

THE INITIAL SEGMENT OF THE FOURIER SERIES OF A  
MODULAR FORM WITH CONSTANT TERM

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Abstract

C. L. Siegel showed that the dimension of the vector space associated to a level one modular form with non-zero constant term is an upper bound on the exponent of the second non-zero Fourier coefficient of the form. C. L. Mallows, A. M. Odlyzko, and N. J. A. Sloane gave sharper bounds for level one theta functions. In the present work, I investigate upper bounds on this exponent for level two modular forms with non-zero constant term. I show that one plus the dimension of the associated vector space is an upper bound on the exponent of the second non-zero Fourier coefficient of a level two modular form with non-zero constant term and weight  $2x - 4$  or  $2x - 6$ . I conjecture that this behavior holds for all positive even weights and that the constant terms of the Fourier expansions of certain meromorphic modular forms satisfy congruences related to their order at poles. I adduce evidence for the second conjecture and show that it implies the first conjecture for positive weights  $h \equiv 2 \pmod{4}$ .