EXISTENCE AND STABILITY OF RELATIVE EQUILIBRIA IN THE N-BODY PROBLEM

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ABSTRACT

We study special periodic solutions of the Newtonian n-body problem known as *relative equilibria*. These solutions consist of configurations of bodies rotating rigidly about their centers of mass. It is generally believed that the set of relative equilibria equivalence classes for a given set of positive masses is finite. However, this result has only been proven for $n = 3$ and for 4 equal masses, and it remains a difficult, open question for $n \geq 5$ and for 4 unequal masses. By finding a continuum of relative equilibria in the five-body problem which (unfortunately) includes one negative mass, we demonstrate the necessity of considering only positive masses in this conjecture.

To investigate the stability of a relative equilibrium we linearize the differential equation about the equilibrium and analyze the eigenvalues of the associated linear system. A necessary condition on the initial positions of the bodies is derived which must be satisfied in order for the corresponding periodic solution to be spectrally stable. This condition is examined in the equal mass case, and it is shown that any relative equilibrium of n equal masses is not spectrally stable, provided $n \geq 24,306$.

We also study the $1 + n$-gon family of relative equilibria, consisting of n equal masses located at the vertices of a regular n-gon with an additional body of mass m at the center, where m is treated as a parameter. We show that when $n \leq 6$, this configuration is linearly unstable. For $n \geq 7$, a value h_n is found such that the configuration is linearly stable if and only if $m > h_n$. This value is shown to increase proportionately to n^3.

Finally, we consider a limiting problem with n "big" masses and p "small" masses, with the small masses being order ϵ. This problem decouples at $\epsilon = 0$. We derive sufficient conditions on the limiting positions of the bodies to insure that the entire family is linearly stable for ϵ sufficiently small. We then apply these conditions to the $1 + n$-gon family showing that it is possible to thicken the ring around the central body.