Room-temperature operation of a nanoelectromechanical resonator embedded in a phase-locked loop

T. Kouh, O. Basarir, and K. L. Ekinci

Aerospace and Mechanical Engineering Department, Boston University, Boston, Massachusetts 02215

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We describe the operation of a phase-locked loop (PLL) that tracks the high-frequency electromechanical resonance of a nanoscale beam resonator. The fundamental in-plane flexural resonance of the beam resonator embedded in the PLL is actuated electrostatically and detected optically. PLL operation is demonstrated by locking stably to the resonance frequency of the beam, and by tracking this resonance with high fidelity as the beam is mass loaded. Our analysis reproduces the observed locking behavior. Feedback control schemes for nanoelectromechanical resonators may offer prospects for miniature timekeeping devices and ultrasensitive sensors.

Microelectromechanical systems (MEMS) are rapidly being miniaturized following the trend in commercial transistor electronics. The emerging nanoelectromechanical systems (NEMS) possess high resonance frequencies, high quality ($Q$) factors and minuscule active masses, and present unique technological opportunities—especially in sensing and timekeeping applications. In resonant mass sensing, for instance, a feedback circuit tracks the resonance frequency of the sensor element and thus provides a frequency shift that is directly proportional to the inertial mass accreted upon the sensor. In this realm, unprecedented NEMS mass sensitivities could ultimately allow weighing individual molecules. In timekeeping, one usually references the resonance frequency of a high $Q$ electromechanical resonator, such as a quartz crystal, in an oscillator circuit. Here, NEMS with on-chip control electronics could obviate the need for externally packaged frequency reference elements. To achieve the full potential of NEMS in both applications, it is necessary to develop low noise frequency control schemes for NEMS operating at room temperature.

In this letter, we describe the room-temperature operation of a NEMS resonator embedded in a phase-locked loop (PLL). The PLL approach achieves the same end result as a self-driven resonant system without the need for thermomechanically limited displacement detection. The work presented here has several other interesting facets in addition to the PLL design, operation, and analysis. First, the NEMS resonator employed has a unique geometry. It is fabricated on a membrane and is accessible from both the top and the bottom sides: by an optical probe from the top, and by a molecular beam from the bottom. Second, detection of in-plane NEMS motion is demonstrated using simple optical reflection. Most optical displacement measurements on NEMS up to date have been sensitive to out-of-plane NEMS motion.

The NEMS resonators used in these experiments were silicon nitride doubly clamped beams. A representative structure is shown in the upper inset of Fig. 1. The electrically isolated side gate is for in-plane capacitive actuation. The devices were fabricated on a silicon wafer with 125-nm-thick silicon nitride coatings on both sides. The first step in our device fabrication was the preparation of a silicon nitride membrane. The beam-gate structure was patterned on the membrane using electron beam lithography, thermal mask deposition, lift-off, and reactive ion etching.

The silicon nitride doubly clamped beam resonator was operated at room temperature inside an ultrahigh vacuum (UHV) chamber. This chamber allowed optical device probing through a quartz viewport. In addition, through the backside opening on the chip, we could thermally evaporate Au atoms upon the beam resonator to change its effective mass. In the experiments, the motion of the beam was actuated capacitively through the side gate. To detect the resulting in-plane motion, we implemented a simple optical reflection measurement. For this, a He–Ne laser was focused upon the device into an optical spot size of 1.2 μm and the intensity of the light reflecting from the device was monitored by a photodetector. In the measurements, we found out that if the optical spot was carefully positioned on the nanomechanical beam, the motion of the highly reflective metatized beam modulated the reflected optical power, and hence, the photodetector current $I$. Figure 1 shows the in-plane fundamental flexural resonance of a 14-μm × 200-μm × 125-nm ($l \times w \times t$) doubly clamped beam with a 100-nm-thick aluminum metallization layer, as measured by the optical reflection technique. The lower inset in the figure displays the displacement responsivity $\partial I / \partial x$ of the technique as a function of the optical spot position on the structure. Here, the optical spot was scanned across the structure along the $x$ axis. In the

![Figure 1](http://apl.aip.org/apl/fig1.png)

**FIG. 1.** (Color online) In-plane fundamental flexural resonance of a beam (upper inset) with $l=14 \mu m$, $w=200 \mu m$, and $t=125 \mu m$ at $\omega_0/2\pi = 7.9440$ MHz for increasing drive amplitudes as measured by the optical reflection technique. The gap between the beam and the side gate is 130 nm. The measured photodiode current $I$ was converted to a beam center displacement $x$ by using the data in the lower inset. Here, $\partial I / \partial x$ was obtained by scanning the optical spot along the structure. The center of the beam is located at $x=0$ and the gate at $x=230$ nm as shown by the shading.

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Author to whom correspondence should be addressed; electronic mail: ekinci@bu.edu

resonance measurements, the optical spot was positioned at the point of maximum $d\omega/2dx$ on the structure, which corresponded roughly to the nanomechanical beam center. Subsequently, the beam was actuated and the photodetector current was monitored. Using this reflection technique, we measured the in-plane fundamental flexural resonances of five beams with dimensions $10 \mu m < l < 20 \mu m$, $100 \text{nm} < w < 500 \text{nm}$, and $t=125 \text{nm}$. The resonance frequencies observed were in the $7 \text{MHz} < \omega_0/2\pi < 20 \text{MHz}$ range.

The block diagram of the PLL is depicted in Fig. 2(a). As the voltage controlled oscillator (VCO), we used a constant amplitude radio frequency (rf) source where the carrier frequency $\omega_c$ could be frequency modulated by a quasistatic control voltage $u(t)$. During operation, $\omega_c$ was set by the operator; this resulted in a NEMS drive signal at the frequency $\omega_c = \omega_0 + K_{\text{VCO}} m$. Here, $K_{\text{VCO}}$ is the VCO gain in units of $s^{-1} V^{-1}$. The optically transduced signal from the NEMS was then mixed with the VCO output and low-pass filtered—to be fed back into the VCO as the control voltage. The low-pass filter bandwidth $\Delta f$ determined the effective bandwidth of the PLL.

Figure 2(b) demonstrates the operation of the PLL. Here, $\omega_c$ was increased by the operator in steps and the frequency of the loop $\omega_{\text{PLL}}$ was monitored by a frequency counter. For small amplifier gains, no locking was observed (see the data in gray). For larger gains, the PLL locked stably to the resonance frequency at $\omega_0/2\pi = 7.9440 \text{MHz}$ of the beam resonator. The lock range in Fig. 2(b) is $7.9245 \text{MHz} < \omega_0/2\pi < 7.9501 \text{MHz}$. Note that the lock frequency $\omega_{\text{PLL}}$ is not exactly equal to $\omega_0$; moreover, it is slightly different for each $\omega_c$. This suggests that there remains a finite but small phase error in the loop.

Within the lock range, the loop stability was monitored as a function of time as shown in the inset of Fig. 2(b). The rms frequency fluctuations $\delta\bar{\omega}$ was $\delta\bar{\omega}/2\pi = 51 \text{Hz}$ for $\Delta f = 100 \text{Hz}$. The observed noise was dominated by the white noise in the amplifier with current noise density $S_f(\omega)$. Using the experimentally determined values $S_f(\omega) = 50 \text{pA}^2/\text{Hz}$. 

In Figure 2(c), we further demonstrate that the PLL tracks the resonance frequency $\omega_0$ of the NEMS resonator during mass loading of the resonator. In this set of measurements, mass loading was accomplished by thermally evaporating Au atoms upon the resonator through the back side of the chip [see the inset of Fig. 2(c)]. The Au flux was measured by a quartz crystal. The technique used here is similar to that described in Ref. 2. Fig. 2(c) shows the frequency shift as a function of the added mass; the slope of the plot gives the mass responsivity, $\delta\omega_0/(2\pi) \Delta m = -1.95 \text{Hz/ag}$ (ag $=10^{-18} \text{g}$). This is close to the value $-2.15 \text{Hz/ag}$ estimated from the dimensions of the resonator. This corresponds to a maximum detectable mass of $-25 \text{ag}$, given the above $\delta\bar{\omega}$.

We now turn to the analysis of the PLL operation. A two-port nanomechanical beam resonator—actuated capacitively and transduced optically—is at the center of the PLL. The resonator’s displacement $x(t)$ in response to a drive around $\omega = \omega_0$ can be determined from the one-dimensional damped oscillator equation as

$$m\ddot{x}(t) + m\frac{\omega_0}{Q}\dot{x}(t) + m\omega_0^2x(t) = K_{\text{d}}A_d\cos[\theta_d(t)].$$

Here, $m$ is the effective mass and $Q$ is the quality factor for the fundamental flexural mode. $K_{\text{d}}$ is the input transducer responsivity (in units of N/V) and $A_d$ is the constant drive voltage amplitude (in units of V). The phase angle $\theta_d(t)$ of the VCO output signal is defined as

$$\theta_d(t) = \omega_0 t + K_{\text{VCO}}\int_0^tu(t')dt'.$$

Note that $\theta_d(t) = \omega_d(t)$. The optical transducer-amplifier cascade with responsivity $K_{\text{out}}$ (in units of V/m) converts $x(t)$ into an output voltage. This output is mixed with the VCO drive signal and low-pass filtered. This results in

$$u(t) = \omega_{\text{LPF}}K_{\text{out}}K_{\text{VCO}}x(t)A_d\cos[\theta_d(t) - u(t)],$$

where $\omega_{\text{LPF}}$ is the cut-off frequency of the low-pass filter (LPF) and $K_m$ is the mixer gain (in units of V$^{-1}$). The output voltage $u(t)$ of the LPF is fed back into the VCO as the control voltage. Table I. Experimentally determined parameters used in the simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$m$</td>
<td>$1.4 \times 10^{-15} \text{kg}$</td>
</tr>
<tr>
<td>$\omega_0/2\pi$</td>
<td>$7.944 \times 10^6 \text{Hz}$</td>
</tr>
<tr>
<td>$Q$</td>
<td>1300</td>
</tr>
<tr>
<td>$A_d$</td>
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<td>$K_{\text{in}}$</td>
<td>$1 \times 10^{-10} \text{N/V}$</td>
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<td>$K_{\text{out}}$</td>
<td>$1.9 \times 10^9 \text{V/m}$</td>
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<tr>
<td>$K_m$</td>
<td>0.1 V$^{-1}$</td>
</tr>
<tr>
<td>$K_{\text{VCO}}$</td>
<td>$6.27 \times 10^3 \text{s}^{-1} \text{V}^{-1}$</td>
</tr>
<tr>
<td>$\omega_{\text{LPF}}/2\pi$</td>
<td>$100 \text{Hz}$</td>
</tr>
</tbody>
</table>

The inset shows the fluctuations in $\omega_{\text{PLL}}$ while the PLL is locked to $\omega_0$. (c) Resonance frequency shift of the NEMS resonator as thermally evaporated gold atoms are absorbed on its surface. The mass is determined from separate quartz crystal and NEMS surface area measurements. The inset shows the experimental geometry.
The lock range $\Delta \omega_L$ as the frequency interval where $\omega_{PLL} = \omega_o$, regardless of the value of $\omega_o$. In the inset of Fig. 3(a), we display $\Delta \omega_L$ as a function of $K$. Larger $K$ appears to favor a broader lock range.

Figure 3(b) shows $\omega_{PLL}$ as a function of $K$. In this set of analyses, $\omega_o$ is set at a value close to $\omega_c$ and $\omega_{PLL}$ is determined as $K$ is increased. At low $K$, $\omega_{PLL}$ stays near $\omega_c$. As $K$ is increased, $\omega_{PLL}$ is observed to flow towards $\omega_o$. We note that for finite $K$ there remains a small error between $\omega_{PLL}$ and $\omega_o$ (also observed experimentally). As $K \rightarrow \infty$, the error approaches zero.

The analysis presented here does not capture all aspects of PLL operation. First, the experimentally observed lock range is not symmetric around $\omega_c$ [cf. Fig. 2(b) and Fig. 3(a)]. Second, we do not observe hysteresis in $\omega_{PLL}$ depending upon the sweep direction of $\omega_o$; such behavior is common in nonlinear systems. We believe that, in going from the dynamic description, i.e., Eqs. (1)–(3), to the steady state approximation, we lose some of the interesting nonlinearities in the system. In an effort to reveal this complex nonlinear behavior, we investigated the numerical time domain solutions of Eqs. (1)–(3). In this analysis (not shown), the expected frequency capture behavior emerged after a period of several milliseconds. However, after long times, we encountered growing truncation errors due to the stiff nature of the system. Implicit methods with very small time steps may result in accurate solutions capturing the interesting physics due to the nonlinearities.

We have described the design, operation, and analysis of a PLL to track the fundamental resonance frequency of a NEMS resonator at room temperature. Such NEMS feedback control schemes may find use in timekeeping and sensor applications.

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10 Nonlinear behavior was observed at a displacement amplitude of $x = 20$ nm.
11 In this approximation, we ignore the small amplitude oscillations sometimes encountered in steady state.