ESTIMATES OF FIRMS’ PATENT RENTS FROM FIRM MARKET VALUE

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Estimates of Patent Rents from Firm Market Value

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Abstract: The value of patent rents is an important quantity for policy analysis. However, estimates in the literature based on patent renewals might be understated. Market value regressions could provide validation, but they have not had clear theoretical foundations for estimating patent rents. I develop a simple model to make upper bound estimates of patent rents using regressions on Tobin’s $Q$. I test this on a sample of US firms and find it robust to a variety of considerations. My estimates correspond well with estimates based on patentee behavior outside the pharmaceutical industry, but renewal estimates might be understated for pharmaceuticals.

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Introduction

Patents are intended to provide an economic incentive for invention by granting the patent holder an exclusive right for a limited period. This right to exclude allows a patent-holding firm to become a monopolist or, perhaps more often, to achieve some lesser degree of market power, either in product markets or in the market for technology licenses. This market power, in turn, permits the firm to earn supra-normal profits, “rents,” and these are the source of the economic incentive to invent.

The value of patent rents is thus an important quantity for evaluating the performance of the patent system and also for understanding firm value. Some researchers have used the observed behavior of patent owners to estimate the private value of patents, which should equal the discounted value of patent rents.\(^1\) Beginning with Pakes and Schankerman (1984), these studies have imputed patent value from observed decisions to pay maintenance fees,\(^2\) decisions to file patents in multiple countries (Putnam 1996), and decisions to sell (re-assign) patents (Serrano 2006).

But these approaches share an important limitation: they do not directly reflect the value of the most valuable patents and, given the skewed distribution of patent values, most of the aggregate value of patents is determined by the relatively small number of highly valuable patents. These studies typically assume a distributional form, such as a log-normal distribution. They then fit that distribution to the observed data and extrapolate to the upper tail. However, if the upper tail diverges significantly from the assumed distribution, then estimates of mean patent value might be too large or too small (although estimates of median patent value obtained from these methods are accurate). In the worst case, the upper tail might be so “heavy” that the actual distribution has an infinite mean as with the Pareto distribution (Scherer and Harhoff 2000). Then estimates of the mean would be unstable and would not converge even at asymptotically large sample sizes.

An alternative might be to use firm market value to estimate patent value, that is, to decompose firm value into its component parts including that part attributed to patents. This way, investor behavior, rather than the behavior of patent owners, might reveal patent value. At the very least, estimates based on firm market value might serve as an important check on the values obtained from data on the behavior of patent owners.

\(^1\) Other researchers have used surveys to assess inventors’ view of patent values (Harhoff et al. 2003a, Gambardella et al. 2006). However, these studies obtain a measure of patent value that is equivalent to the value of patent rents plus the value of the invention realized by other means (see Harhoff et al. 2003a). See section 4.4 below.

\(^2\) See Lanjouw et al. 1998 for a review of this literature and Baudry and Dumont (2006), Bessen (2006), Gustafsson (2005) and Serrano (2006) for more recent estimates of renewal value.
A large number of researchers have run regressions that use firm market value (or Tobin’s $q$, which is firm market value divided by the replacement value of firm assets) as the dependent variable and some measure of patents (patent flows, patent stocks, or patent citation stocks) as an independent variable. Many of these studies build on Griliches’s (1981) “hedonic” model of the firm, where investors are assumed to view the firm as a bundle of characteristics that determine the firm’s value. Patents are assumed to be one of these characteristics.

These studies generally show a positive correlation between a firm’s patents and its market value. In his seminal paper, Griliches estimated that the regression coefficients implied that “a successful patent is worth about $200,000.” However, subsequent studies have generally not calculated the implied value of firm patent rents. Perhaps this is because researchers recognize that the correlation between firm value and patents may involve more than just the direct contribution of patent rents to firm value. Indeed, the inclusion of patent variables in the estimated models is sometimes ad hoc, and, as I show below, patent rents are not fully identified. The correlation between patents and market value partly reflects the value of the patents per se, but patents also proxy for other unmeasured variables, and these account for part of the correlation as well.

But can market value regressions reveal any useful information about the magnitude of firm patent rents? Building on the previous theoretical and empirical literature, this paper develops a formal model of the relationship between patents and firm value. Although a coefficient corresponding to patent rents cannot be fully identified, I show that an approximate upper bound on mean rents per patent can be estimated. This permits me to make some limited inferences about patent rents.

I show that estimates based on this model are robust to a variety of considerations including firm-specific differences in appropriability conditions, other firm characteristics, different specifications and stability over time. I further test whether these estimates appear to be stable in light of the skewed distribution of invention values (Scherer 1965, Scherer and Harhoff 2000, Silverberg and Verspagen 2004). I find that my estimates of mean patent value show definite evidence of convergence to the mean, suggesting that the distribution of patent values does not have an infinite mean (n.b., invention values may be different).

My theoretical model permits the data from previous market value regression studies to be reinterpreted in order to obtain estimates of patent rents. I compare my estimates to these as well as to estimates of patent value from a variety of other sources and using a variety of different methods.

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3 See Hall (2000) for a review of this literature. Some recent additions to this literature include Bosworth and Rogers (2001), Toivanen et al. (2002), Hall (2007), Hall et al. (2005) and Griffiths et al. (2005).
These include estimates based on the renewal behavior of US and European patentees and estimates based on the choices of US patentees regarding re-assignment and international filing. I also compare my estimates of annual patent rents to several rough benchmarks including the net income of large pharmaceutical companies and patent licensing revenues of IBM and US universities.

This exercise demonstrates that although only limited inferences can be drawn from market value regressions, by providing upper bound estimates, they can play a role in evaluating estimates obtained by other methods, possibly confirming those estimates or possibly calling them into question.

1. A Market Value Model of Patent Rents

1.1 Market value regressions using patent data

In theory, the value of patents derives from the rents they generate. Patents provide their owners a degree of market power—either in product markets or in markets for technology—that affects the demand for the owner’s products, allowing them to charge prices that exceed those they could charge in a perfectly competitive market. These supra-normal prices generate supra-normal profits, or rents, and these should contribute to the value of the firm that owns the patents.

I wish to explore the extent to which market value regressions can be used to measure the magnitude of the rents earned per patent. There is a significant literature that performs market value regressions. This research, however, began with a different objective: it sought to measure the “knowledge stock” of firms and patent terms have been included in these regressions as ad hoc proxies of R&D quality. Because of this different objective, the models used in this literature do not lend themselves to the clearest inferences regarding patent rents, as we shall see. I begin by reviewing this literature.

Griliches (1981) launched this line of research in an attempt to use market value regressions to identify the success of firm R&D investments. To do this, he posited that firm value could be modeled:

\[ V = q(A + R) \]

where \( V \) is firm market value, \( A \) is the current value of the firm’s conventional assets (plant, equipment, inventories and financial assets), \( R \) is the current value of the firm’s “knowledge stock,” and \( q \) measures the current valuation of the firm’s assets. This last variable is a generalized version of Tobin’s \( Q \) that reflects, among other things, the firm’s market power or, as Griliches notes, the firm’s “monopoly position.”

The intuition behind this equation is that in a steady-state equilibrium in competitive markets, the
value of a firm should approximately equal the value of its assets. If firm value is greater than the value of assets \((q > 1)\), then firms could profitably invest in more assets, either existing firms that expand or new firms that enter. However, if a firm has market power, then it might persistently have a \(q\) greater than one because it has the ability to constrain output.

But how can one measure the knowledge stock? One might begin by constructing a stock based on investments in knowledge development, that is, investments in R&D. Just as a capital stock of conventional assets is constructed by depreciating and deflating a stream of investments, so an R&D stock can be constructed by depreciating and deflating a stream of R&D expenditures. However, there is an important difference between the two types of assets: while tangible assets predictably deliver a known value, the asset value generated by R&D investments is uncertain. Whether an R&D investment results in a useful “knowledge stock” asset or not depends on whether the R&D effort was “successful” or not. A failed R&D project adds little or nothing to a firm’s knowledge stock.

Griliches recognized that patent variables may improve the measurement of the knowledge stock. Firms tend to obtain more patents ex post when R&D efforts are successful, so patent measures are correlated with the degree of success. In other words, patents serve as a proxy for the ex post quality of a firm’s technical knowledge. He included patent variables in a modified version of (1). Re-arranging and taking logarithms, (1) becomes

\[
\ln Q = \ln \frac{V}{A} = \ln q + \ln \left(1 + \frac{R}{A}\right)
\]

Then, adding a stochastic error term and using the approximation \(\ln(1+x) = x\) for \(x<<1\),

\[
(2) \quad \ln \frac{V}{A} \approx \beta_0 + \beta_1 \frac{R}{A} + \epsilon = \beta_0 + \beta_1 \frac{R_1}{A} + \beta_2 \frac{R_2}{A} \ldots + \epsilon
\]

where the right hand side might include an R&D stock, or, alternatively, a series of lagged values of R&D spending, a patent stock, or lagged numbers of patent applications, and possibly other variables.

Since (2) is similar to the equations used in hedonic price regressions (Court 1939, Griliches 1961), this has been called an “hedonic” model of the firm. The intuition is that investors value the firm as a bundle of its characteristics much like consumers can be thought of as valuing automobiles by bundles of their characteristics. Researchers have included a variety of knowledge stock quality characteristics on the right hand side of market value regressions, including R&D spending and stocks, patent counts, patent stocks, citation counts, and citations stocks, as well as counts of trademark and

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4 This assumption is not appropriate for all data, consequently more recent papers tend to estimate a non-linear regression instead of using this approximation to linearize it.
design applications.

Hall (1993, 2007) has provided a more rigorous theoretical underpinning to this class of regressions by extending the market value model of Hayashi and Inoue (1991). Under some assumptions including perfectly competitive product markets and constant returns to scale, Hayashi and Inoue show that the market value of a firm can be related to an aggregate capital stock, $K$, constructed from multiple capital stocks, $A$ and $R$, as

$$V = qK, \quad K = A + \phi R$$

or

$$\ln \frac{V}{A} = \ln q + \ln \left(1 + \phi \frac{R}{A}\right)$$

where $\phi$ is proportional to the marginal profitability of $R$ divided by the marginal profitability of $A$.

Hall used the stock of conventional assets for $A$ and a declining-balance R&D stock for $R$, both valued in current dollars. This analysis thus provides a formal model for including an R&D stock in the market value regression and it provides a clear interpretation of the coefficient of the R&D term, which Hall (1993) used to analyze changes in R&D productivity over time.

However, the Hayashi-Inoue model does not provide guidance on how to include patents in the regression. Arguing as above that patents and citations can proxy for a measure of R&D success, Hall, Jaffe and Trajtenberg (2005) add patent stock, $P$, and a citation stock $C$, to this equation:

$$\ln \frac{V}{A} = \ln q + \ln \left(1 + \phi \frac{R}{A} + \theta \frac{P}{R} + \delta \frac{C}{P}\right)$$

Although the meaning of these last two terms is not guided by an explicit theoretical model, they use this regression to show that both patent and citation stocks improve the measurement of the knowledge stock.

But can reliable estimates of firm rents per patent be derived from these market value regressions in either form? As I mentioned, Griliches’s initial paper (1981) noted that “a successful patent is worth about $200,000.” But few other market value regression studies have interpreted a coefficient on a patent term as an estimate of patent value and researchers generally recognize that patent rents cannot be fully identified in these regression models.\(^5\)

Indeed, there are two reasons estimates of patent rents might not be sound:

First, the patent coefficients suffer from endogeneity bias as an estimator of patent rents. The

\(^5\) Cockburn and Griliches (1988) mention tentative values implied by their coefficient estimates. Hall (2005) and Hall and MacGarvie (2006) compare estimates of $\theta$ for different sub-samples using equations similar to (3); they infer that higher values of this coefficient imply more valuable patents on average.
logic of including the patent variable is precisely that it reflects the ex post success of R&D investments. This means that the patent coefficient does not independently represent the contribution of patents to firm value—it will be larger than this because it is positively correlated with the unobserved success of R&D, which falls into the error term (I formally model this endogeneity below).

Second, without an explicit model of patent rents, it is difficult to relate the estimated coefficients to patent rents unambiguously. For example, it is well-known that the coefficients of hedonic regressions are difficult to interpret unless one makes some strong assumptions (Rosen 1974). If one were to make the assumption that all investors had uniform preferences regarding the risk of R&D and patents, then perhaps the coefficient could be consistently interpreted, but, even then, this would appear to measure the marginal value of a patent rather than the average value (Rosen p. 44).

Hall’s model in (3) provides a clear interpretation of \( \phi \), the coefficient of the R&D term. This coefficient relates to the marginal increase in firm value associated with an additional R&D dollar spent. But the meaning of the patent coefficient, \( \theta \), is not so clear. Moreover, this model is based on an assumption of perfectly competitive product markets, which would seem inconsistent with an interpretation of this coefficient as a measure of patent rents.

More generally, although we can describe patents loosely as “assets” of the firm, they are different from other assets—because patents affect the firm’s demand curve, they enhance the returns on all other assets. Griliches noted that the variable \( q \) in (1) captures firm market power. If we want to measure the rents from patents, as opposed to measuring the knowledge stock, then we might want to look more closely at this term and use it to explicitly model the ability of patents to earn rents, as I do next. I build a model that incorporates Hall’s model, but which also explicitly represents the roles that patents play in generating rents and in serving as a proxy for R&D quality.

### 1.2 Patent rents

Hayashi (1982) developed a formal model for this intuition about market power. He identified a relationship between Tobin’s \( Q \) and the value of rents for firms with market power. Under assumptions of constant returns to scale and profits as a function of an aggregate capital stock in nominal dollars, \( K \), for the \( j \)th firm at time \( t \),

\[
V_{jt} = q_t(K_{jt} + W_{jt})
\]

where \( V \) is firm market value, \( W \) is the present discounted value of firm rents (Hayashi derives this as a function of the product demand elasticity), and \( q \) is “marginal \( q \)” which reflects short term
disequilibrium in capital markets. Marginal $q$ is assumed to be approximately equal to 1 and, given competitive capital markets, it is equal across firms at any given time.

This equation captures two intuitions. One intuition is Tobin’s original insight that the market value of a firm is related to the replacement cost of its assets. In a competitive market with no market power, firms will add capacity (either new entrants or existing firms) at the replacement cost of capital, driving prices down until, in long run equilibrium, the discounted stream of expected future profits (market value) equals the cost of those assets. The second intuition is that firms with market power earn sustained supra-normal profits reflected in a higher market value than this.

The rents, $W$, consist of rents earned from patents and rents earned by other means, including rents on technical knowledge realized through first-mover advantage, trade secrecy, etc. or they might include other sorts of rents, e.g., from barriers to entry. Hayashi’s model thus provides a framework for thinking about firm value and patent rents distinct from the role patents might serve as a proxy for R&D quality.

But how are patent rents related to total rents, $W$? It is helpful to think about a model of the patent premium as in Arora et al. (2003). At any point in time, the firm has a number of patentable inventions that are a function of its R&D investments and of the relative success of these investments. The profitability of each invention will depend on the patent rents, if the firm chooses to patent the invention. Whether the firm patents or not, firms typically also earn rents from lead time advantage, the sale of complementary products and services, etc. In general, inventions will differ in profitability and the distribution of these values will be highly skewed.

The firm will choose to patent those inventions where the value of rents realizable with the patent minus the cost of patenting exceeds the rents that could be obtained without the patent. A general reduced-form relationship might be expressed in a patent propensity equation,

\[ P_{jt} = R_{jt} G(s_{jt}; c, ...) \]

where the number of patents obtained per R&D dollar increases as a function of R&D quality or “success,” $s$ (in units to be defined below), and is also a function of other parameters including the cost of patenting, $c$, the distribution of rents per patent and the effectiveness of other means of appropriation. Function $G$ is increasing in $s$ because more successful R&D will generate more inventions that have patent rents that exceed patenting costs and therefore will be patented. This specification posits constant returns to scale in R&D productivity and it assumes that the rents from each patent are independent of the other patents the firm owns (that is, it assumes no “patent portfolio”
effects). These assumptions make the exposition simpler, but I will discuss departures from them below.

Given this, the contribution of patents to total rents, \( W \), is \( uP_{jt} \) where \( u \) is the mean rent per patent, the mean taken over the distribution of patented inventions. Then total rents can be written without loss of generality as
\[
W_{jt} = u \cdot P_{jt} + \mu_j K_{jt}
\]
where \( \mu \) is the markup for rents that the firm earns on its assets through other means. My objective is to estimate the variable \( u \), mean rents per patent, and equations (4) and (6) provide a path to insert \( u \) into an equation about firm market value.

### 1.3 R&D quality and the capital stock

The role of \( u \) in firm rents is distinct from the contribution of patents as a proxy for R&D quality. However, (5) implies a relationship between the success of R&D and the patent stock that, in turn, implies a relationship between patents and measurement of the knowledge stock of the firm. This means that patents might enter a market value regression in two different ways.

The notion of patents as a proxy for R&D quality can be formally incorporated into the measure of the aggregate capital stock, \( K \), in (4) above. This aggregate stock includes both the stock of tangible assets and the knowledge stock. Following Hall (1993, 2007), under some assumptions, the aggregate stock can be represented as a weighted sum of these two stocks, where the weights are determined by the marginal productivities of the different capital types. This aggregation remains valid as long as the profits of the firm exhibit constant returns to scale and this will be true if the production function exhibits constant returns and the specification in (5) holds.\(^6\) Below I discuss the significance of departures from the assumption of constant returns. Given this way of aggregating capital stocks, we can write the current value of aggregate capital
\[
K_{jt} = A_{jt} + s_{jt} R_{jt}
\]
where \( A \) is conventional tangible assets in current dollars, \( R \) is the R&D stock in current dollars (calculated by applying the declining balance method to the stream of deflated R&D investments), and \( s \) is a normalized quality adjustment dependent on the firm’s ex post success rate. This success rate reflects the greater marginal productivity of successful R&D and the quantity \( sR \) can be thought of as a quality-adjusted measure of the knowledge stock. If it is true on average that an additional dollar of

\(^6\) This can be shown by incorporating (5) into the Hayashi and Inoue model (1991).
R&D generates as much profit as an additional dollar invested in tangible assets (as it should in long run equilibrium), then \( s \) will be approximately equal to one on average.

Although we do not observe \( s \), Griliches’s insight was that patents serve as a proxy for the quality of R&D. This can be seen by inverting (5). Assuming that the inverse of \( G \) can be approximated as a linear relationship,

\[
(8) \quad s_{jt} = \alpha + \beta \frac{P_{jt}}{R_{jt}}, \quad \alpha = \alpha(\mu_j, u, c), \quad \beta = \beta(\mu_j, u, c)
\]

This equation formally captures the intuition that patenting is related to R&D success. The functional definitions on the right underline that the parameters \( \alpha \) and \( \beta \) will, in general, change along with changes in the value of patents, the cost of patents and other factors that affect patent propensity. This means that differences in patent propensity between groups or over time will likely correspond to changes in the relationship between patents and firm market value, thus limiting what inferences can be safely drawn from comparisons of market value regressions between groups or over time.

Equation (8) could be further enhanced to include some measure of patent citations. Hall et al. (2005) argue that patent citations add significant informational content in a market value regression and patent citations have been frequently used to capture notions of patent quality or invention quality. Below I also estimate an alternative specification where the number of citations received per patent are assumed to capture the “quality” of the patent.

Finally, assuming that the second term of (8) is small implies that \( \alpha \) approximately equals one, given the normalization of \( s \) above. In practice, as Hall (1993) shows, the coefficient on R&D can vary from year to year as technologies obsolesce at different rates. This implies that \( \alpha \) and \( \beta \) may vary year to year as well. As we shall see below, estimates obtained over an extended time period (to average out short term fluctuations in the productivity of R&D) do, indeed, show values of \( \alpha \) close to one and relatively small values for the second term in (8). Below I check the robustness of my estimates to variation in \( \alpha \).

### 1.4 Estimable specifications

Substituting (6) – (8) into (4), I obtain (see Appendix),

\[
(9) \quad \ln \frac{V_{jt}}{A_{jt}} = \ln q_t + \ln (1 + \mu_j) + \ln \left( 1 + \alpha \frac{R_{jt}}{A_{jt}} + \gamma \frac{P_{jt}}{A_{jt}} \right), \quad \gamma \equiv \frac{u}{1 + \mu_j} + \beta
\]

Assuming that all firms have the same rents from other sources (or, alternatively, that variation in \( \mu \) is uncorrelated with other right hand side variables), and adding a stochastic error term, then this equation...
can be estimated using Non-Linear Least Squares:

\[
\ln \frac{V_{jt}}{A_{jt}} = \ln q_t + \ln (1 + \mu) + \ln \left(1 + \alpha \frac{R_{jt}}{A_{jt}} + \gamma \frac{P_{jt}}{A_{jt}}\right) + \epsilon_{jt}, \quad \gamma \equiv \frac{u}{1 + \mu} + \beta, \quad \mu_j \equiv \mu
\]

This is one specification I use below. Clearly, it is quite similar to (3), the specification used by Hall, Jaffe, and Trajtenberg (2005).

However, firm specific effects may be an important source of heterogeneity. Moreover, depending on the nature of the patent propensity equation, \( \mu \) may be correlated with the patent stock. For instance, firms that earn substantial non-patent rents may be less likely to patent successful R&D projects, all else equal. So it is helpful to also have a specification that incorporates firm effects.

Assuming that \( \alpha \) approximately equals one and \( \gamma P \ll K \) (both assumptions supported by the NLLS estimation), and using a Taylor series approximation, I derive (see Appendix)

\[
\ln \frac{V_{jt}}{A_{jt} + R_{jt}} = \ln q_t + \delta_j + \gamma \frac{P_{jt}}{A_{jt} + R_{jt}} - \frac{\gamma^2}{2} \left(\frac{P_{jt}}{A_{jt} + R_{jt}}\right)^2 + \epsilon_{jt},
\]

This specification uses a modified version of Tobin’s \( Q \) as the dependent variable, but it can be estimated using fixed effects, random effects, first differences, or longer differences.

The functional forms of (10) and (11) differ slightly from specifications used in previous research. Below I derive estimates of \( \gamma \) from statistics reported in several other papers by transforming the equations used in those papers to one of these specifications.

1.5 Identification and alternatives

It comes as no surprise that in (9) – (11), mean rents per patent, \( u \), is not identified. Two unknown parameters remain, \( \mu \) and \( \beta \). However, all hope is not lost because we know something about these parameters.

First, on theoretical grounds and much anecdotal evidence, \( \beta \) should be positive—that is, there should be a positive relationship between R&D success and patenting. Second, I show below that \( \mu \) cannot be greater than ten or twenty percent. This means that

\[
1.2 \approx \gamma \geq u
\]

In other words, an estimate of \( \gamma \) serves as an “almost upper bound” on \( u \); a definite upper bound is within ten or twenty percent of the estimate. This means that we cannot derive precise estimates of mean rents per patent from these market value regressions, as is well-recognized. However, it is still possible to use these regressions to check whether estimates based on patentee behavior are
substantially understated. For that inquiry, errors of ten or twenty percent are not material—instead, we want to check merely whether $\gamma$ is two times or so larger than the estimates based on patentee behavior. Given the differences in samples used and the precision of the estimates, we can only make rough comparisons in any case. This is what I do below.

The above derivation made several strong assumptions, so the model needs to be checked against some alternative specifications. I assumed above that the rents from each patent were independent of the rents earned on other patents held by the firm. Alternatively, the size of the firm’s patent “portfolio” could matter. Suppose that the total patent rents of a firm were a concave function, $h(P)$, such that $h(0)=0$. In this case, $h$ could be expanded into a Taylor series,

$$h(P) \approx h_1 P + \frac{1}{2} h_2 P^2 + \ldots$$

But since (11) includes higher order terms in $P$, $\gamma$ would then be an upper bound estimate of $h_1$, the first order coefficient in the Taylor expansion. And because $h$ is concave, $h_1$ itself is an upper bound estimator of the mean rents per patent, so $\gamma$ is still an upper bound estimator of mean patent rents.7

The model above also assumed constant returns to scale in production. The Hayashi 1982 model in (4) uses this assumption to derive the relationship between firm value and the value of the aggregate capital stock. However, this derivation can be readily adapted to permit non-constant returns to scale (Chirinko and Fazzari 1994, Galeotti and Schiantarelli 1991). These changes would not alter estimates of $\gamma$.8 The assumption of constant returns was also used in the Hall-Hayashi-Inoue construction of the aggregate capital stock from separate stocks for tangible and knowledge capital. Departures from this assumption might mean that I mismeasure the aggregate capital stock. Below I run some tests on the composition of the capital stock by hard-coding values of $\alpha$. I find that the estimates of $\gamma$ are not sensitive to these changes. The “true” capital aggregate might also include higher order terms in $A$ and $R$, but it seems unlikely that these would make much difference to estimates of $\gamma$ if the first order terms do not.

7 For a concave function, $h'$ is greatest at $P=0$ and this will be at least as great as the mean rent per patent, $h(P)/P$. But $h'(0) = h_1$.

8 The effect is to add a quasi-rent term to $\mu$, which leaves $\gamma$ unchanged. Note also that with increasing returns to scale, the firm might have a natural monopoly and have no need for patents.
2. Data

2.1 Sample

The data come from two sources, Compustat and a database of patent information from 1969 through 2002. The patent data come from the US Patent and Trademark Office (USPTO) and have been supplemented with information on citations compiled by Bronwyn Hall.9

To match patent data to firms in Compustat, I used a matching program initially developed for another project (Bessen and Meurer 2005). The USPTO provides an assignee name for every assigned patent after 1969. To match the USPTO assignee name to the Compustat firm name, we began with the match file provided by the NBER (Hall et al. 2001). To this we added matches on subsidiaries developed by Bessen and Hunt (2007), we manually matched names for large patenters and R&D-performers, and we matched a large number of additional firms using a name-matching program.10 In addition, using data on mergers and acquisitions from SDC, we tracked patent assignees to their acquiring firms. Since a public firm may be acquired, yet still receive patents as a subsidiary of its acquirer, we matched patents assigned to an acquired entity in a given year to the firm that owned that entity in that year.11 Finally, using a software program, we identified a group of Compustat firms that had unique names that could not be found in the USPTO list of assignees. These were classified as definite non-matches.

This group of firms with match information (either a match or a definite non-match) includes 10,736 patent assignees matched to one of 8,444 owning firms in Compustat, with as many as five different owners matched to each assignee. This group accounts for 96% of the R&D performed by all US Compustat firms, 77% of all R&D-reporting firms listed in Compustat and 62% of all patents issued to domestic non-governmental organizations during the sample period. Sample statistics show that this sample is broadly representative of the entire Compustat sample, although it is slightly weighted toward larger and incumbent firms. Testing our match against a sample of 131 semiconductor industry firms that had been manually matched, we correctly matched 90% of the firms that accounted for 99.5% of the patents acquired by this group.12

9 Downloaded from http://emlab.berkeley.edu/users/bhhall/bhdata.html.
10 A software program determined matches between the two files by identifying firm names that matched after taking into account spelling errors, abbreviations and common alternatives for legal forms of organization.
11 This dynamic matching process is different from that used in the original NBER data set which statically matched a patent assignee to a Compustat firm. These data were developed with the help of Megan MacGarvie, to whom I am indebted.
12 Thanks to Rosemarie Ziedonis, who originally compiled this data, for sharing it with me.
From this group with matching information, I excluded firms that did not have at least four years of non-missing data on key variables and firms that did not perform significant R&D. I kept observations from 1979 through 1997, using the first ten years of data to build stock variables for patents and R&D and eliminating later years because of possible truncation bias in patent application data (see below). Finally, because (10) and (11) involves ratios and consequently any measurement error may be greatly exaggerated in the tails of the distributions of the variables, I trimmed the sample of the 1% tails of Tobin’s $Q$. I also experimented with other screens. I obtained similar coefficient estimates with these screens, but these methods seemed more arbitrary.

This left me with 25,861 observations of 3,451 firms. Sample statistics are shown in Table 1. As can be seen, the sample is broadly representative of R&D-performing firms, including a large portion of small and newly public firms. 85% of the observations are for firms whose primary business is in the manufacturing sector.

2.2 Variables

Key variables are defined as follows:

- The market value of the firm, $V$, consists of the sum of all the claims on the firm, namely, the sum of the value of the common stock, the preferred stock (valued by dividing the preferred dividend by Moody’s Index of Medium Risk Preferred Stock Yields), long term debt adjusted for inflation (see Hall 1990 and Brainard et al. 1980), and short term debt net of current assets.
- The value of assets, $A$, is the sum of the net value of plant and equipment, inventories, accounting intangibles, and investments in unconsolidated subsidiaries all adjusted for inflation using the method of Lewellen and Badrinath (1997).\(^{13}\)
- The R&D stock, $R$, is calculated assuming a 15% annual depreciation rate and an 8% pre-sample growth rate (Hall 1990). I use Bronwyn Hall’s R&D deflator to obtain the current value of $R$ from the stream of past investments.
- The patent stock of the firm, $P$, is based on the number of patent applications each year that resulted in a grant of a patent by 2002. Since there is a lag of possibly several years between the application and grant of a patent (see Hall et al. 2005), I only use data through 1997. I calculate the patent stock using a 15% depreciation rate. I also calculate patent citation stocks (stocks of citations received through 2002), using a 15% depreciation and adjusting for

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13 Thanks to Bronwyn Hall for providing Stata code to compute this. The code was developed by Bronwyn and Daehwan Kim.
truncation using the method described in Hall et al. (2005). In order to interpret the coefficient $\gamma$ in constant ($\$92$) dollars, I multiply the associated variable ($P/A$ in equation 6 and $P/(A+R)$ in equation 7) by the GDP deflator.

- To explore possible strategic interaction, I also develop a measure of rival firms’ patent stocks. I do this using a technology distance measure developed by Jaffe (1986). I calculate the technology distance between two firms as follows. For each firm I construct a vector of the share of its patents that falls into each USPTO technology class (there are over 400 in the 1999 classification I use). The distance measure is then the uncentered correlation between these two vectors (the vector product divided by the product of the standard deviations of each vector). This measure is 1 if the firms distribute their patents identically across classes and zero if they share no patent classes (hence this distance measure might be more appropriately termed a “nearness” measure). I calculate this measure using pooled patent data over three periods from 1979 through 1999. I calculate the measure of rivals’ patents as the sum of the patent stocks of all other firms weighted by the other firms’ distances.

3. Empirical Results

3.1 Regression estimates

Column 1 of Table 2 uses the nonlinear specification in (10), which ignores firm specific effects. The estimate of $\gamma$ is $370,000 in 1992 dollars and the estimate of $\alpha$ is just about equal to one, as predicted. This estimation is made over a 19 year period. Below I explore the possibility that these parameters may shift over time. I also tested the sensitivity of the estimates of $\gamma$ to changes in $\alpha$. I ran a series of regressions (not shown) using different, fixed values for $\alpha$. I found that the resulting changes in $\gamma$ were small for both the specification in (10) and the one in (11). This suggests that the assumption that $\alpha$ approximately equals one is a safe assumption, at least for a sufficiently long sample period. It also suggests that my estimates are not sensitive to the specification of the aggregate capital stock.

The time dummies for this regression (not shown in the table) correspond to the term

$$\ln q_t + \ln (1 + \mu)$$

in equation (9). The mean of these time dummies is 0.24. This allows us to make some rough inferences about the magnitude of $\mu$. We know that $q$ is likely greater than unity in most years for at least two reasons. First, the capital stocks of the majority of firms in our sample are growing fairly rapidly. The mean annual growth rate is 17% (median 9%). Given the quasi-fixed nature of capital—
capital stocks do not adjust instantly and costlessly—this means that \( q \) should be greater than 1 during most years. Second, I have not included all intangibles in my capital calculations, for example, I have not included “brand capital” that might be associated with advertising and marketing investments. Some economists argue that these intangibles significantly elevate \( q \) (Villalonga 2004). Since \( q \) is generally greater than one, this implies that \( \mu \) must be less than \( e^{24} - 1 = .27 \), perhaps not larger than 0.10 or 0.20. As above, this, in turn, implies that \( \gamma \) can be used as a rough upper bound estimate on mean rents per patent.

The remaining columns in Table 2 explore specification (11), which permits firm heterogeneity. Column 2 is estimated using simple Ordinary Least Square without firm effects and Column 3 is estimated using firm fixed effects. The differences are significant and a good deal of the variance is explained by the fixed effects. I also ran the same regressions using random effects. A Hausman test rejected the random effects specification, however. These results indicate that firm heterogeneity is important.

These regressions included the first three terms of the Taylor series expansion used in (11). The coefficients have the predicted signs, but third order terms are not significant, both economically and statistically, so I only use the first two order terms in subsequent regressions. Note, however, that the second order term is quite influential and should not be ignored, as has sometimes been done in the literature. This may seem surprising because this term is small for most of the sample; at the mean it is only about 0.01. However, because some observations have rather high values of \( P/(R+A) \), the second order term is significant for these and estimates of \( \gamma \) show a downward bias if this term is not included.

The estimate of \( \gamma \) in the fixed effects regression is much smaller than the estimate in the simple OLS regression. This may be because of the role of firm fixed effects or it may be because of attenuation—the fixed effects estimate may suffer from the well-known problem of errors in panel data (Griliches and Hausman 1985). Column 4 shows an estimate using four-year differences rather than fixed effects. These estimates should suffer less from problems of errors in the panel data, although at the cost of a smaller sample size. I tested several different lags and found little change in the estimates after a four year lag. This estimate, using four year differences, falls between the OLS and fixed effects estimates and is quite close to the Nonlinear Least Squares estimate in column 1.

Hall et al. (2005) suggest that patent citation data contain additional information about patent or invention quality beyond what is captured in patent count data alone. Unfortunately, current research provides little guidance about how patent citations might affect patent propensity and patent propensity
provides the rationale for including patents in a Tobin’s $Q$ regression in my model. If one supposes that patent citations reflect patent quality (and that this, in turn, affects patent propensity) then citations can be included in my model as follows: in (8) replace $P$ by

$$P_{jt} \left(1 + \delta \frac{C_{jt}}{P_{jt}}\right)$$

where $C$ is the patent citation stock. This leads to a regression as in (11) with the addition of a term $C/(A+R)$ and, possibly, higher order terms. Column 5 of Table 2 shows a specification with just the first order term. In this specification, the citation stock term is not statistically significant and including this term reduces the estimate of $\gamma$ a bit. Although patent citations may be useful in revealing information about invention quality, these data do not appear to be particularly important to the task I address here.

Table 3 conducts some additional robustness checks. Hall (1993) found that the productivity of R&D capital exhibited short term variation relative to other capital assets. This might imply changes in $\alpha$ and $\beta$ over time. Columns 1 and 2 show separate regressions for the first and second halves of my panel using the nonlinear specification. Although the estimates of $\alpha$ change, the estimates of $\gamma$ do not change significantly. Column 3 shows a similar test using the four-year differenced specification and variables interacted with a time period dummy. The later period shows a lower estimate for $\gamma$, although the difference is only significant at the 10% level.

Because these different estimates of $\gamma$ may reflect changes in $\alpha$ and/or changes in $\beta$, they do not imply that patent value was necessarily lower during the 1990s than during the 1980s. Nevertheless, these results do make it seem unlikely that patent value increased substantially during the 1990s. This might seem to contradict the notion that patents have been getting “stronger.” While patents may have gotten stronger during the 1980s, my results are consistent with the view that patent value has not increased substantially during the 1990s and may have even decreased then, although, for reasons mentioned, this evidence is not conclusive.

Column 4 explores the possible role of strategic interaction. Patent rents are realized when patents deliver a degree of market power. This implies, in turn, that other firms lose a degree of market power if they remain in the market. Other firms’ patents might influence the magnitude of patent rents

14 The change in $\alpha$ is consistent with Hall’s (1993) finding that the marginal profitability of R&D relative to that of physical assets declined during the 1980s. Hall interprets this as evidence of underinvestment in R&D during the early 1980s, perhaps due to accelerated obsolescence, that was corrected during the late 1980s.

15 Evidence based on court decisions suggests a pro-patentee shift after the creation of the Court of Appeals for the Federal Circuit in 1982, but the evidence for the 1990s does not indicate a further pro-patentee shift. Instead, some scholars see a modest anti-patent shift during the 1990s (Lunney 2004, Henry and Turner 2006).
and also their interpretation. In column 4, I add a variable that measures the technology distance-weighted size of other firms’ patent portfolios. This has a large and significant (at the 2% level) coefficient—at the sample mean, other firms’ patents are associated with a reduction in firm value of 13%. Megna and Klock (1993) also found a negative relationship in a similar regression for the semiconductor industry.\(^\text{16}\) However, this regression shows that the estimate of \(\gamma\) is little changed by the inclusion of rivals’ patents in the regression. So the consideration of rival patents does not seem to affect the magnitude of the private rents received, although it may suggest that non-patent rents may be adversely affected by rival’s patents, offsetting the incentive that patents provide to perform R&D.

Finally, I consider the rents that can be attributed to US patents specifically; this will be useful for some of the comparisons below. Although the patent stock measure I use is a stock of US patents, firms realize value from sister patents obtained in other countries. Since these other patent counts are not explicitly included in the regressions, the estimates of patent rents implicitly include the rents earned in other countries. It is well-known that firms will patent valuable inventions in multiple countries (Putnam 1996), so worldwide patent stocks will be correlated with US patent stocks. This means, in effect, that the patent stock variable I use serves as a proxy for worldwide patent stocks and my estimates of patent rents should be interpreted as rents on worldwide patents. In column 5 of Table 3, I estimate the domestic share of \(\gamma\) by using a modified regression. I divide the variable \(P/(A+R)\) by the portion of the firm’s profits that derive from domestic operations. Assuming that the domestic share of patent rents roughly equals the domestic share of profits, the coefficient on this term should represent the domestic share of \(\gamma\). Since not all firms report domestic and foreign profits separately, I also include the original variable \(P/(A+R)\) for those observations. The estimate of domestic \(\gamma\) I obtain is $70,000, about a fifth of the estimate of worldwide patent rents.

The regressions explored so far suggest that estimates of \(\gamma\) are reasonably robust to a variety of considerations.

### 3.2 Industry differences

These estimates all use data samples that cover all technologies and industries. Yet some evidence suggests that patent value might vary substantially across technologies or industries. In surveys, research managers in the chemical and pharmaceutical industries rate patents as much more effective than do managers in other industries (Levin et al. 1987, Cohen et al. 2000). Bessen (2006), using the renewal method, finds that patents on chemical entities are about 6 times more valuable than

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\(^{16}\) McGahan and Silverman (2006) explore a much richer set of potential interactions.
the average patent. 17

Table 4 shows separate regressions on different industry groups using the same specification as in Table 2, Column 4. 18 Here, the “Large pharmaceutical” category includes firms in SIC 2834 (“Pharmaceutical preparations”), that employ more than 500 employees and that are not primarily manufacturers of generic drugs. The table breaks out chemical firms, large pharmaceutical firms and firms in other industries; estimates for the latter are broken out in greater detail, but at a loss of some precision.

The estimates of γ show an order of magnitude difference between chemical firms, especially large pharmaceutical firms, and other firms. These differences are similar to those in estimates from the renewal data, but even larger.

As above, there is no guarantee that β is constant across industries so these inter-industry differences not directly reflect differences in patent rents. Nevertheless, such large differences seem likely to reflect underlying differences in patent rents rather than differences in marginal patent propensity.

This is significant for two reasons. First, it suggests that much of aggregate patent value is highly concentrated among a relatively small number of firms and industries. Chemical firms account for a large portion of aggregate patent assets, especially two dozen or so large pharmaceutical companies. The fifth column of Table 4 shows the implied share of aggregate patent value. Chemical and pharmaceutical firms account for over 80%. Even though this percentage may be somewhat overstated, it does suggest that, economically speaking, there appear to be two distinct patent systems: one for chemical entities and one for other technologies and it is the former that receives most of the benefit.

Moreover, these differences do not arise just because some industries spend more on R&D. The last column of Table 4 shows the ratio of implied annual patent rents (assuming a 15% rate of return applied to the aggregate discounted patent rents) to associated R&D spending. Schankerman (1998) calls this ratio the “equivalent subsidy rate,” arguing that it represents an upper bound on the subsidy that patents provide to perform R&D. Clearly, this ratio is much higher for chemical and pharmaceutical firms.

Second, industry/technology is an important determinant of patent rents and this variable may affect estimates. The bottom row of Table 4 shows the mean value of γ using the separate industry

17 Schankerman (1998) and Lanjouw (1998) using European renewal data, however, find pharmaceutical patents to have low or modest value relative to other patents.
18 A similar pooled regression with industry interaction terms had qualitatively similar results and the null hypothesis that the coefficients on these interaction terms were zero was strongly rejected.
estimates in the table. Clearly, the mean value of $798,000 is substantially larger than the estimates obtained in Table 2 using pooled observations. This difference is largely driven by the large pharmaceutical firms that account for 7% of the aggregate patent stock, but over half the aggregate patent value. The influence of these observations (only 2% of the observations) is apparently lost in the pooled regressions of Table 2, but show up here albeit without a high degree of precision.

3.3 Do estimates of mean patent value converge?

Scherer (1965) first pointed out that coefficients of patent stocks in market value regressions might be substantially understated if the distribution of patent values is highly skewed. This is because the coefficient essentially represents an average patent value and if the distribution is highly skewed, then the average value might converge to the true mean only slowly or perhaps not at all. In extreme cases, such as the Pareto distribution, the true mean is infinite, so averages calculated over finite samples will not be representative.

Harhoff et al. (2003b) study the distribution of invention values and conclude that the lognormal distribution, which does have a finite mean and variance, fits the data well. On the other hand, Silverberg and Verspagen (2005) argue that a Pareto distribution provides a better fit for the upper tail. But these results concern the distribution of invention values, not the values of patents. Patents might follow a different distribution because firms may tend to obtain more patents on highly valuable inventions, tending to compress the distribution and thin the upper tail (see Bessen 2006).

Our market value regressions provide a simple test of convergence to the mean for patent values. If patent values do converge to the mean, the variance of the stochastic error term should decrease with the size of a firm’s patent portfolio, all else equal. This is because the sampling variance of the mean patent value is proportional to the inverse of the patent stock. If the distribution of patent values has an infinite mean, then the regression variance should not vary with the size of the patent stock after controlling for other size-related variables.

Table 5 shows regressions where the dependent variable is the square of the residuals from the regression in Table 2, column 4. The first independent variable is the inverse of the patent stock (coded to zero for observations with zero patents). The second variable is a dummy flag for zero patents. The second column adds controls for the log of employment and the log of deflated R&D stock. The statistically significant coefficient on the inverse of the patent stock in both regressions suggests that regression error does decrease with the size of the patent stock, rejecting the hypothesis of no convergence to the mean.
4. Comparing the Estimates

Although the theoretical analysis indicates that estimates of patent rents obtained from market value regressions are upper bound estimates, it is nevertheless informative to compare these estimates to other estimates of patent value. If the biases in market value regressions are not too great, then one would expect the estimates to be roughly similar. These other estimates might be called into question if they were much smaller than the estimates obtained from market value regressions. On the other hand, if the other estimates were roughly similar to the market value estimates, then this would suggest that these other estimates were not substantially understated.

4.1 Comparison to implied estimates from previous market value regressions

I begin by looking at previous studies using market value regressions. Although my regression equations differ from specifications used in the literature, some rough computations can be used to compare previous results to my estimates of $\gamma$. Table 6 shows some approximate calculations of equivalent estimates of $\gamma$ expressed in thousands of 1992 dollars. The details used to calculate these estimates from the original reported coefficients are described in the Appendix.

The three studies cited all use data from US publicly-listed manufacturing firms. The estimates range from $119,000 to $343,000. One study, Megna and Klock (1993), is for semiconductor firms only. Its estimate of $343,000 is not too different my estimate for the electronics industry of $407,000. The other estimates pool data from different industries and are thus most comparable to my preferred estimate from Table 2, Column 4. My estimate is a bit higher, at $351,000, than the previous estimates, but overall these estimates are reasonably consistent.

4.2 Comparison to estimates based on patentee behavior

How, then, do they compare to estimates obtained based on observations of patentee behavior? There are at least two important considerations the need to be taken into account when making such a comparison. First, as I noted above, the estimates from most of the market value regressions are implicitly estimates of the value of worldwide patent rights, while most of the estimates based on patentee behavior (Putnam 1996 is the exception), calculate the value of domestic rights only. Second, patents held by public firms are a select sample. Estimates based on renewal studies suggest that patents held by public firms are worth substantially more, maybe 50% more or so, than other patents.\footnote{In Bessen (2006) the mean value for all patents granted to domestic patentees in 1991 is $78,000; the mean value for patents granted to public manufacturing firms 1985-91 is $113,000, 45% more. This is not surprising for several reasons. For one, public firms may have greater complementary assets with which to utilize patents.}
Most of the estimates based on patentee behavior use samples of all patents, but market value estimates are based only on public firms.

Table 7 compares various estimates using data pooled over all industries. Putnam (1996), using data on international patent applications, estimated the value of worldwide patent rights for patents that were applied for in 1974 in more than one country. Using his figures (see Appendix for calculations), I calculate that the worldwide value for inventions associated with US patent applications in 1974 is about $230,000 in 1992 dollars. Considering that this estimate is for all patents, not just patents owned by publicly listed firms, a comparable estimate (based on a 30-50% premium) for publicly traded manufacturing firm might be $300,000 – 350,000. This suggests that after adjusting for differences in samples, the values of $\gamma$ (using data pooled from different industries) are quite close to the value of patents implied by Putnam’s estimates.

The studies based on renewal and re-assignment data produce estimates for the value of US patents only, not the associated worldwide patent rights. These estimates are $62,000 for patents granted in 1986 (Barney 2002), $47,000 for patents held by small patentees (Serrano 2006), $78,000 for all patents granted in 1991 to domestic patentees (Bessen 2006) and $113,000 for patents granted to publicly listed manufacturing firms from 1985 to 1991 (Bessen 2006).

I compare these numbers to the estimates of $\gamma$ in two ways. First, in the regression shown in Table 3, column 5, I controlled for the domestic share of patent rents using the domestic share of profits and obtained an estimate of $70,000 as an estimate of the domestic rents from patents, reasonably close to the renewal-based estimates.

Second, using Putnam’s data (see appendix), I estimate that the value of domestic US patents runs about 32% of the value of the associated worldwide patent rights. This implies, for example, that the estimate of $\gamma$ from Table 2, column 4 corresponds to a domestic patent value upper bound of about $112,000 for publicly listed firms, quite close to the most comparable renewal estimate of $113,000. For the other studies at the top of Table 6, this thumbnail estimate corresponds to mean domestic patent values from $38,000 to $110,000, quite similar to the estimates derived from renewal data.

Table 8 compares estimates for technology classes (from Bessen 2006) with estimates in this paper by industry from Table 4. These classifications are different, nevertheless, one might expect industry and technology groupings to yield broadly similar results. The estimates outside of pharmaceuticals are quite similar, however, the market value estimates for large pharmaceutical firms
are an order of magnitude larger than estimates for drug and medical patents.\textsuperscript{20} It is possible that renewal value estimates for major drug patents are understated because the drug approval process might affect the way patent value changes over time in a way that conflicts with the renewal models.\textsuperscript{21} In any case, these comparisons taken together suggest that renewal estimates may be reasonably accurate outside of the pharmaceutical industry, but that renewal estimates appear to be substantially understated within this industry.

The ratios of patent rents to associated R&D shown in Table 4 provide another way to compare estimates from market value regressions to estimates obtained using renewal data. Lanjouw et al. (1998) summarize this ratio from renewal studies that use European patent data but pro-rate worldwide R&D.\textsuperscript{22} They find that the estimates fall between 10\% and 15\% for most studies. Arora et al. (2003) estimate a ratio of 17\% using a model employing survey data for the US. The ratio for the total sample in Table 4 is 18\%. Note that the same calculation made using the specification in Table 2, column 1 with pooled data ($370,000), yields an equivalent subsidy ratio of 9\%. Thus, these numbers roughly correspond to the range of earlier estimates.

4.3 Comparison to benchmarks

Another way of checking the market value estimates is to compute the implied annual flow of rents from the estimated value of $u$, which is the discounted value of the future stream of rents. That is, assuming a return on investment of 15\%, then $0.15 \times \gamma \times \text{patent stock}$ gives a rough, upper-bound measure of the flow of patent rents. I compare this to the profits of large pharmaceutical firms, and to the patent licensing revenues of IBM and of universities.

It is widely held that pharmaceutical profits depend heavily on patent rents. For the years 1990-97, using the estimate in Table 4, I find that estimated patent rents accounted for 62\% of the deflated net income of large pharmaceutical firms. If one assumes that firms earn “normal” profits equal to 5\% of net revenues, then the estimated patent rents accounted for 93\% of the total rents of large pharmaceutical firms.\textsuperscript{23} Of course, pharmaceutical firms earn rents from other sources in addition to patents: they earn rents on large marketing expenditures (larger than R&D) and rents from industry

\textsuperscript{20} Lanjouw (1998) and Schankerman (1998) also find relatively low estimates for the value of German and French pharmaceutical patents.
\textsuperscript{21} See the discussion of depreciation and option value in Bessen (2006).
\textsuperscript{22} There is a difference in the way this ratio is calculated. The renewal studies calculate the flow of the value of patent grants to pro-rated R&D used to produce these patents. I use the ratio of the flow of patent rents (on the entire patent stock) to entire R&D stock. These should be equivalent in equilibrium.
\textsuperscript{23} On average, this group of firms had net income of $14.7 billion per year, flows of patent rents of $9.1 billion per year and total rents (net income less 5\% of book value) of $9.8 billion per year all in 1992 dollars.
regulation (generic pharmaceutical companies also make above-average profit margins\textsuperscript{24}). Nevertheless, this suggests that the estimate in Table 4 for large pharmaceutical firms is in the right ballpark.

Another benchmark is IBM’s vaunted patent licensing program, which has been taken as a model of how firms can aggressively extract value from their patents. Beginning in 2000, IBM began reporting licensing and royalty fees. Translated into 1992 dollars, the patent licensing program has earned between $3,700 and $7,600 per year per patent in force.\textsuperscript{25} This figure is gross of the costs of the several hundred patent lawyers that IBM employs. These patent licensing revenues account for 2-3\% of IBM’s income before extraordinary items from 1999-2003. In comparison, estimated patent rents account for 8-13\% of IBM’s income, roughly four times larger.\textsuperscript{26} Considering that IBM earns rents from its ability to exclude rivals from product markets in addition to rents in the form of licensing revenues, this seems to be reasonably consistent. For 1999, the estimated patent rents account for 19-26\% of IBM’s total rents, again calculated assuming normal profits corresponding to a 5\% net margin. Considering that IBM operates in industries known to appropriate substantial returns to innovation by means other than patents, this figure, too, seems quite reasonable.

As another rough check, I compare estimated patent rents to the gross royalties earned on university patents, which include some patents of high social value such as the Cohen-Boyer patent. In 2003, universities realized gross licensing revenues of about $1.1 billion in 1992 dollars (AUTM 2003). This excludes the cost of running technology transfer offices, which apparently consume a substantial portion of this revenue (Thursby and Thursby 2003). Assuming that university patents are of comparable value to the patents of public companies (university patents are more concentrated in biotech and pharma), my estimates of rents from university patents in 2003 range from $1.0 billion to $2.2 billion.\textsuperscript{27} These estimates, too, appear to be in the right ballpark, although perhaps a bit high.

\textsuperscript{24} The mean pretax margin on sales during 1990-97 was 19.7\% for large pharmaceutical firms and was 13.9\% for generic pharmaceutical firms in my sample. For the entire sample, the pretax margin was 8.0\%.

\textsuperscript{25} The figures in IBM’s annual reports have been mis-represented, including the much-hyped “$1.7 billion in licensing revenues.” This figure actually included many other things, including the value of IP in divisions that were spun off and fees from custom software development. The annual reports list “licensing/royalty-based fees.” According to sources at IBM about 60\% of this comes from technology licensing and about 40\% from the pure patent licensing program. I use 40\% of this figure.

\textsuperscript{26} I obtained these estimates using the “Other Industries” patent value of $260,000 from Table 4 and the estimate of $351,000 in Table 2, Column 4 applied to IBM’s patent stock and income from 1995-99.

\textsuperscript{27} I obtained the lower estimate using the value of $351,000 from Table 2 and the higher estimate from the value of $798,000 in Table 4.
4.4 Comparison to a survey estimate of value

Another comparison can be made to the recent PATVAL survey in Europe where inventors were asked to value their patents (Gambardella et al. 2006). Using a random sample of patents granted by the European Patent Office (EPO), inventors were asked at what value they would be willing to sell their patents to rival companies.\textsuperscript{28} The mean value of the responses was $9.6 million, converted to 1992 dollars; for German inventors who had received additional information about the value of their patents to their companies (because of a German law requiring inventor compensation), the mean value was $5.2 million.

These estimates might seem to conflict with almost all of the estimates cited above. However, these figures are not directly comparable for two main reasons. First, EPO patents are likely several times more valuable than their corresponding US patents because of stricter standards and because inventors obtain fewer EPO patents per invention.\textsuperscript{29}

Second, this concept of patent “value” does not directly correspond to the notion of discounted patent rents estimated by the studies above. Selling a patent to a rival means that the firm can no longer practice the invention (see Harhoff et al. 2003a). This means that the firm not only gives up patent rents, but it also gives up rents that it earned on the invention by lead time advantage, learning-by-doing, etc. That is, this notion of value might be described as “invention value” rather than the value of patent rents. A rough calculation comparing my estimate of the mean value of $\mu$ above (0.24) with the mean value of $uP$ (.028), suggests that the value of all of the rents associated with an invention might be an order of magnitude larger than the rents from patents alone.

Finally, the survey responses might be inflated for those cases where there are multiple patents on an invention. Selling just one of these patents to a rival might prevent the firm from practicing the invention at all, so the reservation value might reflect the value of all of the patents covering an invention. Although some survey respondents might mentally prorate this value across all of the patents involved,\textsuperscript{30} many respondents might not make such an adjustment, leading to an inflated average value.

Thus, accounting for differences in the nature of the patents and in the concept of value

\textsuperscript{28} The actual question asked was “Suppose that on the day in which this patent was granted, the applicant had all the information about the value of the patent that is available today. In case a potential competitor of the applicant was interested in buying the patent, what would be the minimum price (in Euro) the applicant should demand?”

\textsuperscript{29} According to Dietmar Harhoff, one of the authors of the PATVAL study, there are 4 US patents corresponding to each invention that also has an EPO patent application and only 70% of these EPO applications result in a grant (email 11/1/2007). He estimates that EPO patents correspond to the most valuable one-third of US patents, suggesting that the mean should be several times larger.

\textsuperscript{30} Dietmar Harhoff also noted that at least one survey respondent who was interviewed in depth did just that.
measured, the survey results do not necessarily conflict with my estimates and may well be consistent with them.

5. Conclusion

The model developed in this paper shows that clear but limited inferences can be drawn about the magnitude of patent rents from market value regressions. Coefficient estimates from carefully specified equations can be interpreted as upper bound estimates of the discounted value of future rents earned by patents. Moreover, I show that these estimates are robust to a variety of considerations including the possibility that a “fat” upper tail in the distribution of patent values may make finite sample estimates unreliable.

Only limited inferences can be drawn using these coefficient estimates because they are known to be biased upwards. For example, comparisons across sub-samples might be difficult because the bias may change by an unknown amount when different sub-samples have different patent propensities.

Nevertheless, upper bound estimates are useful because they can be compared to other estimates of patent value as a check. Economists have inferred patent values from data on patentee behavior using a variety of methods. However, these studies suffer from a common weakness: since they rely on an extrapolation for the “upper tail” of the distribution of patent values, these estimates may be understated if that tail is “fatter” than predicted. In contrast, market value regressions directly reflect the value of the patents in the upper tail.

I compare market value estimates to estimates based on patentee behavior and I find a reasonably close correspondence except for large pharmaceutical firms. Indeed, except for the pharmaceutical industry, I find that a significant number of estimates obtained using a variety of different methods yield roughly equivalent estimates. Although these estimates are not very precise, this correspondence should provide some confidence in their accuracy. This is further supported by some benchmark calculations.

However, large pharmaceutical firms are an important exception. I find that market value regressions for these firms yield much higher estimates of patent value than do estimates based on renewal data. I also find that the estimates based on market value regressions account for the majority of pharmaceutical firm’s profits, supporting the accuracy of my estimates. It might be that the regulatory approval process for drugs affects the evolution of patent value over time in a way that conflicts with the renewal estimates. In any case, I conclude that the renewal estimates are understated for pharmaceuticals.
Of course, large pharmaceutical firms make up only a couple percent of the sample. Nevertheless, because these patents are so valuable, they account for a majority of aggregate patent value among public firms. This highlights the importance of performing separate economic analysis for patents in this industry.

Finally, this analysis only concerns the mean value of the private returns to patents. To the extent that market value regressions validate estimates of mean patent value obtained from studies based on patentee behavior, other statistics of the distribution of patent values derived from these latter studies are also supported.\(^31\) In any case, as noted above, estimates of median patent value from these studies do not suffer from the “upper tail problem.” More important, the private returns from patents are only one element that goes into the policy performance of the patent system. A social welfare calculation needs to also consider the social returns, which might be larger or smaller than the private returns, and the costs of the patent system, both to innovators and to other parties. Nevertheless, to the extent that the literature on private returns can provide some solid estimates, we are one step closer to a complete welfare analysis.

\(^{31}\) It is, of course, possible that the means could correspond but that the distribution of value in the upper tail could be “lumpy” so that estimates of, say, the 99th percentile might be off. This seems unlikely, however.
Appendix

Derivations of (9) and (11)

Substituting (6) into (4)

\[ V_{jt} = q_t(K_{jt} + W_{jt}) = q_t(K_{jt} + u \cdot P_{jt} + \mu_j K_{jt}) = q_t(1 + \mu_j) \left( K_{jt} + \frac{u \cdot P_{jt}}{(1 + \mu_j)} \right). \]

Then, substituting in (7) and (8),

\[ V_{jt} = q_t(1 + \mu_j) \left( A_{jt} + \alpha R_{jt} + \beta P_{jt} + \frac{u \cdot P_{jt}}{(1 + \mu_j)} \right). \]

Introducing \( \gamma \), dividing by \( A \), and taking logs yields (9):

\[ \ln \frac{V_{jt}}{A_{jt}} = \ln q_t + \ln(1 + \mu_j) + \ln \left( 1 + \alpha \frac{R_{jt}}{A_{jt}} + \gamma \frac{P_{jt}}{A_{jt}} \right), \quad \gamma = \frac{u}{1 + \mu_j} + \beta. \]

Since \( \gamma P \ll K \) (as found in the nonlinear estimation), then the last term can be expanded using a Taylor series approximation:

\[ \ln \left( 1 + \frac{\alpha R_{jt}}{A_{jt}} + \gamma \frac{P_{jt}}{A_{jt}} \right) \approx \ln \left( \frac{A_{jt} + \alpha R_{jt}}{A_{jt}} \right) + \gamma \frac{P_{jt}}{A_{jt}} + \frac{1}{2} \left( \gamma \frac{P_{jt}}{A_{jt}} \right)^2 + \ldots \]

Substituting this into (9) and assuming that \( \alpha \approx 1 \), so that \( K^* \equiv A + R \approx A + \alpha R \),

\[ \ln \frac{V_{jt}}{A_{jt}} \approx \ln q_t + \ln(1 + \mu_j) + \ln \left( \frac{K_{jt}^*}{A_{jt}} \right) + \gamma \frac{P_{jt}}{K_{jt}^*} + \frac{1}{2} \left( \gamma \frac{P_{jt}}{K_{jt}^*} \right)^2 + \ldots \]

or

\[ \ln \frac{V_{jt}}{K_{jt}^*} \approx \ln q_t + \ln(1 + \mu_j) + \gamma \frac{P_{jt}}{K_{jt}^*} + \frac{1}{2} \left( \gamma \frac{P_{jt}}{K_{jt}^*} \right)^2 + \ldots \]

which is equivalent to (11).

Imputations used in Table 6

Cockburn and Griliches (1988) use a sample of large, publicly held manufacturing firms. Their regression equation is equivalent to a first-order Taylor series approximation of (9) except that the patent term they use is \( P/A \) rather than \( P/(A+R) \). To obtain the equivalent to \( \gamma \), I multiply their coefficient (.111) times 1.2 (the mean ratio of \( (A+R)/A \) for my sample of large public firms in 1980),
yielding .133, which, deflated to 1992 dollars, is .213.

Megna and Klock (1993) use a similar term, and I use the mean ratio of \((A+R)/A\) for my sample of semiconductor firms for 1979-91 (1.62).

Hall et al. (2005) use a rather different specification that is not so easily compared to (10) or (11). Instead, I calculate the implied increase in firm value from an additional patent, holding all else constant. This is \(0.018*V/R\) at the sample mean and \(0.022*V/R\) at the sample median using the regression coefficients in their column 1 Table 3. Using the reported mean and median values of \(V\) and \(R\), and deflating, this yields equivalent estimates of \(\gamma\) of .093 and .252.

Putnam (1996) reports that the mean value of a family of international patents held in the United States is $245,000 in 1974 dollars. Only 36% of the patents filed in the US are also filed abroad. Putnam also reports that for Germany, the aggregate value of all patents (both those filed internationally and those filed only at home) is 5% greater than the aggregate value of internationally filed patents. This implies that the aggregate value of patents is \(1.05 \times 245,000\) x no. of int’l patents. Therefore the mean value of all patents is aggregate value/total no. of patents = \(1.05 \times 245,000 \times .36 = 92,600\), which, deflated to 1992 dollars, is $230,000. This number is not too sensitive to the 5% figure; e.g., if it is calculated assuming that domestic-only patents add 10% to the aggregate value of patents, then the mean value is $241,000.

Putnam also reports that patents granted in the US and also filed abroad were worth $75,700 in 1974 dollars. Assuming the same relationship between domestic and international patent value as in Germany, the mean value for US patents should be \((75,700 + .05*245,000)\)*no. of international patents/total no. of US patents = $31,700 in 1974 dollars, or $78,800 in 1992 dollars. The ratio of domestic patent value/worldwide patent value is then $78,800/$245,000 = 32%.
References


Court, A.T., 1939. “Hedonic price indexes with automotive examples.” In: General Motors Corporation (Ed.), The Dynamics of Automobile Demand, New York.


Table 1. Sample Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean (million $)</th>
<th>Median (million $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm market value, $V$</td>
<td>1568.0</td>
<td>80.5</td>
</tr>
<tr>
<td>Log $Q$</td>
<td>0.85</td>
<td>0.61</td>
</tr>
<tr>
<td>Patent stock, $P$</td>
<td>86.8</td>
<td>4.5</td>
</tr>
<tr>
<td>R&amp;D stock, $R$</td>
<td>227.9</td>
<td>15.5</td>
</tr>
<tr>
<td>Accounting assets, $A$</td>
<td>917.0</td>
<td>34.9</td>
</tr>
<tr>
<td>Percent observations with no patents</td>
<td>19%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: 25,861 observations for 3,451 firms from 1979 – 97.
Table 2. Basic Specifications

<table>
<thead>
<tr>
<th>Estimation technique</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NLLS</td>
<td>OLS</td>
<td>FE</td>
<td>D4</td>
<td>D4</td>
</tr>
<tr>
<td>Dependent variable</td>
<td>Ln V/A</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
</tr>
<tr>
<td>( \gamma ) ($92 million)</td>
<td>0.370 (0.024)</td>
<td>0.445 (0.037)</td>
<td>0.205 (0.061)</td>
<td>0.351 (0.072)</td>
<td>0.290 (0.084)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.992 (0.023)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((P/(A+R))^2)</td>
<td>-0.070 (0.018)</td>
<td>-0.019 (0.020)</td>
<td>-0.031 (0.007)</td>
<td>-0.031 (0.007)</td>
<td></td>
</tr>
<tr>
<td>((P/(A+R))^3)</td>
<td>0.003 (0.002)</td>
<td>0.001 (0.001)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((C/(A+R)))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. observations</td>
<td>25,861</td>
<td>25,861</td>
<td>25,861</td>
<td>13,317</td>
<td>13,317</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.625</td>
<td>0.066</td>
<td>0.550</td>
<td>0.157</td>
<td>0.157</td>
</tr>
</tbody>
</table>

Notes: Standard error in parentheses are heteroscedasticity-robust. All regressions include year dummies. NLLS regression uses equation (10); other regressions use equation (11).
Table 3. Additional Regressions

<table>
<thead>
<tr>
<th>Estimation technique</th>
<th>1 year&lt;1990</th>
<th>2 year&gt;=1990</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLLS</td>
<td>NLLS</td>
<td>D4</td>
<td>D4</td>
<td>D4</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ ($92$ million)</td>
<td>0.373 (0.034)</td>
<td>0.353 (0.033)</td>
<td>0.462 (0.091)</td>
<td>0.353 (0.072)</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.542 (0.050)</td>
<td>0.737 (0.024)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(P/(A+R))^2$</td>
<td>-0.059 (0.015)</td>
<td>-0.032 (0.007)</td>
<td>-0.019 (0.007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P/(A+R) \times (yr&gt;1989)$</td>
<td>-0.161 (0.098)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(P/(A+R))^2 \times (yr&gt;1989)$</td>
<td>0.041 (0.018)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log rival’s patents</td>
<td>-0.016 (0.007)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(P/(A+R))/ domestic share of profits</td>
<td></td>
<td></td>
<td></td>
<td>0.070 (0.027)</td>
<td></td>
</tr>
<tr>
<td>$(P/(A+R)) \times (no foreign profits reported)$</td>
<td></td>
<td></td>
<td></td>
<td>0.239 (0.063)</td>
<td></td>
</tr>
<tr>
<td>No. observations</td>
<td>13621</td>
<td>12240</td>
<td>13317</td>
<td>13317</td>
<td>13317</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.323</td>
<td>0.350</td>
<td>0.157</td>
<td>0.157</td>
<td>0.156</td>
</tr>
</tbody>
</table>

Notes: Standard error in parentheses are heteroscedasticity-robust. All regressions include year dummies. NLLS regression uses equation (10); other regressions use equation (11). Rival’s patents are the sum of distance-weighted patent stocks; the distance measure is described in the text.
Table 4. Estimates from Separate Industry Regressions

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals, excluding large pharmaceuticals</td>
<td>28</td>
<td>1,465 (344)</td>
<td>1,543</td>
<td>25%</td>
<td>37%</td>
</tr>
<tr>
<td>Large pharmaceutical firms</td>
<td>2834</td>
<td>7,177 (2,801)</td>
<td>272</td>
<td>60%</td>
<td>79%</td>
</tr>
<tr>
<td>Other industries</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machinery Including computers</td>
<td>35</td>
<td>-60 (215)</td>
<td>2,285</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electronics</td>
<td>36</td>
<td>407 (211)</td>
<td>2,304</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Instruments</td>
<td>38</td>
<td>380 (149)</td>
<td>2,225</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other manufacturing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Business services, including software</td>
<td>73</td>
<td>76 (1,041)</td>
<td>645</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other non-manufacturing</td>
<td>243</td>
<td>243 (182)</td>
<td>767</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEIGHTED MEAN</td>
<td></td>
<td>798 (88)</td>
<td>13,317</td>
<td>18%</td>
<td></td>
</tr>
</tbody>
</table>

Note: Estimates are for separate industries using the specification in Table 2, Column 4 for 1979-97. Robust standard errors in parentheses. Bold coefficients are significant at the 5% level or better. The large pharmaceutical category includes firms whose primary business is in SIC 2834, who have over 500 employees and are not identified as primarily manufacturers of generic drugs. The mean and share of aggregate value are weighted by the stock of patent applications in the observation year. The aggregate value calculation ignores the role of $β$ and assumes that $γ$ entirely represents the discounted value of patent rents. The patent rents/R&D ratios are aggregate patent rents (depreciated patent stock times $γ$ times a flow rate of 15%) divided by deflated depreciated R&D stock.
Table 5. Regressions on Squared Residuals

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 / P$</td>
<td>0.355 (0.057)</td>
<td>0.181 (0.056)</td>
</tr>
<tr>
<td>No patents dummy</td>
<td>5.785 (0.469)</td>
<td>3.222 (0.477)</td>
</tr>
<tr>
<td>Ln employment</td>
<td></td>
<td>-2.963 (0.123)</td>
</tr>
<tr>
<td>Ln deflated R&amp;D stock</td>
<td></td>
<td>1.527 (0.126)</td>
</tr>
<tr>
<td>Constant</td>
<td>4.396 (0.165)</td>
<td>0.417 (0.449)</td>
</tr>
</tbody>
</table>

No. observations: 13317, 13317

Adjusted $R^2$: 0.013, 0.064

Note: Heteroscedastic-robust standard errors in parentheses. Dependent variable is square of residuals from regression in Table 2, column 4.
Table 6. Comparison to Other Market Value Regressions

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample</th>
<th>$\gamma$ or equivalent (1000s $$92)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hall et al. (2005), using means</td>
<td>US public manufacturing firms, 1979-88</td>
<td>$119</td>
</tr>
<tr>
<td>Hall et al. (2005), using medians</td>
<td></td>
<td>$322</td>
</tr>
<tr>
<td>This paper</td>
<td>US public firms, 1979-97</td>
<td></td>
</tr>
<tr>
<td>Table 2, Column 1</td>
<td></td>
<td>$370</td>
</tr>
<tr>
<td>Table 2, Column 4</td>
<td></td>
<td>$351</td>
</tr>
</tbody>
</table>
Table 7. Comparison to Estimates Based on Patentee Behavior (pooled data)

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample</th>
<th>Value of worldwide patents (1000s $92)</th>
<th>Value of domestic patents (1000s $92)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US patents granted to US public manufacturing firms, 1985-91</td>
<td>$113</td>
<td></td>
</tr>
<tr>
<td>This paper</td>
<td>US public firms, 1979-97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Table 2, column 4</td>
<td></td>
<td>$351</td>
<td></td>
</tr>
<tr>
<td></td>
<td>“ prorated</td>
<td>$112</td>
<td></td>
</tr>
<tr>
<td>Table 3, column 5</td>
<td></td>
<td>$70</td>
<td></td>
</tr>
</tbody>
</table>

Note: see text and Appendix for details of comparisons. The prorated estimate of domestic patent value is calculated by multiplying $351 by 0.32, a ratio of domestic US patent value to worldwide patent value derived in the Appendix.
Table 8. Comparison to Estimates Based on Renewal Data by Technology/Industry

<table>
<thead>
<tr>
<th>Study</th>
<th>Technology/Industry</th>
<th>Value of domestic patents (1000s $92)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bessen (2006)</td>
<td>Chemical patents</td>
<td>$497</td>
</tr>
<tr>
<td></td>
<td>Drug &amp; medical patents</td>
<td>$120</td>
</tr>
<tr>
<td></td>
<td>Other technologies</td>
<td>$39 - 86</td>
</tr>
<tr>
<td>This paper, Table 4</td>
<td>Chemical industry excluding large</td>
<td>$469</td>
</tr>
<tr>
<td></td>
<td>pharmaceutical firms</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Large pharmaceutical firms</td>
<td>$2,297</td>
</tr>
<tr>
<td></td>
<td>Firms in other industries</td>
<td>$83</td>
</tr>
</tbody>
</table>

Note: Figures in the bottom panel are calculated by multiplying estimates of $\gamma$ from Table 4 by 0.32, a ratio of domestic to worldwide patent value derived in the Appendix.