INNOVATION AND OPTIMAL PUNISHMENT, WITH ANTITRUST APPLICATIONS

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Abstract: This paper modifies the optimal punishment analysis by incorporating investment incentives with external benefits. In the models examined, the recommendation that the optimal penalty should internalize the marginal social harm is no longer valid as a general rule. We focus on antitrust applications. In light of the benefits from innovation, the optimal policy will punish monopolizing firms more leniently than suggested in the standard static model. It may be optimal not to punish the monopolizing firm at all, or to reward the firm rather than punish it. We examine the precise balance between penalty and reward in the optimal punishment scheme.

Keywords: optimal law enforcement, optimal antitrust penalty, monopolization, innovation, internalization, strict liability, static penalty.

JEL Classification: D42, K14, K21, K42, L41, L43
I. Introduction

Strict liability rules are rare in the law. In general, legal rules apply a rough cost-benefit test to determine liability – the negligence rule of torts is the most common example. Indeed, it would be a fair summary to say that courts exempt efficient conduct from punishment, through the use of cost-benefit legal tests, and apply general statutorily-determined penalties to the conduct that violates the law. For example, in American antitrust law, courts use rule of reason analysis to exempt efficient conduct for the most part, while applying statutorily-set penalties or treble damages to the violations.

This basic pattern in the law makes it difficult to apply the lessons from the economic theory of punishment. The literature offers models in which offenders are held strictly liable and assessed with case-specific optimal penalties (depending on the harm and probability of punishment in the particular case). The law, in contrast, applies a case-specific cost-benefit analysis to the determination of liability, and then uses standard off-the-rack penalties (statutory penalties, compensatory damages, or predetermined damage multipliers).

In spite of this difference between the theory of punishment and practice in courts, the theory remains useful as a guide for policy. This paper follows the tradition of the optimal punishment literature by examining optimal penalties as a source of guidelines for legal policy. The specific focus here is innovation and punishment, especially in the context of antitrust.

The prevailing analysis of optimal antitrust penalties holds that in order to avoid over-deterrence of efficient conduct, the optimal penalty would internalize the social losses generated by the offensive conduct (Becker, 1968, at 199). If enforcement is perfect and costless, such a penalty would internalize the transfer from consumers and the deadweight loss (Landes, 1983).
Thus, if the monopolizing firm introduced efficiencies, it might still have an incentive to carry out its monopolizing conduct, as long as the value of the efficiency gain exceeded the deadweight loss.

This analysis is incomplete because it is based on a model that does not incorporate some plausible social benefits from monopolization. The prevailing analysis employs a static model in which the gains from monopolization accrue to the firm and the losses are suffered by society. In contrast, a dynamic model would take into account the social gains from investments made by the firm in its effort to become a monopoly. Investments in market-creating or market-expanding innovation should be incorporated into the economic analysis of punishment.

This paper modifies optimal punishment analysis by incorporating dynamic incentives with external benefits. In the models examined, the recommendation that the optimal antitrust penalty should seek to internalize the marginal social harm, as measured by consumer transfer and the deadweight loss (the static penalty), is no longer valid as a general rule. We explore the recommendations from two simple versions of the dynamic model, one in which the offender invests in market-expanding innovation, and another in which victims invest.

We focus on antitrust applications, although the model can be applied to patent law, tort law, and other areas. The most obvious implication is that in light of the benefits from innovation, the optimal policy will punish monopolizing firms more leniently than suggested in the static model. It may be optimal not to punish the firm at all, or to reward the firm rather than punish it. In this sense, the model provides a Schumpeterian approach to punishment, as well as

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1 Schumpeter (1943) (Chapters 7 and 8) famously criticized static model of competition for ignoring the social benefits of innovation, and the need for firms to gain monopoly power in order to earn a positive return on socially valuable innovation. For a review, see Mason (1951). This paper offers a simple model that incorporates innovation and examines the optimal antitrust penalty.
the groundwork for a positive account of monopolization law, which has been puzzlingly lenient over its history.  

In general, the optimal penalty for monopolization is a function of the consumer harm, the residual consumer surplus after monopolization, the cost of enforcement, and the relative responsiveness of innovation and monopolization to changes in the penalty. The optimal (dynamic) penalty turns out to be a weighted average of the static penalty and an innovation subsidy, with the weights determined by the relative sensitivities of investment and of monopolization to punishment. This has implications for law and punishment policy – both in the positive sense of understanding the law that exists and in the normative sense of reforming it.

Although our focus is antitrust, we discuss other applications. The connection between innovation and punishment is a concern in both antitrust and products liability. In *U.S. v. Microsoft*, the D.C. Circuit Court of Appeals refused to apply a per se liability rule to the firm’s technological integration of the internet browser and operating systems because of its fear that such a rule would discourage innovation. In *Trinko v. Verizon*, the Supreme Court cited the negative innovation effect as a basis for refusing to adopt the essential facilities theory of monopolization. Products liability lawsuits against drug manufacturers have been met with the criticism that their success will deter the development and marketing of new drugs.

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2 On the perceived leniency of monopolization law and its explanation, see Evans and Hylton (2008).
3 253 F.3d 34 (D.C. Cir. 2001).
4 *Id.* at 89-90. In addition, much of the commentary about the Microsoft litigation focused on the implications of antitrust enforcement for innovation in the technology industries, see Evans and Schmalensee (2002).
6 *Id.* at 407.
difficult and unexplored issue in these cases is the precise relationship between the deterrence of offensive conduct and the encouragement of innovation in an optimal punishment scheme.\(^8\)

Parts II.A and II.B set up the static model, which replicates the standard optimal penalty recommendation. Parts II.C.2 and II.C.3 explore punishment in a setting in which the offender makes an investment that provides social benefits, the returns to which are a function of an offense that he may commit in the future as well as the punishment for that offense. Part II.C.4 examines an extension in which the victims make investments, the expected returns to which are reduced by an offense that may be committed by the offender. Part III discusses implications for the law.

II. Model

A. Basic Assumptions

All actors are risk neutral and victims are the only parties who suffer loss. The state apprehends an offender only after an injury has occurred. The state does not attempt to apprehend the offender in each instance of an offense, and therefore the probability of apprehension (equivalently, detection) given an injury is less than one. Once apprehended punishment is certain.

Let \(z\) = probability of apprehension. We assume that there is a lower limit \(z'\) on the feasible values of \(z\), so that \(0 < z' < z \leq 1\). Let \(c\) = the cost to the state of apprehending the

\(^8\) One question that arises is whether the optimal penalty results derived in this paper could be implemented, and if so, how. First, the variables that the optimal penalty incorporates suggest matters that courts take into account in the relevant law on liability. We consider implications of the model for legal doctrine near the end of the text. Second, the optimal penalty requires information on matters such as the probability of detection (or apprehension), the harm to victims, and the residual consumer surplus. Although the law generally does not attempt to estimate optimal case-specific penalties, a move to such a regime, as originally advocated in the Becker article, would not be infeasible.
offender, $c > 0$; $v =$ the loss suffered by a victim, $v > 0$; $F =$ fine imposed on apprehended offender; $M =$ the gross gain to the offender from committing an offense, $M > 0$.

$M$ is governed by the probability distribution function $H$ with corresponding density function $h$, where $h(M) > 0$ for $0 < M < M_u$, and $h(M) = 0$ otherwise. The offender will commit the offense if $M > zF$; therefore the probability that the offender does not commit an offense is $H(zF)$. If $M_u \leq zF$ no crimes will be committed (the offender will not commit an offense when he is indifferent). Thus, $F = M_u/z$ is the minimum level of the fine that achieves complete deterrence.

The offender cannot satisfy his preferences through the market; thus in order to enjoy the gain $M$ he must commit an offense. The time line of events is as follows: the offense occurs with probability $1 – H(zF)$ causing a loss of $v$; enforcement occurs with probability $z$; the offender is apprehended at cost $c$, and then punished with a fine equal to $F$.

**B. Optimal Punishment Policy: Static Case**

The optimal punishment policy is the combination of the fine and the probability of apprehension that minimizes the cost of offenses and the cost of avoiding offenses:

$$C = (1–H(zF))(v + zc) + H(zF)E[M | M < zF].$$

There are, of course, alternative formulations of the social welfare function that yield the optimal policy. An equivalent objective is to maximize the net benefit from offenses $NB(\text{offenses}) = (1–H(zF))E[M–v– zc | M > zF]$, or to maximize the difference between net deterrence benefits and
the costs of enforcement $H(zF)(v - E[M | M < zF]) - (1 - H(zF))zc$. The optimal policy can be stated as follows.

**Proposition 1**: If $M_u > v + z'c$, then the optimal punishment policy is to set the fine so that it satisfies $F = F^* = v/z' + c$, and the probability of apprehension at the minimum level $z'$. If $M_u \leq v + z'c$, then the optimal policy is to set the fine and probability of apprehension so that $zF \geq M_u$ (complete deterrence rule).

This is a familiar result that replicates Becker (1968), and Polinsky and Shavell (1992) with modifications. We provide proofs in the appendix. The intuition is that if the offensive activity is potentially efficient, in the sense that the gain to some offenders exceeds the marginal social cost of the activity, then the optimal penalty internalizes the social cost of the offender’s conduct. In order to minimize enforcement costs the state sets the probability of apprehension at its minimum level. However, if the offender’s activity is not potentially efficient, the optimal policy aims to completely deter it – and, as Becker noted, the internalizing penalty $F^*$ satisfies the complete deterrence goal in this case.

It follows that an antitrust punishment authority should distinguish conduct that is potentially efficient from conduct that is not, and apply the internalization rule in the potentially efficient category, and the complete deterrence rule in the inefficient category. In the static setting the monopolizing firm takes an act that allows it to extract surplus from consumers and may also generate an efficiency gain, as shown in Figure 1. For example, the monopolizing act might be an exclusive dealing contract that both forecloses competition and reduces supply costs. Since $M = T + E$ and $v = T + D$, the optimal antitrust penalty is $F^* = (T + D)/z' + c$, and if no

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9 Punishment in this model occurs through action of an enforcement authority that imposes a discrete fine after the monopolizing act occurs. We do not allow in this model for victims to sue for damages, which could complicate matters in the antitrust context, see Baker (1988).
efficiency gain were possible ($E=0$, e.g., naked price fixing) the optimal policy would eliminate profits ($F \geq T/z'$). $F^*$ would serve this purpose, as would any other penalty greater than $T/z'$.
C. Optimal Punishment Policy in a Dynamic Setting

In the previous part we examined the enforcement model in the context of antitrust, replicating the static penalty (internalize the transfer and deadweight loss). In this part, we extend the model to incorporate investment by the offender.

The reason for taking investment by the offender into account is that it is important in the antitrust setting, especially in monopolization cases. Suppose the monopolizing firm has undertaken investments that benefit consumers, such as creating a new product market. The static enforcement policy may be socially excessive because it might prevent the firm from earning a break-even return on its investments in market creation. This is distinguishable from the static case, where the product market was already in existence and the monopolizing firm took an action that permitted it to gain monopoly power, perhaps also with an efficiency gain. In the dynamic model, the firm creates the market and then monopolizes it.

1. Assumptions

Using Figure 1, the firm invests in the first period creating the market. In the second period, the firm takes an action that monopolizes the market. As in the previous model, the second period action could generate an efficiency gain. When the monopolizing firm creates the market, it generates \( S = T + D + W \). If the firm is deterred from monopolization, consumers get \( S \). If the firm is not deterred from monopolization, consumers get \( W \).

For example, suppose in the first period the firm invests in the design and production of a new artificial tooth that will be ready to market in the second period. The tooth design can be
copied by rivals easily, so the second period market has the potential to be perfectly competitive. However, the firm can reduce competitive pressure by engaging in some exclusionary act at the start of the second period. The ideal exclusionary act would be the attainment of a legal barrier to entry, such as a patent, but suppose such options are not available to the firm. Suppose the firm’s best option for excluding competition is entering into an exclusive dealing contract with a key resource supplier,10 and that in addition to excluding competition the contract reduces supply costs by permitting the resource supplier to better predict demand.11 The returns from the creation of the new artificial tooth depend on the firm’s later success in excluding competition.12 It will have an incentive to monopolize if the gains from monopolization exceed expected antitrust penalties.

In the remainder we will focus on the optimality condition for the penalty. Becker’s suggestion that the probability of enforcement should be set at its minimum level (under reasonable assumptions) has not been disturbed in the subsequent literature (e.g., Polinsky and Shavell, 1992),13 so there is no need to reexamine this issue. We will assume, in parts of the discussion, that the optimal policy will set the enforcement probability at the minimum level.

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10 This hypothetical is based partly on the facts of U.S. v. Dentsply Int’l, Inc., 399 F.3d 181 (3rd Cir. 2005). In Dentsply the exclusive dealing contract was with dealers rather than suppliers. An example involving an exclusivity contract with a key supplier was Alcoa’s contract to purchase electricity on the condition that the seller refrain from selling electric power to any other producer of aluminum. U.S. v. Aluminum Co. of America (Alcoa), 148 F.2d 416, 422 (2d.Cir. 1945).

11 For example, the tying contract in International Salt Co. v. United States, 298 U.S. 131 (1936) was explained by Peterman (1979) as a device that permitted the company to better predict, and thereby take advantage of economies in, the distribution of salt.

12 This story takes the innovation concept as given to the innovator and examines the incentive to carry it out. This is distinguishable from the innovation race in which many firms are attempting to develop a new technology at the same time, in which case perpetual rivalry might, or might not, enhance innovation. See Loury (1979); Lee and Wilde (1980); Aghion, Bloom, Blundell, Griffith, and Howitt (2005). In general, one can distinguish the incentive to search for innovations and the incentive to carry a particular innovation concept out. Competition may reduce the incentive to carry out a particular concept and increase the incentive to search for an innovation.

13 If there is a fixed cost involved in setting the probability of apprehension, the optimal probability of apprehension may be positive for sufficiently small values of the marginal enforcement cost (Polinsky and Shavell, 1992). Also,
2. Optimal Punishment: Offender Investment Case

The potential offender invests in the first period, at cost $k_o$. The private return to the offender is determined by his prospects for monopolization. Thus, the potential offender will invest if the expected return from monopolization, net of the penalty, exceeds his investment cost. Let the investment cost $k_o$ be governed by the probability distribution $\Psi$ with corresponding density $\psi$. The potential offender invests when $k_o < \bar{k}_o = (1-H(zF))[E(M \mid M > zF) - zF]$. The probability of investment is therefore $\Psi(\bar{k}_o)$.

The problem for the social planner is to choose the optimal fine to maximize the net social benefit:

\[
NB = \Psi(\bar{k}_o) \left\{(1-H(zF))E(M \mid M > zF) - E(k_o \mid k_o < \bar{k}_o)\right\} + (1-H(zF))(S-v-\alpha) + H(zF)S
\]

where the first term (in brackets) represents the net gain from investment to the would-be monopolist, the second term represents the net gain to society if investment is followed by monopolization ($S - v = S - T - D = W$), and the third term represents the net gain to society if investment is not followed by monopolization. The first order condition with respect to the fine is

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if there is a fixed upper limit on the fine, the optimal probability of apprehension will not necessarily be at the minimum level. These are variations on the Becker model that we are not exploring.
The first two terms in (4) are the familiar static penalty (e.g., Landes, 1983), which internalizes consumer harm. The remaining terms involve penalties or subsidies for the offender’s conduct. The offender gets a subsidy of $S/z$ in recognition of the potential surplus it delivers to consumers when its investment creates a new product market. This subsidy is divided by the probability of apprehension because the offender gets no subsidy from the state in cases in which it is not punished. The last term reflects the dynamic component of the penalty – the part that is concerned with the investment incentive – and it is a combination of subsidy and penalty. The first component of the numerator reflects a penalty to discourage monopolization, the second a subsidy to encourage investment. The entire last term can be positive or negative (a subsidy) depending on the responsiveness of investment efforts or of monopolization efforts to the penalty.

Overall, however, the subsidy effect dominates in the last two terms of (4). Simplifying, we have
\[ F^* = \frac{v}{z} + c - \frac{S}{z} \left( \frac{\psi(k_o^*)(1-H(zF^*))}{\Psi(k_o^*)h(zF^*) + \psi(k_o^*)(1-H(zF^*))^2} \right) \]  

(5)

and the following recommendation.

**Proposition 2:** The optimal policy is to set the fine so that it satisfies (5) and the probability of enforcement at the minimum level.

Unlike the static case examined previously, the “complete deterrence rule” setting the fine greater than or equal to the monopolist’s gain (inflated by the apprehension probability) is no longer a sensible policy. When the monopolist’s investments make the product market available, its conduct always provides some benefit to society. Given this, it does not make sense to wipe out its entire gain. This is different from the static case, where the monopolist merely transfers wealth from consumers to itself, in which a policy of complete deterrence could be optimal.

Letting \((S/z)\theta\) represent the final term in (5), the optimal penalty is \(F = \frac{v}{z} - (S/z)\theta + c\). Moreover, \(\theta\) is a discontinuous function of \(F\) with the properties \(\theta > 0; \theta = 1\) for \(F \leq 0\); and \(\theta(F) > 0\) for \(F > 0\). The properties of the optimal penalty solution are considered in detail in the appendix.\(^{14}\)

Since \(\theta > 0\), the optimal penalty is unambiguously less than the static penalty \(\frac{v}{z} + c\). In other words, the optimal penalty for monopolization in the innovation setting is unambiguously less than the static penalty that internalizes the consumer harm.

\(^{14}\) See Proof of Proposition 2. In this formulation, the net marginal social harm from the offender’s conduct consists of the difference between the harm to consumers and the marginal innovation benefit, where the innovation benefit is a function of the size of the penalty.
The parameter (actually function) $\theta$ captures the relative sensitivities of the marginal firm’s innovation-investment and monopolization decisions to the penalty. When

$$\frac{h(zF^*)}{1-H(zF^*)} > \frac{\psi(\bar{k}^*) H(zF^*)}{\Psi(\bar{k}^*)}$$

we have $\theta < 1$, the case in which the monopolization incentive exceeds the investment incentive at the margin. The penalty is larger because its shadow price, discouragement of innovation, is relatively low. When the inequality in (6) is reversed (shadow price of investment high) we have $\theta > 1$; and when there is a strict equality $\theta = 1$. The incentive parameter $\theta > 0$, because $\theta = 0$ implies complete deterrence, which is ruled out because it is always desirable for society to enjoy some of the surplus rather than none of it.$^{15}$

Although the optimal penalty is unambiguously less than the static penalty, it can also be a subsidy given that $\theta$ can take the value of 1. We explore its precise form in the appendix (Proposition 2). When the marginal social harm $v + zc$ exceeds the surplus $S$ (or, equivalently, when the expected enforcement cost, $zc$, exceeds the residual surplus $W$) the optimal penalty is always positive; i.e., $F^* = v/z - (S/z)\theta^* + c > 0$. This makes sense because it is desirable to discourage the firm’s conduct to some degree, though the discouragement is not as severe as in the static model. When the marginal harm is less than the surplus (equivalently, expected enforcement cost is less than the residual surplus), the optimal penalty can be either a penalty or the subsidy $F^* = v/z - S/z + c < 0$. The choice between penalty and subsidy depends on whether the extra surplus gained from deterring monopolization is greater than the wealth generated from

$^{15}$ See Proof of Proposition 2 in the appendix.
investment (appendix, Proposition 2), which is a function of the ratio of the consumer harm to the residual surplus from innovation.

Substituting the terms from Figure 1, the optimal penalty can be expressed as

\[ F^* = (1 - \theta) \left( \frac{T + D}{z} \right) + \theta \left( -\frac{W}{z} \right) + c. \] (7)

This is a weighted average of the static monopolization penalty \( (T + D)/z + c \) and an investment subsidy \(-W/z + c\), where the weight is determined by the investment incentive term \( \theta \). The optimal rule penalizes the firm for a portion of the consumer harm and rewards the firm for a portion of the residual surplus. As the relative investment sensitivity parameter increases the optimal penalty puts more weight on the subsidy component.

Optimal punishment depends on the residual surplus, the consumer harm, the investment incentive term \( \theta \), and the cost of enforcement. Punishment becomes more severe, holding other factors the same, as the ratio of consumer harm to residual surplus increases, and as the relative sensitivity of innovation to the penalty falls.\(^{16} \)

The obvious policy implication is that it may not be optimal to punish the firm for monopolization, even when there are no static efficiencies connected to the firm’s monopolizing conduct. For example, consider exclusive dealing. If the exclusivity contract offers no cost advantage whatsoever, and serves the sole purpose of excluding competitors, it still may not be

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\(^{16}\) Baker (2005) argues that antitrust enforcers target their prosecution decisions to cases in which the penalty is more likely to reduce monopolization efforts than to reduce investment efforts. If true, real enforcement could be consistent with the model of this paper. However, this paper’s model suggests that there should be no enforcement when the investment elasticity is greater than or equal to the monopolization elasticity. This suggests that a bit more caution or forbearance on the part of enforcement agents may be optimal.
optimal to impose a penalty on the monopolizing firm. Indeed, the optimal fine for monopolization could be negative. One purpose of the fine is to align private and social incentives for investment. If the private monopolization gain is less than the social gain from investment, the optimal policy will include a subsidy. Suppose the innovating firm monopolizes the market in the second period with probability one. The social gain from the firm’s first-period investment would be the sum of the residual consumer surplus and the monopoly transfer \((W + T)\). The private gain, however, would be equal to the monopoly transfer. Optimal punishment policy trades off deterrence of monopolization with equalizing the private and social gains from investment, and the latter policy requires a bounty based on the residual surplus.\(^{17}\)

Comparative statics in the dynamic setting differ from the static case. First, consider the behavior of the optimal penalty as consumer harm increases. Unlike the static scenario, the optimal penalty in the dynamic setting does not go to infinity as the consumer harm goes to infinity. Assuming a positive penalty is optimal, it converges to a value determined by \(\theta(F^*) = 1\) as \(v\) goes to infinity (appendix, Proposition 2a). The optimal penalty increases with consumer harm to a limit that is consistent with sustaining investment. Moreover, a decline in the probability of apprehension does not necessarily cause the optimal fine to increase; the effect is ambiguous (Proposition 2b). It is not necessarily desirable to have the combination of a low probability of apprehension with a high fine.

Although we have focused on monopolizing conduct that might fall afoul of the antitrust laws, the model applies equally to the case of a patent. The optimal penalty result provides the

\(^{17}\) If the potential surplus is equal to zero the optimal penalty obviously simplifies to the static penalty examined earlier. Since the monopolist can invest, one might ask why the optimal penalty would fail to encourage investment? Where there is no new surplus anticipated from the firm’s investment, the entire investment is devoted to transferring surplus from consumers. The monopolist’s investments are equivalent to those made in one period by a thief solely to facilitate theft in a later period.
optimal “price” that should be charged to a patentee, which could be a monetary fine or a prize. The relevant ratio that should be considered in analyzing patent-antitrust disputes is that between the consumer harm and the residual surplus created by innovation.\(^{18}\)

3. Some General Applications of Offender Investment Model

The offender investment model can be applied to non-antitrust settings. Consider product safety regulation. The firm invests in the first period, creating the market. In the second period, it decides whether to take care to avoid harming the consumer. It takes care only if the cost of taking care \(M\) is less than the expected fine \(zF\). The firm’s investment in the first period is a function of its profit in the second, which is determined in part by the relationship between \(M\) and \(zF\). This example is analogous to the innovation-monopolization problem studied previously. The optimal penalty is therefore of the form \(F = \frac{v}{z} - \left(\frac{S}{z}\right)\theta + c\), where \(v\) is the injury to the consumer and \(S\) is the surplus from innovation. The incentive parameter \(\theta\) captures the relative sensitivities of precaution and innovation, and increases as the shadow price of innovation increases. This is not a weighted average of consumer welfare components (as in the monopolization case) because \(v\) does not have any necessary relationship to \(S\). The optimal product safety fine is unambiguously less than the one that internalizes the consumer injury, and could be a reward.

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\(^{18}\) On ratio tests and the patent-antitrust cases, see Kaplow (1984). The model here suggests a different type of ratio test than the one recommended by Kaplow. Kaplow focuses on the ratio of the monopolist’s reward \((T)\) to the deadweight loss. The relevant ratio suggested in this analysis is that of the residual surplus to the sum of lost and transferred surplus \((T+D)\). One question this generates is whether the case law is more consistent with the ratio test suggested here or Kaplow’s ratio test.
Suppose, instead of a strict liability rule, the punishment authority operates under a negligence standard which imposes punishment only when $M < v$. This case has not been considered in the economic treatments of punishment (Becker, 1968; Polinsky and Shavell, 1992), but it would not have made a significant difference in those analyses. Even under a negligence rule, the optimal (static) fine would still internalize the social costs of the offender’s conduct. Interestingly, a negligence rule would enable the enforcement authority to avoid any enforcement expenditure at all. The offenders for whom $M > v$ would not be deterred by the threat of punishment (because they are exempted under negligence), and those for whom $M < v$ would be deterred. There would never be a need for enforcers to act. In the standard punishment analysis, exemplified by the static model examined earlier in this paper, negligence is superior to strict liability because it avoids expenditures on apprehension and punishment.

In the innovation context examined here, where the offender’s conduct generates a positive externality, the use of a negligence rather than strict liability standard does alter the result for the optimal penalty. If the punishment authority operates under a negligence rule, under which the fine is imposed only when $M < v$, the social objective would be to set the fine to maximize

$$ NB = \Psi(\bar{k}_o) \{(1-H(v))E(M | M > v) + [(H(v)-H(zF))E(M | v > M > zF) - E(k_o | k_o < \bar{k}_o)] \}
+ (1-H(zF))(S-v-zc) + H(zF)S \} \quad (8) $$

where $\bar{k}_o = (1-H(v))E(M | M > v) + (H(v)-H(zF))[E(M | v > M > zF) - zF]$. This objective function reflects the fact that when $M > v$, the punishment authority will apprehend but not punish the
offender; when \( v > M > zF \), the offender will be punished but will not be deterred from the offense; and when \( zF > M \), the offender will be deterred from committing the offense. The optimal fine for the negligence case satisfies

\[
zF^* = v + zc - \frac{\psi(k_{\sigma}^*)(H(v) - H(zF^*))((S - (1 - H(v))(v + zc))}{\psi(k_{\sigma}^*)(H(v) - H(zF^*))^2 + \Psi(k_{\sigma}^*)h(zF^*)},
\]

which is smaller than the static expected penalty \((v + zc)\) when the surplus is sufficiently large relative to the marginal social harm from the offense. When the surplus is small relative to the marginal social harm, the optimal penalty exceeds the static penalty. In effect, the optimal penalty under the negligence rule inflates the static penalty and offers a subsidy as well. The reason for inflating the static penalty is to discourage some investment, given that some offenders will not be punished under the negligence rule even though they have forced society to bear the enforcement costs of apprehending them.\(^{19}\) The optimal penalty under the negligence standard is probably (though not necessarily) larger than the optimal penalty under the strict liability standard (5). The subsidy provided by the optimal penalty is smaller because the negligence rule already exempts some potential offenders from any punishment. The static penalty comes closer to being socially optimal (provided a positive penalty is optimal) when the regulator punishes according to the negligence standard.

4. Optimal Punishment: Victim Investment Case

\(^{19}\) Indeed, if the surplus is zero, the optimal penalty in (8) is greater than the static penalty. The reason, again, is to discourage investment that yields no social surplus, but generates enforcement (litigation) costs from enforcement efforts.
Now potential victims invest in some activity, at cost $k_v$. The gross gain from investment to the potential victim, if there is no offense, is $B$. However, because of the risk that an offense will destroy the value of the investment, the potential victim's expected gain from investment is $[1-\text{prob}(\text{offense})]B$. Assume the victim suffers a direct loss $v$ in the event an offense occurs. We will allow $v$ to differ from $B$ because it is possible that the offense both destroys the value of the victim’s investment and imposes a different direct loss on the victim.

Returning to the terms introduced in the previous section, the expected private gain from investment is $H(zF)B$. Suppose $k_v$ is distributed according to the probability distribution $R$, with corresponding density function $r$, $r(k_v) > 0$ for $k_v > 0$ and $r(k_v) = 0$ otherwise. The potential victim invests whenever $k_v < \bar{k}_v = H(zF)B$, so the probability he invests is $R(\bar{k}_v)$. The expected net benefit from investment is

$$R(\bar{k}_v) E(k_v - k_v | k_v < \bar{k}_v)$$

Differentiating the net benefit function with respect to $F$, we have

$$R(\bar{k}_v)Bzh(zF).$$

The investment benefit from increasing the fine is equal to the product of the marginal reduction in the probability of an offense, $zh(zF)$, and the expected gain among the pool of potential investors $RB$. Increasing the fine, through its deterrent effect, allows more of those who invest to
realize their returns without having them destroyed by the offender. The expected net benefit from investment is maximized when the penalty is set at a level that eliminates the offender’s gain, $F \geq M_u/z$, and minimized when the penalty is set at zero.

Of course, a social planner would not set out to maximize the expected net benefit from investment alone. The social objective is to maximize the net benefits from enforcement, which is the sum of the net gain from investment and the net gain from offenses:

$$NB = NB(\text{investment}) + NB(\text{offenses})$$

$$= R(\bar{k}_v)(\bar{k}_v - E(k|k < \bar{k}_v)) + (1 - H(zF))[E(M|M > zF) - (v + zc)]$$

The solution is to internalize the direct losses of victims ($v$), the marginal enforcement cost ($c$), and the investment gain among the pool of potential victims that is forgone because of the fear of an offense ($RB$).

**Proposition 3:** Let $NB^*$ represent the value of the net benefit from enforcement under the optimal policy, and let $\overline{NB}$ represent the value of the net benefit from enforcement when offenses are completely deterred. Let $\bar{M} > v'c + R(\bar{k}_v)B$ denote the value of $M_u$ such that $NB^* = \overline{NB}$. Then if $M_u > \bar{M}$, the optimal punishment policy is to set the penalty and probability of apprehension so that $F^* = \frac{v}{z'} + c + \frac{R(\bar{k}_v)B}{z'}$. If $M_u \leq \bar{M}$, then the optimal policy is to set the fine and the probability of apprehension so that $zF \geq M_u$. 
The optimal penalty internalizes the investment return forgone as a result due to fear of offenses.\textsuperscript{20} In addition, the circumstances under which a complete deterrence policy is optimal are broader than in the static case. If the maximum gain to offenders ($M_a$) is less than the marginal social cost of an offense, complete deterrence is optimal. However, even if the maximum gain exceeds the marginal social cost of an offense, complete deterrence may still be optimal, because the gain is still insufficient to compensate for the social cost of reduced investment.

This analysis applies to the antitrust setting in which a firm alleges that the exclusionary acts of a dominant firm reduced incentives to innovate. One frequent complaint during the Microsoft litigation was that the firm’s aggressive tactics deterred innovation by smaller rivals in Silicon Valley.\textsuperscript{21} The optimal penalty derived here implies that antitrust enforcement should be more aggressive where the defendant’s conduct discourages innovation. Although this policy recommendation is not easy to translate into legal standards, the general direction is clear. A rule-of-reason analysis should incorporate the rival investment effect as part of the evaluation of the anticompetitive harms of the dominant firm’s conduct.\textsuperscript{22}

\textsuperscript{20} Consider the case where the offender burns down the crops and destroys the houses of farmers in his town. Fearing the loss of their crops, the farmers will be less willing to invest in planting seeds. Suppose, in view of the fear of crop damage, only half of the farmers are planting when the penalty is set at the optimal level; and that they expect to yield $5000. The social gain from investment is $2500. Let the direct harm from setting fire to the house be $10,000, and the enforcement cost be $400. If the minimum apprehension probability is 10 percent, the optimal penalty is ($10,000)/(.10) + ($400) + ($2500)/(.10) = $125,400.

\textsuperscript{21} 253 F.3d 34, 65 (D.C. Cir. 2001).

\textsuperscript{22} It should be clear that the offender-investment and victim-investment models can be combined to yield an optimal penalty formula that balances opposing externalities. In the combined model, the penalty would be reduced relative the benchmark static penalty to the extent that punishment reduced the social gain from investment by the offender, and the penalty would be enhanced to the extent that punishment of the offender encouraged potential victims to invest. The net direction would be ambiguous a priori, but could be simulated in this model under various parameter assumptions. Segal and Whinston (2007) suggest that a policy that protects victims would be preferable because it would frontload profits to new innovators (entrants). These considerations complicate the penalty analysis and raise questions about the ability of an enforcement authority to implement an optimal policy without committing errors.
D. Headline Effects and Penalties

News headlines may alter the investment decisions of offenders or victims. For example, a potential offender may read news headlines about punishments dealt to other offenders. The headlines may lead the offender to believe that the likelihood of apprehension is greater (or less) than it is.

One way to model headline effects in the offender-investment case is to let the perceived probability of apprehension differ from the real probability of apprehension: \( \tilde{z} = z(1+\mu) \). The optimal policy is captured in the equation:

\[
F^* = \frac{1}{1+\mu} \left\{ \frac{v}{z'} + c - \frac{S}{z'} \left( \frac{\psi(\tilde{k}_o^+)(1 - H(z'(1+\mu)F^*))}{\Psi(\tilde{k}_o^+)h(z'(1+\mu)F^*) + \psi(\tilde{k}_o^+)(1 - H(z'(1+\mu)F^*))^2} \right) \right\}
\]

Suppose offenders overestimate the likelihood of apprehension, so that \( \mu > 0 \). The optimal fine can be examined as if it consists of two parts, one influencing incentives in the static setting ((13), first two terms) and the other influencing the investment decision (last term). In the absence of any investment with a social payoff, the optimal fine would be reduced to compensate for the offender’s overestimate of the likelihood of apprehension. The total surplus component complicates matters, though. Because the portion of the penalty regulating the offender’s incentives with respect to the total surplus is ambiguous, offender overestimation of the

For example, a policy of protecting new innovators could be distorted in practice to protect unsuccessful innovators at the expense of successful innovators.
likelihood of apprehension could raise or lower the optimal penalty. Moreover, it is plausible to think the investment effect would be amplified relative to the other terms, because the offender invests in an earlier time period in which he has the least information about the real probability of apprehension.

Now consider the investment decision of the victim. The victim invests while relying on news headlines to project the likelihood that he will reap the rewards of his investment. Suppose the break even cost level for the victim is \( \bar{k}_v = (1 + \eta)H(zF)B \), where \( \eta > 0 \) means that the victim underestimates the likelihood of an offense that destroys his investment. Now the optimal penalty satisfies

\[
F^* = \frac{v}{z} + c + \frac{R(\bar{k}_v)B - \eta(1 + \eta)B^2r(\bar{k}_v)H(zF)}{z},
\]

which implies that the penalty assessed against the offender will be reduced when the victim underestimates the likelihood an offense. The penalty will be increased when the victim overestimates the likelihood of an offense. The reason for reducing the penalty when the victim underestimates is to align the private and social incentives to invest. If the victim thinks that there will not be an offense, he will invest too much in light of the objective return. The penalty is reduced in this case in order to indirectly diminish the investment incentives of the victim.\(^{23}\)

The parameter \( \eta \) is an index of the alarm-to-danger ratio identified by Bentham (1789, at 153). If \( \eta \) is equal to zero, the danger and alarm caused by crime are the same. The

\(^{23}\) A similar problem is encountered in the context of crime and victim precaution. The optimal fine varies in order to control the incentives of both the offender and the victim, see Hylton (1996).
impressions potential victims get from reading newspaper headlines are accurate indicators of the likelihood of an offense. If $\eta$ is positive, the alarm is less than the danger, and if $\eta$ is negative the alarm is greater than the danger. The optimal penalty in (14) implies that the penalty should increase as the alarm increases relative to the danger. The reason for this is that alarm has a discouraging effect on investment by potential victims. Given that, the penalty should be increased in order to internalize the negative investment effect.

III. Discussion

A. Antitrust

The optimal punishment literature does not translate easily into recommendations for the law. As we noted at the outset, the models assume strict liability with case-specific optimal penalties; while the law adopts cost-benefit tests for liability and uses standardized penalties. Still, the results can be used to offer suggestions about factors the law should considered.

The offender investment model suggests that the social payoff from innovation antecedent to monopolization should be part of rule of reason analysis under Section 2 of the Sherman Act. One could argue that the law reflects this recommendation already – that is, that the optimal punishment model of this paper provides a justification for some basic features of Section 2 antitrust doctrine. Monopolization law does not hold monopolies per se liable. The law immunizes firms from liability when they have acted merely as profit-maximizing monopolists (e.g., setting the monopoly price). Liability is imposed under Section 2 for

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24 The legal standard is sometimes described as a balancing test that compares anticompetitive harms with procompetitive benefits, see U.S. v. Microsoft, 253 F.3d 34, 59 (D.C. Cir. 2001).
25 In Alcoa, Judge Learned Hand distinguished monopolies that are passively acquired, and monopolies acquired through superior skill, foresight, and industry, from monopolies that are actively acquired. See Alcoa, 148 F.2d, at
predatory conduct and efforts to exclude rivals. Exploitative conduct is distinguished from exclusionary conduct.

The law is underinclusive in its regulatory reach in comparison to the optimal penalty model, which would not provide a blanket exemption to firms that merely exploit their market power. If the residual consumer surplus generated was small relative to the surplus transferred and destroyed, the firm would pay a fine under the optimal penalty regime even though all it did was exploit its power. However, antitrust law does not make such detailed distinctions. It formally exempts all firms that merely exploit their power.

An example can clarify this point. If a firm acquires its monopoly through exclusionary conduct, it may be held liable under Sherman Act Section 2. On the other hand, if a firm acquires monopoly status because firms supplying key resources to its rivals all go out of business, then it will not be found in violation of the Sherman Act. In both scenarios, the harm to consumers is the same. The law imposes the threat of liability in the first scenario and exempts the firm in the second scenario. The optimal punishment model would generate the same penalty in both scenarios.

Although antitrust law is underinclusive in its reach in comparison to the optimal penalty model, the exemption provided to firms that merely exploit their market power rather than engage in predation can be understood as an attempt by the law to take the welfare gains from innovation into account. One fundamental paradox of antitrust, stressed in Judge Hand’s *Alcoa* opinion, is that cartel pricing is treated as per se illegal, while monopoly pricing is per se

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429-431. Carlton and Heyer (2008) propose the distinction between extraction and extension as normative guideline for monopolization law. Of course, this model distinguishes creation (innovation) from extraction of surplus. The distinctions are not the same, but there is an overlap. In this model, there are cases that might be described as extension where punishment is still unwarranted.
lawful. This apparent paradox is resolved under this paper’s model. Firms do not receive a direct subsidy for innovation under the law, but they are also not punished for merely exploiting the monopoly power that results from innovation.

For firms that engage in exclusionary conduct the law is overinclusive, in the sense that it does not reduce expected penalties for firms that have created or expanded markets through innovation. One exception is *United States v. Jerrold Electronics Corp.* The court held that rule of reason rather than per se liability applied to Jerrold’s tying policies that were instituted at a time when the market for antennae systems was in its infancy. Jerrold had played a big role in the creation of the market. The court exempted Jerrold from antitrust liability for the infancy period of its business, effectively a penalty reduction based on innovation. The optimal penalty model implies that the doctrine of *Jerrold Electronics* should be incorporated into Section 2 law.

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26 The paradox served as a key justification for Judge Learned Hand’s reform of monopolization law in *Alcoa*. Hand argued that monopolies (monopoly pricing) should be viewed in the same way as cartels (cartel pricing). *Alcoa*, 148 F.2d., at 427-29. For a discussion of innovation and its implications for Hand’s argument, see Evans and Hylton (2008).

27 This model provides a theoretical justification for the structure of monopolization case law under Section 2 of the Sherman Act. The static penalty approach fails to provide such a justification. Implementation issues would arise only if the law were to adopt an administrative process that led to the assessment of optimal case-specific penalties. We think such an approach is feasible – it would require estimates of the monopoly welfare components and the incentive parameters of the model – though it would be expensive. At present, the law has not adopted such an approach. Still, the legal doctrine has developed to mirror the issues that would be quantified under the administrative approach.


29 Baker (2005) argues that enforcement authorities are sensitive to innovation effects in their targeting decisions. If enforcement authorities could target only those cases in which the monopolization effect is substantially greater than the investment effect ($\theta$ close to zero in this model), then Baker’s argument suggests that antitrust enforcement may be optimal, and will operate as if the *Jerrold Electronics* doctrine applied generally. But if, under this paper’s model, the two effects are equal or close ($\theta$ close to one), enforcement would reduce society’s wealth.
A symmetrical and opposing conclusion applies where the dominant firm’s conduct discourages innovation by rivals. The rule of reason should take into account credible evidence that the monopolist’s conduct reduced innovation by potential competitors.\(^{30}\)

More generally, the distinction between complete deterrence and internalization as punishment policy goals in this model provides a basis for distinguishing areas of per se and rule of reason liability. When internalization is the optimal approach, a rule of reason analysis would be the proper framework in the law. In that analysis some attempt would be made to compare the social benefits from innovation with the social costs of discouragement of rivals’ innovation. Both are externalities that could tilt the court’s analysis either away or toward a finding of liability in comparison to the static benchmark analysis.

To be more concrete, the rule-of-reason test for monopolization has been described as a balancing test which compares the anticompetitive harms with the procompetitive benefits of the firm’s conduct. Under *U.S. v. Microsoft*, the plaintiff bears the burden of proving a substantial harm to consumer welfare,\(^{31}\) and then the burden then shifts to the defendant to either prove that there was no consumer harm or to proffer a procompetitive benefit. If the defendant cannot disprove consumer harm, he would have to identify static efficiency gains. This paper points to a special category of procompetitive benefit, the residual consumer surplus from innovation, that should be recognized as a counterweight in the balancing test.\(^{32}\)

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\(^{30}\) Of course, counterfactual arguments about innovation that would have occurred are difficult to assess. If courts were to take negative innovation effects into account under the monopolization law, some effort would have to be made to weed out spurious claims.

\(^{31}\) 253 F.3d 34, 59 (D.C. Cir. 2001).

\(^{32}\) The key factor distinguishing the static model from the dynamic (offender investment) model is the existence of a positive externality in the latter model. It happens that the positive externality is connected to innovation. However, any positive externality connected to the firm’s monopolization would require the inclusion of a subsidy in the optimal penalty. The dynamic model is special because it implies sharp limits on the size of the penalty. Although Becker examined innovation as a special case of his model, and noted that a subsidy would be optimal (Becker, at
B. Penalties and Torts Generally

This model has implications for the literature on optimal regulatory fines and tort damages. Polinsky and Shavell (1992), building on Becker (1968), concluded that the “optimal fine equals the harm, properly inflated for the chance of not being detected, plus the variable enforcement cost of imposing the fine.” But the literature on penalties does not incorporate the investment effects of offensive conduct or of punishment.

Consider the case in which offensive conduct discourages investment decisions of potential victims. This was first considered by Bentham, who referred to the “secondary effects” of criminal behavior (Bentham 1789, at 153). Bentham claimed that offensive conduct led to primary and secondary harms to society. The primary harms are the direct and derivative losses, as well as the enforcement costs. The secondary harms are the costs that result from the discouraged investment and extend “either over the whole community, or over some multitude of unassignable individuals” (Bentham, at 153). Secondary effects could include a range of costs generated by changes in behavior in response to the fear of crime. Bentham argued that punishment should be enhanced to reflect the secondary costs of crime. This penalty formula derived here implies, consistent with Bentham, that the optimal penalty should be enhanced to internalize the secondary costs of offensive conduct.

Another type of secondary effect unexamined in the previous literature is the case in which punishment affects productive investment decisions by the offender. This is the case examined in the offender investment model.

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202-204), he did not examine the case examined here, where the offender’s conduct both inflicts injuries and benefits society.
Consider the theory of tort remedies. The damage multiplier approach of Polinsky and Shavell (1998) suggests that the optimal tort damage award will divide the harm by the probability of liability. This is likely to be inadequate as a method of internalizing social costs when there are negative investment effects from offenses.

Conversely, the damage multiplier approach overdeters when the offender’s investment yields a positive externality (e.g., product innovation) and is dependent on profits from a later action that causes harm, as in the innovation-monopolization model. This implies that strict liability, coupled with a damage multiplier, is inappropriate in settings in which the offender is responsible for a significant positive externality.

Suppose the firm invests in a new vaccine in the first period. In the second period, when the product is on the market, it can take precaution to reduce the likelihood of harm to consumers. The precaution could take many different forms: enhancing the warning label, or better monitoring of the production process. The firm decides in the second period whether to take the precaution by comparing the cost of precaution to the expected fine. This description of the vaccine marketing is analogous to the investment-monopolization model examined earlier. It follows that the optimal penalty will depend on several factors: the harm to the consumer, the positive externality to society (consumer surplus from innovation and other beneficial externalities from vaccination), the degree to which an increase in the penalty affects the investment incentive versus the precaution incentive. As in the monopolization problem examined earlier, the optimal punishment may be no punishment at all – or even a subsidy.

Obviously, this has implications for strict products liability. In particular, strict products liability may not be optimal in the innovation setting, in which the firm’s investment has generated a substantial positive externality such as the creation of a market, or in a setting in
which the firm’s product marketing yields beneficial spillovers (e.g., vaccines). As long as the injuries caused by the product are not too large in relation to the surplus created, the negligence standard would be preferable. Products liability litigation has generated a few court decisions consistent with this prescription.33

IV. Conclusion

Using a model of punishment based on strict liability with monetary penalties, we have examined the design of optimal penalties in a dynamic setting where actors make investment decisions that are affected by the penalty. One important scenario examined is that in which the offender invests in an activity that benefits society, and the private return to that activity is a function of the offense he later commits as well as the penalty for that offense. The optimal penalty will internalize the costs of the offender’s conduct and subsidize the offender’s investment. In the monopolization setting, the optimal penalty will be a weighted average of a penalty based on the consumer harm and a subsidy based on the residual surplus created by investment, with the weights depending on the relative responsiveness of investment and monopolization incentives to the penalty. The subsidy component may be greater than the penalty component – generating a reward for monopolization rather than a penalty. Although a reward for monopolization seems counterintuitive initially, and certainly inconsistent with static

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33 In the case of drugs, the law has shown some signs of incorporating this implication, though the record is mixed. Comment k of Restatement (Second) of Torts, Section 402A, suggests that courts should exempt drug makers from strict liability for product defects and defective designs. In Brown v. Superior Court, 44 Cal. 3d 1049 (1988), the California Supreme Court held that comment k insulated all Food and Drug Administration-approved prescription drugs from strict liability for design defect. The Court reasoned that strict liability would deter innovation of new drugs. The California courts later applied the same reasoning to implanted prescription medical devices; see Artiglio v. Superior Court, 22 Cal. App. 4th 1388 (1994); Plenger v. Alza Corp., 11 Cal. App. 4th 349 (1992). However, Brown has been adopted in only a minority of U.S. states.
punishment models, the law on monopolization is actually consistent with the implications of the dynamic punishment model. We explored normative implications for monopolization law as well.

The same result applies to the product safety scenario. If the firm invests in a product that creates a new market, the optimal penalty will include a reward for innovation, and on net may be zero or negative. Since innovation-based exemptions from products liability law are recognized in only a minority of jurisdictions, the implications of this model for products liability law are largely normative.
Appendix

Proof of Proposition 1: The social planner’s problem is to choose $z$ and $F$ to minimize

$$C = H(zF)(E(M | M \leq zF)) + (1 - H(zF))(v + zc)$$

$$= \int_0^{zF} Mh(M)dM + (1 - H(zF))(v + zc)$$

The first-order conditions are:

$$\frac{\partial C}{\partial F} = -zh(zF)(v + zc - zF)$$

$$\frac{\partial C}{\partial z} = -Fh(zF)(v + zc - zF) + (1 - H(zF))c$$

Note that when $z^{*}$ and $F^{*}$ are chosen so that $z^{*}F^{*}$ is greater than $M_{u}$, then the offender is completely deterred and the above equation equals to $E(M)$. We discuss the optimal choice of enforcement rate and penalty below.

(1) $v + z^{*}c \geq M_{u}$

This is the case where the minimum cost from an offense is higher than the maximum benefit to the offender. In this case it is optimal to eliminate offenses by setting $z^{*}F^{*}$ is greater than $M_{u}$. Here is the proof.

$$C = H(zF)(E(M | M \leq zF)) + (1 - H(zF))(v + zc) \geq H(zF)(E(M | M \leq zF)) + (1 - H(zF))M_{u}$$

$$> H(zF)(E(M | M \leq zF)) + (1 - H(zF))(E(M | M \geq zF)) = E(M)$$

In this scenario, $E(M)$ is the lower bound for social cost, as in Figure A1. Given a specific enforcement rate $z$, $F^{*} \in \{F: F \geq M_{u}/z\}$.

(2) $v + c < M_{u}$

Given a specific value of $z$, let $F^{*} = \frac{v}{z} + c$ and $F^{L} = \frac{M_{u}}{z}$. Note here that $F^{*} < F^{L}$.

When $F < F^{*}$, $C$ is decreasing and when $F > F^{*}$, $C$ is increasing until it reaches $E(M)$ for $\forall F \geq F^{L}$. Figure A2 shows the relationship. Thus for a given $z$, the optimal choice of $F$ is $F^{*}$.
Moreover, given $F^*$, $C$ is increasing in $z$ so that it is optimal to set $z^* = z'$. Consequently, the optimal policy is \{ $z^* = z'$, $F^* = \frac{v}{z'} + c$ \} and the corresponding social cost is 
\[ C^* = H(v + z'c)(E(M|M \leq v + z'c)) + (1 - H(v + z'c))(v + z'c). \]
Note that $C^*$ is smaller than $E(M)$.

(3) $v + z'c < M_u \leq v + c$

Using the same logic, we know that given a specific value of $z$, the optimal $F$ is $F^* = \frac{v}{z} + c$. Depending on the value of $z$, now $F^*$ may be smaller or bigger than $F^L$. As for $\forall F \geq F^L$, $C$ remains at the level $E(M)$ so it would be better to have $F^* < F^L$ so that we could reach the cost level less than $E(M)$. Thus in this scenario, the optimal policy is again reached when $z$ is set at its minimal level and $F$ equals to $F^*$, namely, \{ $z^* = z'$, $F^* = \frac{v}{z'} + c$ \}. ☐

![Figure A1](image-url)
**Proof of Proposition 2:** The proof follows in three steps.

First, we show that it is never optimal to set \( z^*F > M_u \). If \( z^*F \geq M_u \), the offender will not make any investment and there will be no social gain. Instead, for any \( z^*F < M_u \), we observe a positive social benefit as

\[
NB > \Psi(k^*_o(1-H(zF^*)))\{(1-H(zF^*)E(M|M > zF^*)-k^*_o+(1-H(zF^*)(S-v-zc)+H(zF^*)S
\]

\[
= \Psi(k^*_o)\{(1-H(zF^*)+S-v-zc)+H(zF^*)S
\]

\[
= \Psi(k^*_o)/\Psi(k^*_o)h(zF^*)+\psi^*(1-H(zF^*))^2S > 0
\]

Second, we need to prove that the optimal penalty \( F^* \) is the unique solution to the global maximum. The first order condition (equation 3 in the paper) can be simplified as follows:
\[
\frac{\partial NB}{\partial F} = (v + c - zF)\left\{\psi(\kappa_o)h(zF) + \psi(\kappa_o)(1 - H(zF))^2\right\} - S\{\psi(\kappa_o)(1 - H(zF))\}
\]
\[
= \frac{P}{z} \left\{\frac{v}{z} + c - \frac{S}{z}\theta\right\}
\]

where \(P = \psi(\kappa_o)h(zF) + \psi(\kappa_o)(1 - H(zF))^2\) and \(\theta = \frac{\psi(\kappa_o)(1 - H(zF))}{P}\). Since \(P > 0\), if there exists \(F^*\) such that

\[
F = \frac{v}{z} + c - \frac{S}{z}\theta(F)
\]

then we will have \(\frac{\partial NB}{\partial F}\bigg|_{F=F^*} = 0\).

To proceed, let’s first take a look at \(\theta\), which is a function of \(F\). When \(F \leq 0\), \(\theta = 1\); when \(F > 0\), \(\theta\) is strictly increasing in \(F\) because (1) \(\frac{\psi(\kappa_o)}{\psi(\kappa_o)}\) is increasing in \(\kappa_o\) and \(\kappa_o\) is decreasing in \(F\); (2) \(\frac{h(zF)}{(1 - H(zF))}\) is decreasing in \(F\); (3) \((1 - H(zF))\) is decreasing in \(F\); (4) both \(\frac{\psi(\kappa_o)}{\psi(\kappa_o)}\) and \(\frac{h(zF)}{(1 - H(zF))}\) are positive.

Let \(\theta_{\text{min}}\) be the value of \(\theta\) when \(F\) is approaching zero. It is straightforward that

\[
\theta_{\text{min}} = \lim_{F \to 0}\theta = \frac{\psi(\kappa_o)}{\psi(\kappa_o)h(zF) + \psi(\kappa_o)} < 1. [\text{The value of } \theta_{\text{min}} \text{ will depend on the shape of the distributional forms of } \psi(\cdot) \text{ and } H(\cdot). \text{ For example if we assume that both of them}}
\]
take an exponential form with mean of $1/\lambda^y$ and $1/\lambda^H$, then
\[
\theta_{\text{min}} = \frac{1}{(\lambda^H/\lambda^y)(e^{\lambda^y/\lambda^H} - 1) + 1} < \frac{1}{2}.
\]

Let’s then consider equation (2B). The left-hand side (LHS) of equation (2B) is $F$ itself, which could be treated as strictly increasing function of $F$. The right-hand side (RHS) then is a nonincreasing function where
\[
RHS = \begin{cases} \frac{v}{z} + c - \frac{S}{z} & \text{if } F \leq 0 \\ \frac{v}{z} + c - \frac{S}{z} \theta & \text{if } F > 0 \end{cases}
\]
and the RHS reaches its maximum at $\frac{v}{z} + c - \frac{S}{z} \theta_{\text{min}}$ when $F \to 0$. The following three cases are explained for the equality of the LHS and RHS.

Case 1: $\frac{v}{z} + c - \frac{S}{z} \geq 0$

It is obvious that in this case, $\frac{v}{z} + c - \frac{S}{z} \theta_{\text{min}} > 0$. There exists a unique solution such that
\[
LHS = RHS \quad \text{and the unique equilibrium is } F^* = \frac{v}{z} + c - \frac{S}{z} \theta^* > 0.
\]

The proof of the global maximum is quite straightforward: when $F < F^*$, $\frac{\partial N\!B}{\partial F} \bigg|_{F<F^*} > 0$; and when $F > F^*$, $\frac{\partial N\!B}{\partial F} \bigg|_{F>F^*} < 0$. This case corresponds to the graph in Figure A3 below.
Before moving to Case 2, we will briefly discuss two properties of the Case 1 solution.

Proposition 2a: As $v$ goes to infinity, the optimal penalty converges to a constant $F^{**}$ that satisfies $\theta(F^{**}) = 1$.

Proof: Recall that the optimal penalty $F^*$ satisfies:

$$RHS = \frac{v}{z} + c - \frac{S}{z} \theta(F^*) = c - \frac{w}{z} + \frac{(v+w)}{z}(1-\theta(F^*)) = F^* = LHS$$

First, it is easy to show that $F^*$ is increasing in $v$. To see why, assume that $v$ is increased to $v'$ and the associated optimal penalty is $F^{*'}$. We need to show that $F^{*'} > F^*$. Assume otherwise that $F^{*'} \leq F^*$, $\theta(F^{*'}) \leq \theta(F^*)$, so that $RHS(F^{*'}) > RHS(F^*)$. However $LHS(F^{*'}) \leq LHS(F^*)$ so that $RHS(F^{*'}) > LHS(F^*)$, a contradiction. Second, we need to show that when $v$ goes to infinity, $F^*$ does not go to infinity. Otherwise if $F^*$ goes to
infinity, \( \theta \) will also go to infinity. As a result, \( RHS(F^*) \) goes to negative infinity, which contradicts the fact that \( LHS(F^*) \) goes to positive infinity. Last, since \( F^* \) is increasing in \( v \) and since \( F^* \) is finite as \( v \) goes to infinity, then it must be the case that \( \theta \) goes to 1 so that the optimal penalty makes \( RHS = LHS \). We have shown that \( \theta \) is increasing in \( F \), so that the optimal penalty is \( F^{**} \) which satisfies \( \theta(F^{**}) = 1 \).

**Proposition 2b:** When \( z \) is decreasing, \( F^* \) is ambiguous.

**Proof:** (1) It is easy to show that when \( z \) is decreasing, \( zF^* \) is decreasing as well. To see why, since \( \frac{v}{z_0} + c - (S/z_0)\theta(z_0F_0^*) = F_0^* \), we have \( v + z_0c - S\theta(z_0F_0^*) = z_0F_0^* \). Now \( z_0 \) is decreased to \( z_1 \) and \( v + z_1c - S\theta(z_1F_1^*) = z_1F_1^* \). If \( z_1F_1^* \geq z_0F_0^* \), we know that \( S\theta(z_1F_1^*) \geq S\theta(z_0F_0^*) \) (as \( \theta \) is increasing in \( zF \)) and so that \( v + z_1c - S\theta(z_1F_1^*) < v + z_0c - S\theta(z_0F_0^*) \). However, if \( z_1F_1^* \geq z_0F_0^* \), \( v + z_1c - S\theta(z_1F_1^*) \geq v + z_0c - S\theta(z_0F_0^*) \) by the definition of the optimum. We see a contradiction here so that \( z_1F_1^* < z_0F_0^* \). The remaining question is whether \( F^* \) is increasing or decreasing. Using the implicit function theorem, \( \frac{\partial F^*}{\partial z} = \left[ c - (1+s\theta')F^* \right] \frac{1}{z(1+s\theta')} \), where \( \theta' = \frac{\partial \theta}{\partial(zF)} \), and this depends on the comparison of \( c \) and \( (1+s\theta')F^* \), which further depends on the function \( \theta \), especially the value of \( \theta' \) at \( F^* \).

**Case 2:** \( \frac{v}{z} + c - \frac{S}{z} < 0 \) and \( \frac{v}{z} + c - \frac{S}{z} \theta_{\min} < 0 \)

Again we obtain a unique optimum \( F^* = \frac{v}{z} + c - \frac{S}{z} < 0 \) for \( LHS = RHS \). \( F^* \) is the global maximum as when \( F < F^* \), \( \left. \frac{\partial NB}{\partial F} \right|_{F<F^*} > 0 \); and when \( F > F^* \), \( \left. \frac{\partial NB}{\partial F} \right|_{F>F^*} < 0 \). This case corresponds to the graph in Figure A2.
Case 3: \( \frac{v}{z} + c - \frac{S}{z} < 0 \) and \( \frac{v}{z} + c - \frac{S}{z} \theta_{min} \geq 0 \)

There exist two \( F^* \) (one for \( F^* < 0 \) and the other for \( F^* \geq 0 \)) such that \( LHS = RHS \):

\[ F_1^* = \frac{v}{z} + c - \frac{S}{z} < 0 \quad \text{and} \quad F_2^* = \frac{v}{z} + c - \frac{S}{z} \theta^* > 0. \]

It can be verified that both are local maximums since \( \frac{\partial NB}{\partial F}_{F<F_1^*} > 0 \) and \( \frac{\partial NB}{\partial F}_{0<F>F_1^*} < 0 \); and \( \frac{\partial NB}{\partial F}_{\theta<F>F_2^*} > 0 \) and
\[
\frac{\partial NB}{\partial F} \bigg|_{F>F^*_2} < 0. \text{ The solutions are illustrated in Figure A5. The global maximum is the F}^* \text{ that provides the higher social benefit.}
\]

Consider the following comparison of the two \(F^*\) solutions:

\[
F_1^* = \frac{v}{z} + c - \frac{S}{z} < 0 \text{ so that } \overline{k}_o = \overline{M} - zF_1^* = \overline{M} + S - v - zc
\]
\( NB_1 = \Psi(\overline{k}_o)\{\overline{M} - E(k|k < \overline{k}_o) + (S - v - zc)\} \\
= \Psi(\overline{k}_o)\overline{F}_o - \int_0^{\overline{F}_o} k\psi(k)dk \\
F_2^* = \frac{v}{z} + c - \frac{S}{z} \theta^* > 0 \) so that \( \overline{k}_o^2 = \int_{zF_2^*} Mh(M)dM - (1 - H(zF_2^*))zF_2^* < \overline{k}_o^1 \\
\[ NB_2 = \Psi(\overline{k}_o)\{(1 - H(zF_2^*))E(M|M > zF_2^*)\overline{M} - E(k|k < \overline{k}_o^2)\}+ (1 - H(zF_2^*))zF_2^*\theta^* + (1 - H(zF_2^*))S \]
\[ = \Psi(\overline{k}_o)\overline{F}_o + (1 - H(zF_2^*))zF_2^* - E(k|k < \overline{k}_o^2)\theta^* + (1 - H(zF_2^*))S \]
\[ = \Psi(\overline{k}_o)\overline{F}_o - \int_0^{\overline{F}_o} k\psi(k)dk + S - S(1 - H(zF_2^*))\theta^* \]
Note that \( \theta^* \) must be smaller than one since \( F_1^* = \frac{v}{z} + c - \frac{S}{z} < 0 \) and \( F_2^* = \frac{v}{z} + c - \frac{S}{z} \theta^* > 0 \). Also note that \( \Psi(\overline{k}_o)\overline{F}_o - \int_0^{\overline{F}_o} k\psi(k)dk \) is strictly increasing in \( \overline{k}_o \) and \( \overline{k}_o^2 < \overline{k}_o^1 \), we have \( \Psi(\overline{k}_o)\overline{F}_o - \int_0^{\overline{F}_o} k\psi(k)dk > \Psi(\overline{k}_o)\overline{F}_o - \int_0^{\overline{F}_o} k\psi(k)dk \). Still, \( NB_2 \) could be greater than \( NB_1 \) given a large enough \( S \). The net subsidy solution \( F_1^* \) is preferable to the penalty \( F_2^* \) if the enhanced wealth from additional investment is greater than the wealth gained from deterring monopolization with the penalty.

Lastly, we show that it is optimal to set \( z = z' \). The first-order condition with respect to \( z \) is as follows,
\[ \frac{\partial NB}{\partial z} = F(v + zc - S - zF)\{\Psi(\overline{k}_o)h(zF) + \psi(\overline{k}_o)(1 - H(zF))^2\} - \psi(\overline{k}_o)(1 - H(zF))cF - FS[\psi(\overline{k}_o)H(zF)(1 - H(zF)) - \Psi(\overline{k}_o)h(zF)] \]
Given that \( F = F^* \), \( \frac{\partial NB}{\partial z} \bigg|_{F=F^*} = -\psi(\overline{k}_o)(1 - H(zF^*))c < 0 \) so that it is optimal to choose the minimum enforcement rate.
Proof of Proposition 3: The social welfare function can be expressed as

\[ NB = R(k_v)(\bar{k}_v - E(k|k < \bar{k}_v)) + (1 - H(zF))\{E(M|M > zF) - (v + zc)\} \]

where \( \bar{k}_v = H(zF)B \).

Assume that \( zF < M_u \). The two first-order conditions for the welfare maximization problem are:

\[ \frac{\partial NB}{\partial F} = R(k_v)Bzh(zF) + zh(zF)(zc + v - zF) \]

where the first term denotes marginal gain from investment; and

\[ \frac{\partial NB}{\partial z} = Fh(zF)(zc + v - zF + R(k_v)B) - (1 - H(zF))c \]

Using the same logic of the basic model (see Proposition 1), it is easy to show that for a given value of \( z \), \( F \) is optimally set so that \( F^* = \frac{v}{z} + c + \frac{R(k_v)B}{z} \). And since \( NB \) is decreasing in \( z \) when \( F^* = \frac{v}{z} + c + \frac{R(k_v)B}{z} \), it is optimal to choose enforcement rate at its minimal level, namely \( z^* = z' \).

Now let \( NB^* \) denote the social welfare evaluated at \((z', F^*)\), namely,

\[ NB^* = R(k_v')(\bar{k}_v' - E(k|k < \bar{k}_v')) + (1 - H(z'F^*))\{E(M|M > z'F^*) - (v + z'c)\} \]

And let \( \overline{NB} \) be the social welfare evaluated when \( F^* \in \{F:F \geq M_u/z\} \). In this latter scenario, the offense is completely deterred and social welfare depends solely on the victim’s net gain from investment, where \( \overline{NB} = R(B)[B - E(k_v|k_v < B)] \).

(1) \( M_u \leq v + z'c + R(k_v')B \)
In this case, it is optimal to set $z^*F^*$ greater than $M_u$ as $NB^*$ is smaller than $\overline{NB}$.

\[
\begin{align*}
NB^* - \overline{NB} &< R(\overline{k_v})(\overline{k_v} - E(k | k < \overline{k_v})) + (1 - H(z^*F^*))(M_u - (v + z'c)) - R(B)[B - E(k_v | k_v < B)] \\
&\leq R(\overline{k_v})(\overline{k_v} - E(k | k < \overline{k_v})) + (1 - H(z^*F^*))R(\overline{k_v})B - R(B)[B - E(k_v | k_v < B)] \\
&= \int_{\Delta}^{\beta} kr(k)dk - B\int_{\Delta}^{\beta} r(k)dk \\
&< B\int_{\Delta}^{\beta} r(k)dk - B\int_{\Delta}^{\beta} r(k)dk = 0
\end{align*}
\]

(2) $M_u > v + z'c + R(\overline{k_v})B$

First, let $\Delta = NB^* - \overline{NB}$ and it is easy to show that $\Delta$ is increasing in $M_u$ as

\[
\frac{\partial \Delta}{\partial M_u} = M_u h(M_u).
\]

Let $\overline{M}$ denote the value of $M_u$ such that $NB^* - \overline{NB} = 0$. Based on case (1), it is obvious that $\overline{M}$ is bigger than $v + z'c + R(\overline{k_v})B$. For $M_u \in \{M_u : M_u > \overline{M}\}$, $NB^* > \overline{NB}$ so that \{z', F^*\} is the optimal policy; for $M_u \in \{M_u : v + z'c + R(\overline{k_v})B < M_u \leq \overline{M}\}$, $NB^* \leq \overline{NB}$ so that $z^*F^* > M_u$ is the optimal policy.\]
References


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