ME712: Applied Mathematics in Mechanics

Problem Set #1

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1. The luminosity of certain giant and supergiant stars varies in a periodic manner. It is hypothesized that the period p depends upon the star's average radius r, its mass m, and the gravitational constant G.

   - Newton's law of gravitation asserts that the attractive force between two bodies is proportional to the product of their masses divided by the square of the distance between them, that is $F = Gm_1m_2/d^2$, where G is the gravitational constant. From this, determine the fundamental dimensions of G.
   - Use dimensional analysis to determine the functional dependence of p on m, r, and G.
   - Arthur Eddington used the theory for thermodynamic heat engines to show that $p = (3\pi/2\gamma G\rho)^{1/2}$, where γ is the ratio of specific heats of the stellar materials, and ρ is the star's average density, how does this differ from your result?

2. When a drop of liquid hits a wetted surface a crown formation appears. It has been found that the number of points N on the crown depends on the speed U at which the drop hits the surface, the radius r and density ρ of the drop, and the surface tension σ of the liquid making up the drop.

   - Use dimensional reduction to determine the functional dependence of N on U, r, ρ, and σ. Express your answer in terms of the Weber number, $We = \rho U^2 r/\sigma$.
   - The value N has been measured as a function of the initial height h of the drop. Express your answer in terms of h by writing U in terms of h and g. Assume the drop starts with zero velocity.
   - Experimental results show a linear dependence of N on h with a slope of about 1/4. Use this result to find the unknown function in the first part of the problem. If we take $r = 3.6$ mm, $\rho = 1.1014$ gm/cm^3, and $\sigma = 50.5$ dyn/cm, what must the initial height of the drop be to produce 80 points?

3. A ball is dropped from a height h_0 and it rebounds to a height h.

   - Identify the parameters you suspect will dictate the rebound height, and find a dimensionally reduced form of h.
   - Assume h depends linearly on h_0 (with $h = 0$ if $h_0 = 0$). How does this reduce your formula for h?

4. The frequency ω of waves on a deep ocean is found to depend on the wavelength λ of the wave, the surface tension σ and density ρ of the water, and gravity.

   - Use dimensional analysis to determine the functional dependence of ω on λ, σ, ρ, and g.
   - In fluid dynamics, it is shown that $\omega = \left(gk + \sigma k^3/\rho\right)^{1/2}$, where $k = 2\pi/\lambda$ is the wavenumber. How does this differ from your result?

5. A ball, when released underwater, will rise towards the surface with velocity v. This velocity depends on the density ρ_b and radius R of the ball, on gravity g, and on the density ρ_f and kinematic viscosity ν of the fluid.

   - Find a dimensionally reduced form for v.
   - In fluid dynamics, using Stoke's Law, it is found that $v = 2gR^2(\rho_b - \rho_f)/9\nu\rho_f$, how does this differ from your result?

6. From bartenders to infamous "Ice King" of Boston, Frederic Tudor, it has been long known that small ice cubes melt faster than large ones, meaning that small ice cubes will cool your drink faster than large ones. The common explanation of this effect is that small cubes has a larger surface area for the same total volume as large cubes. To test this hypothesis, we would expect the time for a drink to cool down will be proportional to L^2. Using dimensional analysis, and recognizing that the relevant parameters are length L, time T, temperature Θ, thermal conductivity $[\kappa] = LT^{-3}\Theta^{-1}M$, and volumetric heat capacity $[s] = L^{-1}T^{-2}\Theta^{-1}M$, show that cooling time is proportional to L^2.