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Ph.D. Thesis: Geometric Methods for Periodic Orbits in Singularly Perturbed Systems

ABSTRACT

This dissertation concerns singularly-perturbed systems of ordinary differential equations and applications to physical problems with multiple spatial or temporal scales, such as the FitzHugh-Nagumo, Hodgkin-Huxley and Gray-Scott equations, predator-prey systems, and excitable membrane systems.

In Part one, a general modular method based on geometric singular perturbation theory to establish the existence of periodic orbits is developed. This method exploits the geometry of the systems' slow manifolds and their normally hyperbolic structure. It transforms the Poincare map problem into a boundary-value problem in a naturally-augmented system, and the periodic orbit is found as the transverse intersection of invariant manifolds. A modified version of the Exchange Lemma is proven, in which the transversality of the tracked manifold and the local stable manifold of the slow manifold is not required.

Independently, a multiple-pulse Melnikov function is constructed for adiabatic, slowly-varying, Hamiltonian systems. It detects homoclinic orbits with multiple, successive fast excursions. Combined with the above-mentioned method, it shows the existence of a wide class of multiple-pulse periodic orbits in adiabatic systems, such as that modeling resonant sloshing in shallow water.

In Part two, the focus is on the development of geometric methods to study the periodic behaviors, mechanisms of frequency control and the functional role of connectivity in small networks of neurons. Motivated by a subnetwork of the crustacean stomatogastric ganglion, I study a three-cell network formed by an intrinsic oscillator, electrically coupled to a bistable element. Both of them make inhibitory synapses (with different time courses) onto an excitable cell, which in turn inhibits the bistable element.

By using geometric singular perturbation theory, I find circumstances under which the electrically coupled pair can effectively be reduced to an intrinsic oscillator. I also find physiologically plausible conditions under which the only possible periodic behaviors of the full network are N:1 rhythms, in which the electrically coupled pair undergoes N oscillations in each cycle of the excitable element. This is shown by constructing a singular Poincare map.