The Game of Nim
The setup:

- 2 players take turns picking circles from each row (we call the rows “heaps”).
- At each turn, at least 1 circle has to be picked.
- A player cannot pick from more than 1 row.
Background

- Variants played since ancient times
  - resemblance to Chinese “picking stones”

- Current name and theory developed by C. Bouton of Harvard in 1901
  - name taken from German *nimmt* meaning “take”
Simple Example

Player 1 takes 2 from heap 2

Player 1 is forced to take the last one

Player 2 wins!

Player 2 takes 1 from heap 1
Theory

- Theory completely solved for any number of heaps/objects by C. Bouton
  - Based upon *binary digital sum* of heap sizes
    - also known as “nim-sum”
Algorithm!

- Write the size of each heap in binary
- Add the sizes without carrying
  - Simple rule of thumb:
    - Column w/ even # of 1’s = 0
    - Column w/ odd # of 1’s = 1
Binary Digital Sum

\[
\begin{array}{c}
1 & 0 \\
1 & 1 \\
\hline
= & 0 & 1
\end{array}
\]

\[
\begin{array}{c}
1 & 0 \\
1 & 0 \\
\hline
= & 0 & 0
\end{array}
\]
Winning strategy: finish each move such that the nim-sum is zero

- If your partner gives you a non-zero nim-sum, it is **always** possible for you to make it into a zero nim-sum.
- If your partner gives you a zero nim-sum, it is **never** possible for you to keep it at a zero nim-sum. You will have to change it into a non-zero nim-sum.
Example

010
011
100
= 101

→

010
011
001
= 000
Endgame

- When the next move will result in heaps of size 1.
  - **Normal play**: Move such that an even number of heaps of size 1 remain. *Here, you will lose with Normal play!*
  - **Misère play**: Move such that an odd number of heaps of size 1 remain.
Any Questions?