ESTIMATES OF FIRMS’ PATENT RENTS FROM FIRM MARKET VALUE

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Abstract: The value of patent rents is an important quantity for policy analysis. However, estimates based on renewal behavior may be understated. I develop a simple model to make upper bound estimates using regressions on Tobin’s $Q$. I test this model on a sample of US firms and find it robust to a variety of considerations. My estimates correspond well with implied estimates derived from previous research and with estimates based on renewal behavior and other patentee behavior. I also find that chemical/pharmaceutical patents are much more valuable than others, likely accounting for a large share of aggregate patent value.
Introduction

Patents are intended to provide an economic incentive for invention by granting the patent holder an exclusive right for a limited period. This right to exclude allows a patent-holding firm to become a monopolist or, perhaps more often, to achieve some lesser degree of market power, either in product markets or in the market for technology licenses. This market power, in turn, permits the firm to earn supra-normal profits, “rents,” and these are the source of the economic incentive to invent.

The value of patent rents is thus an important quantity for evaluating the performance of the patent system and also for understanding firm value. Some researchers have used the observed behavior of patent owners to estimate the private value of patents, which should equal the discounted value of patent rents. Beginning with Pakes and Schankerman (1984), these studies have imputed patent value from observed decisions to pay maintenance fees,\(^1\) decisions to file patents in multiple countries (Putnam 1996), and decisions to sell (re-assign) patents (Serrano 2006).

But these approaches share an important limitation: they do not directly reflect the value of the most valuable patents and, given the skewed distribution of patent values, most of the aggregate value of patents is determined by the relatively small number of highly valuable patents. These studies typically assume a distributional form, such as a log-normal distribution. They then fit that distribution to the observed data and extrapolate to the upper tail. However, if the upper tail diverges significantly from the assumed distribution, then estimates of mean patent value might be too large or too small (although estimates of median patent value obtained from these methods are accurate). In the worst case, the upper tail might be so “heavy” that the actual distribution has an infinite mean as with the Pareto distribution (Scherer and Harhoff 2000). Then estimates of the mean would be unstable and would not converge even at asymptotically large sample sizes.

An alternative might be to use firm market value to estimate patent value, that is, to decompose firm value into its component parts including that part attributed to patents. This way, investor behavior, rather than the behavior of patent owners, might reveal patent value. At the very least, estimates based on firm market value might serve as an important check on the values obtained from data on the behavior of patent owners.

A large number of researchers have run regressions that use firm market value (or Tobin’s \(q\), which is firm market value divided by the replacement value of firm assets) as the dependent variable.

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1 See Lanjouw et al. 1998 for a review of this literature and Baudry and Dumont (2006), Bessen (2006), Gustafsson (2005) and Serrano (2006) for more recent estimates of renewal value.
and some measure of patents (patent flows, patent stocks, or patent citation stocks) as an independent variable.\(^2\) Many of these studies build on Griliches’s (1981) “hedonic” model of the firm, where investors are assumed to view the firm as a bundle of characteristics that determine the firm’s value. Patents are assumed to be one of these characteristics.

These studies generally show a positive correlation between a firm’s patents and its market value. In his seminal paper, Griliches estimated that the regression coefficients implied that “a successful patent is worth about $200,000.” However, subsequent studies have generally not calculated the implied value of firm patent rents. Perhaps this is because researchers recognize that the correlation between firm value and patents may involve more than just the direct contribution of patent rents to firm value. Indeed, the inclusion of patent variables in the estimated models is sometimes ad hoc, and, as I show below, patent rents are not fully identified. The correlation between patents and market value partly reflects the value of the patents per se, but patents also proxy for other unmeasured variables, and these account for part of the correlation as well.

But can market value regressions reveal any useful information about the magnitude of firm patent rents? Building on the previous theoretical and empirical literature, this paper develops a formal model of the relationship between patents and firm value. Although a coefficient corresponding to patent rents cannot be fully identified, I show that an upper bound on mean rents per patent can be estimated. This permits me to make some limited inferences about patent rents and patent value.

I show that these estimates are robust to a variety of considerations including firm-specific differences in appropriability conditions, other firm characteristics, different specifications and stability over time. I further test whether these estimates appear to be stable in light of the skewed distribution of invention values (Scherer 1965, Scherer and Harhoff 2000, Silverberg and Verspagen 2004). I find that my estimates of mean patent value show definite evidence of convergence to the mean, suggesting that the distribution of patent values does not have an infinite mean (n.b., invention values may be different).

My estimates correspond well with estimates derived from previous market value regressions on US data. And my estimates also correspond well with estimates of patent value based on patent renewal data and international filing data. Using a sample of publicly listed US firms, I find that upper bound estimates of worldwide mean patent rents correspond reasonably well to Putnam’s (1996) estimates of worldwide patent value of US inventions obtained using data on international patent filings. I also

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\(^2\) See Hall (2000) for a review of this literature. Some recent additions to this literature include Bosworth and Rogers (2001), Toivanen et al. (2002), Hall et al. (2005) and Griffiths et al. (2005).
obtain upper bound estimates of domestic patent value that are reasonably close to estimates based on renewal data.

Thus although only limited inferences can be drawn from these market value regressions, they do tend to confirm estimates of patent value derived from the observed behavior of patent owners, suggesting that these other estimates do not seriously understate mean patent value. And they appear to provide upper bound estimates that do not seriously overstate patent value.

1. Model

Following Griliches (1981), a substantial number of studies include firm patents as a right hand side variable in firm market value regressions. These studies do not measure the value of patent rents per se. The difficulty with extracting estimates of patent rents from these regressions is that the relationship between patents and firm value involves more than just the contribution of patent rents to firm value. Specifically, patents may also serve as a proxy for the quality of a firm’s technical knowledge. Empirical economists have used a capitalized measure of firm R&D spending as a proxy for firm knowledge:

It is widely understood that the R&D conducted by private firms is an investment activity, the output of which is an intangible asset that can be labeled as the firm’s “knowledge stock.” If this asset is known to contribute positively to the firm’s future net cash flows, then the size of a firm’s knowledge stock should be reflected in the observed market value of the firm. This implies that a firm’s R&D investments should be capitalized in the firm’s market value. Further, since the output of the R&D investment process is stochastic, some of the R&D will result in the creation of more valuable knowledge capital; if this success is observable, then it should be reflected in greater market value bang for the R&D buck. (Hall et al. 2005)

Because patents tend to be correlated with “successful” R&D, patents are correlated with the quality of that R&D and the contribution of R&D assets to the value of the firm. This has prompted the inclusion of patent measures on the right hand side of firm value regressions as a quality measure. But note that this relationship between firm value and patents is distinct from (although related to) the direct role of patents in providing rents. In the following model, I spell out the separate contributions to firm value of patent rents and of patents as a quality measure.

1.1 Patent rents

Hayashi (1982) identified a relationship between Tobin’s $Q$ and the value of rents for firms with market power. Under assumptions of constant returns to scale and profits as a function of an aggregate capital stock in nominal dollars, $K$, for the $j$th firm at time $t$, 

$$Q_j = \frac{V_j}{K_j}$$

where $V_j$ is the value of the firm at time $t$. This relationship implies that the value of a firm is a function of its market value relative to its capital stock, indicating the presence of monopoly rents. A typical empirical specification of the Tobin’s $Q$ model is:

$$Q_j = \alpha + \beta R^*_j + \gamma P_j + \epsilon_j$$

where $R^*_j$ represents the rents from patent activities, $P_j$ represents other types of rents, and $\epsilon_j$ is the error term.

This specification suggests that the market value of a firm is a function of the rents generated from patent activities, as well as other types of rents. The coefficient $\beta$ measures the sensitivity of the firm's market value to the rents from patent activities.

Additional controls for firm characteristics such as size, profitability, and industry are often included to further refine the model. The inclusion of such controls helps to isolate the specific impact of patents on firm value.

In the context of the model presented, the rents from patent activities, $R^*_j$, are an important component of the firm's value. The empirical analysis aims to estimate the magnitude of these rents and their contribution to the firm's overall market value.

The relationship between Tobin’s $Q$ and patent rents highlights the importance of understanding the role of patents in generating value for firms. By modeling patent rents as a function of patent characteristics and firm-specific attributes, researchers can better assess the economic impact of patents on firm value and market performance.

In summary, the empirical analysis of the relationship between Tobin’s $Q$ and patent rents provides valuable insights into the economic significance of patents. Through careful specification and estimation, the model captures the complex interactions between patents, firm characteristics, and market value, offering a nuanced understanding of the economic implications of patent activities.
\[ Q_{jt} = \frac{V_{jt}}{K_{jt}} = q_j \left( 1 + \frac{W_{jt}}{K_{jt}} \right) \]

where \( V \) is firm market value, \( W \) is the discounted value of firm rents, and \( q \) is “marginal \( q \)” which reflects short term disequilibrium in capital markets. Marginal \( q \) is assumed to be approximately equal to 1 and, given competitive capital markets, it is equal across firms at any given time.

This equation captures two intuitions. One intuition is Tobin’s original insight that the market value of a firm is related to the replacement cost of its assets. In a competitive market with no market power, firms will add capacity (either new entrants or existing firms) at the replacement cost of capital, driving prices down until, in long run equilibrium, the discounted stream of expected future profits (market value) equals the cost of those assets. The second intuition is that firms with market power earn sustained supra-normal profits reflected in a higher market value.

The discounted rents, \( W \), consist of rents earned from patents and rents earned by other means. Patents earn rents by their power to exclude other firms from the market or, at least, to exclude firms from using processes or product features that are patented. It is natural then to decompose \( W \)
\[ W_{jt} = u_j P_{jt} + \mu_j K_{jt} \]

where \( u \) is the mean value of rent per patent, \( P \) is the patent stock, and \( \mu \) is the markup for rents that the firm earns on its assets through other means. These other means of earning supra-normal profits might include rents on technical knowledge realized through first-mover advantage, trade secrecy, etc. or they might include other sorts of rents, e.g., from barriers to entry.

### 1.2 R&D quality

I assume that the aggregate capital stock is a sum of depreciated quantities of different asset types.\(^3\) For physical assets, the replacement value is predictably related to the original investment—the current value of the asset can be calculated using appropriate adjustments for depreciation and changing asset prices.

But the relationship between R&D investment and the replacement value of technical knowledge is less predictable.\(^4\) R&D is a highly uncertain process. Consider R&D projects that cost $1 million and which have a 10% chance of “success,” but where the technical knowledge developed is worthless for an unsuccessful project. The mean replacement value for an R&D investment is 10% of the original investment, ignoring depreciation. But the specific replacement value for any given firm depends on its

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\(^3\) Hayashi and Inoue (1991) argue that this should be a weighted sum where the weights are determined by the relative marginal productivity of the different capital types.

\(^4\) The difference is one of degree because depreciation of a physical asset is also often stochastic.
realized success rate. An unsuccessful project has zero replacement value; a successful project has a replacement value of $10 million—that is, a firm would have to spend $10 million in order to develop technical knowledge with an expected value equal to the value of the successful $1 million investment.

This suggests that the aggregate capital stock can be represented

\[ K_{jt} = A_{jt} + s_{jt} R_{jt}, \quad E[s] \approx 1 \]

where \( A \) is physical assets, \( R \) is the R&D stock (calculated by applying the declining balance method to the stream of deflated R&D investments), and \( s \) is a quality adjustment dependent on the firm’s \textit{ex post} “success” rate. \( s \) is normalized so that it is approximately equal to unity.

A key insight of the empirical literature on firm value is that a firm’s patent stock (or patent citation stocks) might serve as a proxy for the quality of its R&D stock. This relationship arises for a simple reason: a firm is likely to obtain more patents for a “successful” R&D project than it will obtain for an unsuccessful one. And this occurs because patent protection is relatively more valuable than non-patent alternatives for successful projects. In other words, the idea of using patents as a proxy for R&D quality implicitly involves a model of a trade-off—patents increase the profits from a successful R&D project (relative to non-patent alternatives), but patents also involve some additional cost. The premium that patents provide on more successful projects is greater than the premium on relatively less successful projects, so more successful projects will be patented more frequently.

Arora et al. (2005) provide one specific model of the patent premium trade-off. A general reduced-form relationship might be expressed in a \textit{patent propensity} equation,

\[ P_{jt} = R_{jt} G(s_{jt}, \mu, u, c) \]

where the number of patents obtained per R&D dollar increases as a function of the measure of R&D quality, \( s.\)\(^5\) This relationship is also affected by parameters affecting the cost of patenting, \( c, \) the mean value per patent, \( u, \) and the effectiveness of other means of appropriation, \( \mu. \) These parameters are involved because they affect the relative profitability of obtaining a patent compared to earning profits without patents. Inverting this relationship and assuming, as a first order approximation, a simple linear form, the R&D quality measure relates to patent stock as:

\[ s_{jt} = \alpha + \beta \frac{P_{jt}}{R_{jt}}, \quad \alpha = \alpha(\mu, u, c), \quad \beta = \beta(\mu, u, c). \]

This equation formally captures the intuition of Hall et al. (2005) that “the additional informational value of patents once R&D has already been factored in must reside in the number of

\(^5\) This specification assumes constant returns to scale in R&D productivity.
patents per dollar R&D: if the yield of R&D in terms of patents is higher than average that may indicate that the R&D project succeeded beyond expectations, and conversely if the patent yield is low.” However, the formal representation in (4) goes a bit beyond this informal observation in two ways. First, as I show in the next section, (4) leads to a slightly different functional specification than that chosen by Hall et al. Second, my derivation from a patent propensity equation highlights that the parameters $\alpha$ and $\beta$ will, in general, change along with changes in the value of patents, the cost of patents and other factors that affect patent propensity. This means that differences in patent propensity between groups or over time will likely correspond to changes in the relationship between patents and firm market value, thus limiting what inferences can be safely drawn from comparisons of market value regressions between groups or over time.

These parameters might also vary from firm to firm along with the degree to which each firm earns supra-normal profits without patents, $\mu_j$. In the regressions below, I ignore this firm level heterogeneity in $\alpha$ and $\beta$, but I do capture the direct effect of $\mu_j$ on firm value (an intercept term).

Finally, assuming that the second term of (4) is small implies that $\alpha$ approximately equals one, given the normalization above. As we shall see below, estimates obtained over an extended time period (to average out short term fluctuations in the productivity of R&D) do, indeed, show values of $\alpha$ close to one and relatively small values for the second term in (4). Below I check the robustness of my estimates to variation in $\alpha$.

Equation (4) could be further enhanced to include some measure of patent citations. Hall et al. (2005) argue that patent citations add significant informational content in a market value regression and patent citations have been frequently used to capture notions of patent quality or invention quality. Below I also estimate an alternative specification where the number of citations received per patent are assumed to capture the “quality” of the patent.

1.3 Estimable specifications

Substituting (2) – (4) into (1),

$$V_{jt} = q_t \left( 1 + \mu_j + \frac{uP_{jt}}{K_{jt}} \right) \left( 1 + \alpha \frac{R_{jt}}{A_{jt}} + \beta \frac{P_{jt}}{A_{jt}} \right).$$

Assuming that $\alpha$ approximately equals one, and ignoring second order terms, this can be re-arranged,

$$\ln \frac{V_{jt}}{A_{jt}} = \ln q_t + \ln(1 + \mu_j) + \ln \left( 1 + \alpha \frac{R_{jt}}{A_{jt}} + \gamma \frac{P_{jt}}{A_{jt}} \right), \quad \gamma \equiv \frac{u}{1 + \mu_j} + \beta.$$

If we ignore firm fixed effects, assuming that $\mu$ is negligible, then this equation can be estimated using
Non-Linear Least Squares. This is one specification I use below.

However, firm specific effects may be an important source of heterogeneity. Moreover, depending on the nature of the patent propensity equation, \( \mu \) may be correlated with the patent stock. For instance, firms that earn substantial non-patent rents may be less likely to patent successful R&D projects, all else equal. So it is helpful to also have a specification that incorporates firm effects.

Using the Taylor series expansion of (6),

\[
\ln \frac{V_{jt}}{A_{jt}} = \ln q_t + (1 + \mu_t) + \ln \left(1 + \alpha \frac{R_{jt}}{A_{jt}}\right) + \gamma \frac{P_{jt}}{K_{jt}} - \frac{\gamma^2}{2} \left(\frac{P_{jt}}{K_{jt}}\right)^2 + \ldots
\]

Then, maintaining the assumption that \( \alpha \) approximately equals one, and ignoring second order terms, this can be re-arranged as

\[
(7) \quad \ln \frac{V_{jt}}{A_{jt} + R_{jt}} = \ln q_t + \delta_j + \gamma \frac{P_{jt}}{A_{jt} + R_{jt}} - \frac{\gamma^2}{2} \left(\frac{P_{jt}}{A_{jt} + R_{jt}}\right)^2 + \ldots + \epsilon_{jt}
\]

This specification uses a modified version of Tobin’s \( Q \) as the dependent variable, but it can be estimated using fixed effects, random effects, first differences, or longer differences.

Under the normalization that \( E[\mu] = 0 \), the estimated coefficient of the patent term in both regressions is

\[ \hat{\gamma} = u + \beta. \]

Since \( \beta \) is positive (assuming a positive relationship between R&D success and patenting), the estimated coefficient is thus an upper bound on \( u \), the value of patent rents. In addition, with specification (7), any measurement error in \( R \) will tend to bias the coefficient estimates upwards as well. This means that these estimates are biased estimates of patent value. Moreover, given evidence about changing patent propensity (e.g., Hall and Ziedonis 2001, Bessen and Hunt 2006), it seems likely that \( \beta \) may change over time and across groups.

The functional forms of (6) and (7) differ slightly from specifications used in the literature. Below I derive estimates of \( \gamma \) from statistics reported in several other papers. However, in general, these other specifications generate coefficients on patent terms that involve combinations of \( u \) and \( \beta \), often with some additional variables as well. So these coefficient estimates are likely to suffer from the same limitation.

Nevertheless, upper bound estimates are useful. Here I intend to use them as a check on estimates

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6 This normalization follows from the assumption that average “normal” returns on capital do not include positive rents. If one assumed, instead, that firms earned non-patent rents on average, then estimates of the upper bound might be 10-20% higher.
of patent value obtained by other methods.

2. Data

2.1 Sample

The data come from two sources, Compustat and a database of patent information from 1969 through 2002. The patent data come from the US Patent and Trademark Office (USPTO) and have been supplemented with information on citations compiled by Bronwyn Hall.\(^7\)

To match patent data to firms in Compustat, I used a matching program initially developed for another project (Bessen and Meurer 2005). The USPTO provides an assignee name for every assigned patent after 1969. To match the USPTO assignee name to the Compustat firm name, we began with the match file provided by the NBER (Hall et al. 2001). To this we added matches on subsidiaries developed by Bessen and Hunt (2006), we manually matched names for large patenters and R&D-performers, and we matched a large number of additional firms using a name-matching program.\(^8\) In addition, using data on mergers and acquisitions from SDC, we tracked patent assignees to their acquiring firms. Since a public firm may be acquired, yet still receive patents as a subsidiary of its acquirer, we matched patents assigned to an acquired entity in a given year to the firm that owned that entity in that year.\(^9\) Finally, using a software program, we identified a group of Compustat firms that had unique names that could not be found in the USPTO list of assignees. These were classified as definite non-matches.

This group of firms with match information (either a match or a definite non-match) includes 10,736 patent assignees matched to one of 8,444 owning firms in Compustat, with as many as five different owners matched to each assignee. This group accounts for 96% of the R&D performed by all US Compustat firms, 77% of all R&D-reporting firms listed in Compustat and 62% of all patents issued to domestic non-governmental organizations during the sample period. Sample statistics show that this sample is broadly representative of the entire Compustat sample, although it is slightly weighted toward larger and incumbent firms. Testing our match against a sample of 131 semiconductor industry firms that had been manually matched, we correctly matched 90% of the firms that accounted

\(^7\) Downloaded from http://emlab.berkeley.edu/users/bhhall/bhdata.html

\(^8\) A software program determined matches between the two files by identifying firm names that matched after taking into account spelling errors, abbreviations and common alternatives for legal forms of organization.

\(^9\) This dynamic matching process is different from that used in the original NBER data set which statically matched a patent assignee to a Compustat firm. These data were developed with the help of Megan MacGarvie, to whom I am indebted.
for 99.5% of the patents acquired by this group.\textsuperscript{10}

From this group with matching information, I excluded firms that did not have at least four years of non-missing data on key variables and firms that did not perform significant R&D. I kept observations from 1979 through 1997, using the first ten years of data to build stock variables for patents and R&D and eliminating later years because of possible truncation bias in patent application data (see below). Finally, because (6) and (7) involves ratios and consequently any measurement error may be greatly exaggerated in the tails of the distributions of the variables, I trimmed the sample of the 1% tails of Tobin’s $Q$. I also experimented with other screens. I obtained similar coefficient estimates with these screens, but these methods seemed more arbitrary.

This left me with 25,861 observations of 3,451 firms. Sample statistics are shown in Table 1. As can be seen, the sample is broadly representative of R&D-performing firms, including a large portion of small and newly public firms. 85% of the observations are for firms whose primary business is in the manufacturing sector.

2.2 Variables

Key variables are defined as follows:

- The market value of the firm, $V$, consists of the sum of all the claims on the firm, namely, the sum of the value of the common stock, the preferred stock (valued by dividing the preferred dividend by Moody’s Index of Medium Risk Preferred Stock Yields), long term debt adjusted for inflation (see Hall 1990 and Brainard et al. 1980), and short term debt net of current assets.
- The value of assets, $A$, is the sum of the net value of plant and equipment, inventories, accounting intangibles, and investments in unconsolidated subsidiaries all adjusted for inflation using the method of Lewellen and Badrinath (1997).\textsuperscript{11}
- The R&D stock, $R$, is calculated assuming a 15% annual depreciation rate and an 8% pre-sample growth rate (Hall 1990). I use Bronwyn Hall’s R&D deflator to obtain the current value of $R$ from the stream of past investments.
- The patent stock of the firm, $P$, is based on the number of patent applications each year that resulted in a grant of a patent by 2002. Since there is a lag of possibly several years between the application and grant of a patent (see Hall et al. 2005), I only use data through 1997. I calculate the patent stock using a 15% depreciation rate. I also calculate patent citation

\textsuperscript{10} Thanks to Rosemarie Ziedonis, who originally compiled this data, for sharing it with me.

\textsuperscript{11} Thanks to Bronwyn Hall for providing Stata code to compute this. The code was developed by Bronwyn and Daehwan Kim.
stocks (stocks of citations received through 2002), using a 15% depreciation and adjusting for
truncation using the method described in Hall et al. (2005). In order to interpret the coefficient \( \gamma \)
in constant ($92) dollars, I multiply the associated variable \( \frac{P}{A} \) in equation 6 and \( \frac{P}{(A+R)} \) in
equation 7) by the GDP deflator.

- To explore possible strategic interaction, I also develop a measure of rival firms’ patent
stocks. I do this using a technology distance measure developed by Jaffe (1986). I calculate the
technology distance between two firms as follows. For each firm I construct a vector of the
share of its patents that falls into each USPTO technology class (there are over 400 in the 1999
classification I use). The distance measure is then the uncentered correlation between these two
vectors (the vector product divided by the product of the standard deviations of each vector).
This measure is 1 if the firms distribute their patents identically across classes and zero if they
share no patent classes (hence this distance measure might be more appropriately termed a
“nearness” measure). I calculate this measure using pooled patent data over three periods from
1979 through 1999. I calculate the measure of rivals’ patents as the sum of the patent stocks of
all other firms weighted by the other firms’ distances.

3 Empirical Results

3.1 Regression estimates

Column 1 of Table 2 explores the nonlinear specification in (6), which ignores firm specific
effects. The estimate of \( \gamma \) is $370,000 in 1992 dollars and the estimate of \( \alpha \) is just about equal to one, as
predicted. This estimation is made over a 19 year period. Below I explore the possibility that these
parameters may shift over time. I also tested the sensitivity of the estimates of \( \gamma \) to changes in \( \alpha \). I ran a
series of regressions (not shown) using different, fixed values for \( \alpha \). I found that the resulting changes
in \( \gamma \) were small for both the specification in (6) and the one in (7). This suggests that the assumption
that \( \alpha \) approximately equals one is a safe assumption, at least for a sufficiently long sample period.

The remaining columns in Table 2 explore specification (7), which permits firm heterogeneity.
Column 2 is estimated using simple Ordinary Least Square without firm effects and Column 3 is
estimated using firm fixed effects. The differences are significant and a good deal of the variance is
explained by the fixed effects. I also ran the same regressions using random effects. A Hausman test
rejected the random effects specification, however. These results indicate that firm heterogeneity is
important.
These regressions included the first three terms of the Taylor series expansion used in (7). The coefficients have the predicted signs, but third order terms are not significant, both economically and statistically, so I only use the first two order terms in subsequent regressions. Note, however, that the second order term is quite influential and should not be ignored, as has sometimes been done in the literature. This may seem surprising because this term is small for most of the sample; at the mean it is only about .01. However, because some observations have rather high values of \( P/(R+A) \), the second order term is significant for these and estimates of \( \gamma \) show a downward bias if this term is not included.

The estimate of \( \gamma \) in the fixed effects regression is much smaller than the estimate in the simple OLS regression. This may be because of the role of firm fixed effects or it may be because of attenuation—the fixed effects estimate may suffer from the well-known problem of errors in panel data (Griliches and Hausman 1985). Column 4 shows an estimate using four-year differences rather than fixed effects. These estimates should suffer less from problems of errors in the panel data, although at the cost of a smaller sample size. I tested several different lags and found little change in the estimates after a four year lag. This estimate, using four year differences, falls between the OLS and fixed effects estimates and is quite close to the Nonlinear Least Squares estimate in column 1.

Hall et al. (2005) suggest that patent citation data contain additional information about patent or invention quality beyond what is captured in patent count data alone. Unfortunately, current research provides little guidance about how patent citations might affect patent propensity and patent propensity provides the rationale for including patents in a Tobin’s \( Q \) regression in my model. If one supposes that patent citations reflect patent quality (and that this, in turn, affects patent propensity) then citations can be included in my model as follows: in (4) replace \( P \) by

\[
P_{jt} \left( 1 + \delta \frac{C_{jt}}{P_{jt}} \right)
\]

where \( C \) is the patent citation stock. This leads to a regression as in (7) with the addition of a term \( C/(A+R) \) and, possibly, higher order terms. Column 5 of Table shows a specification with just the first order term. In this specification, the citation stock term is not statistically significant and including this term reduces the estimate of \( \gamma \) a bit. Although patent citations may be useful in revealing information about invention quality, these data do not appear to be particularly important to the task I address here.

Table 3 conducts some additional robustness checks. Hall (1993) found that the productivity of R&D capital exhibited short term variation relative to other capital assets. This might imply changes in \( \alpha \) and \( \beta \) over time. Columns 1 and 2 show separate regressions for the first and second halves of my
panel using the nonlinear specification. Although the estimates of $\alpha$ change, the estimates of $\gamma$ do not change significantly. Column 3 shows a similar test using the four-year differenced specification and variables interacted with a time period dummy. The later period shows a lower estimate for $\gamma$, although the difference is only significant at the 10% level.

Because these different estimates of $\gamma$ may reflect changes in $u$ and/or changes in $\beta$, they do not imply that patent value was necessarily lower during the 1990s than during the 1980s. Nevertheless, these results do make it seem unlikely that patent value increased substantially during the 1990s. This might seem to contradict the notion that patents have been getting “stronger.” For example, Jaffe and Lerner argue (2004) that patents have gotten “stronger” during recent decades, although most of the evidence that they cite only pertains to the 1980s.\textsuperscript{12} Evidence based on court decisions suggests a pro-patentee shift after the creation of the Court of Appeals for the Federal Circuit in 1982, but the evidence for the 1990s does not indicate a further pro-patentee shift but, instead, a possible modest anti-patent shift during the 1990s (Lunney 2004, Matthew and Turner 2006). Unfortunately, my data include only three years before 1982, making it difficult to evaluate the period when the pro-patentee shift may have occurred. My results are consistent with the view that patent value has not increased substantially during the 1990s and may have even decreased, although, for reasons mentioned, this evidence is not conclusive.

Column 4 explores the possible role of strategic interaction. Patent rents are realized when patents deliver a degree of market power. This implies, in turn, that other firms lose a degree of market power if they remain in the market. Other firms’ patents might influence the magnitude of patent rents and also their interpretation. In column 4, I add a variable that measures the distance-weighted size of other firms’ patent portfolios. This has a large and significant (at the 2% level) coefficient—at the sample mean, other firms’ patents are associated with a reduction in firm value of 13%. Megna and Klock (1993) also found a negative relationship in a similar regression for the semiconductor industry. This does not necessarily mean that rivals’ patents cause this loss in value, although that is a distinct possibility and a subject for future research. However, this regression shows that the estimate of $\gamma$ is little changed by the inclusion of rivals’ patents in the regression. So the consideration of rival patents

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\textsuperscript{12} Jaffe and Lerner cite the rate of pro-patentee verdicts at the district court and appellate level. This shows a pro-patentee shift during the early 1980s, but a continuous series shows a decline, if anything, during the 1990s (Matthew and Turner 2006). They also argue that increased use of jury trials has made patents stronger. However, even a series of win rates that includes jury trials shows no positive shift after the mid-1980s (Moore 2000). Finally, they argue that patent law has grown more favorable to patentees by providing stronger remedies and a decreased risk of invalidation. This argument is based on their reading of the case law, but all of the case law cited regarding stronger remedies occurred during the 1980s.
does not seem to affect the magnitude of the private rents received, although it may suggest that non-patent rents may be adversely affected by rival’s patents, offsetting the incentive that patents provide to perform R&D.

The regressions explored so far suggest that estimates of $\gamma$ are reasonably robust to a variety of considerations.

### 3.2 Interpreting the estimates

Although the regressions I have run differ from specifications used in the literature, some rough computations can be used to compare previous results to my estimates of $\gamma$. Table 4 shows some approximate calculations of equivalent estimates of $\gamma$ expressed in thousands of 1992 dollars. The details used to calculate these estimates from the original reported coefficients are described in the Appendix.

The three studies cited all use data from US publicly-listed manufacturing firms. One study, Megna and Klock (1993), is for semiconductor firms only. The estimates range from $119,000 to $343,000. My preferred estimate from Table 2, Column 4 falls a bit above the upper end of this range, at $376,000. But overall, these estimates are reasonably consistent.

How, then, do they compare to estimates obtained based on observations of patentee behavior? There are at least two important considerations the need to be taken into account when making such a comparison. First, the value of $u$ in these market value regressions is implicitly the value of worldwide patent rights. Although the patent stock variable is only the stock of US patents, the rents associated with each patent implicitly include the rents earned in other countries as well because none of the regressions specifically control for foreign patenting. It is well-known that valuable inventions will be patented in multiple countries, earning rents. In effect, US patents act as a proxy for the entire international family of patents associated with each US patent.

Second, patents held by public firms are a select sample. Estimates based on renewal studies suggest that patents held by public firms are worth substantially more, maybe 50% more or so, than other patents. This is not surprising for several reasons. Public firms may have greater complementary assets with which to utilize patents. Also, enforcement costs per patent are substantially lower for public firms (Lanjouw and Schankerman 2004).

Putnam (1996) estimated the value of worldwide patent rights for patents that were applied for in

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13 In Bessen (2006) the mean value for all patents granted to domestic patentees in 1991 is $78,000; the mean value for patents granted to public manufacturing firms 1985-91 is $116,000, 49% more.
1974 in more than one country. Using his figures (see Appendix for calculations), I calculate that the world-wide value for inventions associated with US patent applications in 1974 is about $230,000 in 1992 dollars. Considering that this estimate is for all patents, not just patents owned by publicly listed firms, a comparable estimate for publicly traded manufacturing firm might be $300,000 – 350,000. This suggests that after adjusting for differences in samples, the values of $\gamma$ are not all that much larger than the value of patents implied by Putnam’s estimates.

The studies based on renewal and re-assignment data produce estimates for the value of US patents only, not the associated worldwide patent rights. These estimates are $62,000 for patents granted in 1986 (Barney 2002), $47,000 for patents held by small patentees (Serrano 2006), $78,000 for all patents granted in 1991 to domestic patentees (Bessen 2006) and $113,000 for patents granted to publicly listed manufacturing firms from 1985 to 1991 (Bessen 2006).

I compare these numbers to the estimates of $\gamma$ in two ways. First, using Putnam’s data, I estimate that the value of domestic US patents runs 34-37% of the value of the associated worldwide patent rights. This implies, for example, that the estimate of $\gamma$ from Table 2, column 4 corresponds to a domestic patent value upper bound of about $135,000 for publicly listed firms, modestly larger than the most comparable renewal estimate of $113,000. For the other studies, this thumbnail estimate corresponds to mean domestic patent values from $43,000 to $116,000, quite similar to the estimates derived from renewal data.

Second, I estimate the domestic share of $\gamma$ by using a modified regression in Table 3, column 5. I divide the variable $P/(A+R)$ by the portion of the firm’s profits that derive from domestic operations. Assuming that the domestic share of patent rents roughly equals the domestic share of profits, the coefficient on this term should represent the domestic share of $\gamma$. Since not all firms report domestic and foreign profits separately, I also include the original variable $P/(A+R)$ for those observations. The estimate of domestic $\gamma$ I obtain is $82,000.

The overall picture that emerges from these various estimates is encouraging. Although the figures are not precise, estimates based on firm market value appear to be only modestly higher than comparable estimates obtained from data on patentee behavior. This rough correspondence across estimates using a variety of data sources and statistical methods is encouraging. It suggests that the methods using data on various sorts of patentee behavior do not substantially understate patent values. And it also suggests that estimates using market value regressions do not seriously overstate patent value.
3.3 Industry differences

These estimates all use data samples that cover all technologies and industries. Yet some evidence suggests that patent value might vary substantially across technologies or industries. In surveys, research managers in the chemical and pharmaceutical industries rate patents as much more effective than do managers in other industries (Levin et al. 1987, Cohen et al. 2000). Bessen (2006), using the renewal method, finds that chemical patents are about 6 times more valuable than the average patent.14

Table 5 shows separate regressions on different industry groups using the same specification as in Table 2, Column 4.15 Here, the “Large pharmaceutical” category includes firms in SIC 2834 (“Pharmaceutical preparations”), that employ more than 500 employees and that are not primarily manufacturers of generic drugs. The estimates of $\gamma$ show an order of magnitude difference between chemical firms, especially large pharmaceutical firms, and other firms, similar to the renewal data.

It is tempting to interpret these differences as comparably large differences in patent value, however, there is no guarantee that $\beta$ is constant across industries. Indeed, if one believes that “evergreening” is more prevalent in the pharmaceutical industry, then $\beta$ might well be substantially larger in this industry. This means that the estimates of $\gamma$ in Table 5 might be particularly overstated if interpreted as estimates of patent value.

A rough calculation suggests that this overstatement might not be too great, however. Assuming a return on investment of 15%, then $0.15 \times \gamma$ gives a rough, upper-bound measure of the flow of patent rents. This aggregate flow of rents for the pharmaceutical firms in the sample comes to 62% of the aggregate deflated net income after taxes and before extraordinary items reported in the firms’ financial statements. Of course, net income also includes the normal returns to assets plus rents from other sources, including rents from pharma’s large marketing expenditures (larger than R&D) and rents from industry regulation (generic pharmaceutical companies also make above average profit margins). If one assumes that the normal rate of profit is 5% on the deflated book value of assets, then patent rents come to 107% of rents in aggregate for the large pharmaceutical firms in our sample. This suggests that $\gamma$ is, indeed, too high as an estimate of patent value for large pharmaceutical firms, but, nevertheless, it may have the right order of magnitude.

And this is an order of magnitude larger than the estimates of patent value outside the chemical

14 Schankerman (1998) and Lanjouw (1998) using European renewal data, however, find pharmaceutical patents to have low or modest value relative to other patents.
15 A similar regression constraining the year dummies to be equal across all industries had qualitatively similar results.
industries. This has two important implications. First, chemical firms account for a large portion of aggregate patent assets, especially two dozen or so large pharmaceutical companies. The fifth column of Table 5 shows the implied share of aggregate patent value. Chemical firms account for over 80%. Even though this percentage may be somewhat overstated, it does suggest that, economically speaking, there appear to be two distinct patent systems: one for chemical entities and one for other technologies and it is the former that receives most of the benefit.

Second, the previous estimates of $g$ may suffer from omitted variable bias because they have not explicitly controlled for this large difference in value. The bottom row of Table 5 displays the mean estimate of $g$, weighted by each firm’s stock of patent applications. This is clearly much larger than the values shown in Table 4. The mean value is almost surely overstated, but it nevertheless serves as an upper bound. As a rough check, the rents associated with this value can be compared to the gross royalties earned on university patents, which include some patents of high social value such as the Cohen-Boyer patent. Using a 15% rate of return, the mean rent from public firm patents comes to $102,000 in 1992 dollars. The gross 2003 royalties of university patents come to $28,000 in 1992 dollars (AUTM 2003). Corporate patents may be more valuable than university patents and/or this value estimate may be overstated, but these estimates are in the same ballpark.

As a final point of comparison, the last column of Table 5 shows the implied “equivalent subsidy ratios” for different industries. Following Schankerman (1998), the ratio of patent rents to R&D can be thought of as an upper bound on the gross subsidy that patents provide to perform R&D. As above, I calculate the upper bound of patent rents as $g$ times the patent stock times a rate of return of 15%. This yields ratios similar to estimates of this ratio in the literature: Lanjouw et al. (1998) summarize the ratio from renewal studies at 10-15%; Arora et al. (2003) estimate a ratio of 17% using a model employing survey data. The average in Table 5 is 16%, although large pharmaceutical firms earn 59%. Note that the same calculation made using the preferred specification in Table 2 ($376,000), yields an equivalent subsidy ratio of 9%. This rough correspondence again suggests that the estimates of $g$ in Table 5 serve as reasonably good upper bound estimates of patent value that are not highly overstated.

3.4 Do estimates of mean patent value converge?

Scherer (1965) first pointed out that coefficients of patent stocks in market value regressions
might be substantially understated if the distribution of patent values is highly skewed. This is because
the coefficient essentially represents an average patent value and if the distribution is highly skewed,
the average value might converge to the true mean only slowly or perhaps not at all. In extreme cases,
such as the Pareto distribution, the true mean is infinite, so averages calculated over finite samples will
not be representative.

Harhoff et al. (2003) study the distribution of invention values and conclude that the lognormal
distribution, which does have a finite mean and variance, fits the data well. On the other hand,
Silverberg and Verspagen (2005) argue that a Pareto distribution provides a better fit for the upper tail.
But these results concern the distribution of invention values, not the values of patents. Patents might
follow a different distribution because firms may tend to obtain more patents on highly valuable
inventions, tending to compress the distribution and thin the upper tail (see Bessen 2006).

Our market value regressions provide a simple test of convergence to the mean for patent values.
If patent values do converge to the mean, the variance of the stochastic error term should decrease with
the size of a firm’s patent portfolio, all else equal. This is because the sampling variance of the mean
patent value is proportional to the inverse of the patent stock. If the distribution of patent values has an
infinite mean, then the regression variance should not vary with the size of the patent stock after
controlling for other size-related variables.

Table 6 shows regressions where the dependent variable is the square of the residuals from the
regression in Table 2, column 4. The first independent variable is the inverse of the patent stock (coded
to zero for observations with zero patents). The second variable is a dummy flag for zero patents. The
second column adds controls for the log of employment and the log of deflated R&D stock. The
statistically significant coefficient on the inverse of the patent stock in both regressions suggests that
regression error does decrease with the size of the patent stock, rejecting the hypothesis of no
convergence to the mean.

4 Conclusion

Market value regressions can be used to obtain estimates that correspond to an upper bound on
the mean private value of a publicly listed firm’s patents. These upper bound estimates are reasonably
close to estimates of patent value obtained from renewal data, international filing data, re-assignment
data and a variety of thumbnail calculations. Moreover, analysis of the market value regressions
suggests that the distribution of patent values has a finite mean and variance.

This suggests that although market value regressions themselves do not directly provide an
unbiased estimate of mean private patent value, they do confirm that the estimates obtained using other data do not substantially understate the private value of patents. And the estimates derived from market value regressions may not seriously overstate patent value.

These regressions also suggest that a very large share of patent rents accrue to firms in chemical industries, especially to large pharmaceutical firms. This highlights the importance of treating this industry carefully, for both research and policy.
Appendix. Imputations used in Table 4

Cockburn and Griliches (1988) use a sample of large, publicly held manufacturing firms. Their regression equation is equivalent to a first-order Taylor series approximation of (6) except that the patent term they use is \( P/A \) rather than \( P/(A+R) \). To obtain the equivalent to \( \gamma \), I multiply their coefficient (.111) times 1.2 (the mean ratio of \((A+R)/A\) for my sample of large public firms in 1980), yielding .133, which, deflated to 1992 dollars, is .213.

Megna and Klock (1993) use a similar term, and I use the mean ratio of \((A+R)/A\) for my sample of semiconductor firms for 1979-91 (1.62).

Hall et al. (2005) use a rather different specification that is not so easily compared to (6) or (7). Instead, I calculate the implied increase in firm value from an additional patent, holding all else constant. This is \( .018* V/R \) at the sample mean and \( .022* V/R \) at the sample median using the regression coefficients in their column 1 Table 3. Using the reported mean and median values of \( V \) and \( R \), and deflating, this yields equivalent estimates of \( \gamma \) of .093 and .252.

Putnam (1994) reports that the mean value of a family of international patents held in the United States is $245,000 in 1974 dollars. Only 36% of the patents filed in the US are also filed abroad. Putnam also reports that for Germany, the aggregate value of all patents (both those filed internationally and those filed only at home) is 5% greater than the aggregate value of internationally filed patents. This implies that the aggregate value of patents is \( 1.05 \times 245,000 \times \) no. of int’l patents. Therefore the mean value of all patents is aggregate value/total no. of patents = \( 1.05 \times 245,000 \times .36 = 92,600 \), which, deflated to 1992 dollars, is $230,000. This number is not too sensitive to the 5% figure; e.g., if it is calculated assuming that domestic-only patents add 10% to the aggregate value of patents, then the mean value is $241,000.
References


Hall, B. H. 1993. “Industrial Research during the 1980s: Did the Rate of Return Fall?” Brookings


Table 1. Sample Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean (million $)</th>
<th>Median (million $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm market value, $V$</td>
<td>1568.0</td>
<td>80.5</td>
</tr>
<tr>
<td>Log $Q$</td>
<td>0.85</td>
<td>0.61</td>
</tr>
<tr>
<td>Patent stock, $P$</td>
<td>86.8</td>
<td>4.5</td>
</tr>
<tr>
<td>R&amp;D stock, $R$</td>
<td>227.9</td>
<td>15.5</td>
</tr>
<tr>
<td>Accounting assets, $A$</td>
<td>917.0</td>
<td>34.9</td>
</tr>
<tr>
<td>Percent observations with no patents</td>
<td>19%</td>
<td></td>
</tr>
</tbody>
</table>

Notes: 25,861 observations for 3,451 firms from 1979 – 97.
Table 2. Basic Specifications

<table>
<thead>
<tr>
<th>Estimation technique</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td>Ln V/A</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
</tr>
<tr>
<td>γ ($92 million)</td>
<td>0.370 (0.024)</td>
<td>0.436 (0.039)</td>
<td>0.181 (0.065)</td>
<td>0.376 (0.081)</td>
<td>0.313 (0.090)</td>
</tr>
<tr>
<td>α</td>
<td>0.992 (0.023)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P/(A+R))^2</td>
<td>-0.062 (0.021)</td>
<td>-0.010 (0.022)</td>
<td>-0.036 (0.012)</td>
<td>-0.037 (0.011)</td>
<td></td>
</tr>
<tr>
<td>(P/(A+R))^3</td>
<td>0.001 (0.002)</td>
<td>0.001 (0.002)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C/(A+R))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.004 (0.002)</td>
</tr>
<tr>
<td>No. observations</td>
<td>25,861</td>
<td>25,861</td>
<td>25,861</td>
<td>13,317</td>
<td>13,317</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.625</td>
<td>0.066</td>
<td>0.550</td>
<td>0.157</td>
<td>0.157</td>
</tr>
</tbody>
</table>

Notes: Standard error in parentheses are heteroscedasticity-robust. All regressions include year dummies. NLLS regression uses equation (6); other regressions use equation (7).

Table 3. Additional Regressions

<table>
<thead>
<tr>
<th>Estimation technique</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td>Ln V/A</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
<td>Ln V/(A+R)</td>
</tr>
<tr>
<td>γ ($92 million)</td>
<td>0.373 (0.034)</td>
<td>0.353 (0.033)</td>
<td>0.482 (0.094)</td>
<td>0.378 (0.081)</td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>1.542 (0.050)</td>
<td>0.737 (0.024)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P/(A+R))^2</td>
<td>-0.063 (0.016)</td>
<td>-0.037 (0.012)</td>
<td>-0.020 (0.010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P/(A+R) * (yr&gt;1989)</td>
<td>-0.182 (0.106)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P/(A+R))^2 * (yr&gt;1989)</td>
<td>0.046 (0.021)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log rival’s patents</td>
<td></td>
<td>-0.016 (0.007)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P/(A+R)) / domestic share of profits</td>
<td>0.082 (0.030)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(P/(A+R)) * (no foreign profits reported)</td>
<td>0.245 (0.069)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. observations</td>
<td>13621</td>
<td>12240</td>
<td>13317</td>
<td>13317</td>
<td>13317</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.323</td>
<td>0.350</td>
<td>0.157</td>
<td>0.157</td>
<td>0.156</td>
</tr>
</tbody>
</table>

Notes: Standard error in parentheses are heteroscedasticity-robust. All regressions include year dummies. NLLS regression uses equation (6); other regressions use equation (7). Rival’s patents are the sum of distance-weighted patent stocks; the distance measure is described in the text.
Table 4. Comparison to Other Results

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample</th>
<th>( \gamma ) or equivalent (1000s $92)</th>
<th>Value of Domestic Patents (1000s $92)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimates based on firm market value regressions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hall et al. (2005), using means</td>
<td>US public manufacturing firms, 1979-88</td>
<td>$119</td>
<td></td>
</tr>
<tr>
<td>Hall et al. (2005), using medians</td>
<td></td>
<td>$322</td>
<td></td>
</tr>
<tr>
<td>This paper, table 2, column 4</td>
<td></td>
<td>$376</td>
<td>$135</td>
</tr>
<tr>
<td>This paper, table 3, column 5</td>
<td></td>
<td>$82</td>
<td></td>
</tr>
<tr>
<td>Estimates based on patentee behavior</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: see text and Appendix for details of comparisons.
Table 5. Estimates from Separate Industry Regressions

<table>
<thead>
<tr>
<th>Industry</th>
<th>SIC Code</th>
<th>( \gamma ) ($92 \text{ thousands})</th>
<th>No. observations</th>
<th>Share of aggregate value</th>
<th>Equivalent subsidy ratio (1997)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals, excluding large pharmaceuticals</td>
<td>28</td>
<td>1,498 (352)</td>
<td>1,543</td>
<td>31%</td>
<td>38%</td>
</tr>
<tr>
<td>Large pharmaceutical firms</td>
<td>2834</td>
<td>5,289 (2,184)</td>
<td>272</td>
<td>52%</td>
<td>59%</td>
</tr>
<tr>
<td>Machinery Including computers</td>
<td>35</td>
<td>-110 (226)</td>
<td>2,285</td>
<td>-2%</td>
<td>-3%</td>
</tr>
<tr>
<td>Electronics</td>
<td>36</td>
<td>378 (218)</td>
<td>2,304</td>
<td>10%</td>
<td>12%</td>
</tr>
<tr>
<td>Instruments</td>
<td>38</td>
<td>395 (156)</td>
<td>2,225</td>
<td>6%</td>
<td>15%</td>
</tr>
<tr>
<td>Other manufacturing</td>
<td>19</td>
<td>324</td>
<td>3,276</td>
<td>1%</td>
<td>0%</td>
</tr>
<tr>
<td>Business services, including software</td>
<td>73</td>
<td>218 (1,038)</td>
<td>645</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td>Other non-manufacturing</td>
<td>219</td>
<td>198</td>
<td>767</td>
<td>2%</td>
<td>9%</td>
</tr>
<tr>
<td>WEIGHTED MEAN</td>
<td></td>
<td>678 (72)</td>
<td>13,317</td>
<td>16%</td>
<td></td>
</tr>
</tbody>
</table>

Note: Estimates are for separate industries using the specification in Table 2, Column 4 for 1979-97. Robust standard errors in parentheses. Bold coefficients are significant at the 5% level or better. The large pharmaceutical category includes firms whose primary business is in SIC 2834, who have over 500 employees and are not identified as primarily manufacturers of generic drugs. The mean and share of aggregate value are weighted by the stock of patent applications in the observation year. The aggregate value calculation ignores the role of \( \beta \) and assumes that \( \gamma \) entirely represents the discounted value of patent rents. The equivalent subsidy ratios are aggregate patent rents (depreciated patent stock times \( \gamma \) times a flow rate of 15%) divided by deflated depreciated R&D stock.

Table 6. Regressions on Squared Residuals

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 / P )</td>
<td>0.355 (0.057)</td>
<td>0.181 (0.056)</td>
</tr>
<tr>
<td>No patents dummy</td>
<td>5.785 (0.469)</td>
<td>3.222 (0.477)</td>
</tr>
<tr>
<td>Ln employment</td>
<td>-2.962 (0.123)</td>
<td></td>
</tr>
<tr>
<td>Ln deflated R&amp;D stock</td>
<td></td>
<td>1.526 (0.126)</td>
</tr>
<tr>
<td>Constant</td>
<td>4.396 (0.165)</td>
<td>0.420 (0.449)</td>
</tr>
</tbody>
</table>

| No. observations     | 13317              | 13317              |
| Adjusted \( R^2 \)   | 0.013               | 0.064               |

Note: Heteroscedastic-robust standard errors in parentheses. Dependent variable is square of residuals from regression in Table 2, column 4.