Consider the sentence in (1), which contains a 3-place predicate \textit{gave} and three quantificational arguments. This sentence is multiply ambiguous.

(1) A professor gave every student a book

\begin{center}
\begin{tikzpicture}
    \node (S) {S};
    \node (VP) [below of=S] {VP};
    \node (DP) [left of=VP] {DP
      \node (a_professor) [below of=DP] {a professor};
    \node (V) [left of=VP] {V
      \node (gave) [below of=V] {gave};
      \node (DP) [right of=gave] {DP
        \node (every_student) [below of=DP] {every student};
      \node (DP) [right of=every_student] {DP
        \node (a_book) [below of=DP] {a book};
    \end{tikzpicture}
\end{center}

The syntactic structure above is a simplified representation of the structure of (1). It will do for our purposes. This sentence has six possible translations into Predicate Logic. By far the easiest way to figure out the translations is to adopt the Quantifier Raising (QR) analysis proposed in class, in which quantificational noun phrases move to a position above the sentence node S. This movement is covert; in other words, it is not reflected in the phonology of the sentence. There are four steps involved.

Step 1. Translate the verb \textit{give} as a 3-place predicate \textit{GIVE}(x, y, z). This is an open formula that stands for \textit{x gave y z}. In other words, \textit{he give him it}.

Step 2. There are three quantified expressions in this sentence, namely \textit{a professor}, \textit{every student} and \textit{a book}. Each quantified expression translates into a formula with three parts: a quantifier, a restrictor, and a nuclear scope.

\begin{equation}
\exists x \quad \left[ \text{PROF}(x) \quad \land \quad \_ \_ \_ \right]
\end{equation}

The \textbf{Restrictor} limits the range of \textit{x} and the \textbf{Nuclear Scope} describes a property that \textit{x} satisfies. In other words, there is an entity \textit{x}, \textit{x} is a professor and \_\_\_ (where the blank is a property of \textit{x}). A property of \textit{x} is simply an open formula that has a free occurrence of \textit{x} in it.

\begin{center}
\begin{tikzpicture}
    \node (S) {S};
    \node (VP) [below of=S] {VP
      \node (gave) [below of=VP] {gave
        \node (DP) [left of=gave] {DP
          \node (a_professor) [below of=DP] {a professor
            \node (exists_professor) [below of=a_professor] {\exists x \quad \left[ \text{PROF}(x) \quad \land \quad \_ \_ \_ \right]}
        \node (DP) [right of=gave] {DP
          \node (every_student) [below of=DP] {every students
            \node (forall_students) [below of=every_student] {\forall y \quad \left[ \text{STUD}(y) \quad \rightarrow \quad \_ \_ \_ \right]}
        \node (DP) [right of=forall_students] {DP
          \node (a_book) [below of=DP] {a book
            \node (exists_book) [below of=a_book] {\exists z \quad \left[ \text{BOOK}(z) \quad \land \quad \_ \_ \_ \right]}}
    \end{tikzpicture}
\end{center}
Step 3. Quantifier Raising (QR) is an operation that moves each quantified expression to a position above the sentence and leaves behind a pronoun (as a placeholder). The pronoun represents a variable that the quantified expression binds. For example:

\[ S \]

\[ DP_x \]

a professor

\[ S \]

\[ DP_z \]

a book

\[ S \]

\[ DP_y \]

every student

\[ he_x \]

\[ S \]

\[ VP \]

\[ V \]

gave

\[ him_y \]

\[ it_z \]

So the pronoun \( he_x \) gets interpreted as \( x \) and the quantified expression \( DP \ a \ professor \), contains the quantifier \( \exists x \). The fact that these are both \( x \)’s is very important. It means the quantified expression binds this position in the predicate. Similarly, for \( him_y \) and \( it_z \).

You can move the quantified expressions into any order. QR is unrestricted in this sense. There are in fact 6 different results, each signifying a different translation and interpretation. For example, the tree above represents the order \( DP \ a \ professor > DP \ a \ book > DP \ every \ student \). There is also a tree where \( DP \ every \ student > DP \ a \ book > DP \ a \ professor \) etc.

Step 4. Focus on the tree given in step 3. Replace each term in the tree with its translation in Predicate Logic. Then replace each blank ___ with the S next to it. (The details are shown below.)

\[ S \]

\[ \exists x \ [ PROF(x) \land ____ ] \]

\[ S \]

\[ \exists z \ [ BOOK(z) \land ____ ] \]

\[ S \]

\[ \forall y \ [ STUD(y) \rightarrow ____ ] \]

\[ S = GAVE(x,y,z) \]

\[ GAVE \]

\[ x \]

\[ y \]

\[ z \]
Step 5. Compose each quantifier with its sister S, working up the tree until you are done. In this way, the translation gets built up successively and its complexity grows in a predictable manner.

Composition 1

```
S
\exists x \ [ \text{PROF}(x) \land \_ \_ \_ ]
S
\exists z \ [ \text{BOOK}(z) \land \_ \_ \_ ]
S
\forall y \ [ \text{STUD}(y) \rightarrow \_ \_ \_ ]
\text{GAVE}(x,y,z)
```

Composition 2

```
S
\exists x \ [ \text{PROF}(x) \land \_ \_ \_ ]
S
\exists z \ [ \text{BOOK}(z) \land \_ \_ \_ ]
\forall y \ [ \text{STUD}(y) \rightarrow \_ \_ \_ ]
\text{GAVE}(x,y,z)
```

Composition 3

```
S
\exists x \ [ \text{PROF}(x) \land \_ \_ \_ ]
\exists z \ [ \text{BOOK}(z) \land \_ \_ \_ ]
\forall y \ [ \text{STUD}(y) \rightarrow \_ \_ \_ ]
\text{GAVE}(x,y,z)
```

Result

```
\exists x \ [ \text{PROF}(x) \land \exists z \ [ \text{BOOK}(z) \land \forall y \ [ \text{STUD}(y) \rightarrow \text{GAVE}(x,y,z) ] ] ]
```

Keep in mind that this entire derivation is just for one tree and there are six different trees.
Following this procedure, there are 6 possible translations into Predicate Logic, one for each order of the quantified expressions. These are presented in (2) with a paraphrase for the meaning of each one.

(2)  

a. DP a professor > DP every student > DP a book

\[ \exists x \left[ \text{PROF}(x) \land \forall y \left[ \text{STUD}(y) \rightarrow \exists z \left[ \text{BOOK}(z) \land \text{GAVE}(x,y,z) \right] \right] \right] \]

There is a professor and that professor gave every student a potentially different book.

b. DP a professor > DP a book > DP every student

\[ \exists x \left[ \text{PROF}(x) \land \exists z \left[ \text{BOOK}(z) \land \forall y \left[ \text{STUD}(y) \rightarrow \text{GAVE}(x,y,z) \right] \right] \right] \]

There is a professor and there is a book such that that professor gave every student that book.

c. DP a book > DP a professor > DP every student

\[ \exists z \left[ \text{BOOK}(z) \land \exists x \left[ \text{PROF}(x) \land \forall y \left[ \text{STUD}(y) \rightarrow \text{GAVE}(x,y,z) \right] \right] \right] \]

There is a book and there is a professor such that that professor gave every student that book.

d. DP a book > DP every student > DP a professor

\[ \exists z \left[ \text{BOOK}(z) \land \forall y \left[ \text{STUD}(y) \rightarrow \exists x \left[ \text{PROF}(x) \land \text{GAVE}(x,y,z) \right] \right] \right] \]

There is a book such that for every student a potentially different professor gave the student that book.

e. DP every student > DP a book > DP a professor

\[ \forall y \left[ \text{STUD}(y) \rightarrow \exists z \left[ \text{BOOK}(z) \land \exists x \left[ \text{PROF}(x) \land \text{GAVE}(x,y,z) \right] \right] \right] \]

For every student, there is a potentially different book and a potentially different professor who gave the student that book.

f. DP every student > DP a professor > DP a book

\[ \forall y \left[ \text{STUD}(y) \rightarrow \exists x \left[ \text{PROF}(x) \land \exists z \left[ \text{BOOK}(z) \land \text{GAVE}(x,y,z) \right] \right] \right] \]

For every student there is a potentially different professor who gave the student a potentially different book

(For each of my translations there are many equivalent variations. As long as the quantifiers are in the same order, and the connectives and variables are in the appropriate place, the meaning is preserved. Here are some variations for the translation in (2a).)

\[ \exists x \left[ \text{PROF}(x) \land \forall y \exists z \left[ \text{STUD}(y) \rightarrow \left[ \text{BOOK}(z) \land \text{GAVE}(x,y,z) \right] \right] \right] \]

\[ \exists x \forall y \left[ \text{PROF}(x) \land \left[ \text{STUD}(y) \rightarrow \exists z \left[ \text{BOOK}(z) \land \text{GAVE}(x,y,z) \right] \right] \right] \]

\[ \exists x \forall y \left[ \text{PROF}(x) \land \exists z \left[ \text{STUD}(y) \rightarrow \left[ \text{BOOK}(z) \land \text{GAVE}(x,y,z) \right] \right] \right] \]

\[ \exists x \forall y \exists z \left[ \text{PROF}(x) \land \left[ \text{STUD}(y) \rightarrow \left[ \text{BOOK}(z) \land \text{GAVE}(x,y,z) \right] \right] \right] \]
Turning now to the entailment patterns of these readings. Here are the different scopes again.

(2') a. a professor > every student > a book
b. a professor > a book > every student
c. a book > a professor > every student
d. a book > every student > a professor
e. every student > a book > a professor
f. every student > a professor > a book

Whenever an existential quantifier takes scope over another existential, without anything intervening, the truth-conditions are the same. In other words, ∃ professor > ∃ book is identical to ∃ book > ∃ professor as long as ∀ every student doesn’t come between them. This is summarized below.

(2b) is identical to (2c): If they are identical, that means they entail each other. There is a professor and there is a book = there is a book and there is a professor.
(2e) is identical to (2f): Exactly the same reasoning as above.

There are less trivial entailments. These entailments arise because sentences where ∃ > ∀ entail the sentence with ∀ > ∃. Based on the identities above, I will ignore (2c) and (2f) and just talk about (2b) and (2e).

(2b) entails (2a): If it’s true that there is a particular professor (say Bob) and a particular book (say Moby Dick) such that every student was given that book by that professor, then it is necessarily true that for every student, there is a particular professor who gave the student a book.

(2b) entails (2d): If it’s true that there is a particular professor (say Bob) and a particular book (say Moby Dick) such that every student was given that book by that professor, then it is necessarily true that there is a particular book, such that every student was given that book by a professor.

(2a) entails (2e): If it’s true that there is a particular professor (say Bob) such that every student was given some book by that professor, then it is necessarily true that every student was given a book by a professor.

(2d) entails (2e): If it’s true that there is a particular book (say Moby Dick), such that every student was given that book by some professor, then it is necessarily true that every student was given a book by a professor.

(2b) entails (2e): This follows from the transitivity of entailment.

(2a) does not entail (2d)
(2d) does not entail (2a)
The entailment relations follow a very particular pattern, illustrated in below.

The most specific statements about the outside world are made by (2b) and (2c), which require that particular professor give every student a particular book. These entail (2a) and (2d) which do not entail each other. The propositions in (2a) and (2d) make less specific claims about the world. These entail (2e) and (2f). The last two propositions make the least specific claims about the outside world. As long as every student was given some book by some professor, (2e) and (2f) are true.

The truth-conditions for these translations follow on the next page. I do not show the steps, except for (2a) at the very end.
Here are the truth-conditions for the translations in (2a)-(2f). You only had to have one of these. You had to show some of the work to get the full marks.

(3)  

a. \[ \exists x[\text{PROF}(x) \wedge \forall y[\text{STUD}(y) \rightarrow \exists z[\text{BOOK}(z) \wedge \text{GAVE}(x,y,z)]]] \]  
\[ M,g = 1 \iff \text{for some } e \in D: e \in [\text{PROF}]^{M,g} \text{ and} \]
\[ \text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \text{ or} \]
\[ \text{for some } h \in D: h \in [\text{BOOK}]^{M,g} \text{ and } \langle e, f, h \rangle \in [\text{GAVE}]^{M,g} \]

b. \[ \exists x[\text{PROF}(x) \wedge \exists z[\text{BOOK}(z) \wedge \forall y[\text{STUD}(y) \rightarrow \text{GAVE}(x,y,z)]]] \]  
\[ M,g = 1 \iff \text{for some } e \in D: e \in [\text{PROF}]^{M,g} \text{ and} \]
\[ \text{for some } h \in D: h \in [\text{BOOK}]^{M,g} \text{ and} \]
\[ \text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \text{ or } \langle e, f, h \rangle \in [\text{GAVE}]^{M,g} \]

c. \[ \exists z[\text{BOOK}(z) \wedge \exists x[\text{PROF}(x) \wedge \forall y[\text{STUD}(y) \rightarrow \text{GAVE}(x,y,z)]]] \]  
\[ M,g = 1 \iff \text{for some } h \in D: h \in [\text{BOOK}]^{M,g} \text{ and} \]
\[ \text{for some } e \in D: e \in [\text{PROF}]^{M,g} \text{ and} \]
\[ \text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \text{ or } \langle e, f, h \rangle \in [\text{GAVE}]^{M,g} \]

d. \[ \exists z[\text{BOOK}(z) \wedge \forall y[\text{STUD}(y) \rightarrow \exists x[\text{PROF}(x) \wedge \text{GAVE}(x,y,z)]]] \]  
\[ M,g = 1 \iff \text{for some } h \in D: h \in [\text{BOOK}]^{M,g} \text{ and} \]
\[ \text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \text{ or} \]
\[ \text{for some } e \in D: e \in [\text{PROF}]^{M,g} \text{ and } \langle e, f, h \rangle \in [\text{GAVE}]^{M,g} \]

e. \[ \forall y[\text{STUD}(y) \rightarrow \exists z[\text{BOOK}(z) \wedge \exists x[\text{PROF}(x) \wedge \text{GAVE}(x,y,z)]]] \]  
\[ M,g = 1 \iff \text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \text{ or} \]
\[ \text{for some } h \in D: h \in [\text{BOOK}]^{M,g} \text{ and} \]
\[ \text{for some } e \in D: e \in [\text{PROF}]^{M,g} \text{ and } \langle e, f, h \rangle \in [\text{GAVE}]^{M,g} \]

f. \[ \forall y[\text{STUD}(y) \rightarrow \exists x[\text{PROF}(x) \wedge \exists z[\text{BOOK}(z) \wedge \text{GAVE}(x,y,z)]]] \]  
\[ M,g = 1 \iff \text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \text{ or} \]
\[ \text{for some } e \in D: e \in [\text{PROF}]^{M,g} \text{ and} \]
\[ \text{for some } h \in D: h \in [\text{BOOK}]^{M,g} \text{ and } \langle e, f, h \rangle \in [\text{GAVE}]^{M,g} \]
At this point, you might have noticed how similar these are. This speaks to the expressive power of QR and the principle of compositionality. For additional help, here’s a derivation of the truth-conditions for (3a).

\[
\exists x[\text{PROF}(x) \land \forall y[\text{STUD}(y) \rightarrow \exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]]^{M,g} = 1
\]

iff for some \(e \in D: [[\text{PROF}(x) \land \forall y[\text{STUD}(y) \rightarrow \exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]]]^{M,g[x\leftarrow e]} = 1

iff for some \(e \in D: [\text{PROF}(x)]^{M,g[x\leftarrow e]} = 1 \quad \text{and} \quad [\forall y[\text{STUD}(y) \rightarrow \exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]]^{M,g[y\leftarrow f]} = 1

iff for some \(e \in D: [x]^{M,g[x\leftarrow e]} \in [\text{PROF}]^{M,g[x\leftarrow e]} \quad \text{and} \quad
\text{for every } f \in D: [[\text{STUD}(y) \rightarrow \exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]]^{M,g[x\leftarrow e, y\leftarrow f]} = 1

iff for some \(e \in D: e \in [\text{PROF}]^{M,g} \quad \text{and} \quad
\text{for every } f \in D: [y]^{M,g[x\leftarrow e, y\leftarrow f]} \notin [\text{STUD}]^{M,g[x\leftarrow e, y\leftarrow f]} \quad \text{or} \quad
\text{for some } h \in D: [[\exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]]^{M,g[x\leftarrow e, y\leftarrow f, z\leftarrow h]} = 1

iff for some \(e \in D: e \in [\text{PROF}]^{M,g} \quad \text{and} \quad
\text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \quad \text{or} \quad
\text{for some } h \in D: [\exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]^{M,g[x\leftarrow e, y\leftarrow f, z\leftarrow h]} = 1

iff for some \(e \in D: e \in [\text{PROF}]^{M,g} \quad \text{and} \quad
\text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \quad \text{or} \quad
\text{for some } h \in D: [\exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]^{M,g[x\leftarrow e, y\leftarrow f, z\leftarrow h]} = 1

iff for some \(e \in D: e \in [\text{PROF}]^{M,g} \quad \text{and} \quad
\text{for every } f \in D: f \notin [\text{STUD}]^{M,g} \quad \text{or} \quad
\text{for some } h \in D: h \in [\exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]^{M,g[x\leftarrow e, y\leftarrow f, z\leftarrow h]} = 1

\text{This is equivalent to}

for some \(e \in D: e \in [\text{PROF}]^{M,g} \quad \text{and} \quad
\text{for every } f \in D: \text{if } f \in [\text{STUD}]^{M,g} \text{ then} \quad
\text{for some } h \in D: h \in [\exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]^{M,g[x\leftarrow e, y\leftarrow f, z\leftarrow h]} = 1

also

for some \(e \in D, \text{ for every } f \in D, \text{ and for some } h \in D: \quad e \in [\text{PROF}]^{M,g} \quad \text{and} \quad \text{either } f \notin [\text{STUD}]^{M,g} \text{ or both } h \in [\exists z[\text{BOOK}(z) \land \text{GAVE}(x,y,z)]]^{M,g[x\leftarrow e, y\leftarrow f, z\leftarrow h]} = 1