

RANK-R KP SOLUTIONS WITH SINGULAR RATIONAL SPECTRAL  
CURVES

ALEX KASMAN

Boston University, Graduate School, 1995

Major Professor: Emma Previato, Associate Professor of Mathematics

Abstract

The solutions to the KP hierarchy with rank one bundles of eigenfunctions over their spectral curves are now well understood. However, the solutions having bundles of eigenfunctions of rank  $r > 1$  remain mysterious, despite their significance to theoretical physics and algebraic-geometry. This thesis develops a unified theory for studying KP solutions of arbitrary rank in the case that the spectral curve is a singular rational curve. It is shown that a set of finite dimensional dual grassmannians is sufficient to describe all solutions to the KP hierarchy with singular rational spectral curves. This construction is applied separately to the case of rank one and rank  $r > 1$ .

In the rank one case, the dual construction is applied to the study of the well known soliton and rational solutions in the subgrassmannian  $Gr_1$ . The action of the KP flows on the dual parameters is determined explicitly, allowing for the disjoint decomposition of the finite dimensional grassmannians into KP orbits. It is shown that a tau function corresponds to a rank one bispectral wave function if and only if it is of bounded degree in all time variables. The action of the bispectral involution on dual parameters is computed. The main result of this chapter is the proof that the bispectral involution acting on Calogero-Moser particle systems is a linearizing map, essentially the same as the standard linearizing map, sigma.

The dual construction is extended to the case of a solution of arbitrary rank with rational spectral curve. The tau function theory of the Previato-Wilson grassmannian is developed. This construction is used to determine the relationship between the “fake” rank  $r$  solutions with trivial vacuum to their “true” rank one form. The dual parameters corresponding to rational solutions of Veselov type with constant vacuum and to Grunbaum’s rank two solution of Airy type are determined. Explicit rational and non-rational examples are constructed and examined in the rank two case with constant and Airy vacuum operators.