Structures

- **strength**: resist permanent deformation → plastic yielding; brittle fracture } material
- **Stiffness**: resist deformation/deflection } (linear) geometry
- **stability**: ability to maintain an equilibrium configuration } (nonlinear) geometry

Approach:
1. Method of Equilibrium
   - i. equilibrium equations: $\Sigma F_i = 0$
   - ii. kinematic equations: deformations
   - iii. constitutive equations: $\sigma = E\varepsilon$
2. Energy Methods
   - i. strain energy

Energy

- shape of a deformed object is the shape that minimizes the total potential energy.
  - → ① Lagrangian Mechanics: $\mathcal{L} = T - V$ (kinetic, potential)
  - ② Hamiltonian Mechanics: $\mathcal{H} = T + V$

Total Potential Energy

$$V = \mathcal{U} + P$$

$\mathcal{U}$: Strain energy in the material/structure

P: Potentials from external loads

Strain Energy

$$\mathcal{U} = \int_V \sigma\varepsilon \, dV = \int_V \sigma_{ij}\,\varepsilon_{ij}\, dV = \int_V \sigma^{ij}\,\varepsilon_{ij}\, dV$$

$[\sigma] = \frac{N}{m^2} = Pa$

$\left[\int_V dV\right] = m^3$

$\left[\int \sigma\varepsilon \, dV\right] = \frac{N}{m^2} m^3 = N\cdot m$

Potential of External Loads

$$P = -W$$

$$dW = \vec{f}\cdot d\vec{u} = f_i u_i \longrightarrow W = \int_A^B dW = \int_A^B f_i u_i$$

- forces distributed over volume: $W = \int_V \vec{f}\cdot\vec{u}\, dV$
- forces distributed over surface: $W = \int_S \vec{t}\cdot\vec{u}\, dS$
- concentrated forces: $W = \sum_{i=1}^{N} \vec{f}_i\cdot\vec{u}(x_i)$