Mathematics as a Liberal Art

Sam Vandervelde

I. The Place of Mathematics in the Liberal Arts Discussion

I am grateful to the Center for School Improvement for inviting me to speak this morning, because it has prompted me to think more carefully about the intersection of liberal arts and mathematics, such as it may be. In fact, I was recently visiting my wife's sister, who has some background in math education and knew that I was busy preparing a talk along these lines. She politely inquired as to the title of the conference. "Let's see," I responded, "I believe that it's 'The Power of Liberal Arts in the Classroom.'"

"Oh no," she exclaimed, "liberal arts, in a math class!? What are you going to say?"

This question was at the top of my list, too—how mathematics fits into the liberal arts discussion, that is. Consider the following two quotes from past presidents of Swarthmore College, one of the finest small liberal arts colleges in the country. At the laying of the cornerstone for College Hall, which would later bear his name, Edward Parrish announced in 1866 that "We claim a higher mission for Swarthmore College than that of fitting men and women for business—it should fit them for life, with all its possibilities." To this Courtney Smith added at his inauguration in 1953, "It is not enough to develop intellect, for intellect by itself is essentially amoral, and capable of evil as well as of good. We must develop the character which makes intellect constructive, and the personality which makes it effective." I could not agree more, but so far no mention of the role of mathematics.

According to W.R. Connor, a historian who delivered the keynote address at a conference sponsored by the American Academy for Liberal Arts, the term "liberal" has little to do with politics, but rather is derived from the Greek word *eleutherios*, meaning free, commonly used in reference to a free man as opposed to a slave. Therefore "liberal arts" comprised the basic skills and knowledge with which a citizen should be equipped to function as a contributing member of the state. These originally encompassed oratorical skills, but were soon expanded to include arithmetic, geometry, and astronomy.

Carol Barker underlined how early institutions in the U.S. were based on this prototype in her paper "Liberal Arts Education for a Global Society," which grew out of a 1999 meeting of the Carnegie Corporation of New York. There she states that the goal of an undergraduate liberal arts education is to provide students with knowledge, values, and skills that will prepare them for active and effective participation in society. Drawing on this prototype, undergraduate colleges in the U.S. have sought, with varying degrees of commitment and success, to endow students with the capacity to learn, to reason, and to communicate with proficiency.

These comments are clearly applicable at the school level as well. I doubt that anyone disagrees that mathematics should play an important role in developing these capacities. The debate, of course, revolves around which topics best accomplish this goal, at what grade level and in what order they should be introduced, whether to group students homogeneously or by demonstrated ability level, and so on. These issues are important and deserve thoughtful consideration. However, in light of the objectives of educating students liberally, I firmly believe that the methods we as math teachers employ to present each topic and the means by which we engage our students in discovering, understanding, applying, and communicating mathematics are most central to a successful mathematics education. Therefore I would like to devote some of the remaining time to exploring this issue.

II. The Liberal Arts: An Uncommon Approach

But first, let's examine why one of the most common approaches to teaching mathematics does not achieve our stated goal of a liberal arts education. Unfortunately—and here I am speaking from my own experience—it seems that the teaching philosophy adopted by default in the math classroom is a march-through-the-textbook style in which students learn how to solve a well-defined set of problems, demonstrate to what degree they have absorbed the new techniques on the inevitable test, and then move on to the next chapter. In my case it was the default for many reasons: it was the model I saw in the classroom as I grew up, it allowed me to quickly assemble four different preps, it required the fewest number of external sources beyond the textbook which was handed to me, and it provided a supply of questions for quizzes and tests which could be efficiently written and graded. By sprinkling exciting review games and some other fun non-math related activities throughout the year and by being a generally entertaining, compassionate, and fair sort of fellow I managed to make the whole approach quite palatable. However, at best my students from those days will remember that their teacher was a good guy, even if the subject material didn't attain any lasting value.

Earlier I suggested that teaching from a liberal arts perspective should develop within students the capacity to learn, to reason, and to communicate with proficiency. The style of teaching just described falls short on all three counts. Consider learning, which may be defined briefly as the internally motivated process of asking productive questions and effectively seeking answers. Rather than becoming more adept at learning, the student becomes better at performing; that is, meeting externally prescribed goals and expectations. The latter capability is certainly worth attaining. However, it suffers from the shortcoming that the curriculum (whether in school or beyond) is only as worthwhile as the external source that prescribes it. Furthermore, the skills developed by the performer regularly become obsolete as the goals and expectations change. In the case of the student, this occurs approximately every two or three weeks when the next chapter arrives. In contrast, the ability to learn is a broadly applicable life skill which never expires.

It may seem that at the very least students will engage in reasoning in the mathematics classroom, but upon closer inspection it becomes clear that this is not necessarily the case either. We must distinguish between applying algorithms without making errors and drawing logical inferences from known statements. Multiplying two binomials such as $(2a + 1)(2b + 1)$ is an example of the former, while explaining why the product of two odd numbers is also an odd number involves the latter. The proof, by the way, implements the aforementioned algebra, so it could easily be included in a lesson on binomial products. Unfortunately, it rarely is. There is one subject, however, which does rely heavily on mathematical deduction; namely, geometry. In my mind, this fact alone provides strong justification for its inclusion in the math curriculum at the school level.

At the other end of the spectrum, it may seem that developing the capacity to communicate with proficiency is a lost cause within a mathematical setting and is best left to other subjects better suited to such purposes. But this despair is premature; there is a powerful teaching technique available to math teachers, one that not only improves a student's ability to communicate cogently and persuasively, but also develops their ability to reason clearly. I am speaking of mathematical writing, a practice that is all but extinct at the school level. Even the remnant that persisted in the proving of geometric propositions was weakened by the advent of the two-column proof, which eliminated the use of full sentences. This was done, no doubt, because clear, rigorous writing is difficult and takes time and practice to master. As most of those who teach exposition know, in order for students to become competent at writing they must see good examples to emulate, they must be given manageable assignments with which to hone their skills, and they must be attended with much patience, encouragement, and feedback in the process. The results are easily worth the effort, though, since students will obtain or refine a skill which will be useful for the rest of their lives.

III. A Case Study with Logarithms

In order to illustrate how a liberal arts approach might look in action, I believe that it will be instructive to consider a concrete lesson plan. So, good morning, class. Today I will be lecturing on the common logarithm. If you are anything like the typical student, you are probably thinking to yourself that this does not sound like the most promising way to spend the morning. Those who teach logarithms know that students often find this topic oppressive, for the following reason: it does not lend itself well to march-through-the-textbook style of teaching. There are simply too many non-intuitive "laws of logarithms" to keep straight, too many different techniques to neatly categorize them into a list of "types of problems to know for the test," and seemingly too little external motivation from past experience to make the study of this abstract function worthwhile. Nevertheless, let's see what can be done with the common logarithm.

To begin, I would have students split into groups of two or three, have them find the log button on their calculators, and write down the output their calculator gives for log 0, log 1, log 2, all the way up to log 10. Then I would ask them to take the next five minutes to make as many observations as possible about the numbers they just wrote down. There are quite a few waiting to be made, it turns out. For example, log 0 gives an error message, log 10 equals 1, the sum of log 2 and log 3 is precisely log 6, and the sharp-eyed even notice that log 8 is exactly three times log 2. When we reconvened as a class I would record their findings and then challenge them to make predictions involving larger numbers based on their findings, which we could then check on the spot with our calculators. This exercise simultaneously acquaints students with a potentially scary topic and engages them in the process of mathematical inquiry.

Most importantly, the students obtain a vested interest in the formal definition and proof of the laws of logarithms which will follow, since these will explain the observations made by the class. After giving a precise definition of the common logarithm I would then prove the law that explains why the sum of log 2 and log 3 equaled log 6. This proof would be written out on the board in full sentences and copied down by the students, so that they have an example of sound reasoning and clear writing on hand. Finally, their assignment for that evening would be a worksheet of around twelve questions, perhaps half of which could be checked on their calculators. In addition, I would have them write out a proof of that law of logarithms which explains why log 8 is exactly three times log 2. Naturally, the majority of the time students spent learning about logarithms would still be devoted to becoming familiar with the nuts and bolts of how these functions behave and are applied. The liberal arts approach does not aim to eliminate algebraic skills from the mathematics curriculum, but to help motivate them, set them in a proper context, and teach more fundamental skills through them.

For example, early on during the three weeks or so that are typically allotted for an introduction to logarithms, I would also relate the story of the invention of logarithms by the Scotsman John Napier, who spent twenty years constructing the theory, in all likelihood as an aid to performing computations while compiling tables of trigonometric values. The English mathematician Henry Briggs realized the significance of Napier's breakthrough and introduced a table of common logarithms to the European scientific community which, according to Laplace, "doubled the life of the astronomer," since such a large percentage of the astronomer's work involved time-consuming computations by hand. I would also be sure to introduce them to a working slide rule, which essentially provides an efficient means for storing and using the data from a table of logarithms.

Providing such background information to supplement the more standard applications found in most textbooks (such as the use of logarithmic scales to measure earthquake or sound intensity) takes only moments, but brings mathematics and the people who have shaped it to life. It is important for students to begin to sense that the subject they are studying is not only an abstract set of self-consistent logical statements, or even just a powerful tool for studying the natural world, but also a living endeavor, advanced by men and women throughout history and today, united by their common desire to map out new branches of a subject that is both a beautiful field in its own right and a versatile means for scientifically describing the world around us.

IV. The Role of Standards within a Liberal Arts Education

In closing, I believe that it would be worthwhile to examine how the principles of a liberal arts education might operate against the backdrop of the Mathematics Curriculum Framework and the Massachusetts Comprehensive Assessment System—the MCAS. So far I have elaborated on the goals of a liberal arts education and detailed how they might be realized within the setting of a math class. These principles inform our method and purpose in teaching, but do not prescribe the mathematical content for the year. This, of course, is the job of the curriculum framework. Neither does our liberal arts discussion speak directly to ensuring that the curriculum is adequately covered; this task is delegated to the MCAS. Ideally this trivium creates a healthy tension between depth, encouraged by the liberal arts philosophy, and breadth, as set forth by the framework and held accountable by the MCAS.

In practice, though, these forces can sometimes act at cross-purposes. Consider the task of making lesson plans for our chapter on logarithms. We have already seen one possible plan for such a unit designed with a liberal arts model in mind. Now let's take a look at what the framework has to say about logarithms. In the current edition we find a comprehensive list of learning standards for students in grades eleven and twelve under the heading "Patterns, Relations, and Algebra." Among other things, we find that students should demonstrate an understanding of logarithmic functions: they should be able to recognize this function from data or graphs, they should be able to solve a variety of equations (or systems of equations) which involve logarithmic functions or utilize them in their solution, they should be able to solve these equations using algebraic, numerical, or graphical methods, they should be able to solve everyday problems that can be modeled using logarithms, and finally, they should be able to predict the effect on the graph of a logarithmic function caused by changing parameters within its formula.

A teacher's instinctual response to reading such a list of requirements will probably range from deliberately plotting a course which covers all of these topics in an efficient manner to scrambling to figure out what they even mean and what sorts of questions are likely to appear on the MCAS to test them. Either way, the wonderful opportunities for stimulating and engaging students with a liberal arts program quickly get lost in the shuffle. For this reason it is imperative to approach lesson planning with the higher purposes of teaching mathematics firmly in mind in order to ensure that the curriculum framework and the MCAS play their appointed role.

Put succinctly, the mathematics framework and the MCAS can potentially vie with the liberal arts philosophy for control of the underlying motivation behind each math class. I believe that the liberal arts approach provides an attractive model; however, as I continue to discover, the default "march through the textbook" routine easily surfaces and is difficult to shake midstream. To avoid this style requires believing, and acting on the belief, that to create a curriculum bent on mechanically covering the topics outlined in the framework, motivated by the looming MCAS, has a detrimental effect on students and instructors alike.

The consequence for students is clear: paring mathematics down to a set of rules and techniques robs it of its significance and reduces a beautiful subject with an informative history and a broad range of applications to a chore that must be done well in order to certify their mathematical aptitude to colleges or convince future employers of their ability to master new job skills quickly and effectively. I am not arguing against the existence of such a certification; I am just claiming that the purpose of math education at the school level cannot afford to be defined so narrowly.

The stresses imposed on teachers are also clear: a task whose best-case scenario is to avoid anxious parents and the administrative spotlight for one more year will inevitably drain even the most resilient of teachers. Paradoxically, although teaching classes which reflect the mission of a liberal arts education takes more time and effort at first, the joy of kindling a student's interest and desire to understand and explore mathematics provides the fuel which sustains lifetime teachers. As an analogy, consider two hikers who are headed for a destination an unknown distance away. The first follows what appears to be the most direct route, but eventually tires of the endless progression of trees. The second takes a more laborious trail which climbs hills and plunges into valleys but periodically gives way to breathtaking views or crosses lovely streams. The second traveler, encouraged by these vistas, will stay the course and be uplifted by the journey.

C.S. Lewis was fond of saying that "When first things are put first, second things are not suppressed but increased." Embracing a liberal arts perspective in the math classroom is just such a first thing. Out of this commitment will come students whose horizons have been expanded, whose capacity to learn, reason, and communicate has been increased, and, by the way, whose algebraic and problem solving skills are up to snuff. This is the sort of education that will serve our students.