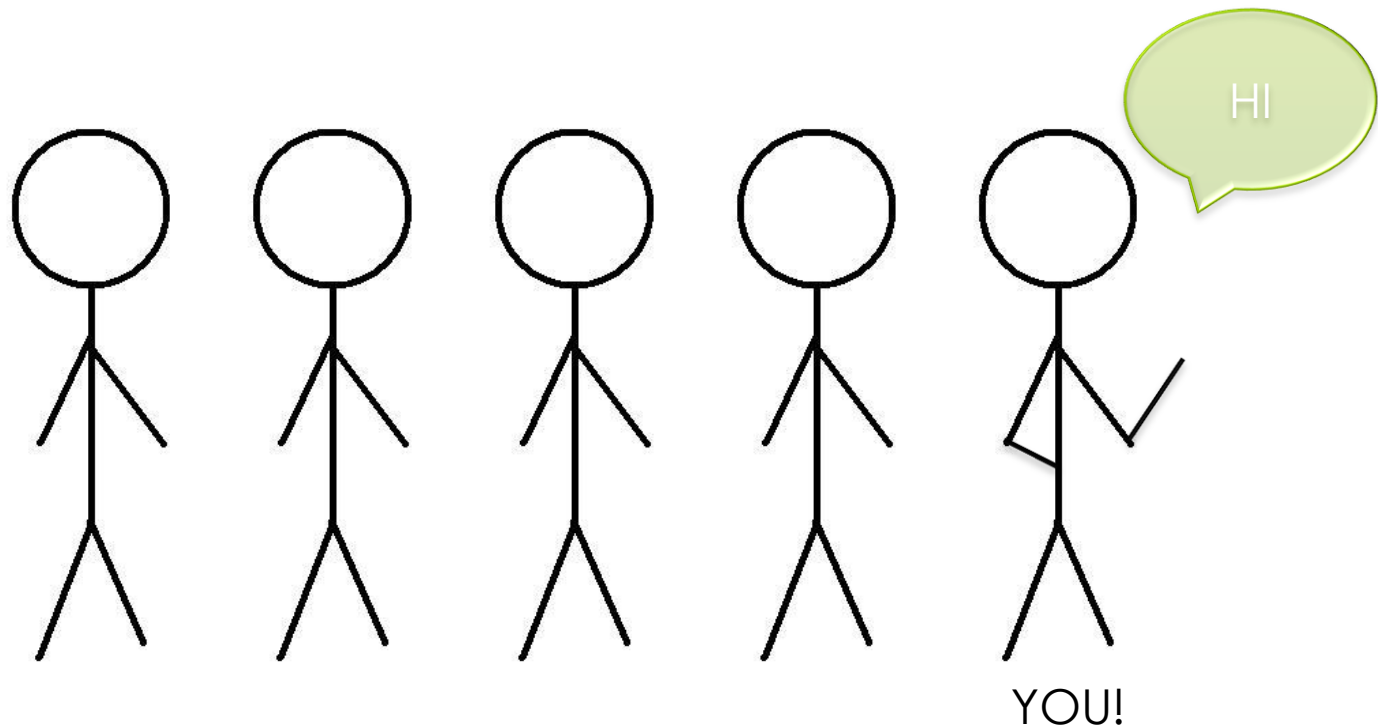




# Recursion

# Suppose You Are Waiting in Line...



# ... A Very Long Line



You want to determine how many people are in front of you, but you **cannot see** and you're **not allowed to move**. You are **only** allowed to **speak to the person in front of you and behind you**. How do you do it?



# Recursion

- A programming technique that breaks down a complex problem into smaller, manageable pieces
- Recursive solutions solve a problem by applying the same algorithm to each piece and then combining the results.

# The General Formula

**#people in front of person in front of you +**

**person in front of you =**

**# people in front of you**

# The Solution

1. Tap the shoulder of the person in front of you and ask how many people are in front of him/her
2. Wait for his/her response and add 1
1. If someone asked, tell them how many people are in front of you

# A Diagram

- Ask and wait
  - Ask and wait
    - Ask and wait....
      - .... Reached first in line. Tell person behind it is 0.
      - Tell person behind it is  $0+1$
      - Tell person behind it is  $1+1...$
    - Tell Person behind it is  $x + 1$



# Recursive Algorithms

- There are two main components to recursive algorithms
  - 1) Base Case
  - 2) The Recursive Case

# Recursive Algorithms

- There are two main components to recursive algorithms
  - 1) Base Case: The point where you stop applying the recursive case
  - 2) Recursive Case: The set of instructions that will be used over and over

# In the Queue Problem...

- Recursive case is
  - Tap person in front of you. Ask #people in front of him. Wait for his answer and add 1.
- Base case is
  - person 0. You do not do execute the above.

# Recursion and Programming

- A recursive function is a function that calls itself

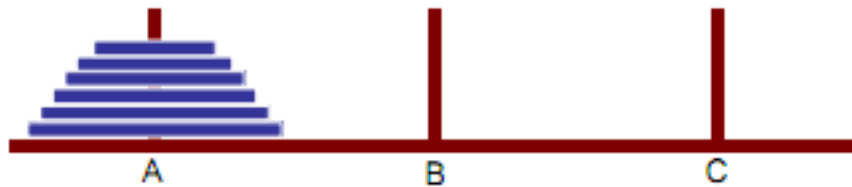
```
numberOfPeopleInFront(person){  
  If (there is no one to tap)  
    return 0  
  Else  
    tap person in front of you (F)  
    #ppl in front of F = numberOfPeopleInFront(F)  
    return #ppl in front of F + 1  
○ }
```

# Pseudo-code diagram

- Ask and wait
  - Ask and wait
    - Ask and wait....
      - .... Reached first in line. Return 0.
    - Return  $0+1$
  - Return  $1+1...$
- Return  $x + 1$

# Towers of Hanoi

- A prominent recursive problem
- Starting Configuration:



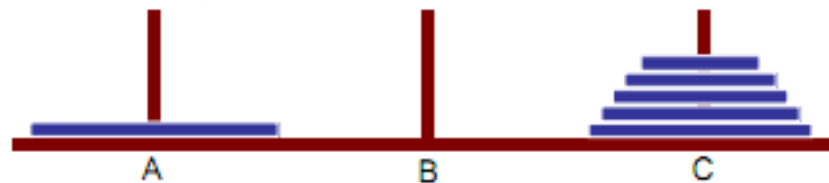
- Goal: Move tower from A to B

# Rules

- Move one disk at a time
- A larger disk cannot be placed on top of a smaller disk
- We can use some needles as temporary storage

# Subgoals

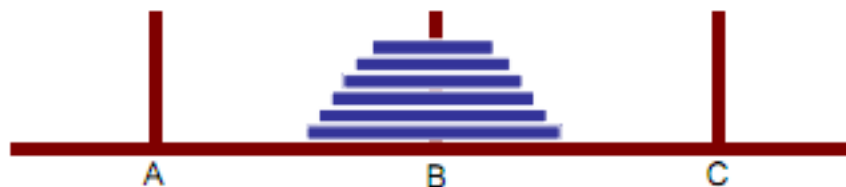
- Get top  $x-1$  disks from A to C



- Get bottom disk from A to B



- Move top  $x-1$  disks from C to B



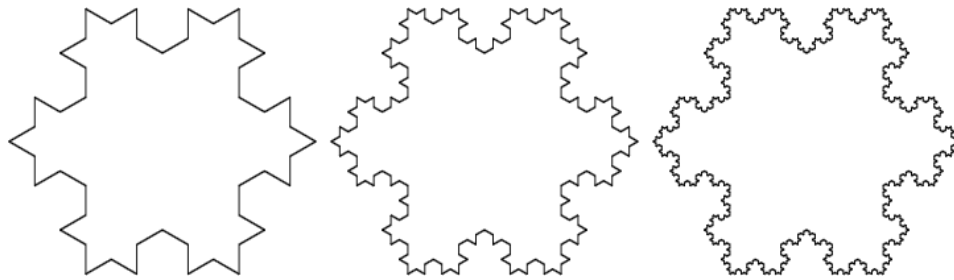


# Recursion Behind Towers

- Base Case: Moving the Largest Disk to Needle B
- Recursive Case: Do same for the  $x - 1$  disk above it
- <http://www.mazeworks.com/hanoi/>
- Fun Fact: It takes at least  $2^n - 1$  moves to solve the puzzle

# Fractals

- A rough or fragmented geometric shape that can be split into parts, each of which is (at least an approx. of) a reduced copy of the whole
- Base case: Starting shape
- Recursive case: Repeating shape in different sizes



Koch Snowflake