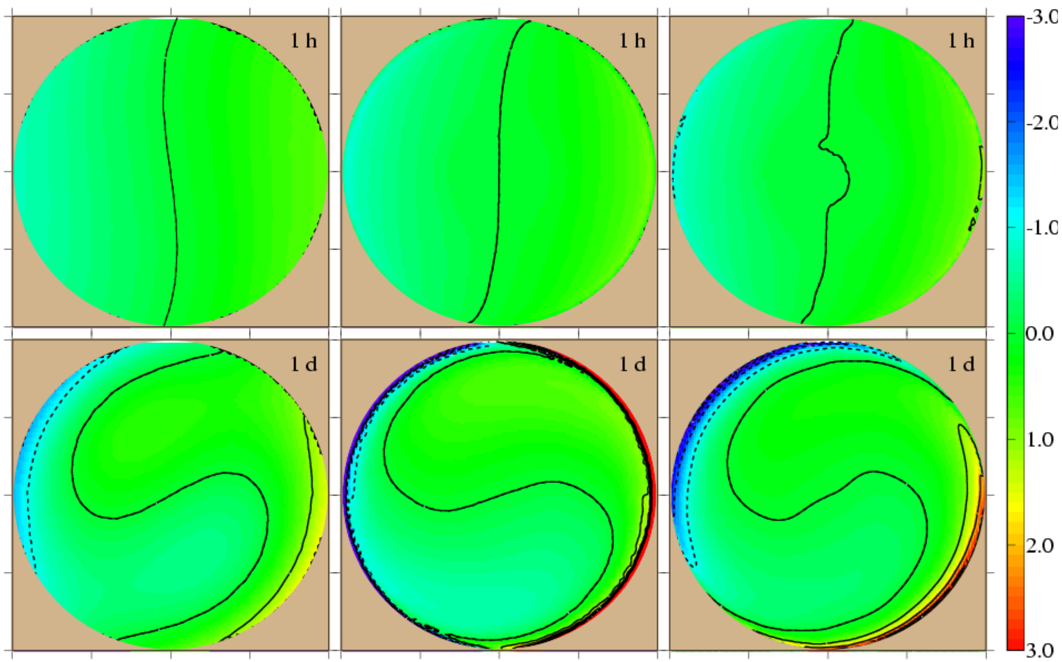
Radial mode: k=1, 2: gravity waves, k=3: topographic wave

Birchfield and Hickie (1977) JPO





Analytical | FVCOM (5 km) | POM (2.5 km)



1 h

1 d

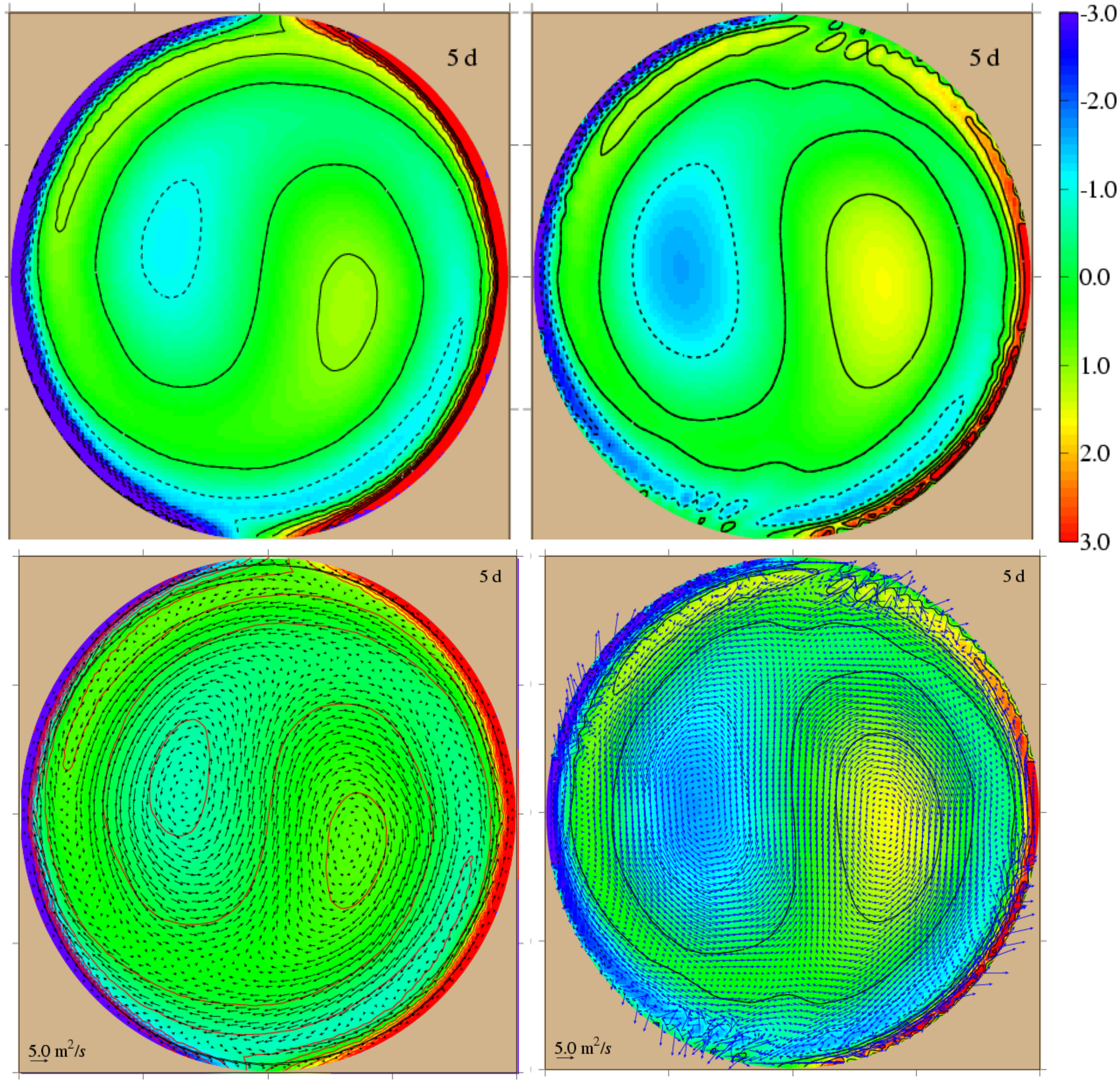
5 days

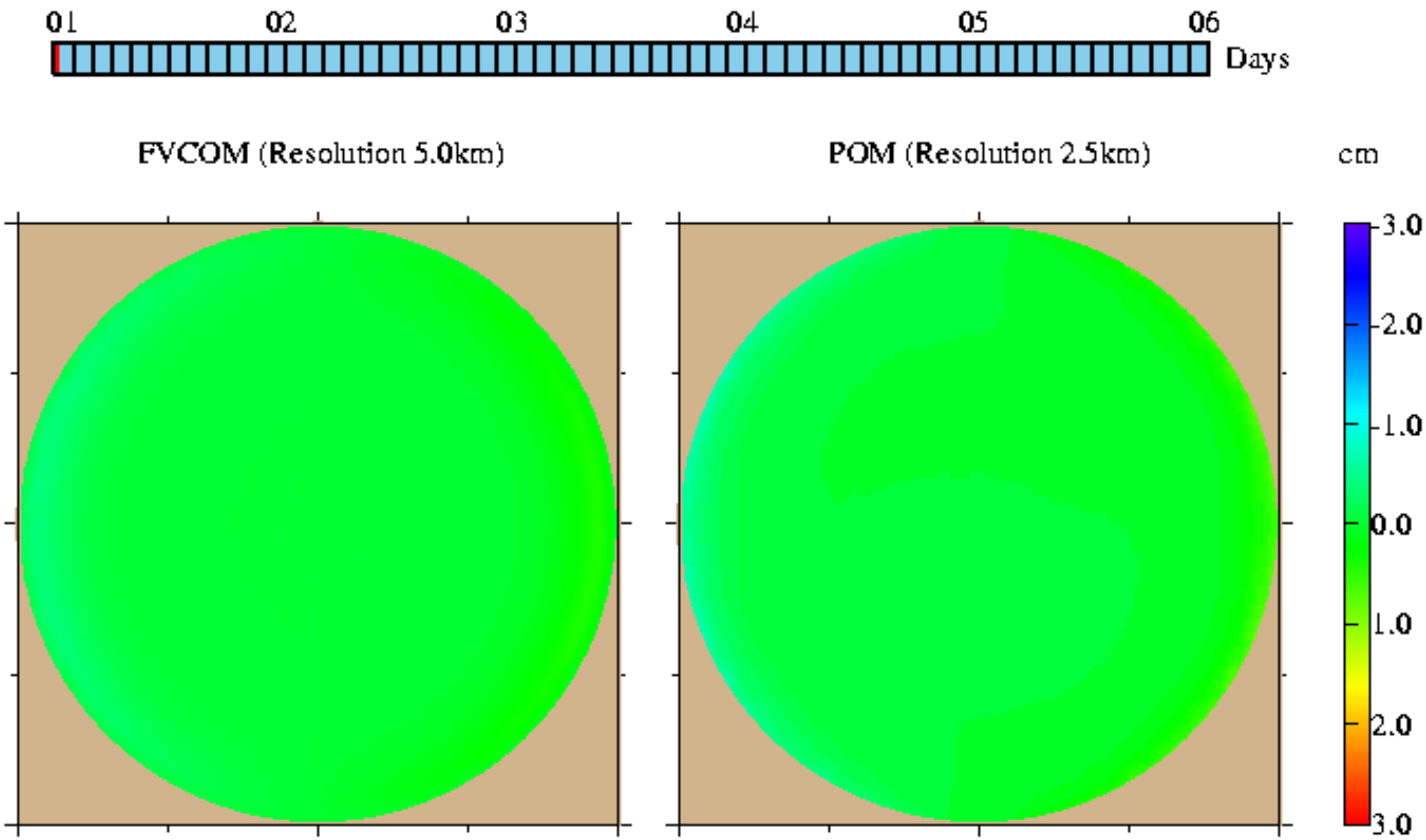
FVCOM (5 km)

POM (2.5 km)

ζ: Elevation

$\vec{V}$: Currents





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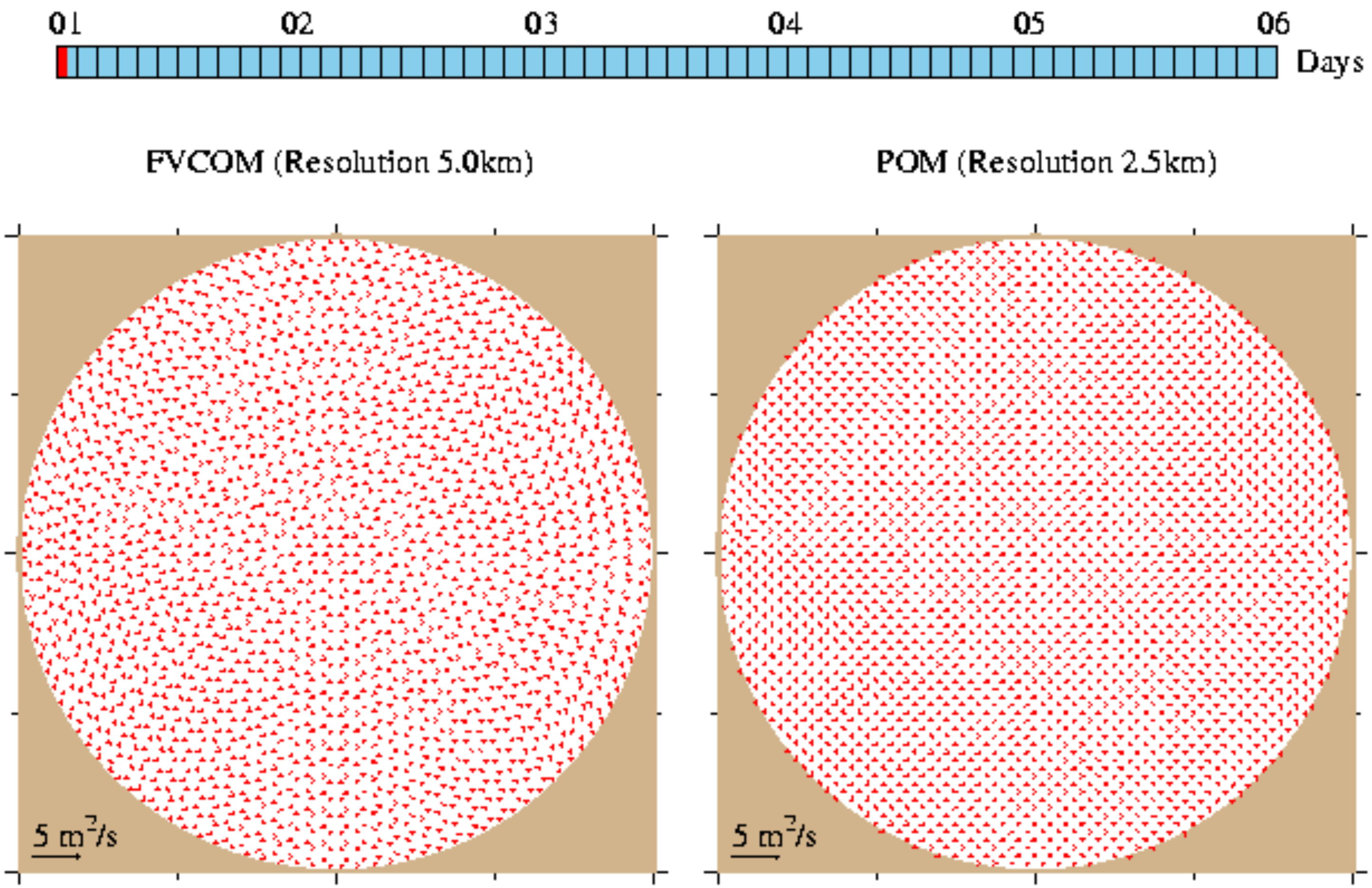
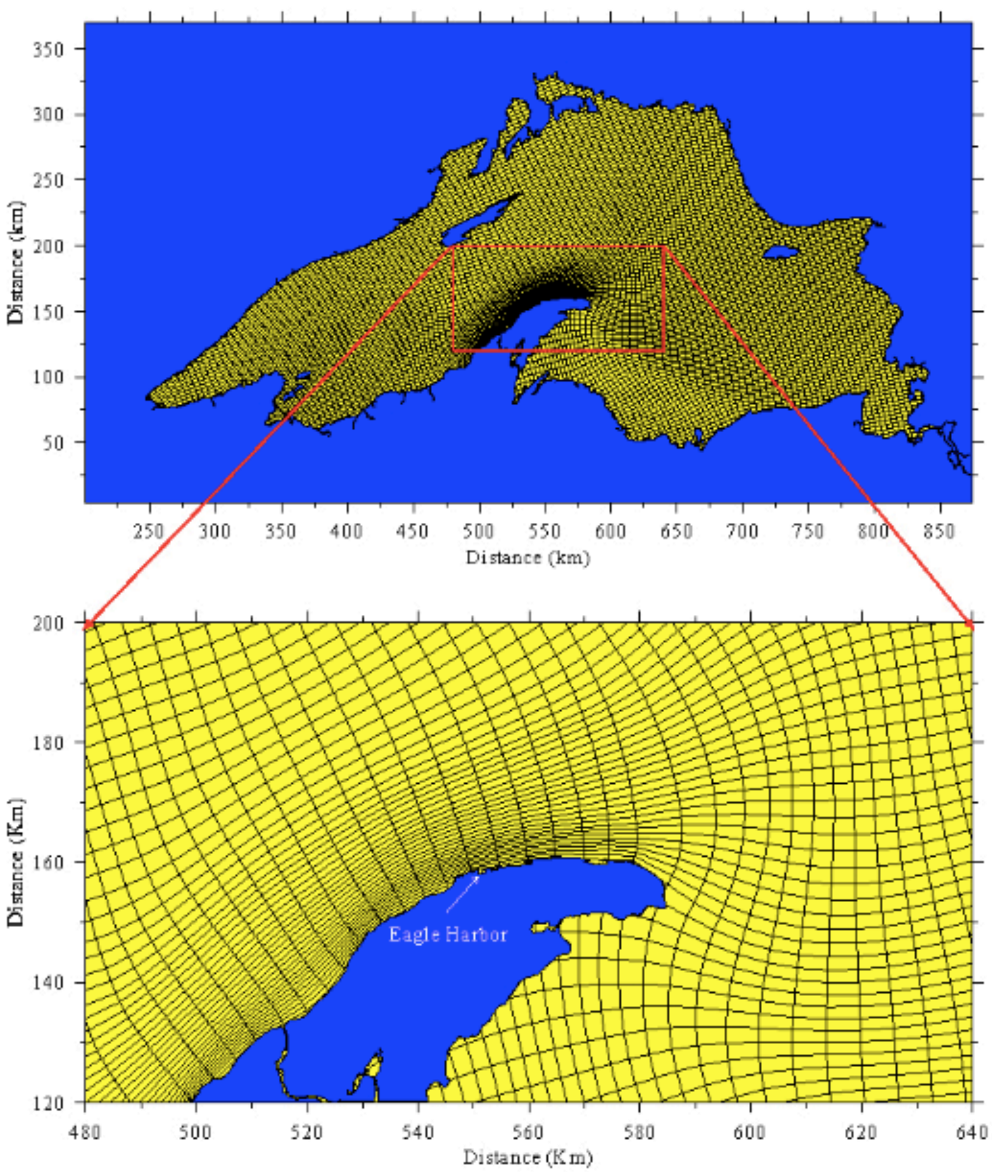
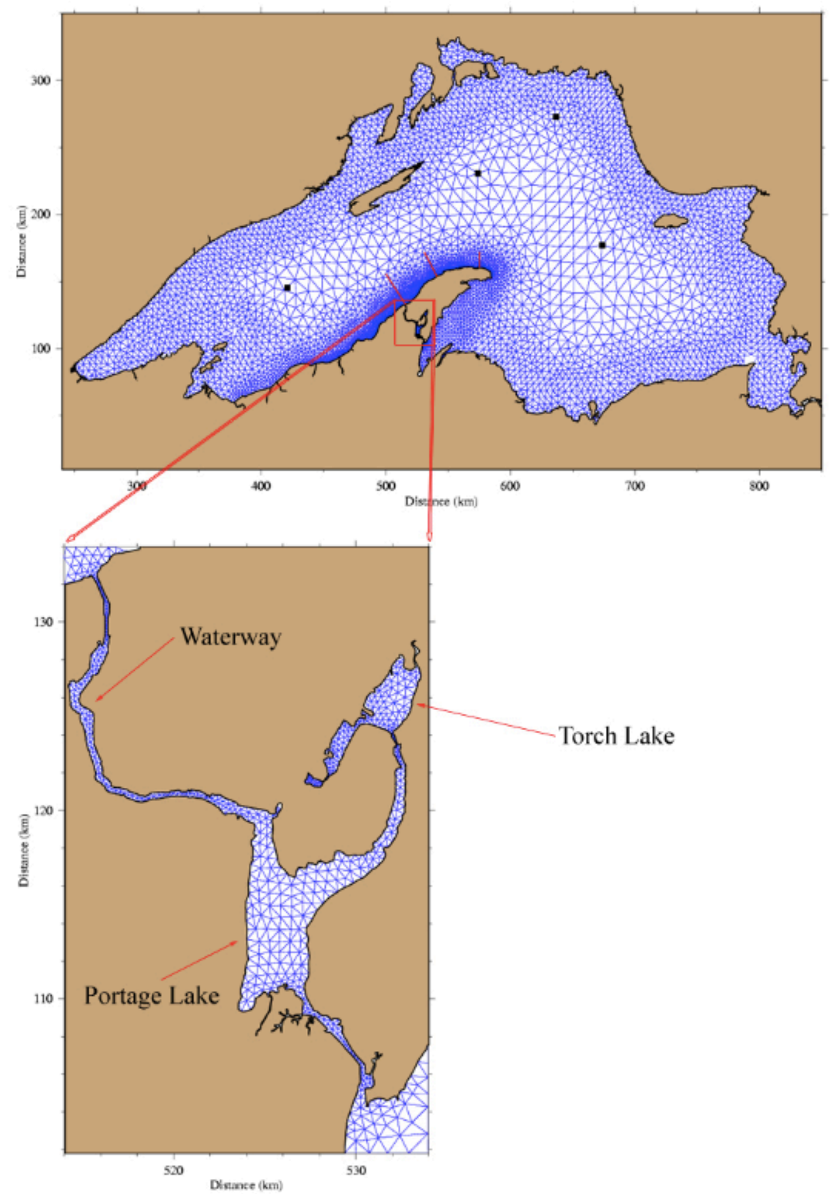
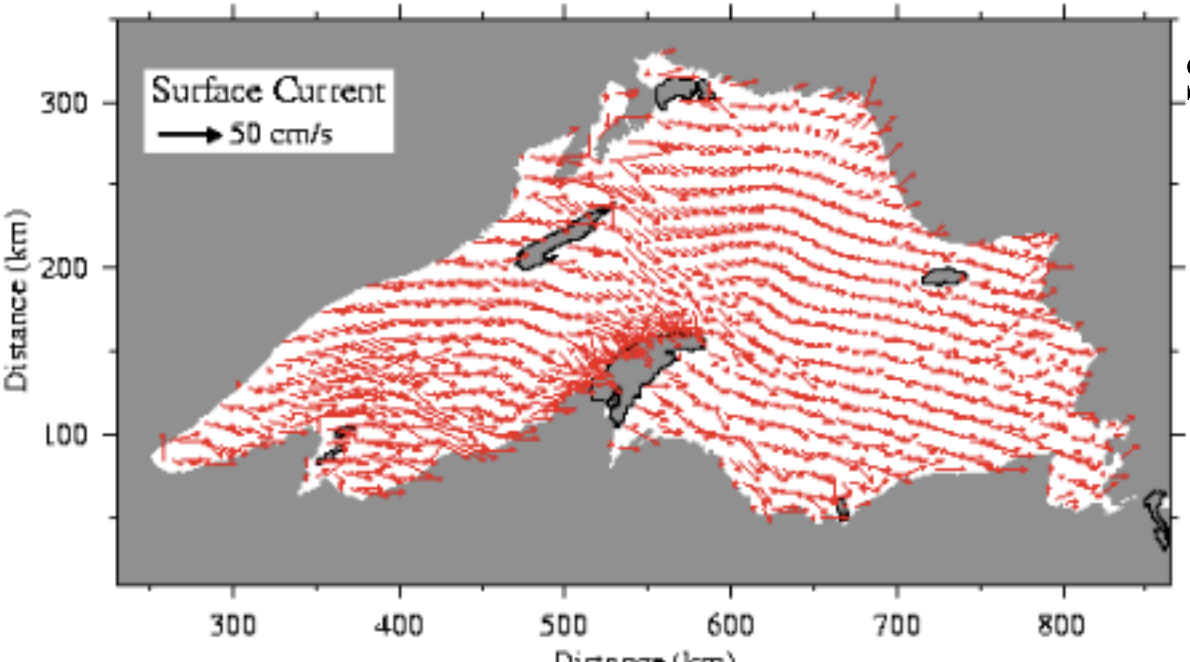

01
02
03
04
05
06
Days
FVCOM (Resolution 5.0km)
POM (Resolution 2.5km)
5 m²/s
5 m²/s

Structured (POM) Unstructured (FVCOM)

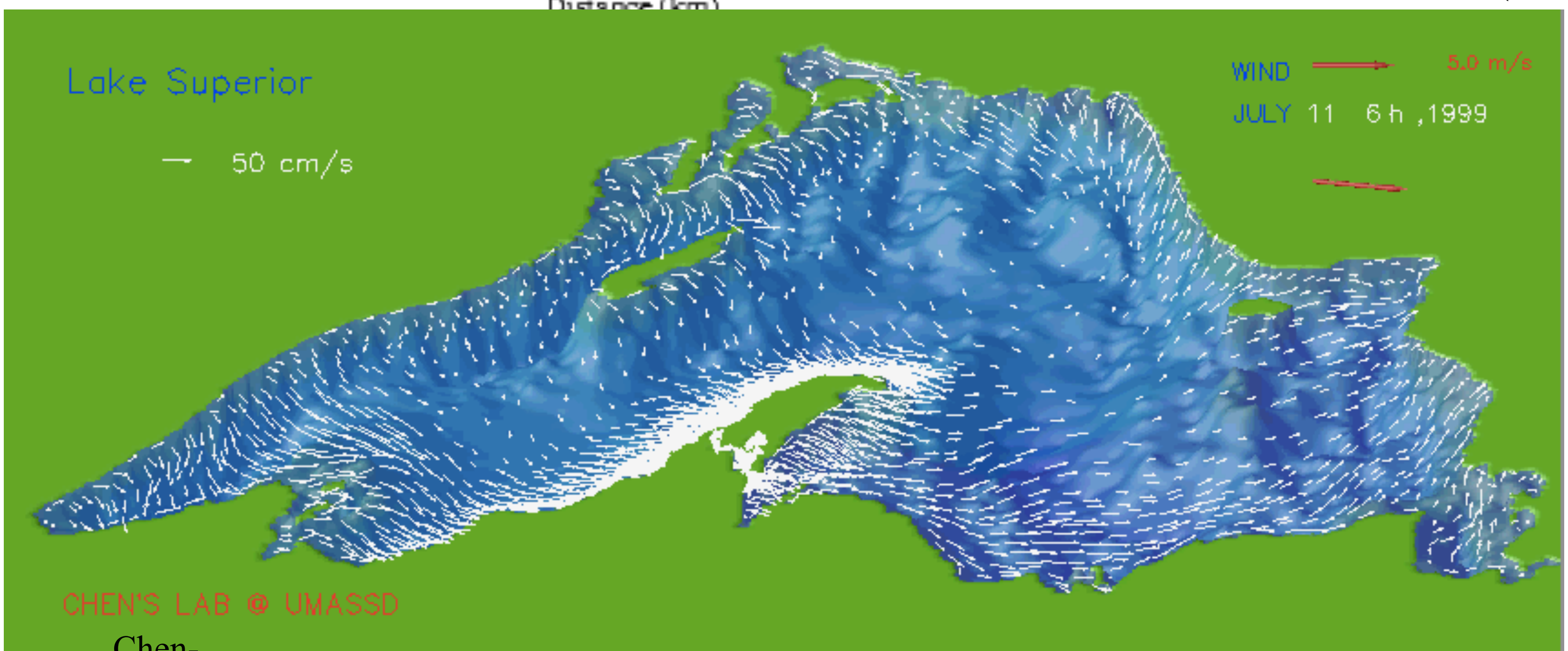






Structured (POM)

Unstructured (FVCOM)



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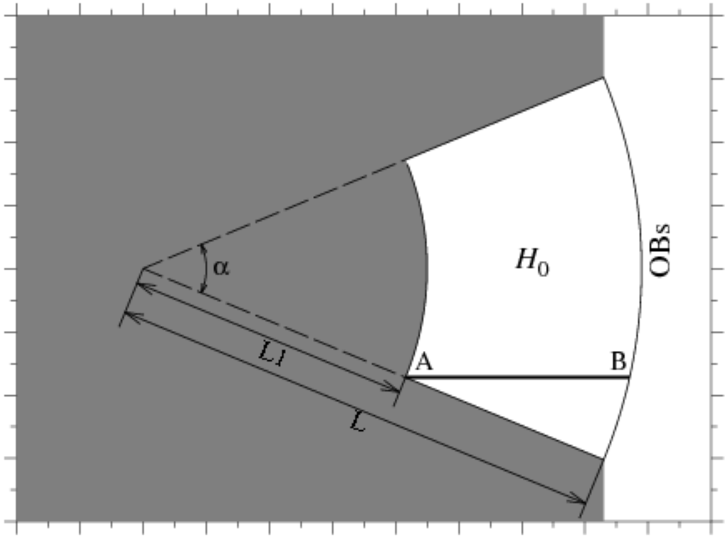
Tidal Resonance in A Semi-closed Channel

Consider a 2-D linear, non-rotated initial problem such as

$$\begin{cases} \dfrac{\partial V_r}{\partial t} + g\dfrac{\partial \eta}{\partial r} = 0 \qquad \dfrac{\partial V_\theta}{\partial t} + g\dfrac{\partial \eta}{r\partial \theta} = 0 \\ \dfrac{\partial \eta}{\partial t} + \dfrac{\partial r V_r H_0}{r\partial r} + \dfrac{\partial V_\theta H_0}{r\partial \theta} = 0 \end{cases}$$



The solution:

$$\eta_0(r,\theta) = [c_1 J_{\gamma_m}(r\frac{\omega}{\sqrt{gH_0}}) + c_2 Y_{\gamma_m}(r\frac{\omega}{\sqrt{gH_0}})]\cdot\cos[\frac{m\pi(\theta+\alpha/2)}{\alpha}]$$

where

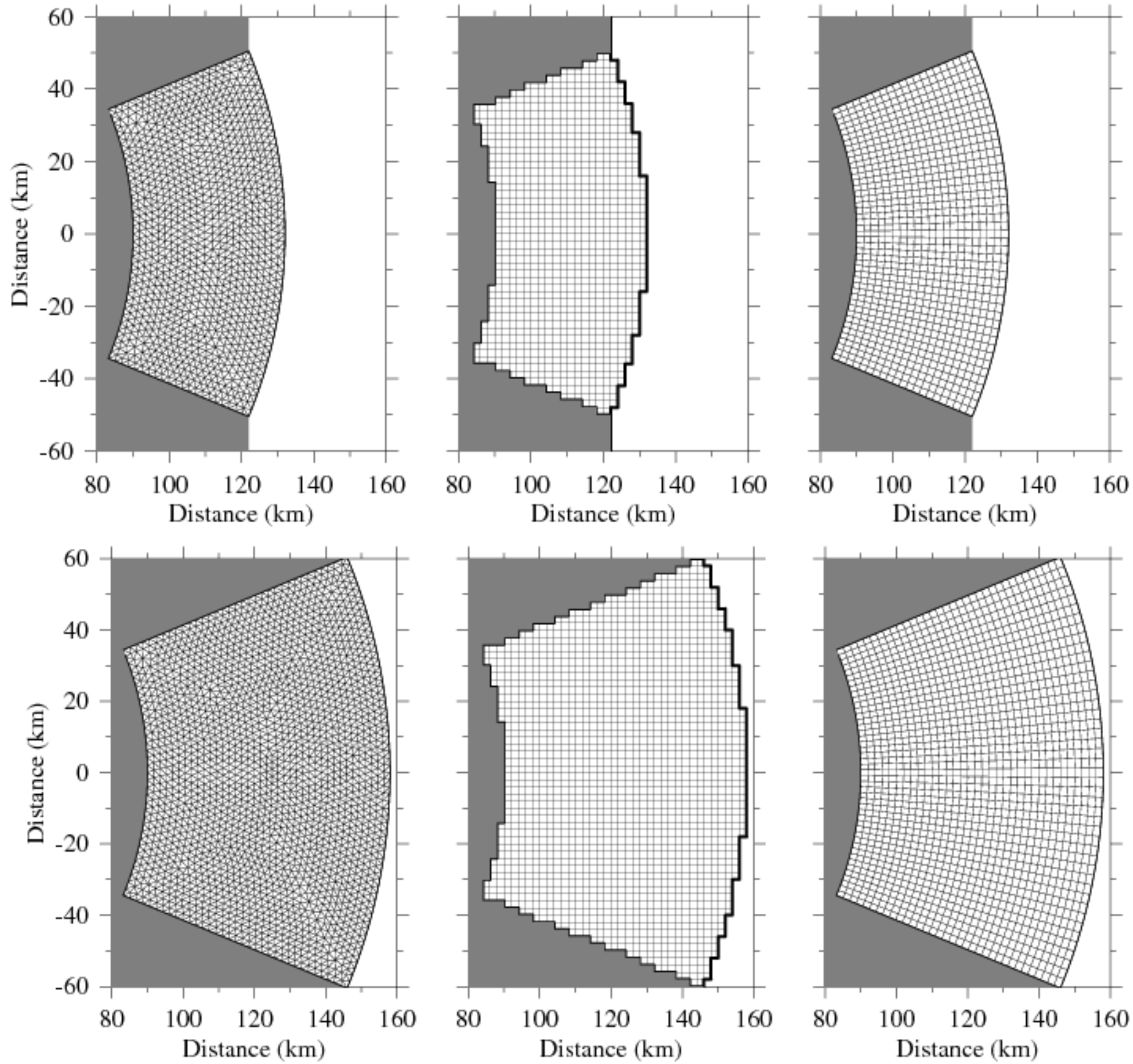
$$c_1 = A\cdot Y'_{\gamma_m}(L_1\frac{\omega}{\sqrt{gH_0}})/[J_{\gamma_m}(L\frac{\omega}{\sqrt{gH_0}})Y'_{\gamma_m}(L_1\frac{\omega}{\sqrt{gH_0}}) - J'_{\gamma_m}(L_1\frac{\omega}{\sqrt{gH_0}})Y_{\gamma_m}(L\frac{\omega}{\sqrt{gH_0}})$$

$$c_2 = -A\cdot J'_{\gamma_m}(L_1\frac{\omega}{\sqrt{gH_0}})/[J_{\gamma_m}(L\frac{\omega}{\sqrt{gH_0}})Y'_{\gamma_m}(L_1\frac{\omega}{\sqrt{gH_0}}) - J'_{\gamma_m}(L_1\frac{\omega}{\sqrt{gH_0}})Y_{\gamma_m}(L\frac{\omega}{\sqrt{gH_0}})$$

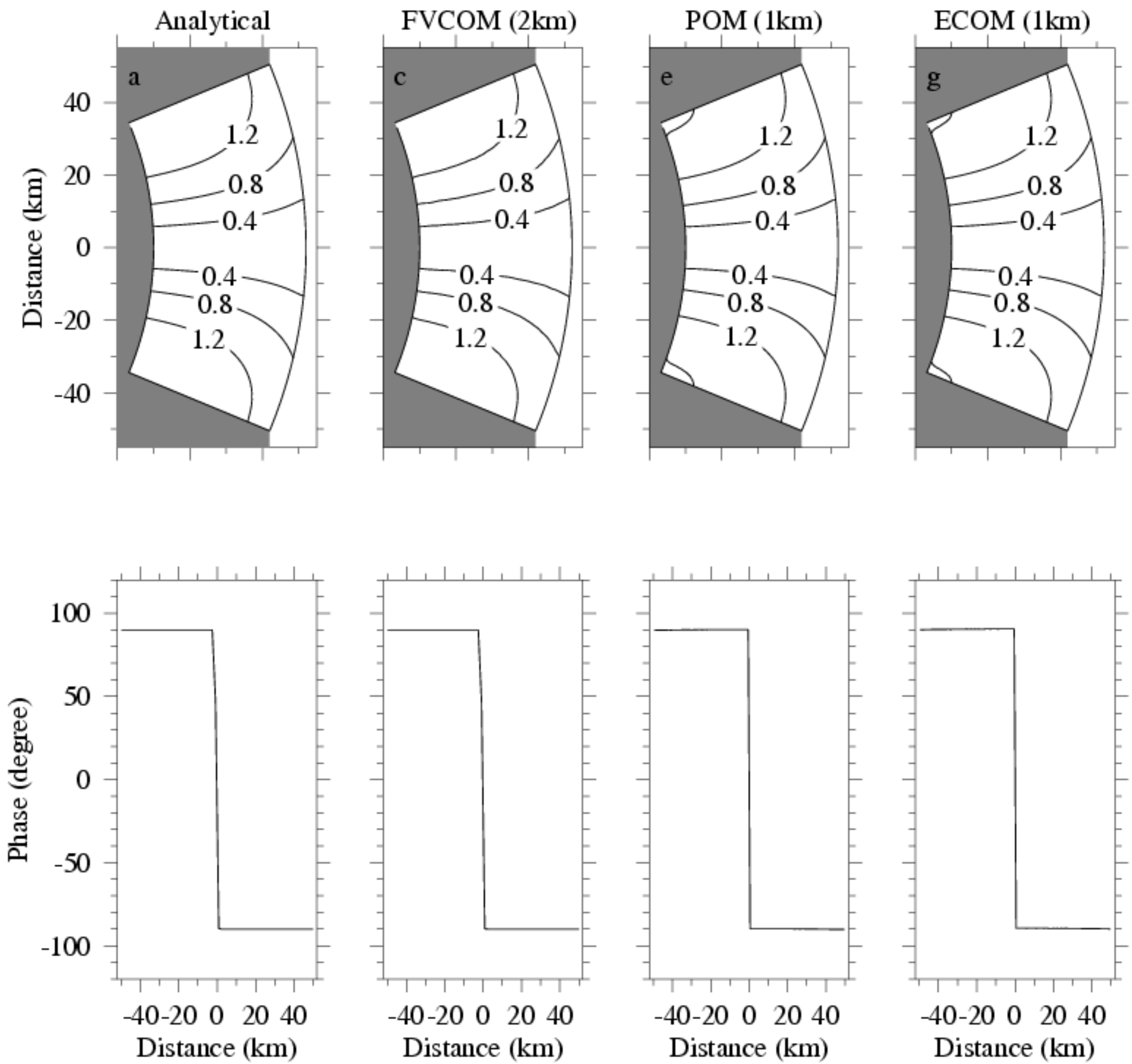
$$\gamma_m = m\pi/\alpha$$

1. Normal condition (non-resonance)

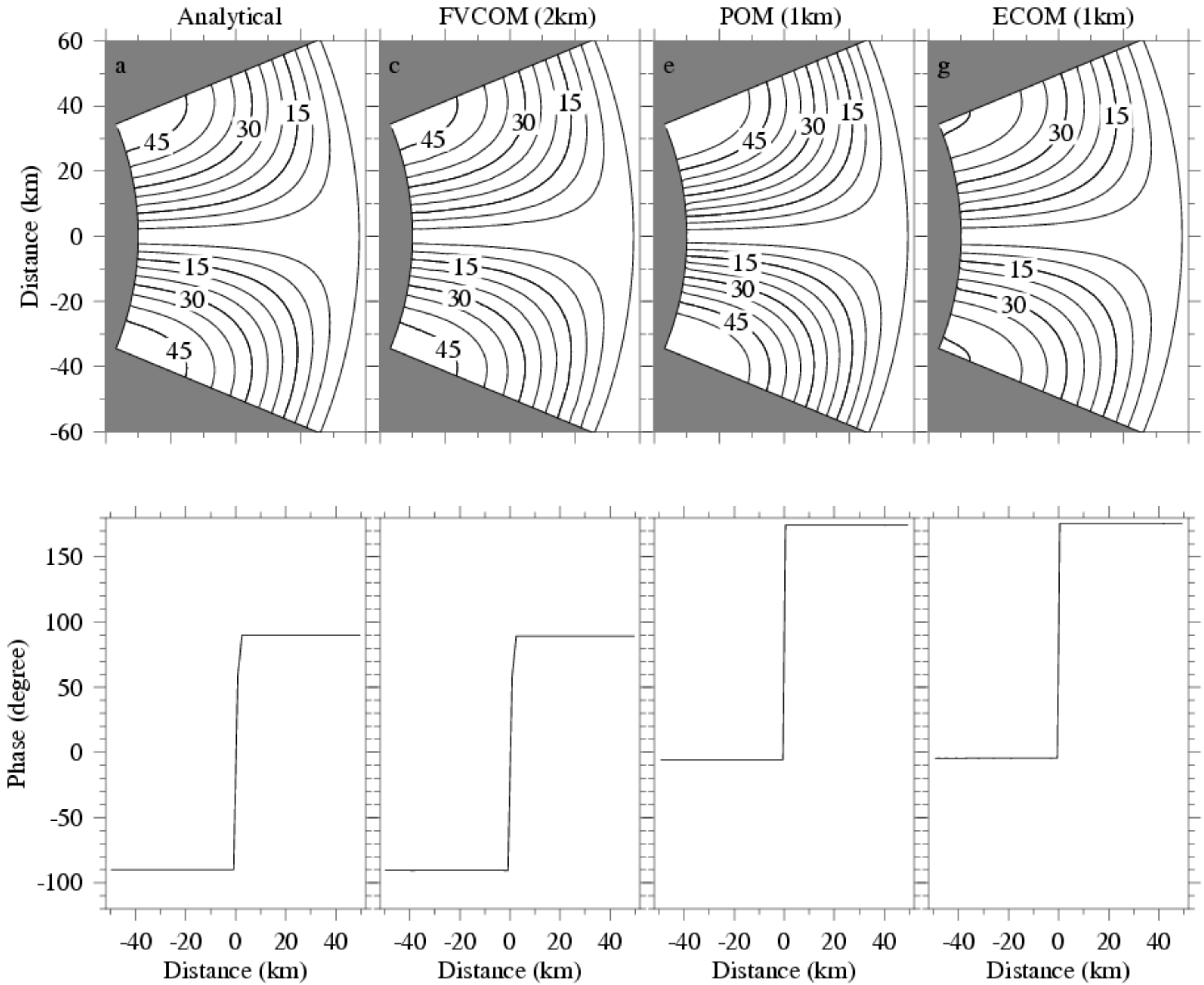
2. Near-resonance condition



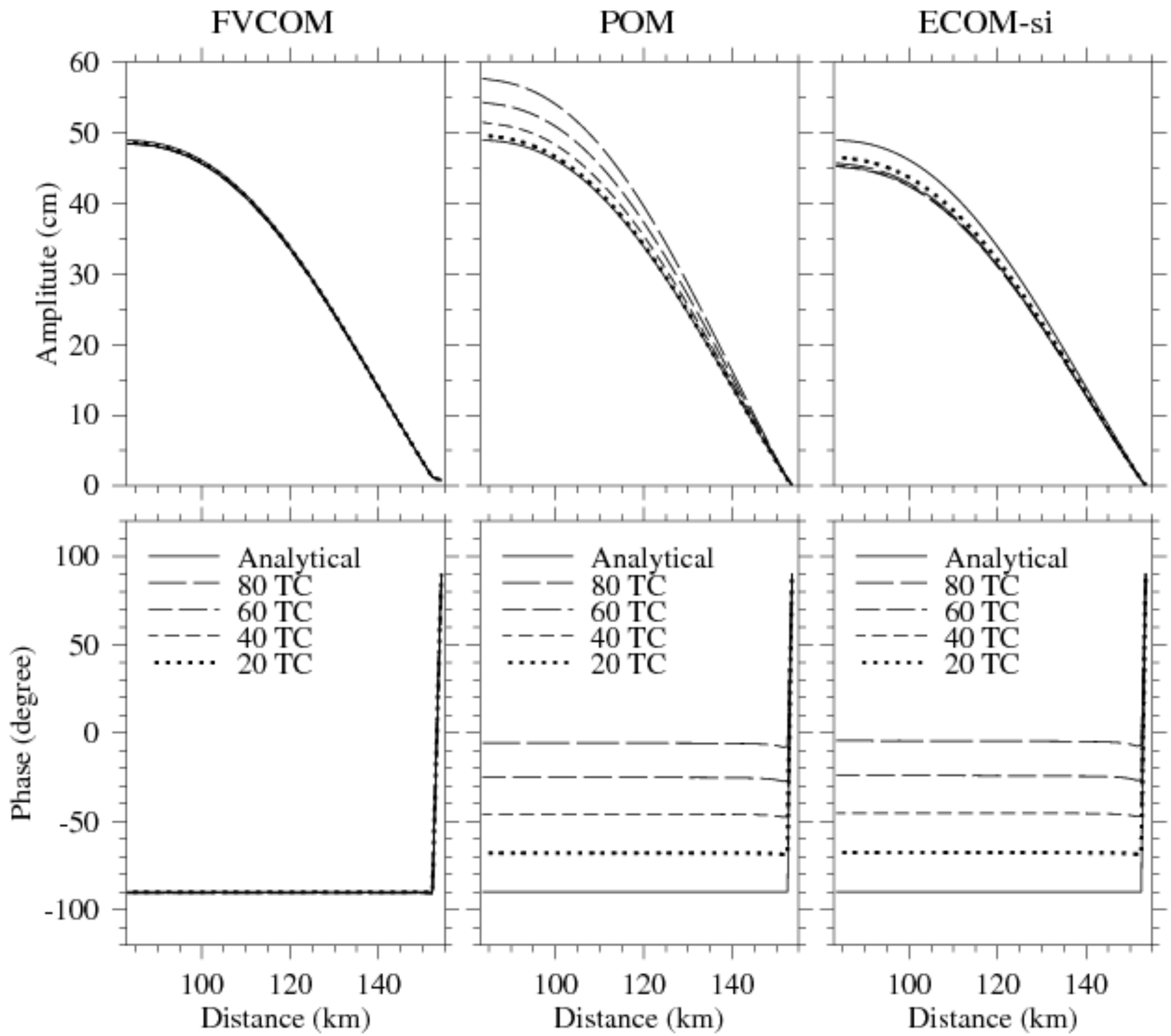
A Normal Case



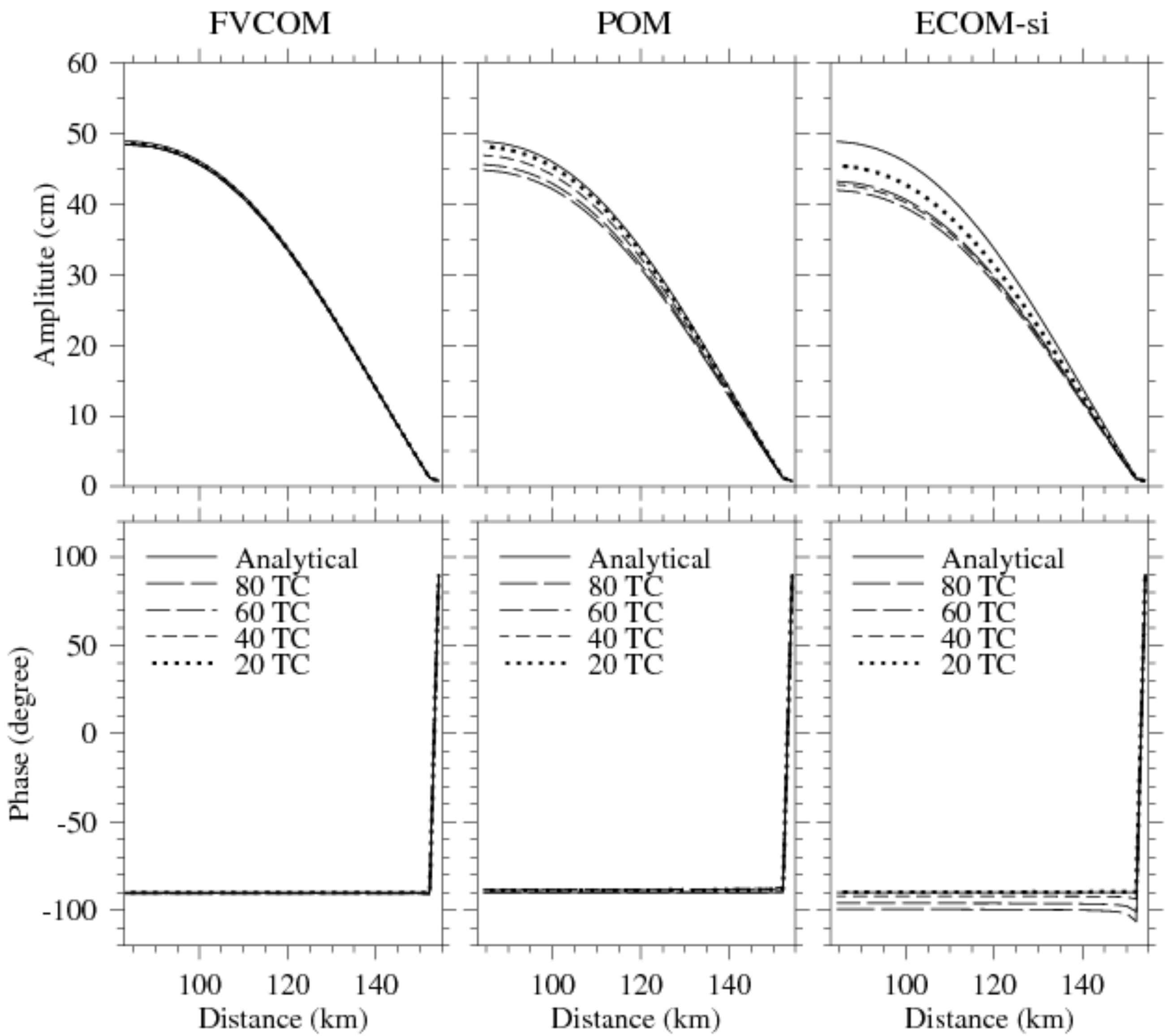
A Near-resonance Case

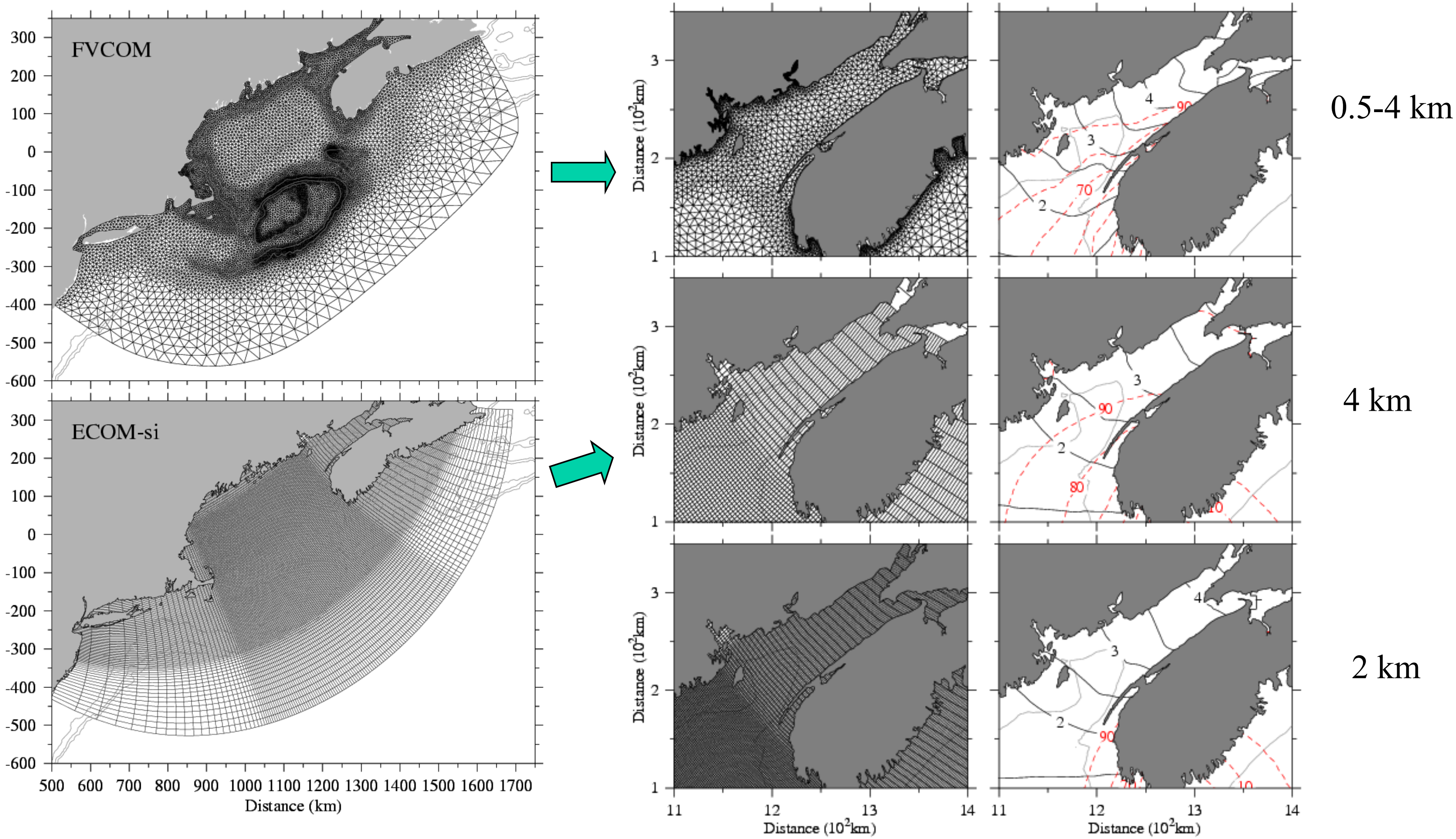


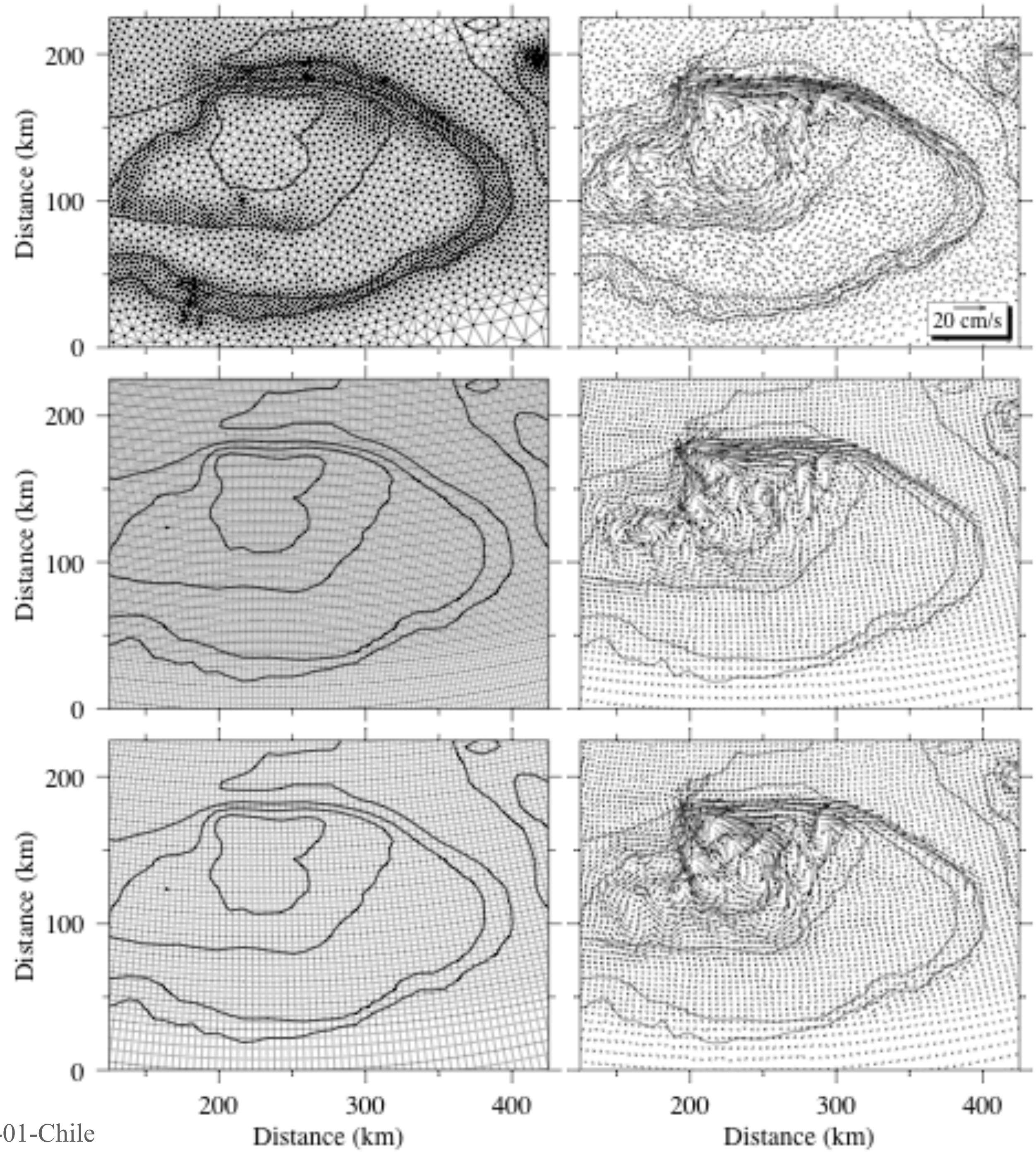
Near-resonance, 2 km (FVCOM), 1 km (POM&ECOM-si)



Near-resonance, 2km, Curvilinear




FVCOM
ECOM-si
Distance (km)
Distance (10^2km)
0.5-4 km
4 km
2 km



Slope topography fitting

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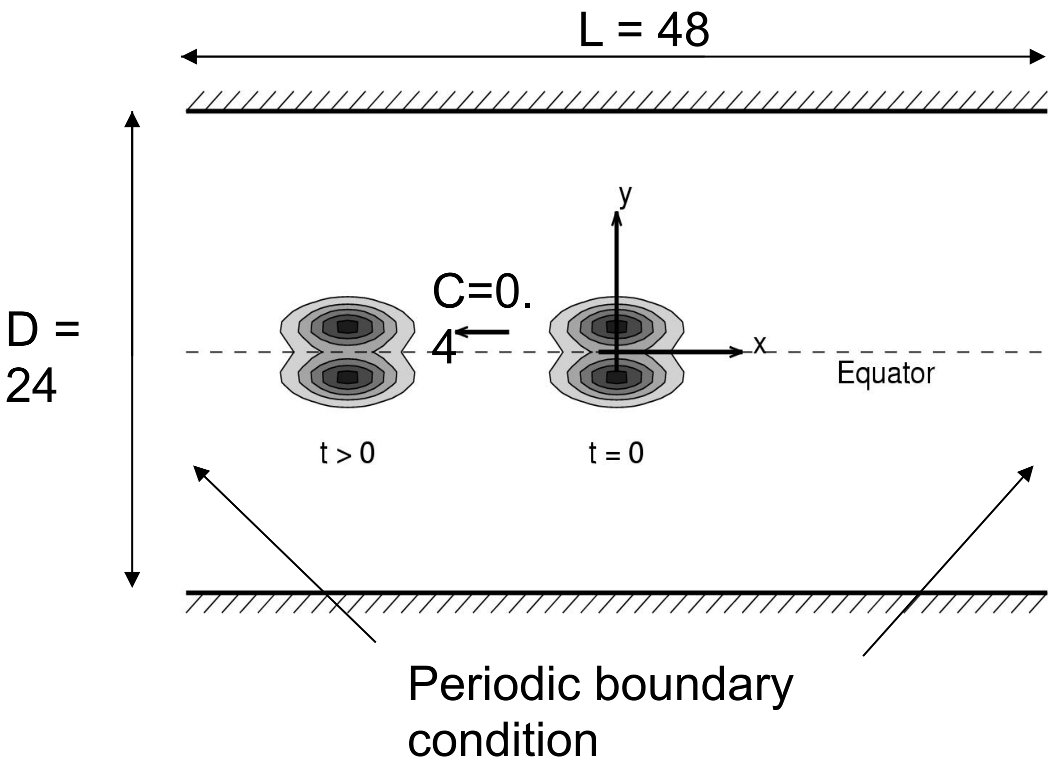
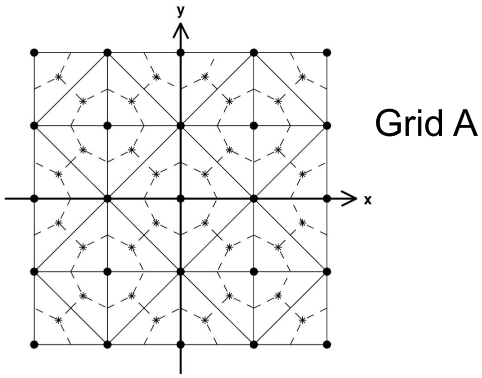
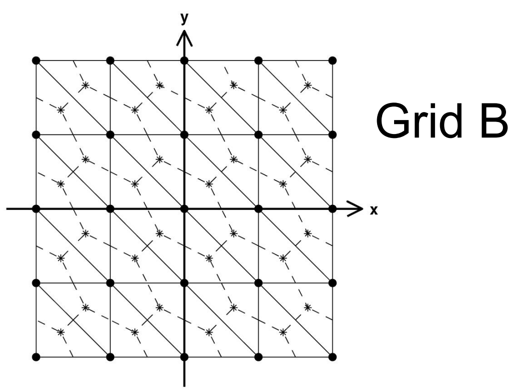
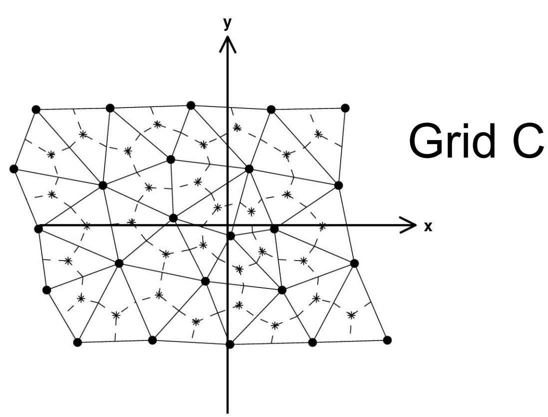
Equatorial Rossby Soliton

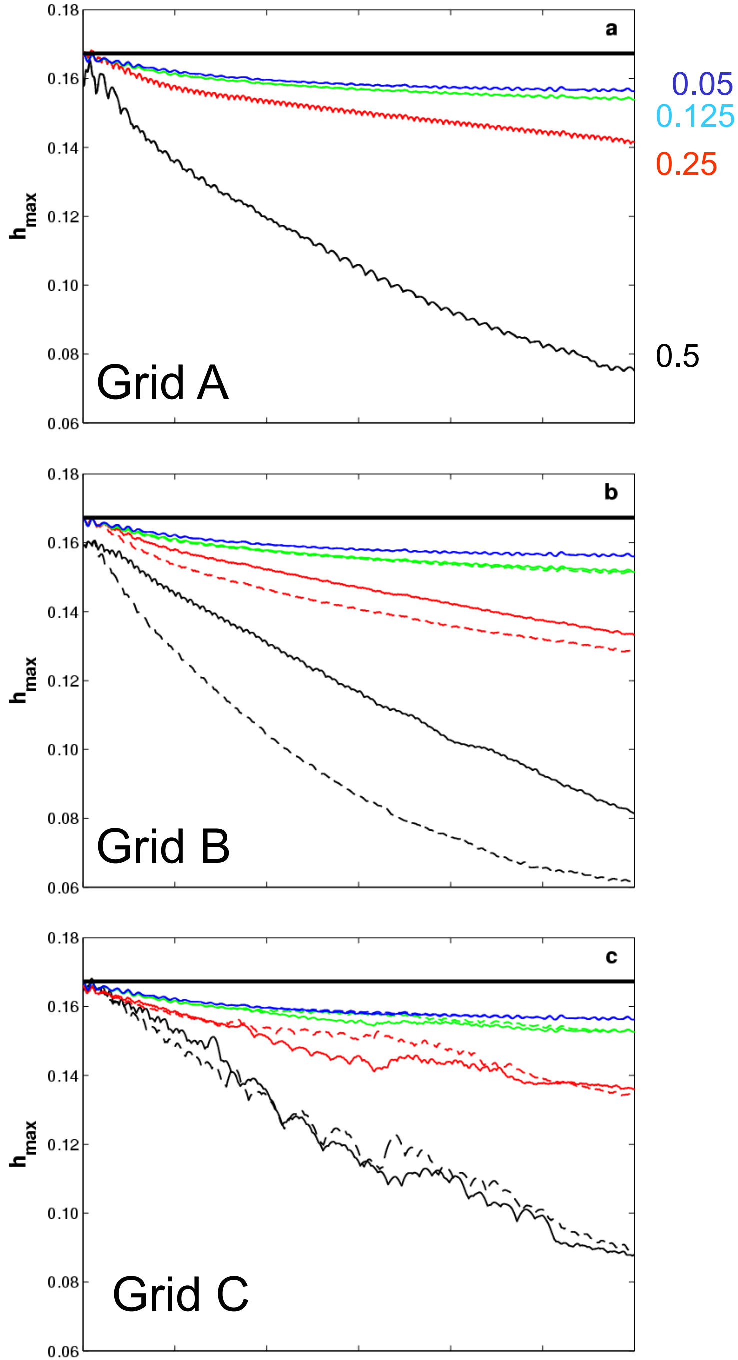








1. Nonlinear shallow water equation in equatorial β-plane
2. Inviscid flow
3. Asymptotic solutions available to zero and first orders (Boyd 1980,1985)



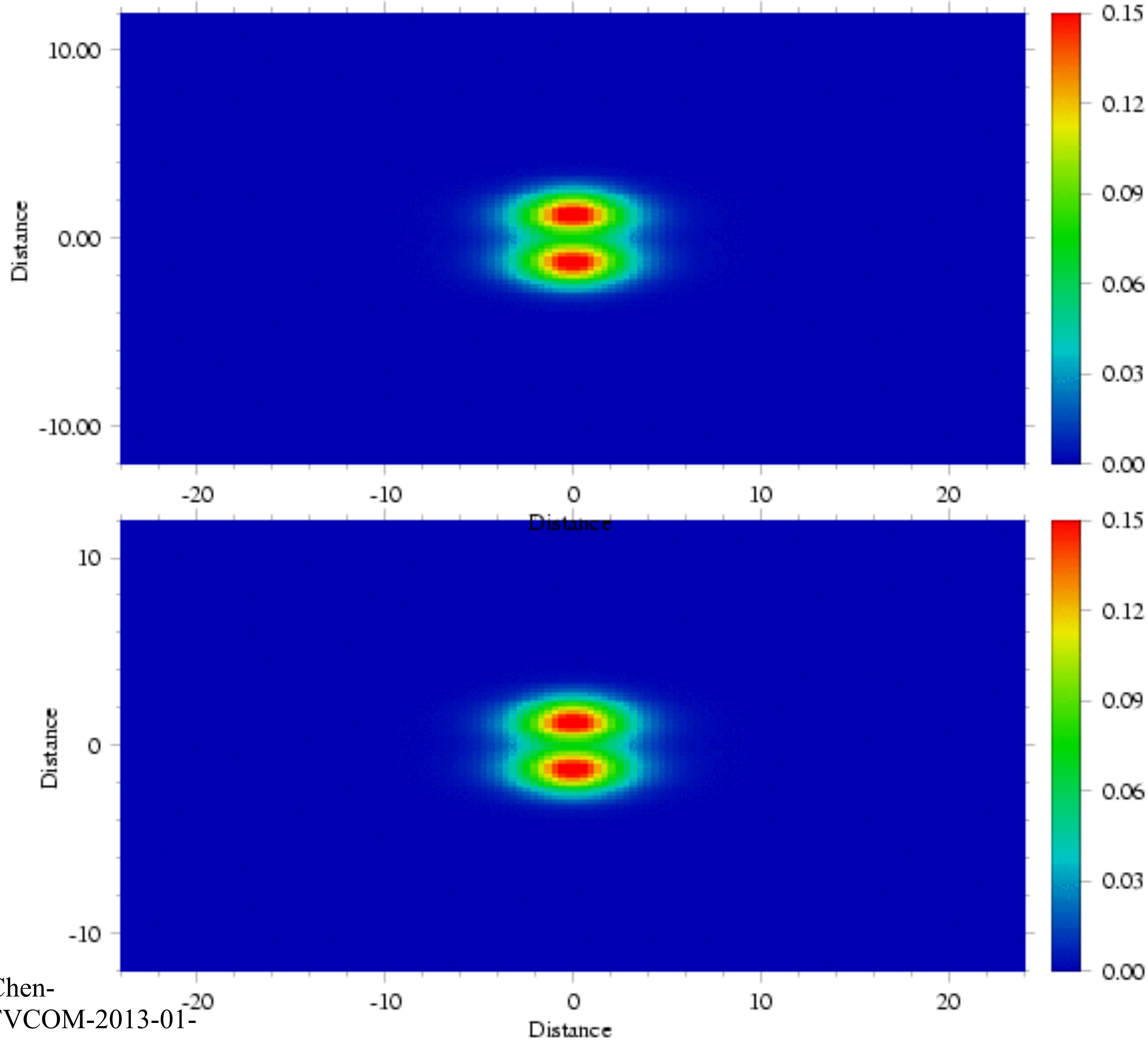
Δx (ND)	FVCOM (2^{nd})		ROMS (4^{th})		SEOM ($7\text{-}9^{th}$)	
	h_n/h_t	C_n/C_t	h_n/h_t	C_n/C_t	h_n/h_t	C_n/C_t
0.5	0.472	0.917	0.884	1.088	0.923	0.98
0.25	0.846	0.984	0.926	0.993	0.929	0.99
0.125	0.92	0.984	0.923	0.986	0.937	0.989
0.05	0.935	0.983	0.936	0.983	0.915	0.98

h_n: Computed peak of the sea surface elevation at 120 units
h_t: Analytical peak of the sea surface elevation at 120 units
C_n: Computed average speed
C_t: Analytical averaged speed.

Comments:

1. Analytical solution only represents the zero and 1st modes, while the numerical solution contains a complete set of higher order modes. This is not surprised to see numerical models can not exactly reach the analytical solutions.

2. FVCOM shows a fast convergence with increase of horizontal resolution.



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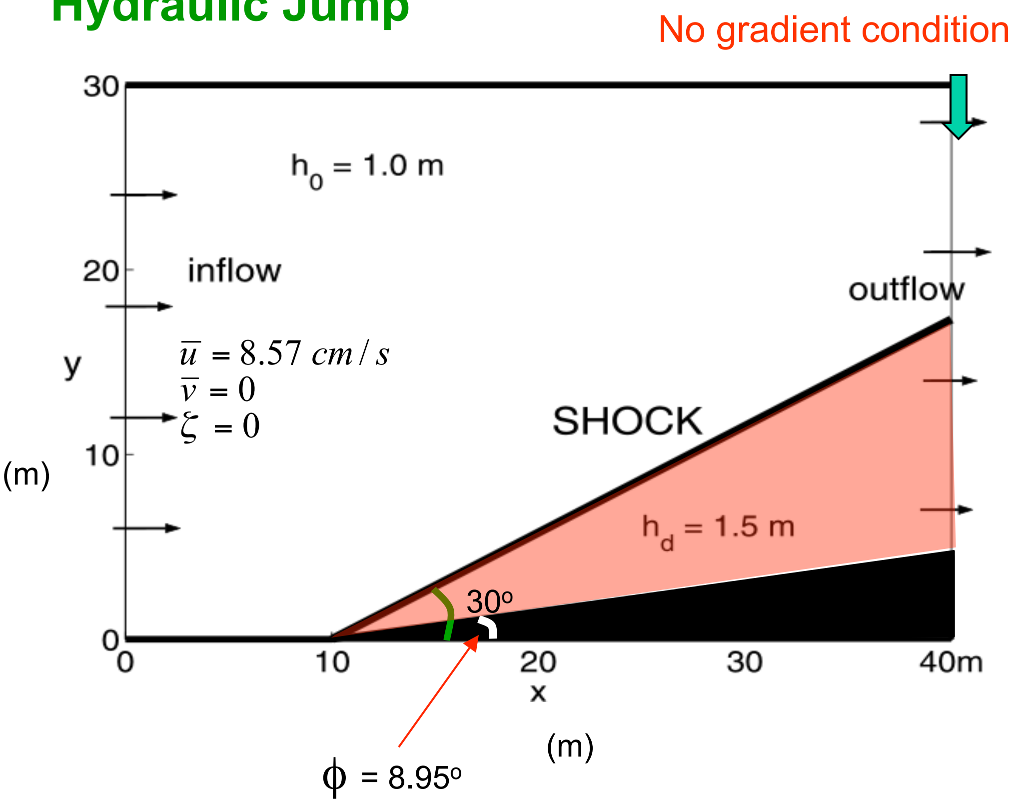
Hydraulic Jump



Analytical solution:

Maximum sea level: $\zeta_{\max} = 0.5$ m

Minimum sea level: $\zeta_{\min} = 0$ m

Mean sea level: $\zeta_{mean} = 0.5$ m

Mean velocity: $\overline{u} = 7.956\ m/s$

Mean Froude #: $Fr = \dfrac{\overline{u}}{\sqrt{gD}} = 2.075$

Shock angle: $\alpha = 30^0$

Thickness: $\delta = 0$ m

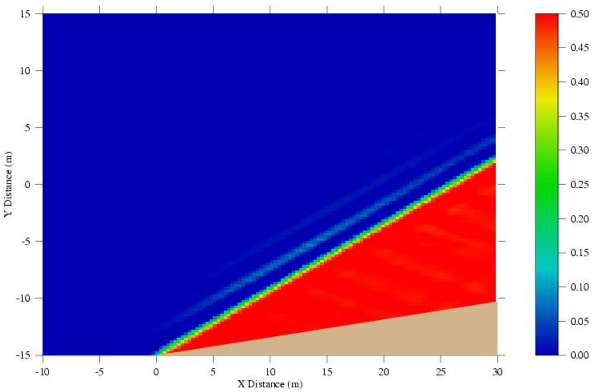
Mean deviation: $|dy| = 0$ m

Characteristics:

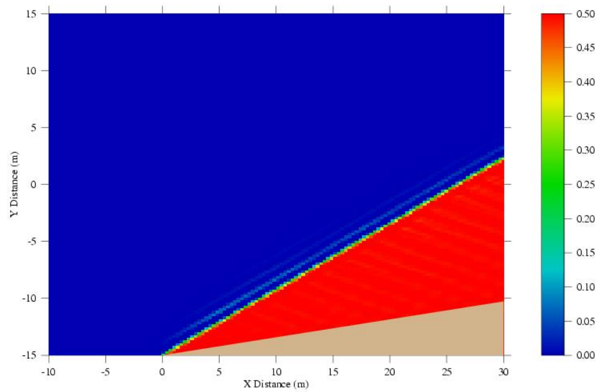
- Barotropic shallow water equations
- No rotation considered, i.e. $f = 0$, $\beta = 0$
- Steady analytical solutions for u, ζ and the jump angle relative to the x axis.

The case with no horizontal diffusion: FVCOM quickly reaches steady status.

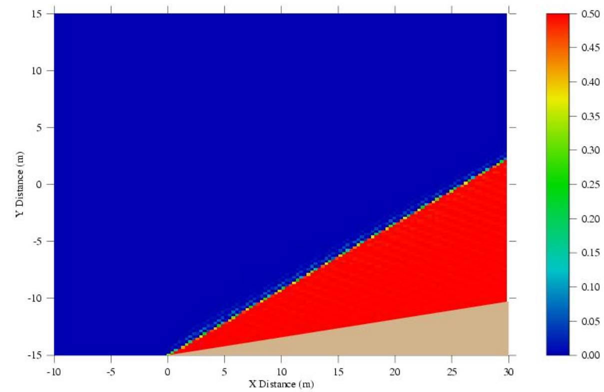
Δx = 0.5 m, Δt = 0.002



Δx = 0.25 m, Δt = 0.001

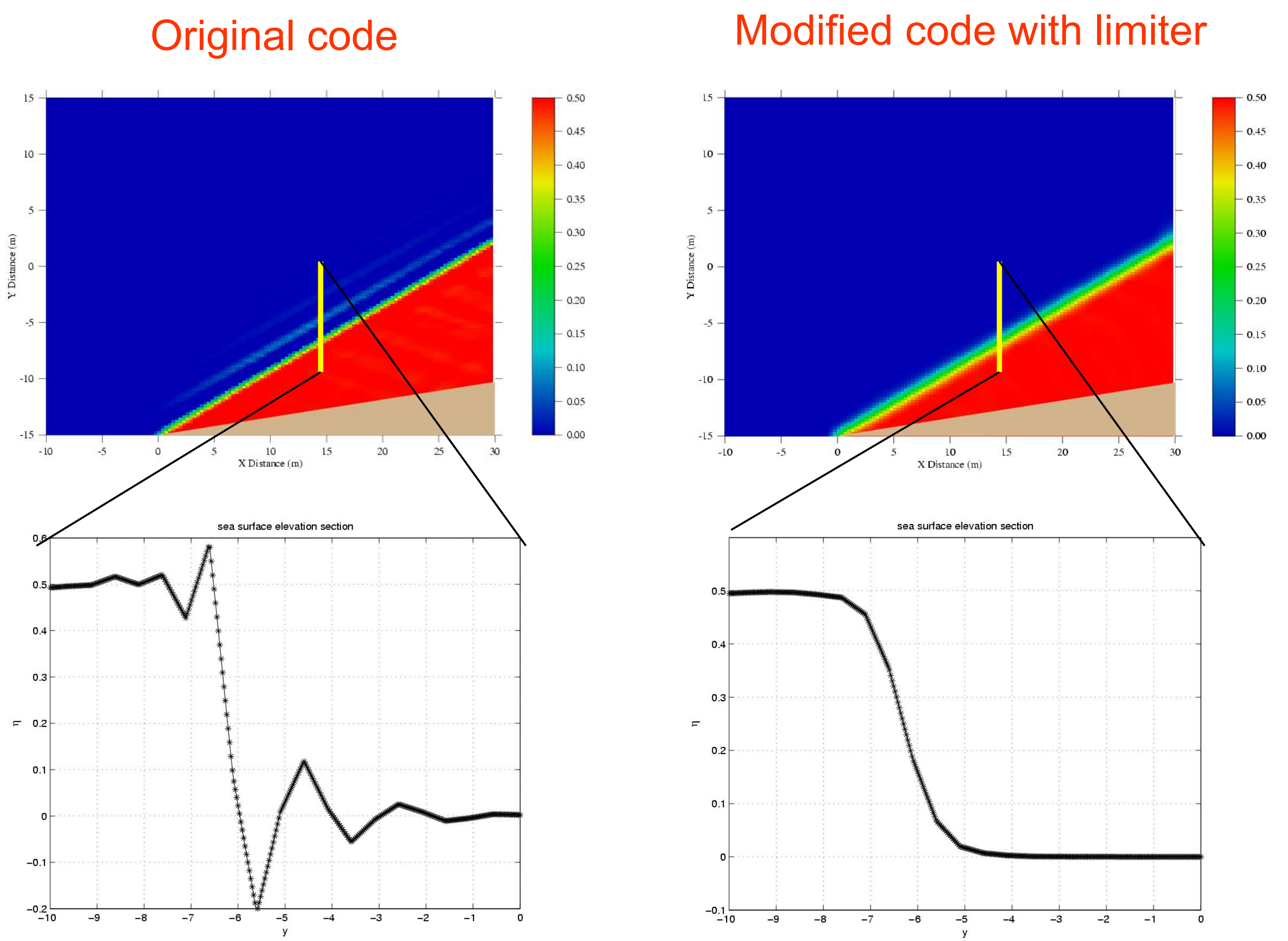


Δx = 0.125 m, Δt = 0.0005



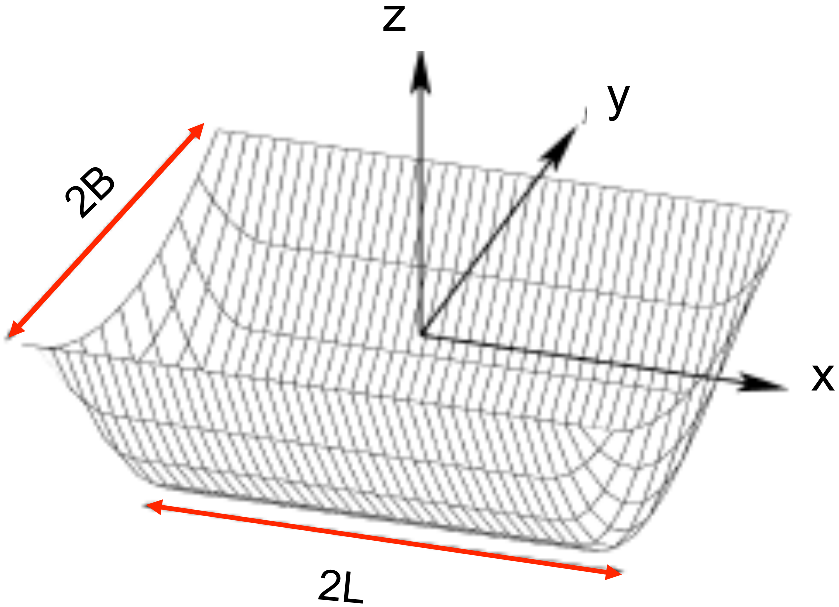
Model	grids	Δt	ζ_{max}	ζ_{min}	ζ_{mean}	$\bar{u}$	F_r	α	δ	$\lvert dy \rvert$
True			0.5	0	0.5	7.956	2.075	30	0	0
FVCOM	80 X 60	0.002	0.688	-0.269	0.5	7.949	2.072	29.952	0.111	0.305
	160 X 120	0.001	0.697	-0.268	0.499	7.951	2.073	30.030	0.063	0.151
	320 X240	0.0005	0.696	-0.272	0.5	7.951	2.073	30.029	0.037	0.076
ROMs	Reach an oscillatory solution without horizontal diffusion.									

Over shocking can be reduced by introducing a slope limiter method (Hubbard, *J. Comput. Phys.*, 1999).



3-Dimensional Wind-Driven Flow in an Elongated, Rotating Basin

Winant (*J. Phys. Oceano.* 2004)



Length: $2L$; width: $2B$, and bathymetry:

$$h = h_0\{0.08 + 0.92 * [X(x/L)(1-(y/B)^2)]\}$$

where $X(x)$ is a function in the form of

$$X(x) = \begin{cases} 1, & |x| < 1-\Delta x \\ 1-[\dfrac{|x|-1+\Delta x}{\Delta x}]^2, & |x| \geq 1-\Delta x \end{cases}$$

Δx is a constant specified as 0.3% of the total length of the basin.

$$-L \leq x \leq L, -B \leq y \leq B$$

Governing equations:

$$\begin{cases} \nabla \cdot \vec{v} + w_z = 0 \\ f\vec{k} \times \vec{v} = -g\nabla\eta + K_m \dfrac{\partial^2 \vec{v}}{\partial z^2} \end{cases}$$

B.Cs:

$$\begin{cases} \vec{v}_z = \dfrac{\vec{\tau}_s}{\rho K_m} & w = 0 \quad at\ z = 0 \\ \vec{v} = 0 & w = 0 \quad at\ z = -h \end{cases}$$

Steady status analytical solution for this linear equation system is given as:

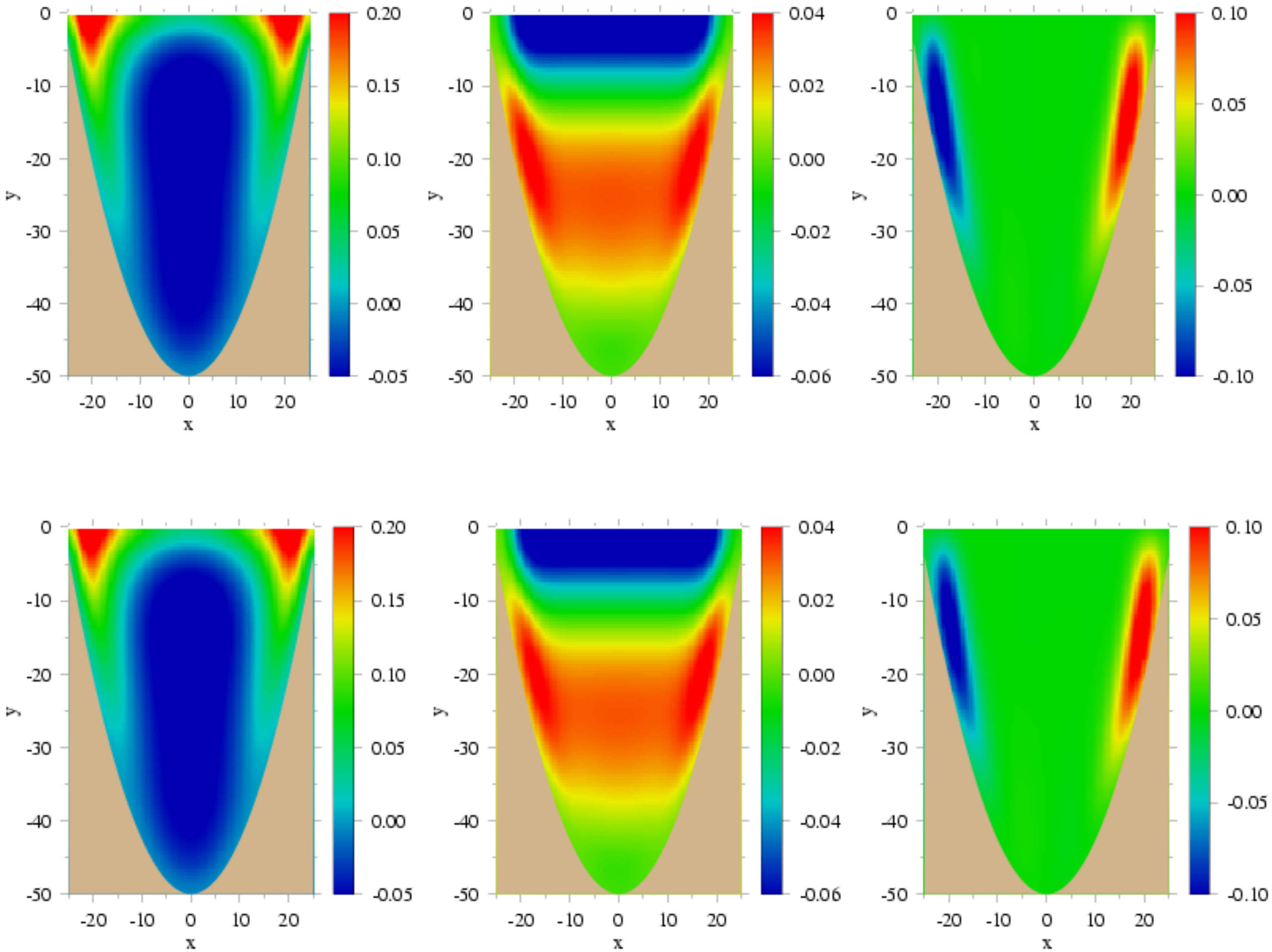
$$u + iv = \frac{\sinh[\alpha(z+h)]}{\alpha\cosh(\alpha h)} - \frac{\eta_x + i\eta_y}{\alpha^2}[1 - \frac{\cosh(\alpha z)}{\cosh(\alpha h)}]$$

$$w = -\int v_y dz$$

where $\alpha^2 = 2i\delta^{-2}$ and $\delta = (2E)^{1/2}$ (E: Ekman number).

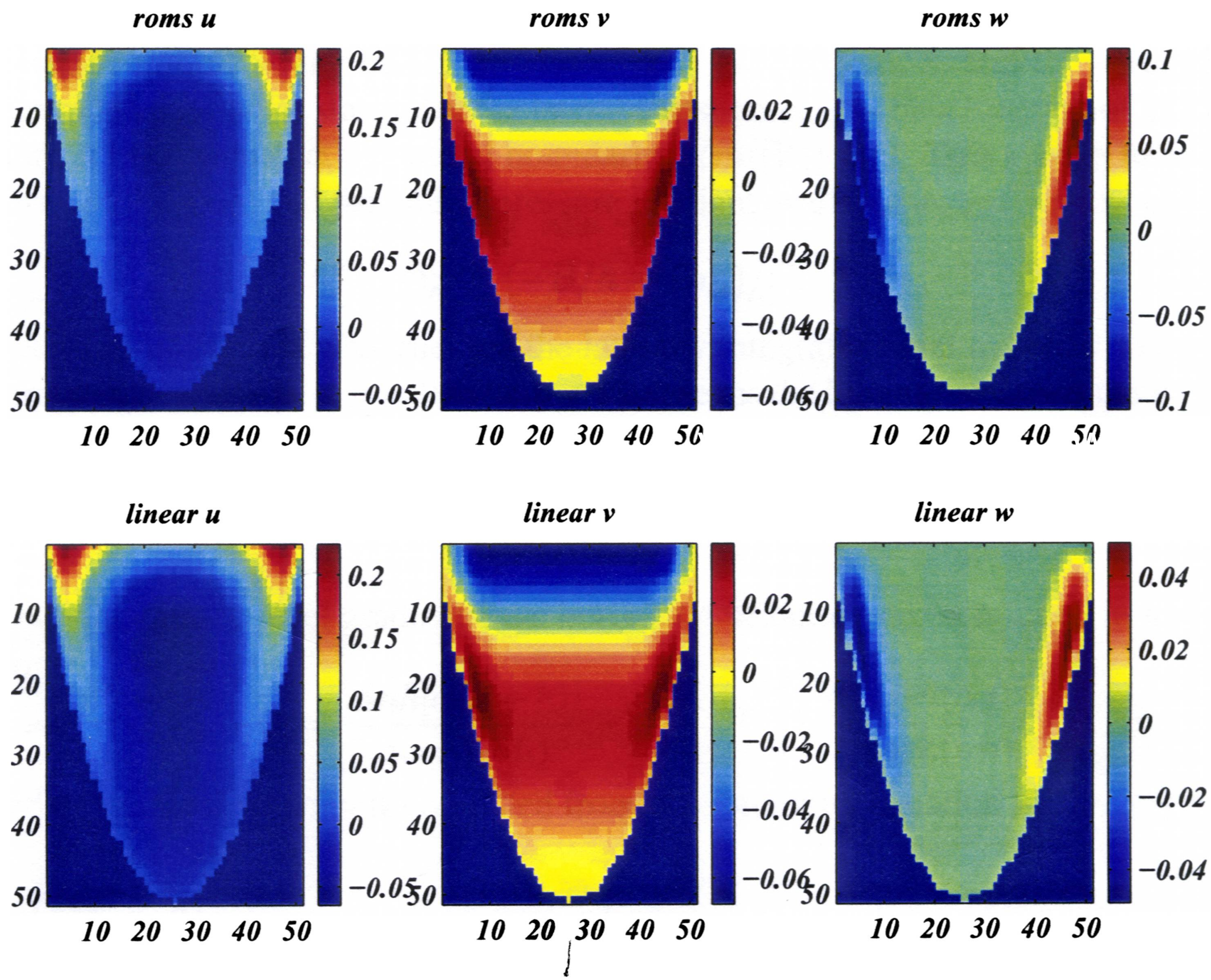
Analytic

FVCOM



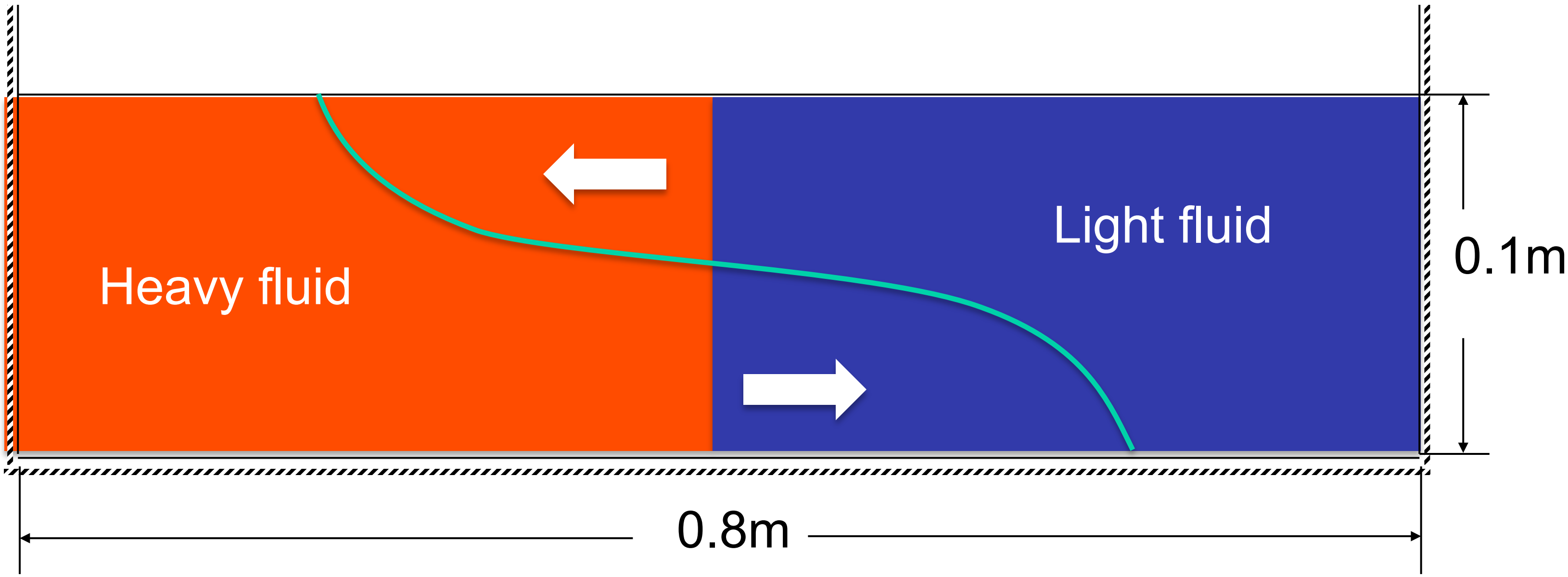
ROMs

Analytic



Be aware that ROMs underestimates *u* and overestimates *w* (color bar scales are different for analytical and ROMs' solutions). This figure is scanned from Winant's working note.

A non-hydrostatic problem: Lock-exchange flow



Model setup:

- Initially $g' = g\Delta\rho / \rho_0 = 0.01 m/s^2$
- Vertical 100 sigma layers
- Horizontal 400×5 nodes, dx = 0.002(m)
- no bottom friction and viscosity/diffusivity

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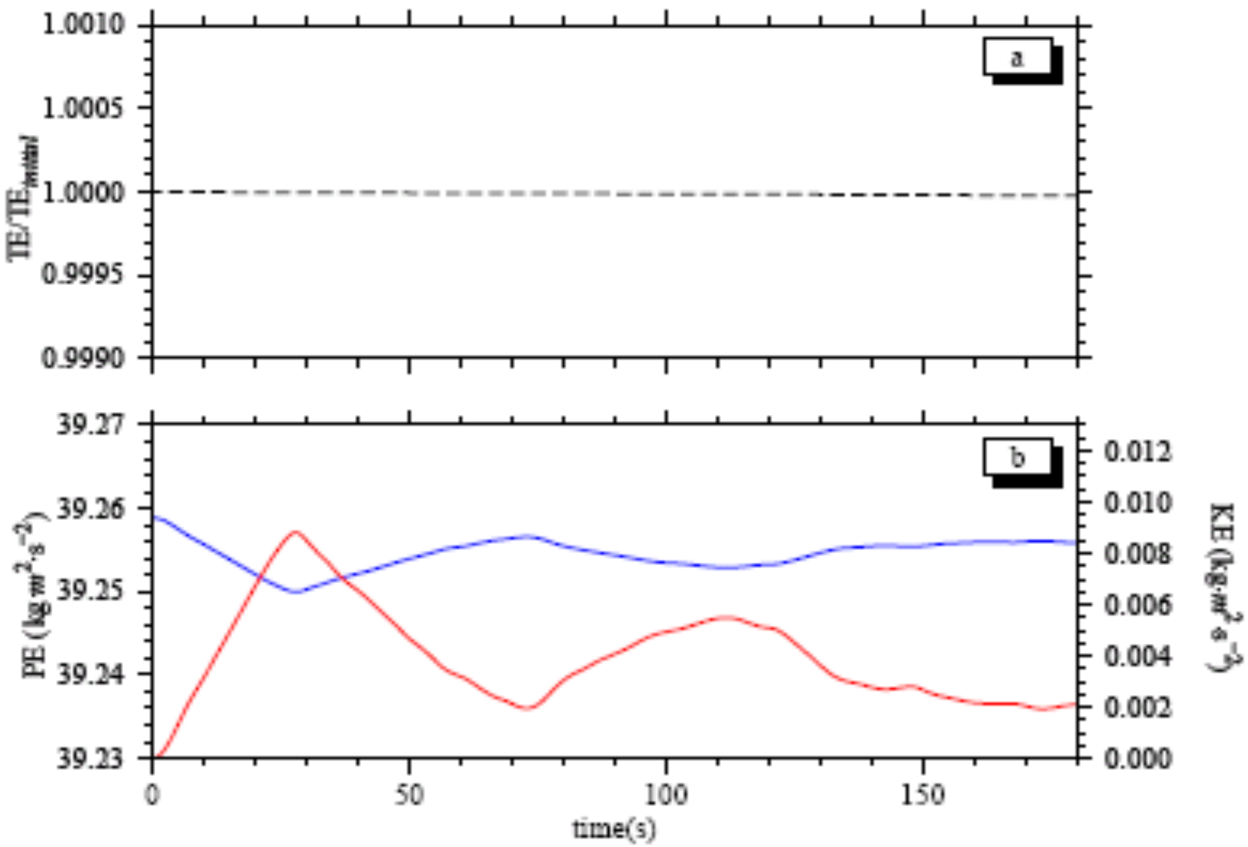
Is the total energy conservative?

Potential Energy (P_E) $= \int_{-L/2}^{L/2}\int_{0}^{H} \rho g z \, dx \, dz$

Kinetic Energy (K_E)$= \int_{-L/2}^{L/2}\int_{0}^{H} \frac{1}{2}\rho V(u^2+v^2)\, dx\, dz$

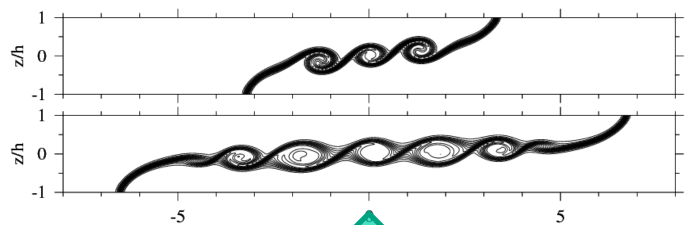
Under an inviscid condition, the total energy is conservative at an order of 10^{-4}.



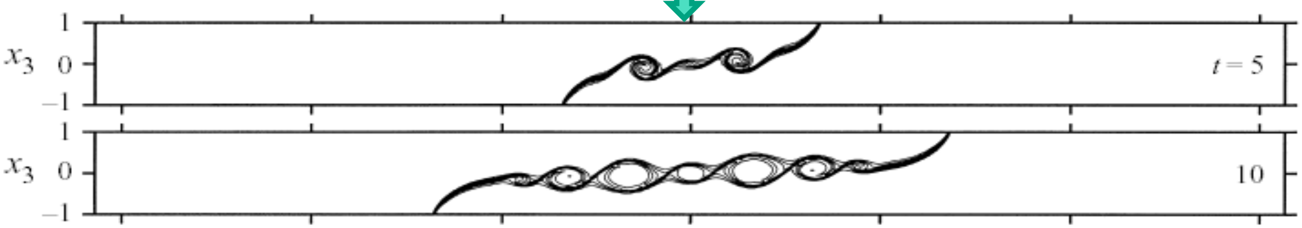
Blue line: PE

Red line: KE

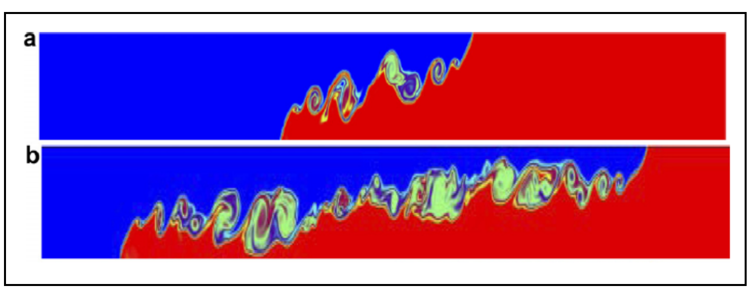
Comparison of FVCOM-NH with a high-order direct numerical simulation (DNS) method, with constant horizontal and vertical eddy viscosity and tracer diffusivity 1×10^{-6} m^2/s:



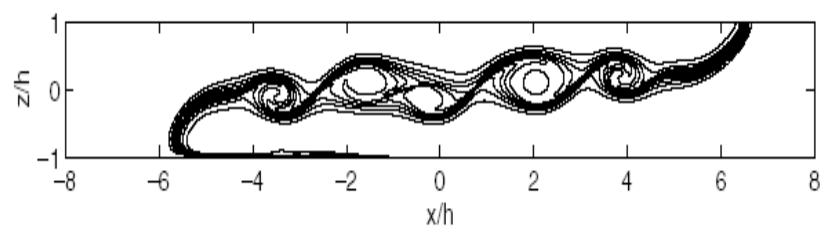
FVCOM-NH



(DNS results from Härtel et al., 2000)

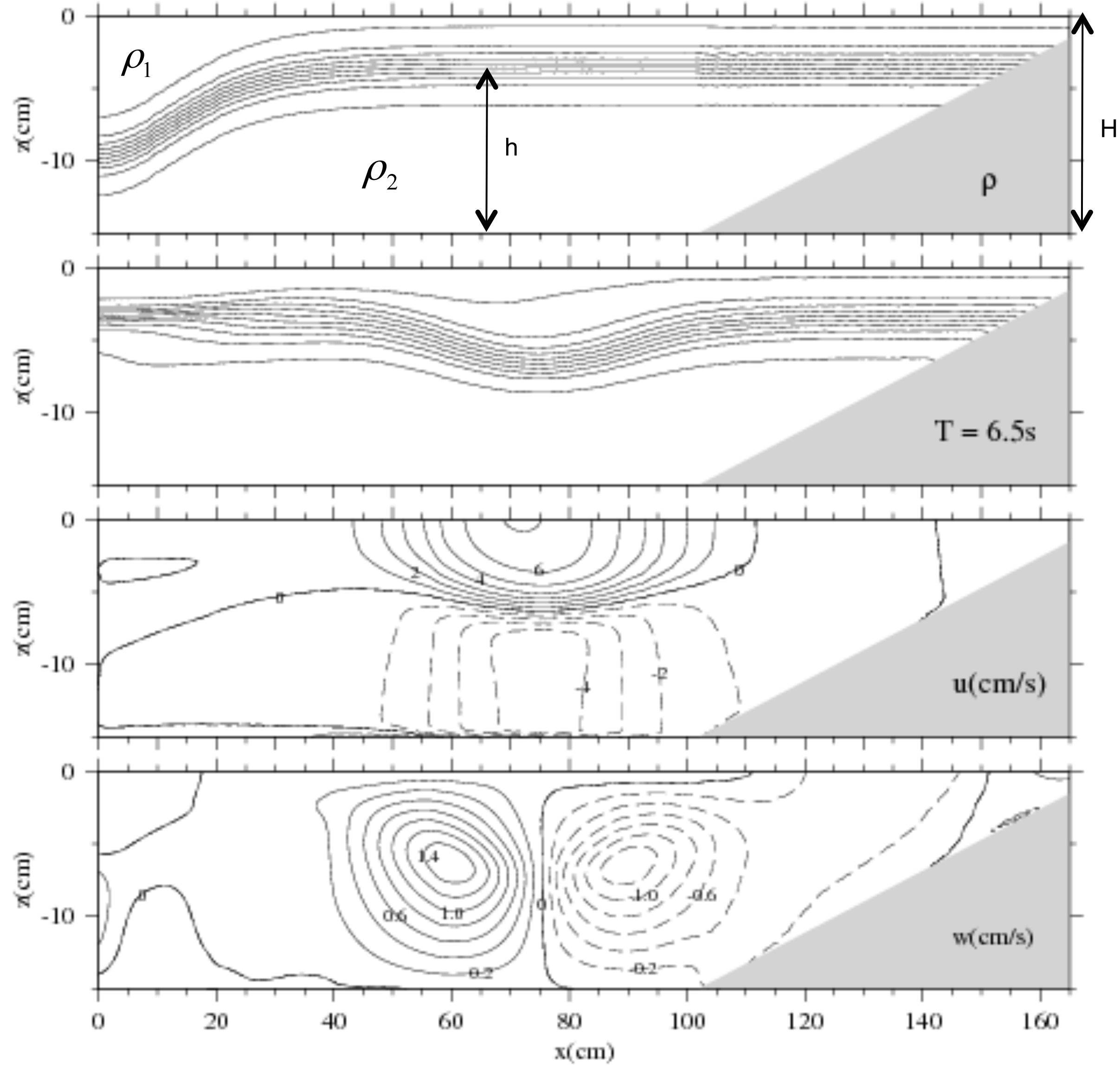


ROMS's results presented by Kanarska et al. (2007)

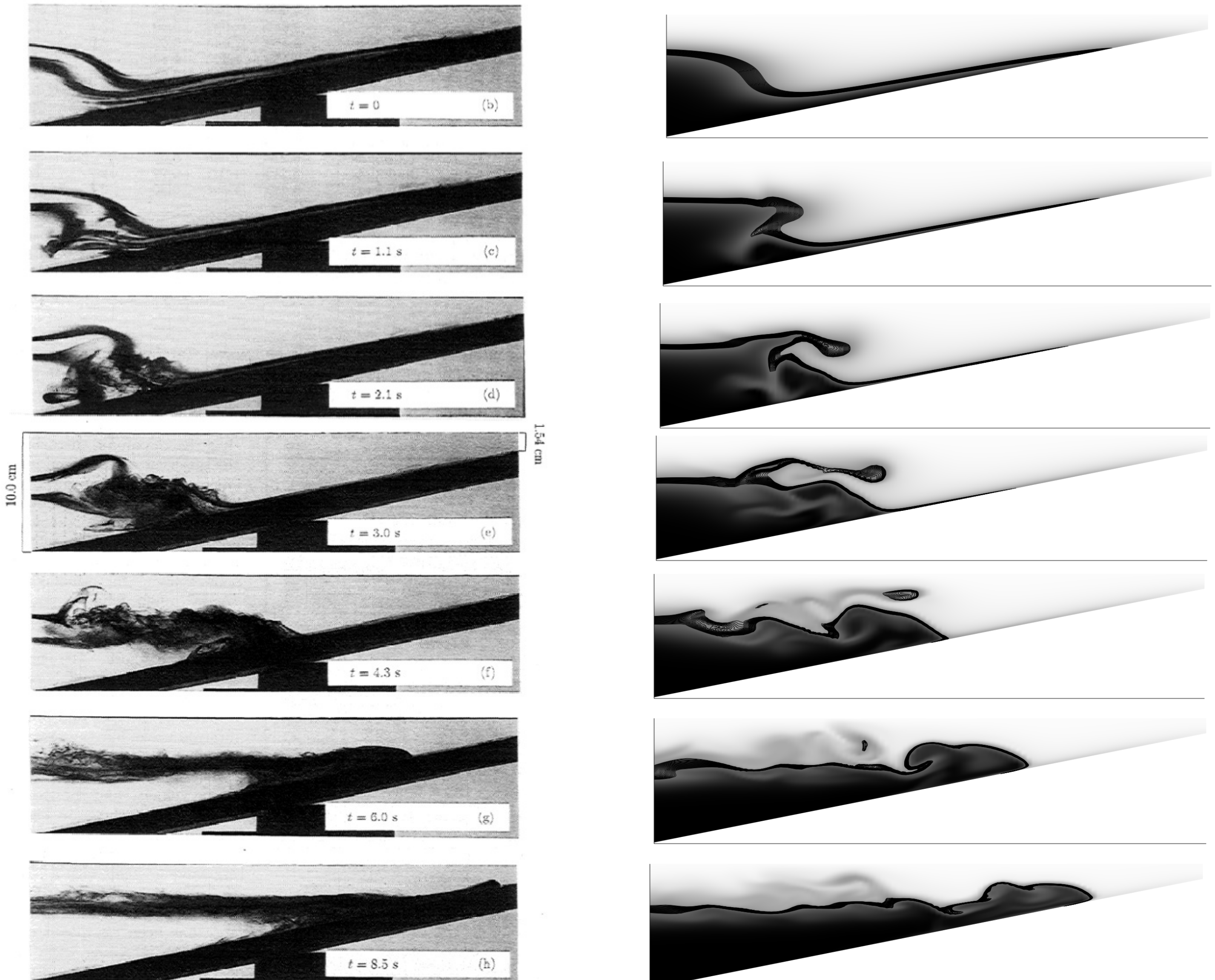


SUNTRAN's results presented by Fringer et al. (2006) with a first-order finite-volume scheme.

Internal solitary waves breaking on a linear varying slope

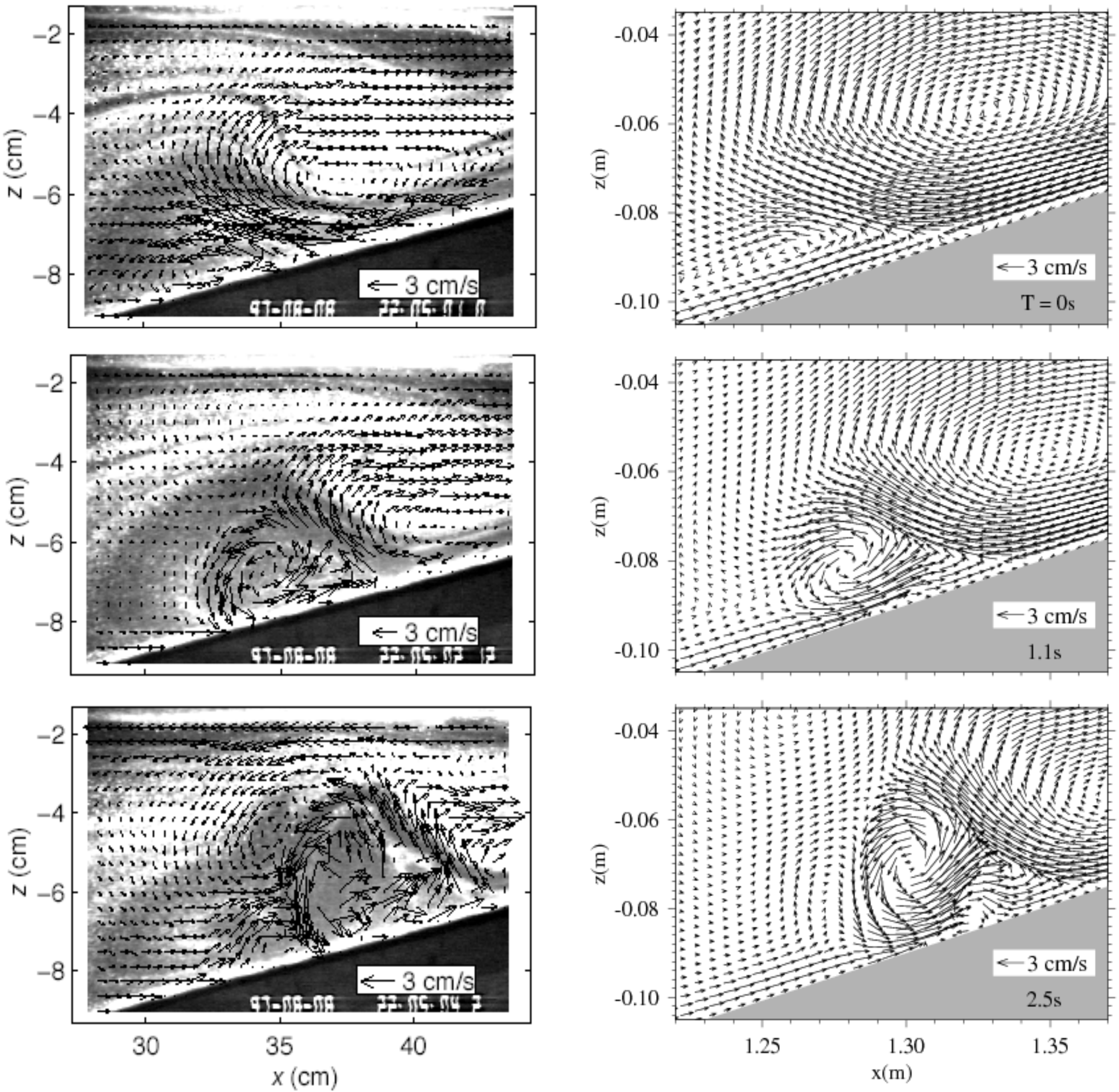


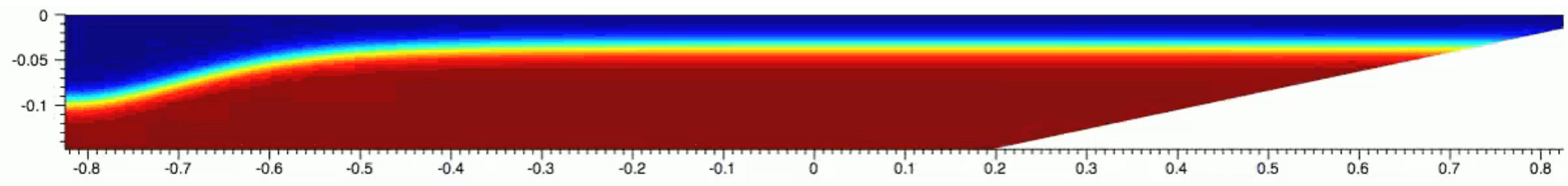
Photography (Michallet and Ivey ,1999) **FVCOM-NH**



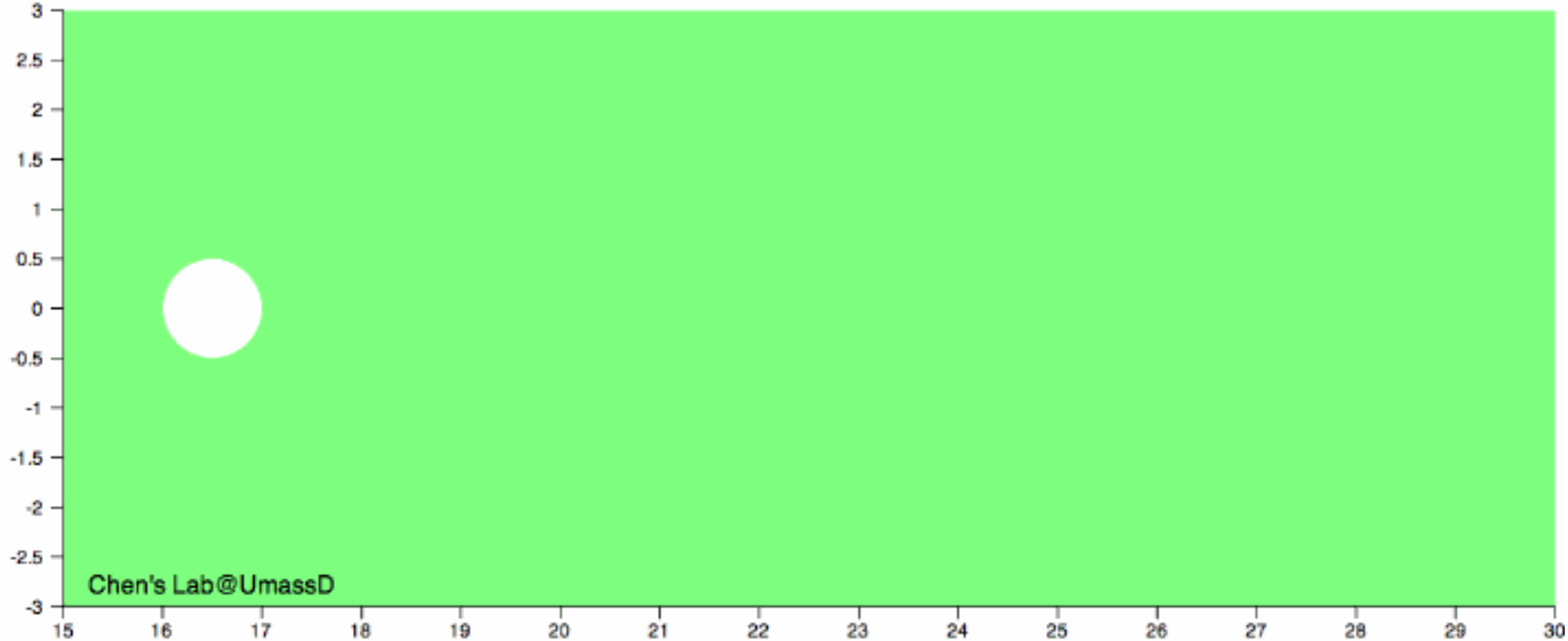
Photography (Michallet and Ivey ,1999)

FVCOM-NH

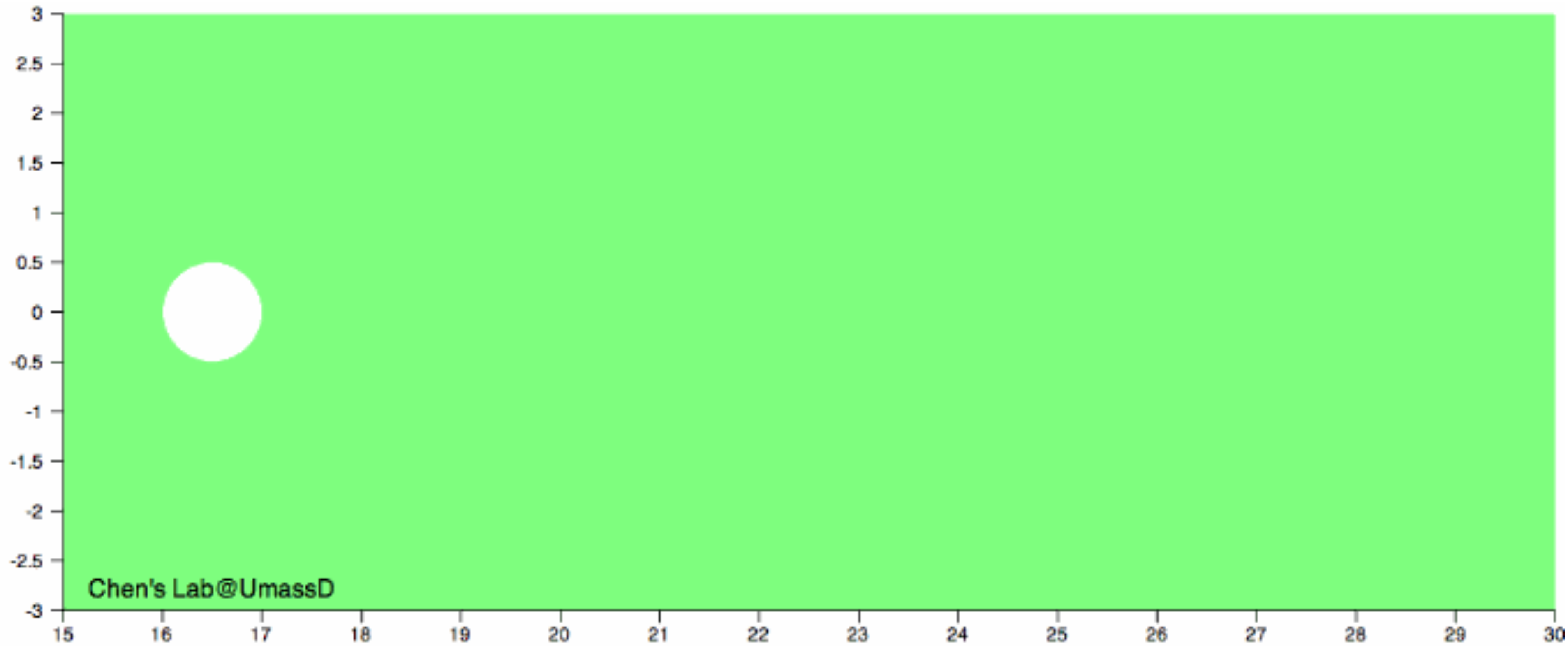


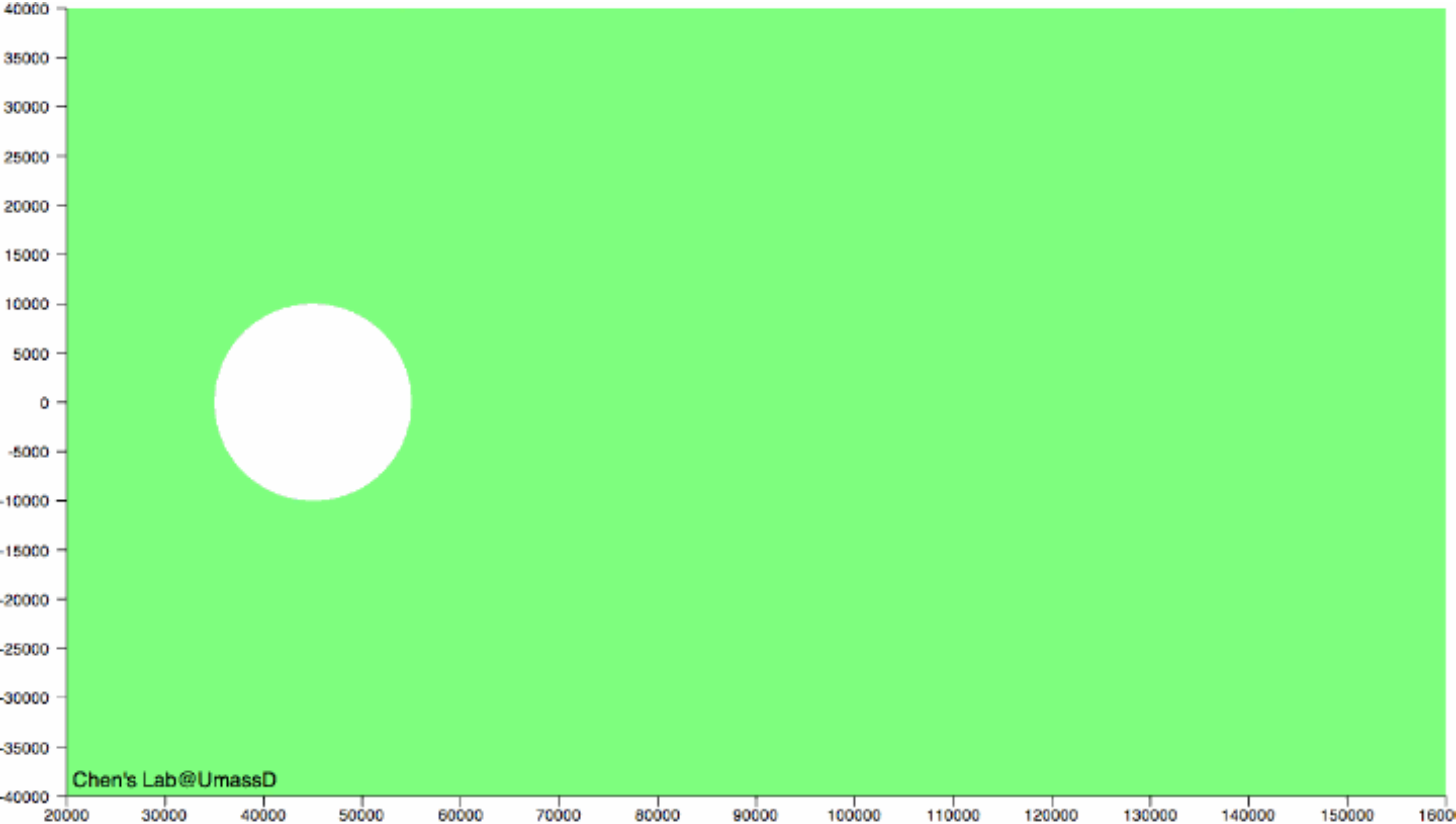

0
-0.05
-0.1
-0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8

Under shear instability condition



Under the vortex condition




40000
35000
30000
25000
20000
15000
10000
5000
0
-5000
-10000
-15000
-20000
-25000
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-35000
-40000
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20000
30000
40000
50000
60000
70000
80000
90000
100000
110000
120000
130000
140000
150000

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Chen, C. H. Huang, R. C. Beardsley, H. Liu, Q. Xu and G. Cowles, 2007. A finite-volume numerical approach for coastal ocean circulation studies: comparisons with finite difference models. *Journal of Geophysical. Research*. 112, C03018, doi:10.1029/2006JC003485.

Huang, H, C. Chen, G. W. Cowles, C. D. Winant, R. C. Beardsley, K. S. Hedstrom and D. B Haidvogel, 2008. FVCOM validation experiments: comparisons with ROMS for three idealized barotropic test problems. J. Geophys. Res., 113, C07042, doi: 10.1029/2007JC004557.

Lai, Z., C. Chen, G. Cowles, and R. C. Beardsley, 2010. A Non-Hydrostatic Version of FVCOM, Part I: Validation Experiments, J. Geophys. Res.-Oceans, 115, doi:10.1029/2009JC005525.