What Differentiates Expert Teachers from Others?

Carol R. Findell

I was asked to write a paper about what makes expert teachers different from others. What follows is anecdotal evidence rather than a research report. It is a report of the evidence I have gathered from over 40 years of observing and teaching pre- and in-service teachers.

What are the qualities of an expert teacher? I will list some of the attributes I consider important and then give examples of what I mean by these attributes. The examples will be in mathematics for two reasons: because mathematics is the field that I know best, and because mathematics is a source of difficulty for many students and well-educated adults.

An expert teacher:

- identifies key ideas, presents them in several ways, and highlights connections among key ideas;
- makes careful plans, but remains flexible;
- listens to students and asks questions to help them make sense of their own understanding of key ideas;
- provides "low threshold, high ceiling" problems;
- helps students think for themselves; and
- never stops learning.

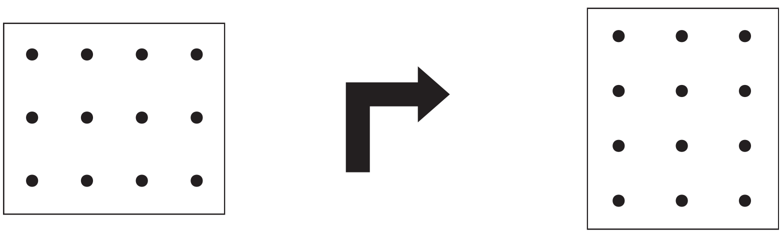
An expert teacher identifies key ideas, presents them in several ways, and highlights connections among key ideas.

First of all, let's define what I mean by a "key idea." A key idea of mathematics is a fundamental idea that has roots in the mathematical sciences, is interconnected with other key ideas, and can be developed from informal childhood exploration through formal school and college study (Steen, 1990). It is part of the "root system of mathematics"—deep ideas that nourish the growing branches of mathematics (Steen, p. 3).

Are key ideas easy to recognize? Are textbooks written so as to highlight the key ideas? No. As a matter of fact, almost all mathematics textbooks have more or less the same format. Lessons are presented in a two-page spread, and then students are asked to practice the skill or concept learned from these two pages. Sometimes there is a "real-world" problem included in the examples, but often the practice is computation without context. The difficulty with this approach is that each concept or skill is given the same amount of space in the book, and thus all concepts and skills are thought to have equal importance. This does not provide an easy way for teachers to distinguish the key ideas from those of lesser importance. Our mathematics curriculum has often been described as "a mile wide, but only an inch deep." One look at the number of different items in the table of contents of a current mathematics textbook confirms that this statement has a ring of truth.

How can this problem be fixed? The National Council of Teachers of Mathematics has helped to solve the problem with the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* published in 2006. This document points out the ". . . important mathematical topics for each grade level, pre-K–8. These areas of instructional emphasis can serve as organizing structures for curriculum design and instruction at and across grade levels" (NCTM, p. 5). Expert teachers need to work together to help teachers at each grade level prioritize the lessons they teach so that all these key ideas are addressed and those of lesser importance are left to the end of the year to be covered if time allows. These key ideas should be addressed at different degrees of complexity at each grade level, allowing students to see the same idea applied to a more complex situation. Students should not continually be given the same thing done in exactly the same way year after year. Our spiral curriculum continues to present information in the same way year after year, with the only difference being the size of the numbers involved. Teachers now have a resource to help identify the key ideas and thus can work together to plan ways to teach them so that students understand the concepts the first time they are presented.

Here is an example of a key idea of multiplication. In order to communicate about multiplication, we need to agree on what we mean by symbols and laws. Many kids have trouble with the commutative law for multiplication. That is, for example, that 3×4 gives the same result as 4×3. We interpret 3×4 as 3 groups of 4 things. The addition sentence that tells this story is $4 + 4 + 4 = 12$. We interpret 4×3 as 4 groups of 3 things. The addition sentence that tells this story is $3 + 3 + 3 + 3 = 12$. It is not obvious why these two addition sentences give the same result. Kids don't see why $4 + 4 + 4$ gives the same result as $3 + 3 + 3 + 3$. A picture can help them see the connection.



Three groups of four turned ninety degrees become four groups of three.

Not only does the picture convince students that the two sums, and thus the two products, give the same result, but this visual approach helps students learn their multiplication facts. Kids often call the facts 7×9 and 9×7 "turn-arounds." For most children, the nines tables are easier than the sevens tables, so knowing 7×9 is 63 can be used to recall that 9×7 is also 63. Notice that this approach highlights the connections between addition and multiplication, and also makes the connections that explain the commutative law of multiplication. Expert teachers present carefully planned lessons that help kids see these connections.

An expert teacher makes careful plans, but remains flexible.

Every good teacher makes careful plans for teaching each concept. These plans typically consist of identification of the mathematical concepts or skills being presented, several different ways of representing these concepts or skills, multiple examples and non-examples, connections among these concepts and skills and others that they have previously learned, and exercises and problems that illustrate and use these concepts or skills.

Working with other teachers to refine lesson plans leads to better learning for students. When teachers work together, they are more likely to identify the points of confusion and plan a sequence of events that help avoid student misconceptions. The only difficulty that can arise from a very carefully planned lesson is that students don't always react in the way the teacher had planned. Expert teachers are able to adapt their careful plans to accommodate the needs of the students in the class.

In all my years of teaching, I don't think that I have ever had a class that went exactly the way I planned it. Students often ask questions that were not anticipated. Sometimes students' questions lead to discussions of related topics. Expert teachers know when to allow these sidetracks and when to keep to the topic at hand. Sometimes the most relevant understanding comes from these conversations. This leads us to the next attribute of an expert teacher.

An expert teacher listens to students and asks questions to help them make sense of their own understanding of key ideas.

Sometimes listening to students' reasoning as they solve problems tells you more about their understanding than a quiz or test grade. What follows is an example of listening to students and analyzing their reasoning.

The Division Problem:
The question was, "What is $42 \div 7$?"
Here is a second grader's solution.

I don't know how many 7s are in 42, but I do know:

$40 \div 10 = 4$

$4 \times 3 = 12$

$12 + 2 = 14$

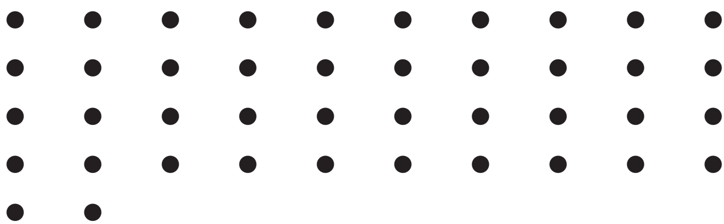
$14 \div 7 = 2$

$2 + 4 = 6$

The answer is 6.

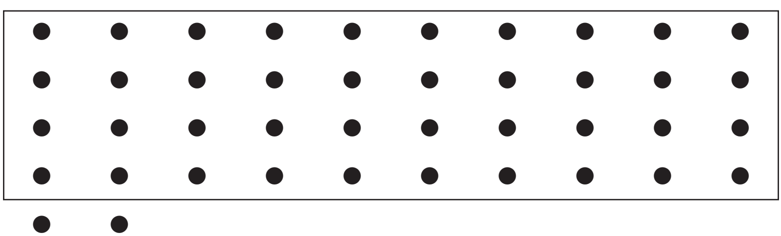
Most people, including math teachers, will look at this reasoning and say, "How did he do that? It is just lucky that he got the right answer." Let's analyze this reasoning to see if the student just made a lucky guess, or really has an understanding of the concept of division and could use this method on other problems.

Here is what the student "saw" in his head. He pictured an array with four rows of 10 dots and 2 dots in the fifth row.



He said: $40 \div 4 = 10$.

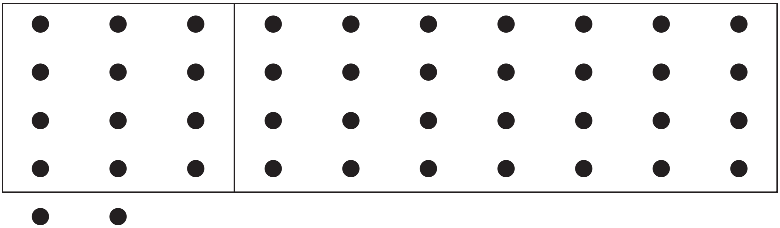
Here are the 4 tens—he is ignoring the extra 2 for now.



But he was not counting tens. He was counting sevens. If there are 4 tens, there are certainly 4 sevens. So how many are left over from the 4 sevens?

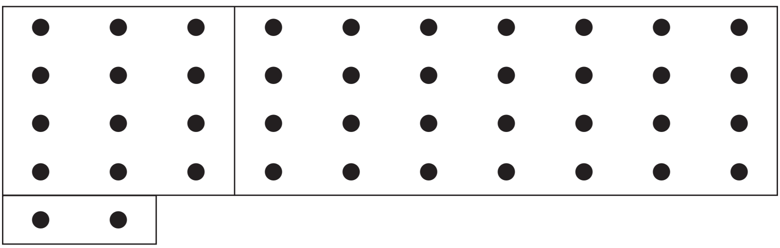
He said: 4 × 3 = 12. That told him how many were left over from the 4 sevens.

This is 4 × 3 = 12 This is 4 sevens



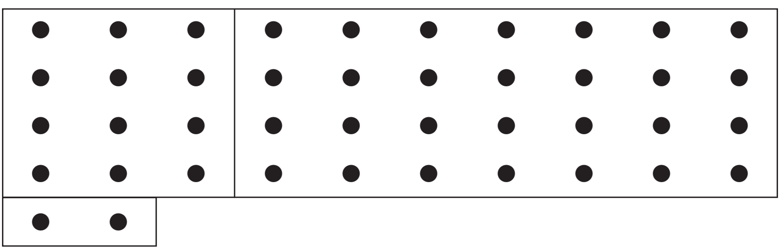
He said: 12 + 2 = 14. That told him how many were left in all.

4 × 3 + 2 = 14 This is 4 sevens



He said: 14 ÷ 7 = 2. That told him there were two more sevens.

This is 2 sevens This is 4 sevens



The last thing he said was: 2 + 4 = 6, the answer is 6. That told him there are 6 sevens in all.

This method is mathematically sound and will work for all divisors less than 10. This student has an incredibly mature understanding of the concept of division, even though he has never been taught how to divide, and probably doesn't yet know the basic facts of multiplication.

Here is what this student would probably say when given the problem $56 \div 8$.

I don't know how many 8s are in 56, but I do know:

$50 \div 10 = 5$
$5 \times 2 = 10$
$10 + 6 = 16$
$16 \div 8 = 2$
$2 + 5 = 7$

The answer is 7.

Although this student's solution process shows a deep understanding of the concept of division, this understanding is not easily apparent. Teachers without a deep understanding of mathematics would be likely to tell this child that his process was wrong. The result might be that the child then doubts all of the mathematical concepts he thought he understood. This child could become math-phobic because he doesn't understand what is wrong with his reasoning. We need to be careful to listen to children's reasoning and ask questions until we understand their processes and can either confirm their alternative methods, or help them with their misconceptions.

We often equate facility with basic facts with deep understanding. The previous student's solution process shows us that it is possible to have deep understanding without knowing basic facts. This does not mean that students should not be required to learn the basic facts. They should! It is meant to point out that knowledge of basic facts should not be the only criteria for judging student understanding. We have found that there are many students who know the facts really well, but have a very shallow understanding of the concepts. An example of this follows.

When asked to reduce the fraction $\frac{16}{64}$, a student immediately said $\frac{16}{64} = \frac{1}{4}$. When asked to explain how he got the answer, he said, "I just crossed out the two sixes, and that left $\frac{1}{4}$. Here's what this looks like:

His answer is correct.
His reasoning is incorrect.

$$\boxed{\frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}}$$

One way to correct this misconception is to ask some leading questions. A good next question for this student would be, "What happens if you use this process on $\frac{12}{24}$?" You hope the student would notice that $\frac{12}{24} = \frac{1}{2}$, but the process used before will give the answer $\frac{12}{24} = \frac{1}{4}$. Once the student is convinced that the process doesn't always work, you can then try to introduce a process that will always work. It is clear that this student has learned that sometimes you can "cross out" two numbers that are the same, but has no idea when that process is appropriate. This is one of the hazards of teaching students shortcuts and tricks without showing why they work and when they are appropriate.

If you have some gifted students in the class, a good next question for these children would be, "Find two other fractions for which this incorrect process gives the correct answer." This leads us to the next attribute of an expert teacher.

An expert teacher provides "low threshold, high ceiling" problems. Low threshold, high ceiling problems are designed as the terms suggest. Every student can get in the door, thus can succeed in solving part of the problem, but only the best will be able to reach the ceiling, that is to finding a general solution to the problem. What follows is such a problem, with a second grader's solution and then a mathematically sophisticated solution.

The Odd Number Problem:
What happens when you add two odd numbers?
Here is the second grader's solution:

Even numbers are lots of twos.
Odd numbers are lots of twos with one left over.

Here is a picture.

I don't know how many twos are under the blobs, but it doesn't matter.

When I put the two odd numbers together, the two left-over dots will match up and make another even number.

So, when you add two odd numbers, you always get an even number.

This second grader clearly understands the problem. The solution shows deep understanding. It is not, however, presented in mathematical language. How could we "mathematize" this argument?

Since odd numbers can all be written as one more than a multiple of two, we could call the first odd number $2n + 1$, and the second odd number $2m + 1$, where n and m are integers.

Rearrange the terms	$(2n + 1) + (2m + 1) = (2n + 2m) + (1 + 1)$
Now add $1 + 1$	$(2n + 2m) + (1 + 1) = 2n + 2m + 2$
Now factor out a 2	$2n + 2m + 2 = 2(n + m + 1)$

Since I can write the number as 2 times an integer, $2(n + m + 1)$ is an even number.

Notice that his process helps students learn to think for themselves.

An expert teacher helps students think for themselves.
Expert teachers routinely ask students to justify their reasoning. They do not always give answers to problems, but ask students to convince themselves that their answer is true. When students attempt to justify their answers, they often look in the back of the book, ask the teacher, ask another student, or recite a rule that they learned to show that the

answer is correct. When asked to supply reasons, many students think that showing a few examples is sufficient. Only the very best know that they need to provide a convincing argument.

We need to help students understand that they need to question ideas and use mathematical arguments to justify these ideas. It is hard to remember things that you do not fully understand. Students who can provide convincing arguments about mathematical ideas are much more apt to remember these ideas and thus be able to apply their knowledge to the solution of problems. The examples shown in this article provide exemplars of logical student thinking. Expert teachers help students learn to think logically and to question answers. Students need to decide for themselves whether something makes sense and not accept something as true just because a book or a teacher says it is true. Expert teachers provide a role model for this type of behavior by always providing justification for their reasoning, and by continuing to learn more about their subject.

An expert teacher never stops learning.

There are lots of different ways to keep learning. In addition to reading, going to conferences, taking advantage of other professional development opportunities, and taking courses, good teachers learn by listening to their students. I know that I have learned a great deal from listening to my students and to smart kids as they solve problems. What follows is a smart second grader's solution to a problem.

The Chicken and Dog Problem:
Chickens and dogs are running around outdoors.
Together they have 35 heads and 94 feet.
How many dogs are there?
Here is the second grader's solution:

> Imagine all the chickens are standing on one leg and all the dogs are standing on their hind legs.
> Now there are 94 ÷ 2, or 47 feet on the ground.
> There are 12 extra legs.

The chickens are all on one leg.
The extra legs belong to dogs.
There are 12 dogs.

Can you follow this reasoning? Most people need more explanation than this. One of the key parts of gifted students' solutions to problems is that they leave out lots of intermediate steps. While most of us reason from A to B to C to D, many gifted thinkers reason from A directly to D or beyond.

Here is further explanation. Remember that chickens have two legs and dogs have four legs.

The student says:

Imagine all the chickens are standing on one leg and all the dogs are standing on their hind legs.

So now, each chicken has one leg on the ground and each dog has two legs on the ground. That means there are half as many legs as before. So, the student says:

Now there are 94 ÷ 2, or 47 feet on the ground.

The student knows that there are 35 heads. Then there are 47 – 35, or 12 extra legs. The student says:

There are 12 extra legs.
The chickens are all on one leg.
The extra legs belong to dogs.
There are 12 dogs

The student knows that 35 heads with one leg each would use 35 legs. The heads with one leg on the ground are chickens. The 12 extra legs can be added to 12 of the heads. The heads with two legs on the ground are dogs. There are now 12 heads with two legs on the ground. Those are the dogs. The other 35 – 12, or 23 heads belong to chickens.

Here is another smart student's solution to a problem:

The Square and Triangle Problem:
The third grader's solution:

$$\square + \square + \square + \triangle = 47$$
$$\square - \triangle = 1$$

$$\square = \underline{\qquad\qquad}$$
$$\triangle = \underline{\qquad\qquad}$$

From the second equation, I know that square is one more than triangle.
(This is very sophisticated reasoning. He reasoned that since the difference between the square and triangle is 1, the square must be 1 more than the triangle.)
Then I imagined that the triangle in the first equation was a square.
Then the sum would be one more, so the 4 squares would equal 48.
(This also is sophisticated reasoning. If the square is one more than the triangle, then changing the triangle to a square would make the sum one more than before.)
The square is 12 and the triangle is 11.
(Since four squares add to 48, then one square is 48 /4, or 12, and the triangle is one less than 12, or 11.)

Listening to these children and others like them provides a multitude of solution paths for the same mathematics problem. Sharing these different solutions with other students gives them a more robust understanding of that problem as well as the ability to solve many other problems.

Summary

In conclusion, what differentiates expert teachers from others is that expert teachers understand their subject matter and they also understand the students that they are teaching. This understanding allows teachers to provide rich problems, to adapt these problems so that they can be used at many levels of difficulty, to listen to student reasoning to help assess student understanding, to use good questioning techniques that help students reflect on their own reasoning processes, and to adapt lessons to maximize the learning for all students.

All of the skills discussed in this article are within the reach of every teacher, but too few seem to integrate them on a daily or even regular basis. How can a beginning teacher become an expert teacher? Beginning teachers should not try to do all of this at once. Beginning teachers might choose one of the six ideas listed, and try to incorporate that idea into their plans. When they see changes in their students' understandings, they can be sure that they are growing as teachers. After they are comfortable with this one idea, they could slowly try others. This list is not meant to be exhaustive. There are many more traits that make teachers experts. Most of all, expert teachers know their subject and their students, and use a variety of methods to help students understand and connect complex ideas of mathematics.

References

National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. Reston, VA: NCTM.

Steen, Lynn, Ed. (1990). *On the shoulders of giants: New approaches to numeracy*. Washington, D.C.: National Research Council.