Dissertation Abstract:
"Wavelet-based Multifractal Spectra Estimation: Statistical Aspects and Applications"

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Multifractal processes have become a successful, widely used modelling tool with applications in areas such as turbulence, Internet traffic and biomedical engineering. Their appeal as a modelling tool stems from the fact that these processes exhibit intricate patterns of locally varying scaling behavior similar to that encountered in real data-sets. A multifractal spectrum (MFS) of a process quantifies the presence of the possibly multiple local scaling behavior from sample paths of these processes. The wavelet transform (WT) has been successfully employed in the estimation of the MFS. In this dissertation we explore statistical properties of estimators of MF spectra base on the discrete WT for the case of fractional Brownian motion (fBm). By developing appropriate tools to study the across-scale covariance structure of functions of wavelet coefficients of fBm, we show the asymptotic normality of new and standard wavelet-based estimators.

Furthermore, since estimators of MFS are based on a log-linear relationship between scale-dependent quantities and scales, the standard estimators are obtained using a regression procedure with potentially correlated error structure. In order to obtain information on the regression error structure, we employ classical arguments based on Hermite polynomials and find bounds for such covariance structure. These bounds correspond to an approximate AR(1) covariance structure. Using information on this asymptotic error structure, we propose an estimator that uses the bounds as a proxy for the covariance matrix in a generalized least squares procedure. We observe, via a simulation study, a sizable increase in efficiency of this new estimator when compared to the standard estimators which are based on ordinary least squares, and weighted least squares.

As an application to the field of biomedical engineering, we study the difference in MFS for center-of-pressure traces of healthy and Parkinson patients. We develop interpretable summaries from the MFS in a multivariate repeated measurement design, which leads to significant discrimination between healthy and Parkinson patients in terms of these MFS-based new measures. Furthermore, these measures correlate with standard clinical measures of disease severity, suggesting that the MFS-based procedure may be of use in a clinical setting.