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Could HPS Improve Problem-Solving?

Ricardo Lopes Coelho

Abstract It is generally accepted nowadays that History and Philosophy of Science (HPS) is useful in understanding scientific concepts, theories and even some experiments. Problem-solving strategies are a significant topic, since students' careers depend on their skill to solve problems. These are the reasons for addressing the question of whether problem solving could be improved by means of HPS. Three typical problems in introductory courses of mechanics—the inclined plane, the simple pendulum and the Atwood machine—are taken as the object of the present study. The solving strategies of these problems in the eighteenth and nineteenth century constitute the historical component of the study. Its philosophical component stems from the foundations of mechanics research literature. The use of HPS leads us to see those problems in a different way. These different ways can be tested, for which experiments are proposed. The traditional solving strategies for the incline and pendulum problems are adequate for some situations but not in general. The recourse to apparent weights in the Atwood machine problem leads us to a new insight and a solving strategy for composed Atwood machines. Educational implications also concern the development of logical thinking by means of the variety of lines of thought provided by HPS.

1 Introduction

Mach's historical and critical approach to Mechanics had a significant influence on the development of physics, as it is well known. The growth of this journal is a good example of the usefulness of History and Philosophy of Science (HPS) in science teaching. Rieß's contribution to the experimental history of science enables us to verify what Coulomb, Oersted, Joule, Hertz, among many others, could have observed and measured in their famous experiments. The European HIPST Project shows us how coordinating conceptual and experimental history of science, teaching and learning can be improved (Höttecke et al. 2010). The reader might be thinking of other cases that similarly show the usefulness of

R. L. Coelho (✉)
Faculty of Sciences, University of Lisbon, Campo Grande C4, 1749-016 Lisbon, Portugal
e-mail: RLC@fc.ul.pt

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HPS. All this is the reason for the present question: Could problem solving also be improved by means of HPS?

The attempt made in the present study to address this question takes as object three typical problems in introductory courses of mechanics: the inclined plane, the simple pendulum and the Atwood machine. The solving strategies of these problems in the eighteenth and nineteenth century constitute the historical component of the study. Its philosophical component stems from the foundations of mechanics research literature.

The main ideas of the philosophical component will be presented next. This is followed by the incline, pendulum and Atwood machine problems, which will be dealt with in the following way:

- a problem taken from a textbook is presented and analyzed;[1]
- a brief account of the solving strategies of this problem in the eighteenth and nineteenth century is given;
- a comparison and discussion of the different problem solving strategies takes place.

Finally, educational implications will be considered.

2 Foundations of Mechanics: A Heuristic Point of View

The solving strategies of the incline, simple pendulum and Atwood machine problems are based on Newton's second law. This law is often expressed by or even identified with the fundamental equation of dynamics $F = ma$.[2] The force involved in these problems is weight. Hence, the elements drawn from the foundations of mechanics research literature concern the concept of weight.

French (1983) pointed out that in free fall problems, in introductory courses, the magnitude mg is taken in two senses: as the gravitational force acting on the falling body and as the weight of the body. He highlights that if the body's weight is the weight measured by a spring scale, this weight is slightly less than the gravitational force (French 1983, p. 528). These two meanings of the concept of weight reappear in the distinction between 'nominal' and 'operational' definition of weight (Galili 2001; Galili and Tseitlin 2010). Weight as the acting force that causes the falling of a body is taken as the nominal definition of weight. According to the operational definition, the weight of a body is the result of weighing. More recently, the nominal definition of weight was explained as a logical consequence of the law of inertia (Coelho 2011). This means that, if the law of inertia is accepted, acceleration becomes a *sufficient condition* for force.[3] As the free fall motion is accelerated, this acceleration would be the reason for the acting force 'weight'. If

[1] The incline, pendulum and Atwood machine problems are dealt with in a very similar way in contemporary textbooks (see for example Alonso and Finn 1992; Cutnell and Johnson 1992; Knudsen and Hjorth 1996; Daniel 1997; Young, Freedman and Sears 2004; Nolting 2005; Faughn et al. 2006; Fließbach 2007; Greiner 2008; Kuypers 2010). Hence, to take a problem from a textbook is not to make of it a particular problem.

[2] This equation is usually called 'Newton's second law'. However, it does not appear anywhere in Newton's *Principia* (Newton 1972 [1726]). Moreover, Euler (1750) presented this equation as a *new principle of mechanics*. On the other hand, historians of science do not agree with each other concerning an equation for Newton's second law (Maltese 1992, p. 26).

[3] Helmholtz (1911) had already pointed out that "nothing can be stated about force, which is not already known from acceleration" ("Man kann daher von der Kraft nichts aussagen, was man nicht bereits von der Beschleunigung weiss") (Helmholtz 1911, p. 24). Bergmann and Schaefer (1998) also express a similar point of view: "acceleration is the only sign we have for force" (Bergmann and Schaefer 1998, p. 114).

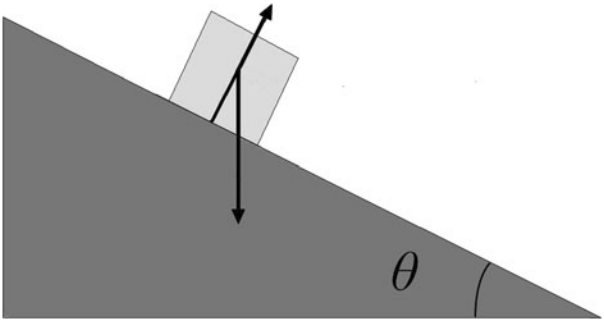


Fig. 1 Two forces act on the body: the weight **w** and the normal force $\mathbf{F}_n$

this is the case, this force is of a logical nature, which should have consequences in problem solving.

The motion along an inclined plane is accelerated. This conclusion is based on the observation of the motion. If acceleration is a sufficient condition for force, force is added to the observable data by anyone who approaches the problem within the conceptual framework based on the law of inertia. If this is really the case, and weight plays here a mere theoretical role, it should be possible to solve the incline problem without the body's weight. We shall see whether this is the case. The same approach holds *mutatis mutandis* for the pendulum problem.

Mechanics teaches us that the Atwood machine moves due to the weights that hang on it. In fact, if these weights do not differ from each other, there will be no motion at all, which shows their significant role in the phenomenon. All this is consistent with the traditional foundation of mechanics. A historical experiment shows us, however (Sect. 5.1), that these weights only hold, if the machine is at rest. They do not exist, if the machine moves. In this case, the bodies do have "apparent weights". This concept creates in us the idea that they are not as important as real weights. Indeed, they are not even mentioned in the usual solving strategies of the problem. However, according to the operational definition of weight, they are simply weights, since they are measurable. They will be used in the present study. We shall see whether the recourse to them is useful regarding problem solving.

3 Inclined Plane

The typical image of an inclined plane problem is the following (Fig. 1).

It represents the falling body being acted on by two forces: the moving force and the force exerted by the incline. Some authors called the moving force 'weight' and others 'gravitational force' but the magnitude indicated is in both cases mg. Let us consider an example taken from a textbook.

Tipler presents the following problem:

> "Find the acceleration of a block of mass m that slides down a fixed, frictionless surface inclined at an angle θ to the horizontal".

The problem is solved as follows. Firstly, two forces are introduced:

> "There are only two forces acting on the block, the weight **w** and the normal surface $\mathbf{F}_n$ exerted by the incline".

This is followed by the decomposition of the weight vector into the direction of the motion and the direction perpendicular to it:

$\mathbf{F}_n$ is in the y direction, and the weight $\mathbf{w}$ has the components
$w_x = w \sin\theta = mg \sin\theta$
$w_y = -w \cos\theta = -mg \cos\theta$

As there is no acceleration along the direction perpendicular to the motion, the respective force is zero and the final result is obtained:

From Newton's second law and the fact that $a_y = 0$,

$$\Sigma F_y = F_n - mg\cos\theta = ma_y = 0$$

and thus

$$F_n = mg\cos\theta$$

Similarly, for the x components,

$$\Sigma F_x = mg\sin\theta = ma_x$$

$a_x = g\sin\theta$ (Tipler 1991, p. 95)

This last equation is the solution for the problem posed. If we compare this result with the problem solving strategy, we can verify that, first, the calculation regarding the y-direction has no effect on the solution and, secondly, the weight and the mass of the body do not appear in the solution. Let us consider these two topics.

The y-direction force is eliminated by Tipler based on "Newton's second law and the fact that $a_y = 0$". A motion along the y-direction is, however, excluded by the formulation of the problem itself. According to this formulation, the surface on which the motion takes place is "fixed". Therefore, the surface is not able to move the sliding body in any way. Thus, there is only one direction to be taken into account: this is the x-direction.

Whereas the calculation regarding the y-direction could be avoided, due to the formulation of the problem, the weight and the mass of the body cannot. The author applies the fundamental equation of dynamics to solve the problem. As this equation requires force and mass, the weight of the body and its mass are needed to complete the equation. Therefore, these magnitudes are necessary due to the strategy of solving the problem. This raises the question of whether a different strategy could dispense with these magnitudes.

The motion on the incline is along the x-direction, as we have seen. Therefore, if there is any change in the velocity of the body—this is the question asked, it can only take place along this direction. At this point, we can write in an intuitive way "$\ddot{x} = \ldots$". What we now need is the right-hand side of this equation. This part of the problem solving requires, according to French's remark (Sect. 2), the input of experimental data: local acceleration. The component of this acceleration along the direction of the motion on the incline is $g\sin\theta$. As this is the acceleration along the x-direction, our equation can now be completed: $\ddot{x} = g\sin\theta$.

This way of solving the problem is based on acceleration. Let us call it kinematic, only to distinguish from the usual one, which will be called dynamical, since it is based on force.

3.1 From History

In the eighteenth century, authors agreed with the following:

– the *explanation* of the motion on an inclined plane
– the *representation* of it
– the presentation of the *result*.

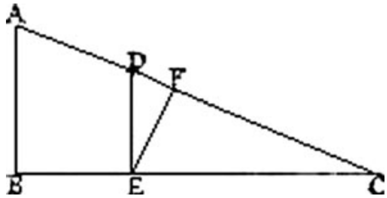


Fig. 2 Wood (1796, p. 146)

The explanation of the motion was: the force of gravity is the cause of falling. The motion was represented by means of a triangle, whose length represents the length of the incline and the height, its height. The result was presented by the proportion between the two magnitudes, height and length, and the proportion between the two accelerations, on the incline and in free falling.

Wood (1796) (Fig. 2) represents the position D of a body, the height DE and the length DF, which the body covers in the same time as it would cover DE, if there were no incline.

The result is presented as follows: the proportion between *accelerating force* (which is the acceleration of the body on the incline) and the *accelerating force of gravity* (the acceleration in free fall) is the same as the height of the plane to its length; in a short form[4]

$$\frac{accel.force}{accel.gravity} \approx \frac{height}{length}.$$

Representing the *height* of the incline by H, its *length* by L and taking 'acceleration of gravity' $= 1$, "the accelerating force on the inclined plane is represented by $\frac{H}{L}$" (Wood 1796, p. 147).

Later on, the same result was expressed by[5]

$$\frac{a}{g} = \frac{h}{l}.$$

This is the typical form of presentation of the result towards the end of the nineteenth century. The way of obtaining it presents, however, some variety.[6]

[4] "From the point D draw DE parallel to AB, and take DE to represent the force of gravity; from E draw EF perpendicular to AC. Then the whole force DE is equivalent to the two DF, FE, of which FE is perpendicular to the plane, and consequently, is supported by the plane's reaction (Art. 116); the other force DF, not being affected by the plane, is wholly employed in accelerating or retarding the motion of the body in the direction of the plane; therefore, the accelerating force: the force of gravity :: DF : DE :: ([…]) AB : AC" (Wood 1796, p. 147).

Similar presentation of the problem can be found in Gravesande (1747, pp. 56–57), Rutherforth (1748, pp. 106–108), Rowning (1779, pp. 28–29), Helsham (1793, p. 148 et seq.), Adams (1794, pp. 154–157), Emerson (1800, p. 45 et seq.). Some authors show the connection between free falling and incline through the terminology: the accelerating force on the incline is called 'relative gravity' and the accelerating force in free falling 'absolute gravity' (see for example Desaguliers 1719, p. 21 and Adams 1794, p. 157).

[5] See for example Lodge (1885, p. 135) and Crew (1900, pp. 81–82). Lommel uses the common representation at that time as well as the currently common one. Using g' for the acceleration on the incline, he writes $g' = g\frac{h}{l} = g \sin a$ (Lommel 1899, p. 33). In this form, these two equations can still be found nowadays (Bergmann and Schaefer 1998, p. 103).

[6] Kater and Lardner use the parallelogram of forces to deal with the incline problem (Kater and Lardner 1830, p. 93 et seq.); Avery (1885, p. 59) speaks of force and uses the same geometrical strategy as those who speak of acceleration; Lommel (1899, pp. 32–33) solves the incline problem as an application of the parallelogram of forces; Henderson and Woodhull (1901, pp. 81–83) presents two methods of solving the problem: virtual velocities and decomposition of weight. On incline experiments in textbooks, see Turner (2012).

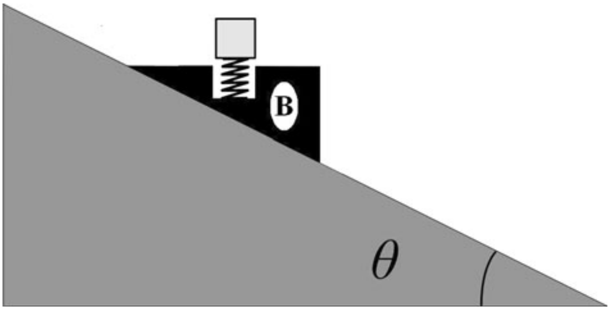


Fig. 3 *Box B* slides frictionless on the incline and supports a weight sensor by means of which the body's weight in motion is measured

3.2 Comparison of the Approaches

Older and modern solving strategies of the incline problem agree with the explanation of the phenomenon: the acceleration on the incline is caused by the force of gravity. They differ regarding the means by which the solution is obtained. The older approach establishes a proportion between the accelerations in free fall and on the incline by means of the height and the length of the incline. In this, the free fall acceleration is taken as the acceleration of reference, sometimes taken equal to 1. The modern approach uses a value for g and starts with the force mg. Due to this, the representations of the problem differ from each. In older textbooks, the geometric representation holds for all bodies on the same incline. The modern representation depends on the body's mass. The kinematic approach does not use any causal explanation but rather previous knowledge about local free falling.

Regarding force, an evolution took place. Older and modern textbooks present the same foundation for their treatment of the incline problem but whereas the older ones do not use this force in the problem solving explicitly, the modern solving strategy starts with this force ($m\mathbf{g}$), being therefore more consistent with the causal explanation. In comparing the dynamical and kinematic strategies, the weight of the former appears as superfluous, since local acceleration is enough to solve the problem.

It could be argued that weight can be dispensed with in a simple problem as considered above but, if friction, for instance, is to be taken into account, then the local acceleration is insufficient. In such cases, the traditional strategy solves the two problems at once. Let us see whether this holds in general.

3.3 Second Problem

The solving strategy of the incline problem is based on the decomposition of the weight vector mg. According to French's remark, g should be the local g. We are able to measure the body's weight on the incline by means of the set up represented in Fig. 3. Box B, that supports a weight sensor, can slide on the incline but is firstly at rest. The weight read off on the weight sensor is, therefore, the normal weight of the body at that place. The question is now of whether this weight is maintained when the body falls.

If in motion, box B moves along x-direction with acceleration $g \sin \theta$. This vector can be decomposed into two components: one in the direction of the local vertical and the other in

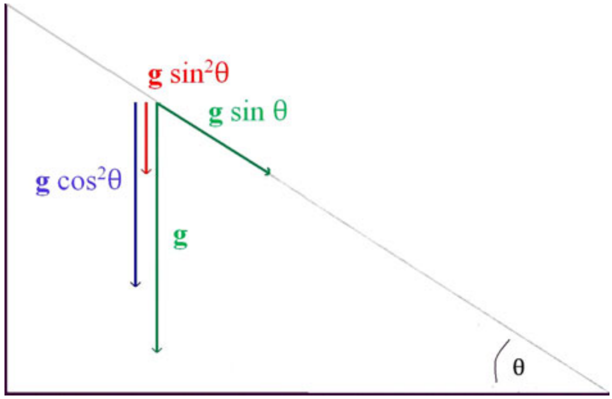


Fig. 4 The vertical component of the acceleration of the box is represented in red and inside the *box* in *blue*. (Color figure online)

the direction perpendicular to this. The magnitude of the former component is then g_{local} $\sin^2\theta$ (see Fig. 4). Thus (as in the case of the elevator), the weight of the body inside B is given as $m\ (g_{local} - g_{local}\ \sin^2\theta)$. Therefore, the body's weight changes from mg_{local} to $mg_{local}\cos^2\theta$ due to its motion. This is then the weight that could be read off on the sensor in the box.

For an observer in this box, this weight is the weight of the body. He is not able to measure any other. Based on this, it could be thought that the component of this weight vector along the *y*-component corresponds to the force that the body would be able to exert on the surface perpendicular to this direction. This reasoning is similar to that of the traditional solving strategy. In both cases the *y*-component of the weight vector indicates the force exerted on the incline. Only the weight vectors are different.

The question arises, therefore, of whether the body's weight to be used in order to calculate the force exerted by the sliding body on the incline is its weight at rest or its weight in motion. If the first hypothesis holds, then a conceptual justification would be desirable, since it is difficult to see how a body exerts a force due to a weight that it does not have. If the second holds, then the incline problem consists of two problems—the acceleration on the incline and the force exerted on it. In this case, the problem cannot be solved as traditionally presented.

4 Pendulum

Historical, philosophical and educational aspects related with the pendulum were studied in the well known book *The Pendulum* (Matthews et al. 2005). Newburgh presents in this a kinematic description of the motion of the pendulum. Observing the motion, he picks out the elements necessary for the equation of this motion: $\theta = A\cos(Kt)$. In order to give a physical explanation, Newburgh uses Newton's law (Newburgh 2004, p. 299). The differential equation of the pendulum can also be obtained by the observation of the motion, as we shall see.

The following image is typical in the presentation of the simple pendulum (Fig. 5).

The bob is acted on by two forces, the weight or gravitational force and the tension of the thread. The acting force is decomposed into two components, from which the tangential

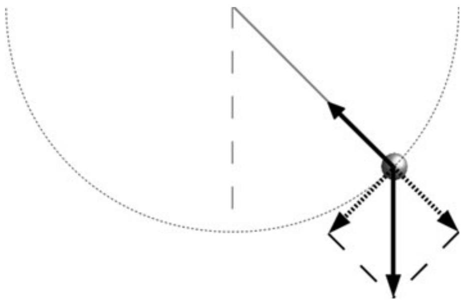


Fig. 5 The forces of tension and weight on the bob

is decisive for the pendulum motion (Nolting 2005, pp. 143–144; Strauch 2009, pp. 46–47).

Serway and Jewett 2004 start with the two forces but move on immediately to the decisive component. They write:

> The forces acting on the bob are the force **T** exerted by the string and the gravitational force $m\mathbf{g}$. The tangential component mg sin θ of the gravitational force always acts towards $\theta = 0$, […] we can apply Newton's second law for motion in the tangential direction:
> $F_T = -mg\sin\theta = m\frac{d^2s}{dt^2}$ (Serway and Jewett 2004, p. 468).

Assuming that the angle is small, the equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

(Serway and Jewett 2004, p. 468, eq. 15.24) is obtained, where L stands for the length of the thread.

Since the normal component is dispensed with because it does not play any role concerning the problem posed, the question could be asked of whether the weight of the body could also be dispensed with for the same reason. As a matter of fact, in the equation $L\ddot{\theta} = -g\theta$ the mass of the bob does not appear. The answer to this question is similar to that question concerning the incline problem.

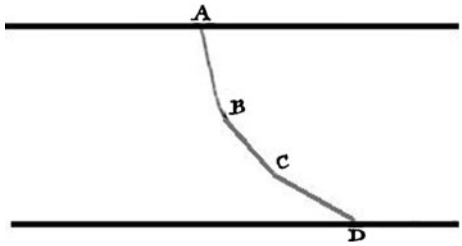
The solving strategy of the pendulum problem is based on the fundamental equation of dynamics. This equation requires a force on the left-hand side and a mass on the right-hand side. Under these circumstances, the weight of the bob and its mass are necessary. It follows, therefore, that weight and mass are required by the solving strategy used. Concerning the application of this strategy to the pendulum, a further problem was highlighted. This concerns the distinction between inertial and gravitational mass.

Hecht (1994) obtains the tangential component of the acceleration as usual but comments on the last step of the solution as follows.

> To get Eq. (12.14) [$a_T = -g\sin\theta$], we set ma_T equal to $-mg$ sin θ and then canceled the mass. Well, that really was more significant than it might seem. The mass in $F = ma$ is the **inertial mass**, the mass associated with the object's tendency to resist changes in its motion. It seems to have nothing to do with gravity and perhaps might even be labeled m_i to underscore the difference in its conceptual origins. On the other hand, the weight of a body is determined by a physical property it possesses called **gravitational mass**, m_g. That property is proportional to the gravitational interaction between objects and seemingly has nothing to do with inertia. Thus, assuming these masses to be different (since they certainly seem to define different characteristics), $T \approx 2\pi\sqrt{m_i L/m_g g}$. But experiments […] all confirm […] that the period is independent of the mass of the bob, which implies that $m_i = m_g$ (Hecht 1994, pp. 413–414).

Let us consider if these difficulties could be overcome.

Fig. 6 We have here the inclines AB, BC and CD (Gravesande 1747, plate 15, fig. 4)



The bob of a pendulum is constrained to move along an arc. Due to this, its displacement can be given by $Ld\theta$. As a consequence, its acceleration can only be $L\ddot{\theta}$. What we still need to know to solve the problem is the local acceleration. The bob would have acceleration g_{local} if there were no constraint. As there is a constraint, the acceleration of the bob cannot be g but rather a component of it. This is $-g \sin \theta$. The question of how the angular acceleration changes due to acceleration $-g \sin \theta$ leads us to the equation of both accelerations: $L\ddot{\theta} = -g \sin \theta$. This approach avoids Hecht's objection, since mass does not appear in it. Let us look briefly at older solving strategies.

4.1 From History

Gravesande (1747) gave us an example of solving the pendulum problem at that time. He moves from the inclined plane to the pendulum in taking the motion along an arc as a sequence of motions along very small inclines.[7] The following images (Figs. 6, 7), from his book, exemplify this in an intuitive way.

As a bob in the pendulum describes a curve, the explanation of this motion follows in a similar way (Gravesande 1747, pp. 91–97).[8]

Towards the end of the nineteenth century, the representation of the phenomenon and the presentation of the result changed. The following one is from Crew (1900) (Fig. 8):The result is presented in the form

$$\frac{a}{g} = -\frac{x}{l}$$

where x represents the displacement along the x-direction and l the length of the pendulum.[9]

[7] The connection between inclined plane and pendulum problems in textbooks of the early eighteenth century is addressed by Gauld (2004, pp. 322–330).

[8] "a Body acquires the same Velocity, in falling from a certain Height, whether it falls directly or comes down along an inclin'd Plane. But a Body may also run down along several Planes differently inclin'd, and even along a Curve ([…]) and the Celerity will be the same when the Height is equal" (Gravesande 1747, p. 86, § 393). In the case that a body runs down along several planes the following remark is made: "we must observe, that the passing from one Plane to another must be without a Shock, for by it the Velocity of the Body would be diminished" (Gravesande 1747, p. 86, § 394).

Similar approaches can be found in Rutherforth (1748, pp. 117–125), Gibson (1755, pp. 54–59), Rowning (1779, pp. 43–44, see also p. 31), Helsham (1793, pp. 153–154, 161), Adams (1794, pp. 206–213), Emerson (1800, p. 45 et seq.), Kater and Lardner (1830, pp. 145–159).

[9] This equation is also similar to that of the incline. Authors presented the following sequence in justifying the pendulum: free falling, inclined plane and pendulum. In all these cases, height is what matters. Similar results based on the same approach or on a slightly different one can be found towards the end of the nineteenth century. The presentation of the results concerning the pendulum by equations represents the main difference between textbooks of this time and earlier ones (see for example Adams 1794; Kater and Lardner 1830; Lommel 1899; Henderson and Woodhull 1901).

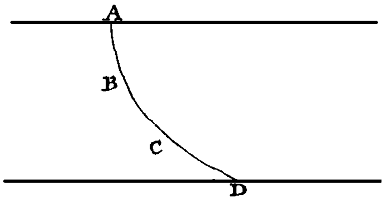


Fig. 7 Here we have a *curve* that goes through the points A, B, C, and D (Gravesande 1747, plate 15, fig. 5)

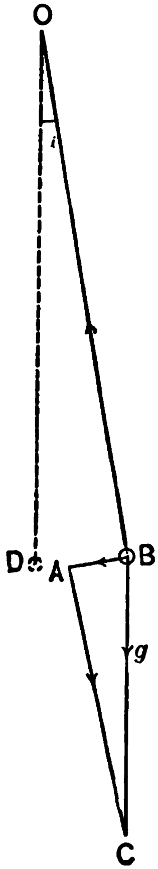


Fig. 8 Crew (1900, p. 84, fig. 43)

4.2 Comparison and Discussion

Older and modern textbooks agree with the explanation of the motion: the force of gravity is the active cause of the motion of the bob. In solving problems, older authors used the acceleration g, whereas modern ones use the force mg. This has as a consequence that in contemporary textbooks the representation of the problem varies with the body, since the vector force depends on its mass, whereas in older textbooks the same representation holds for any body under the same circumstances.

A comparison between the dynamical and the kinematic approaches shows the following. If the simple pendulum is taken as a dynamic problem and, consequently, the fundamental equation of mechanics is applied, two of the three magnitudes—weight, mass, and acceleration—are necessary to solve the problem. If the pendulum is taken as a kinematic problem, only acceleration is needed. If approached kinematically, the cumbersome question of the distinction between inertial and gravitational mass is avoided. On the basis of this approach, nothing can be proved or falsified concerning these masses, since this motion does not depend on mass. The dynamical strategy, on the contrary, needs to make recourse to the equivalence principle to justify the equation of motion.[10]

5 The Atwood Machine

The contemporary version of the Atwood machine consists of two bodies connected by a thread which goes through a pulley, as represented below (Fig. 9).

The typical problem asks the question of the acceleration of the bodies in the machine. According to contemporary textbooks, there are two strategies for solving this problem. The most common takes the two bodies independently and connects them by the tension of the thread. The other takes the two bodies and the thread as one body. Let us begin with the most common solving strategy.

Halliday et al. (1993) present the following problem:

> two blocks connected by a cord that passes over a massless, frictionless pulley. Let m = 1.3 kg and M = 2.8 kg. Find the tension in the cord and the (common) acceleration of the two blocks.

The core of the solution is the following:

> Applying Newton's second law to the block with the mass *m*, which has acceleration *a* in the positive direction of the *y* axis, we find
>
> $$T - mg = ma. \tag{1}$$
>
> For the block with mass M, which has an acceleration of $-a$, we have
>
> $$T - Mg = -Ma, \tag{2}$$
>
> […]
> By adding Eqs. […] we obtain
>
> $$a = \frac{M - m}{M + m} g. \tag{3}$$
>
> Substituting this result in […] and solving for T, we obtain
>
> $$T = \frac{2mM}{M + m} g. \tag{4}$$

Next, the given data of the problem are inserted and a remark on the tension is added:

> we obtain
>
> $$a = \frac{M - m}{M + m} g = \frac{2.8\,\mathrm{kg} - 1.3\,\mathrm{kg}}{2.8\,\mathrm{kg} + 1.3\,\mathrm{kg}} (9.8\,\mathrm{m/s^2}) = 3.6\,\mathrm{m/s^2}$$
>
> and
>
> $$T = \frac{2Mm}{M + m} g = \frac{2(2.8\,\mathrm{kg})\,(1.3\,\mathrm{kg})}{2.8\,\mathrm{kg} + 1.3\,\mathrm{kg}} (9.8\,\mathrm{m/s^2}) = 17\,\mathrm{N}.$$

[10] According to Dransfeld et al. (2001) the fact that the period of the pendulum is independent of the bob is only understandable if the equivalence principle holds (Dransfeld et al. 2001, p. 94). See also Fishbane et al. (1996, pp. 130–131) and Nolting (2005, p. 144).

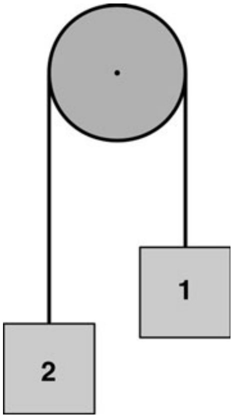


Fig. 9 Schema of the Atwood machine in introductory courses

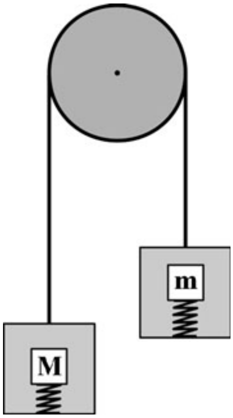


Fig. 10 This schema illustrates the idea presented in the text. For this aim, not only the pulley is frictionless and the thread massless but also the weight sensors and the boxes that carry the bodies and sensors are massless

> You can show that the weights of the two blocks are 13 N (=mg) and 27 N (=Mg). So the tension (=17 N) does indeed lie between these two values (Halliday et al. 1993, p. 116).

Tension T can be related with the weights of the bodies in a different and more precise way. Let us consider each body as a body in an elevator with acceleration a and $-a$ (see Fig. 10). Indeed, for each of these bodies, it does not matter if each one is going up or down in an elevator or in the Atwood machine.

For our purpose, a more precise value for a is needed (Eq. 3)

$$a = \frac{M - m}{M + m} g = \frac{2.8\,\mathrm{kg} - 1.3\,\mathrm{kg}}{2.8\,\mathrm{kg} + 1.3\,\mathrm{kg}} 9.8\,\mathrm{ms}^{-2} = 3.58537\,\mathrm{ms}^{-2}$$

As the smaller body moves upwards, its apparent weight is

$$W'_m = m(g + a) = 1.3\,\mathrm{kg}(9.8 + 3.58537)\,\mathrm{ms}^{-2} = 17.401\,\mathrm{N}. \tag{5}$$

(In this paper, g is taken as positive and the signs of a take care of the direction.) As the other body moves downwards, its apparent weight is

$$W'_M = M(g - a) = 2.8\,\mathrm{kg}(9.8 - 3.5853)\,\mathrm{ms}^{-2} = 17.401\,\mathrm{N}. \tag{6}$$

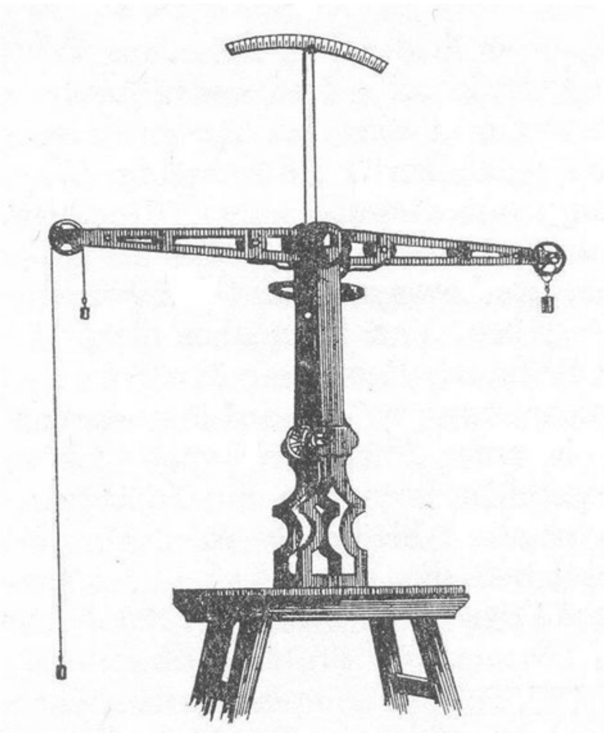


Fig. 11 Mach (1902, p. 207, fig. 135c)

The apparent weight of both bodies is therefore the same. This coincides with tension T

$$T = \frac{2Mm}{M+m} g = \frac{2 \cdot 2.8\,\mathrm{kg} \cdot 1.3\,\mathrm{kg}}{2.8\,\mathrm{kg} + 1.3\,\mathrm{kg}} 9.8\,\mathrm{ms}^{-2} = 17.401\,\mathrm{N}$$

Therefore, T does not only lie between the two normal weights (mg and Mg) but also coincides with the apparent weight of the bodies.

This conclusion has the strange consequence that the total weight of the bodies decreases if the machine is in motion, since that total weight is greater than $2T$. Is this correct?

5.1 From History

In a paper presented to the Academy of Sciences of Berlin, 1853, Poggendorff pointed out that the total weight of the bodies in the Atwood machine decreases during the motion. In order to show this experimentally, he describes an apparatus, which corresponds to the following one, taken from Mach's *Mechanics* (1902).[11]

The left-hand side of this apparatus (Fig. 11) consists of an Atwood machine. Through a thread of negligible mass, the two bodies do not move and the scale is balanced. If this

[11] Poggendorff's paper was also published in the *Annalen der Physik und Chemie*, 1854. Neither in this nor in the 1853 paper is there any picture of the apparatus described. Mach's picture agrees with the description of the instrument given by Poggendorff (1854, pp. 181–182).

thread is burned, the Atwood machine starts moving and the equilibrium is lost. The left-hand side of the apparatus becomes lighter.

This can now be presented in a very simple way. Taking the values of the last problem we have:

– before the motion, the total weight is

$$(M+m)g = (2.8+1.3)\,\mathrm{kg}\cdot 9.8\,\mathrm{ms}^{-2} = 40.18\,\mathrm{N};$$

– during the motion the total apparent weight is

$$W'_m + W'_M = 2\cdot 17.4\,\mathrm{N} = 34.8\,\mathrm{N};$$

– as the total weight of the moving bodies is less than the total weight of the bodies when they were at rest, the inclination of the scale is justified.

It could be argued that this justification is only partially acceptable because the calculation agrees with the general fact that the left-hand side is lighter than the right-hand but there is still no guarantee that the difference between the weights at rest and in motion is the calculated one, since no measurement has been carried out yet. Such a difference between the weights in motion and at rest was determined in a precise way by Graneau and Graneau (2006). They carried out the Poggendorff experiment with the measurement of the total weight of the bodies in motion. The pulley of the Atwood machine was hung on a very precise weighing device and the total weight of the bodies in motion determined. The bodies have the mass 0.2 and 0.3 kg. With the beginning of the motion, the total weight 4.9 N decreased to 4.704 N (Graneau and Graneau 2006, p. 160). That this weight is the sum of the two weights in motion can be shown as follows.

The acceleration of the machine is in this case (Eq. 3)

$$a = \frac{0.1\,\mathrm{kg}}{0.5\,\mathrm{kg}} 9.8\,\mathrm{ms}^{-2} = 1.96\,\mathrm{ms}^{-2}.$$

The apparent weights of the moving bodies are (Eqs. 5, 6)

$$W'_1 = 0.2\,\mathrm{kg}\cdot(9.8+1.96)\,\mathrm{ms}^{-2} = 2.352\,\mathrm{N}$$
$$W'_2 = 0.3\mathrm{kg}\cdot(9.8-1.96)\,\mathrm{ms}^{-2} = 2.352\,\mathrm{N}.$$

Therefore,

$$W'_1 + W'_2 = 2\cdot 2.352\,\mathrm{N} = 4.704\,\mathrm{N}$$

which agrees perfectly with the results obtained by Graneau and Graneau.

5.2 Further Development

Both Poggendorff's experiment and Graneau and Graneau's show that the total weight of the bodies decreases if they are in motion. They do not show, however, how the weight of each body changes. As we shall see, the following holds:

Proposition I *the apparent weights of the bodies on an ideal Atwood machine are equal.* In an ideal Atwood machine, there is no friction, pulleys and threads are massless and the threads do not stretch, so that the accelerations of the bodies connected by the same thread are equal in magnitude.

Taking any two bodies m_2 and m_3, where $m_3 > m_2$, the acceleration a is given as (Eq. 3)

$$a = \frac{m_3 - m_2}{m_3 + m_2} g \tag{7}$$

Since m_2 must accelerate upward and m_3 downward, their apparent weights are given as (Eqs. 5, 6)

$$\begin{aligned} W_2' &= m_2(g + a) \\ W_3' &= m_3(g - a), \end{aligned} \tag{8}$$

where again the signs of a take care of the directions. If (7) is substituted in (8), we obtain

$$\begin{aligned} W_2' &= m_2\left(g + \frac{m_3 - m_2}{m_3 + m_2} g\right) = \frac{2m_2 m_3}{m_3 + m_2} g, \\ W_3' &= m_3\left(g - \frac{m_3 - m_2}{m_3 + m_2} g\right) = \frac{2m_3 m_2}{m_3 + m_2} g. \end{aligned} \tag{9}$$

The apparent weights of these bodies are therefore equal to each other.

From the experimental point of view, this equality can only be reached as a limiting case. To answer the question of how much the weight of each body changes experimentally, it is necessary to measure the weight of each body in motion. This can be done by introducing weight sensors as represented in Fig. 12.

Unlike the weight sensor in Graneau and Graneau's experiment, these sensors influence the motion. They reduce the acceleration in the machine.

To determine the role of these extra masses experimentally, we can reduce the acceleration in the machine further by adding more extra masses, as represented in Fig. 13.

Thus, we obtain a sequence of experimental results with different extra masses. This sequence leads us to think of what would happen if the weight sensors were almost massless. In the limiting case, the bodies on the Atwood machine would weigh the same.

Using, for instance, the masses 0.2 and 0.3 kg, and the extra weights 1, 2 and 3 N, the following results could be obtained (Fig. 14).

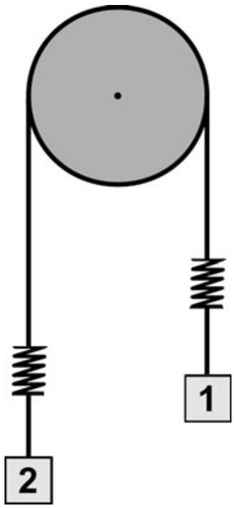


Fig. 12 Schema of Atwood machine with weight sensors

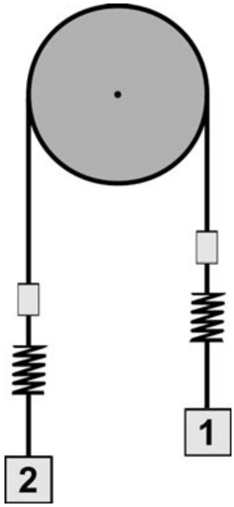


Fig. 13 Schema of Atwood machine with weighing devices and further extra masses

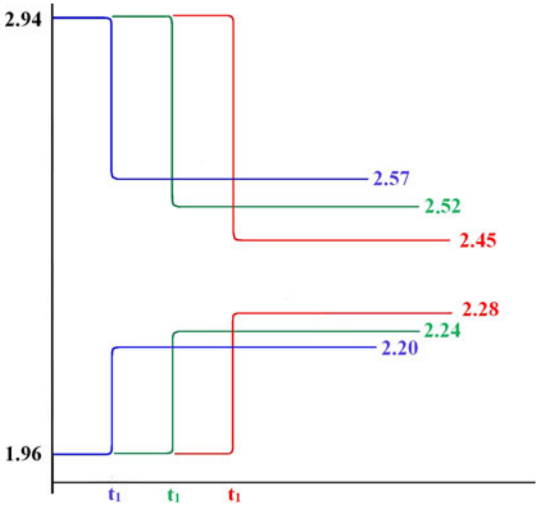


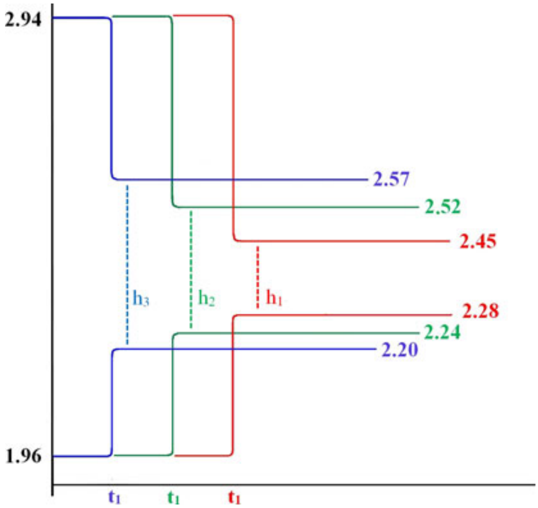
Fig. 14 The *x*-axis represents time, where t_1 stands for the beginning of the motion. The *y*-axis represents weight. When the bodies begin to move, their weights change. This change depends on the extra weights used: 3, 2 and 1 N

If we focus on the distance between the apparent weights, *h* (see Fig. 15), we can verify that this distance decreases as the extra weights decrease. This leads us to think that if the extra masses are almost inexistent, that distance will be almost zero. If $h = 0$ than the two apparent weights of the bodies are equal to each other.

5.3 The Second Problem-Solving Strategy

The second strategy of solving the Atwood machine problem takes the blocks and the thread as a single body. The mass of this body is the sum of the masses of the blocks. Halliday et al. (1993) write:

Fig. 15 The distance h stands for the difference between the two apparent weights of the bodies. This difference decreases if extra weights decrease



Newton's second law in component form for motion along u is

$$\Sigma F_u = Mg - mg = (M + m)a$$

which yields

$$a = \frac{M - m}{M + m} g,$$

as previously […] (Halliday et al. 1993, p. 117).

In this solving strategy, the fundamental equation of dynamics is applied to the machine as a whole and acceleration is calculated without making recourse to tension. This solving strategy is closer to the historical one than the first.

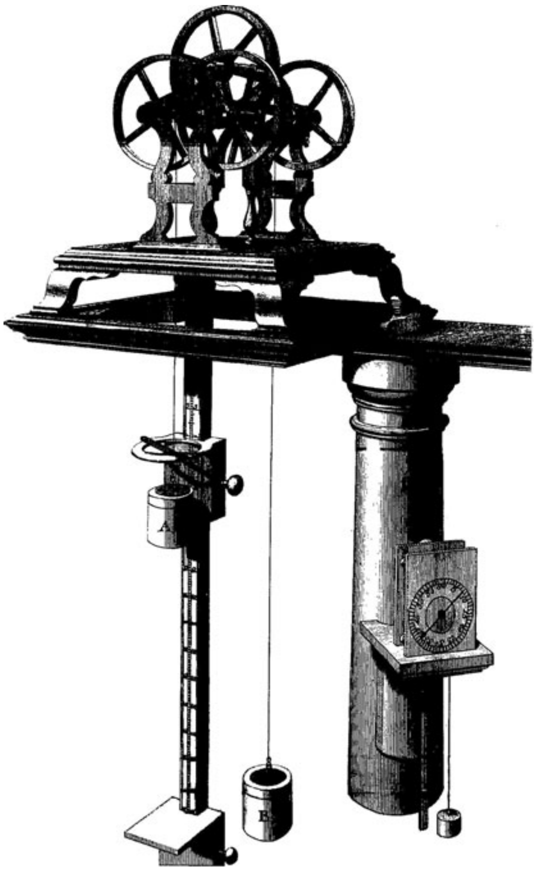
5.4 From History

In 1784, Atwood published *A Treatise on rectilinear motion and rotation of bodies: with description of original experiments relative to the subject*. One of these experiments is the origin of the "Atwood machine" in textbooks. Figure 16 represents Atwood's machine.

On the top of the machine, the main wheel can be seen as well as the others that help to reduce friction on the axle. A thin thread of silk goes through the main wheel. On the extremities of this, two boxes A and B hang. A moves along a scale which allows the space covered to be determined. The time is measured by the clock of the machine, represented on the right-hand side of the image.

In order to simplify calculations, Atwood took ¼ oz as standard weight in the experiment and represented it by m. Thus, for instance, the piece of *4 m* weighs *1 oz* (Fig. 17). The boxes A and B each weigh *6 m* and contain some of those pieces, so that the total weight of each of them is *25 m*. By experiments, it was determined that the wheel requires *11 m* to be kept in motion. The whole weight involved is, therefore, *25 m* (box A) + *25 m* (box B) + *11 m* (wheel) = *61 m*.

Fig. 16 Atwood (1784, fig. 83)



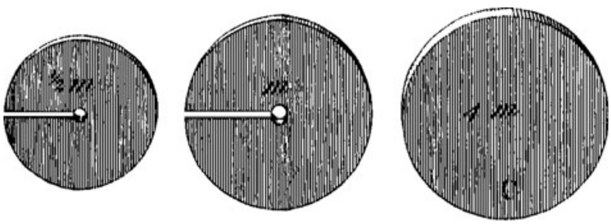


Fig. 17 Three pieces used to vary the weight of the boxes (Atwood 1784, fig. 81, 80, 79). The symbol "m" on the pieces does not represent mass but rather weight, m = ¼ oz

In the first experiment with the machine, as it is presented in the *Treatise*, a piece of *1 m* is added to each box. The total weight becomes *63 m*. Another piece of *1 m* is added to *A* and the machine starts to move. The result is explained as follows: the piece of *1 m* moves the total *64 m*; the acceleration is equal to *1/64 g*, where *g* is the local acceleration.

Let us consider two more examples. Three pieces are added to *A*, this means, the total weight is now *28 m* (*A*) + *25 m* (*B*) + *11 m* (wheel) = *64 m*. The acceleration is equal to *3/64 g*.

In another experiment, pieces of ½ m are used. The total weight is now $25.5\ m + 25.5\ m + 11\ m = 62\ m$. A piece of $2\ m$ is added to A. The acceleration is in this case $2/64\ g$ or $1/32\ g$.[12]

In sum, the difference of weight between the A-side and the B-side moves the total weight and the acceleration a is given by

$$a = \frac{W_A - W_B}{W_A + W_B + W_{wheel}} g \tag{10}$$

This equation form will be used later on.

5.5 Comparison and Discussion

If we apply Atwood's procedure to Halliday, Resnick and Walker's problem, we have for the acceleration a

$$a = \frac{2.8 - 1.3}{2.8 + 1.3 + 0} g \tag{11}$$

The 0 in the denominator indicates that no weight is ascribed to keeping the pulley in motion. This is expressed in contemporary textbooks by saying that the pulley is frictionless and the thread massless. This assumption is not introduced due to any historical reason. It is rather needed to justify the most common solving strategy.

If instead of Eqs. (1) and (2), we take

$$T_1 - m_1 g = m_1 a \tag{12}$$

and

$$T_2 - m_2 g = -m_2 a \tag{13}$$

we would have three unknowns (T_1, T_2 and a) in the two equations, and the problem could not be solved. Taking $T_1 = T_2$, this difficulty is overcome. In order to ensure this equality, that assumption is made.[13]

The second problem-solving strategy is simpler than the first, since it does not require the tension to determine acceleration. Furthermore, it agrees in a simple way with the most

---

[12] Atwood's machine was used to determine local acceleration. In order to achieve this goal, the acceleration of the bodies in the machine was determined. For this, the machine was equipped with a ruler and a clock. Later on, local acceleration was determined by other means and the machine appeared in a simplified form (Kater and Lardner 1830, plate to p. 104; Lodge 1885, p. 69). This was however not general in the nineteenth century (see Avery 1885, pp. 59–62).

[13] Bueche and Jerde, for instance, write: "The two masses […] are tied to opposite ends of a massless rope, and the rope is hung over a massless and frictionless pulley". They explain: "We specify that the rope and pulley be massless so that we can neglect their inertias. Because the pulley is both massless and frictionless, the tension in the rope is the same on both sides of the pulley" (Bueche and Jerde 1995, p. 85). Serway and Jewett write: "In problems such as this in which the pulley is modeled as massless and frictionless, the tension in the string on both sides of the pulley is the same" (Serway and Jewett 2004, p. 129).

The equality of tensions T_1 and T_2 can be justified by the apparent weights. In comparing Eqs. (12) and (13) with Eqs. (5) and (6), it follows that $T_1 = W'_m$ and $T_2 = W'_M$. Therefore, if $W'_m = W'_M$, then $T_1 = T_2$.

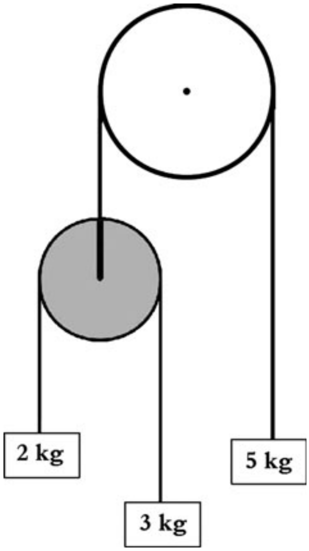


Fig. 18 Sketch of the composed Atwood machine

common interpretation of the fundamental equation of dynamics. In this equation, F represents the force which, acting on a body of mass m, produces acceleration a.[14] If this point of view is transferred to the equation used in the solving strategy, it can be said that force $Mg–mg$ acting on masses $M + m$ causes acceleration a. From a formal point of view, this is very intuitive:

	Acting force	Body acted upon	Acceleration caused
FED	F	m	a
Atwood machine	Mg–mg	M + m	a

There is, however, a difficulty with this interpretation of the phenomenon.

The Poggendorff experiment shows that the sum of the weights decreases during the motion of the machine. If this sum decreases, it cannot be asserted that one of the bodies weighs Mg and the other mg. Thus, it can neither be asserted that Mg and mg make up the acting force because these weights no longer exist. This prevents that interpretation of the second solving strategy. The same holds mutatis mutandis for the most common solving strategy of this problem.

5.6 A Composed Atwood Machine

Newburgh et al. (2004) presented a solution for a composed Atwood machine problem, i.e., for an Atwood machine that includes another Atwood machine. The mechanical situation posed by the problem is expressed in the title of their paper, “When equal masses don’t balance”. On one side of the main Atwood machine hangs a mass of 5 kg and on the other, a smaller Atwood machine with two masses of 2 and 3 kg. Although there are 5 kg on each side, there is no balance (see Fig. 18).

[14] This is the most common definition of force (Planck 1916, p. 10; Westphall 1959, p. 7; Blatt 1989, p. 53; Knudsen and Hjorth 1996, p. 28; Nolting 2005, p. 109, among many others).

The authors calculate the acceleration of body 5, in the inertial frame, and of both bodies 2 and 3, in the non-inertial and in the inertial frame of reference. These same results can be obtained in a simple way by using the apparent weights of the bodies, as we shall see.

If the small Atwood machine is prevented from moving, the masses 2 and 3 kg, on one side, and 5 kg, on the other, balance. If not, the apparent weights play their role and both machines move. Let us follow the steps that lead up to the solution starting with the motion of the small machine.

1. The small Atwood machine moves due to the difference between the weights W_3 and W_2. This difference leads to acceleration a_I of this machine. Using Atwood's Eq. (10),

$$a_I = \frac{W_3 - W_2}{W_3 + W_2} g = 1.96\,\mathrm{ms}^{-2}.$$

2. Due to this acceleration, the weights of the bodies change, as we have seen above. Their apparent weights are now equal to each other (Proposition I). As

$$W_2' = 2\,\mathrm{kg} \cdot (9.8 + 1.96)\,\mathrm{ms}^{-2} = 23.52\,\mathrm{N}$$

the two bodies weigh together

$$2W_2' = 47.04\,\mathrm{N}.$$

On the other side of the main Atwood machine, the body weighs

$$W_5 = 5\,\mathrm{kg} \cdot 9.8\,\mathrm{ms}^{-2} = 49\,\mathrm{N}.$$

3. Due to the difference between these weights, the main Atwood machine moves. In this case, its acceleration a_{II} is given as (Eq. 10)

$$a_{II} = \frac{49\,\mathrm{N} - 47.04\,\mathrm{N}}{49\,\mathrm{N} + 47.04\,\mathrm{N}} 9.8\,\mathrm{ms}^{-2} = 0.2\,\mathrm{ms}^{-2}. \tag{14}$$

4. Due to this acceleration, the apparent weight of body 5 is

$$W_5' = 5\,\mathrm{kg} \cdot (9.8 - 02)\,\mathrm{ms}^{-2} = 48\,\mathrm{N}$$

The apparent weight on the other side is the same, according to Proposition I,

$$W_2'' + W_3'' = 48\,\mathrm{N}.$$

Since the apparent weights on each side of the small machine are equal, it follows from proposition I that

$$W_2'' = W_3'' = 24\,\mathrm{N}.$$

5. These latter apparent weights can be given, as in the previous cases (Eqs. 5, 6, 8), as a product of the form $m(g_{local} \pm a)$. In this case, g_{local} is the acceleration inside the non-inertial frame in which both bodies 2 and 3 move. As this side of the main Atwood machine moves upwards with an acceleration of 0.2 ms^{-2}, $g_{local} = (9.8 + 0.2)$ ms^{-2} = 10 ms^{-2}. Since body 2 moves upwards, its apparent weight is

$$W_2'' = 2\,\mathrm{kg}(10\,\mathrm{ms}^{-2} + a_2') = 24\,\mathrm{N}.$$

Therefore, $a_2' = 2\,\mathrm{ms}^{-2}$. Since both bodies 2 and 3 have the same acceleration in magnitude, body 3 moves downwards with the acceleration $a_3' = 2\,\mathrm{ms}^{-2}$. Thus, the accelerations of the bodies in the non-inertial frame are

$$a'_2 = -2\,\mathrm{ms}^{-2}$$
$$a'_3 = 2\,\mathrm{ms}^{-2}$$

Regarding the inertial frame, the accelerations of the bodies are given as

$$a''_2 = (-0.2 - 2)\,\mathrm{ms}^{-2} = -2.2\,\mathrm{ms}^{-2}$$
$$a''_3 = (-0.2 + 2)\,\mathrm{ms}^{-2} = 1.8\,\mathrm{ms}^{-2}$$
$$a''_5 = 0.2\,\mathrm{ms}^{-2}.$$

These results agree perfectly with Newburgh et al.'s (2004, pp. 290–291).

6 Some Educational Implications

There has been much research on the concepts of weight, force and mass in HPS and in Science Teaching.[15] These topics deserve special attention due to the considerable amount of research literature but also because a distinction is missed in the educational literature between teaching problems in a strict sense and difficulties in learning caused by the theory itself. Indeed, if there are conceptual problems in mechanics, it is to be expected that students will have difficulties in understanding it. In what follows, only some aspects related with the present study will be considered.

Carson and Rowlands (2005, p. 479) and Matthews (2009, p. 706) pointed out that force as such cannot be experienced. This creates a difficulty in problem solving in introductory courses, which it can be exemplified by means of the incline problem. What a student is able to observe is the motion of a body on an incline. To solve the problem, he needs the acting force. Hence, what would be useful in applying the traditional strategy is not available to one's observation. This inconvenience can now be overcome. It can be shown that, on one hand, the acting force stems from the theory and, on the other hand, the question of the acceleration on the incline problem can be solved without using that force (Sect. 3).

Poincaré (1897) pointed out that to say that force is the cause of acceleration is talking metaphysics.[16] Matthews (2009) adds to this: "as every physics class talks of force being the cause of motion, then there is metaphysics lurking in every classroom, just waiting to be exposed" (Matthews 2009, p. 706). The carrying out of this task is significant, since it avoids that a student is led to see in phenomena what does not come from there. That task could now be carried out experimentally. If a teacher presents the Atwood machine to students and calls their attention to the Poggendorff experiment, he is able to show that

[15] See for example Galili and Bar (1992), Galili (1993, 1995, 2001), Bliss and Ogborn (1994), Hijs and Bosch (1995), Jammer (1997 [1961], 1999 [1957], 2001), Carson and Rowlands (2005), Roche (2005), Hecht (2006), Rowlands et al. (2007), Gönen 2008, Coelho (2010, 2012) and Kalman (2011a). Discussion on the concepts of mass and weight also appears in textbooks. See for instance French (1971), Kleppner and Kolenkow (1976), Hestenes (1987), Halliday et al. (1993), Ohanian (1994) and Greiner (2008).

[16] "Quand on dit que la force est la cause d'un mouvement, on fait de la métaphysique, et cette définition, si on devait s'en contenter, serait absolument stérile. Pour qu'une définition puisse servir à quelque chose, il faut qu'elle nous apprenne à *mesurer* la force; cela suffit d'ailleurs, il n'est nullement nécessaire qu'elle nous apprenne ce que c'est que la force *en soi*, ni si elle est la cause ou l'effet du mouvement" (Poincaré 1897, p. 734). This passage also appears in Poincaré's *Science and Hypothesis* (Poincaré 1952, p. 98).

force as the cause of acceleration does not exist, since the weights that constitute that cause no longer exist, if the machine is in motion. This can be shown in the classroom by means of experiments similar to those referred to above (Sect. 5.2).

The use of apparent weights enables a teacher to give an unexpected insight into the Atwood machine: in this machine, there is always equilibrium. 'Equilibrium' means here that the weights of the bodies are equal to each other. If the weights at rest are equal to each other, the machine is at rest. If the weights at rest differ from each other, the machine is in motion. The first case is a trivial one, a static equilibrium; the second is that of dynamical equilibrium.

The importance of apparent weights regarding problem solving emerges clearly in the composed Atwood machine problem. Steps 1 and 3 of this problem solving use what a student learnt with the simple Atwood machine. Steps 2 and 4 use what a student learnt with the elevator problem. The composed problem could, therefore, find its way into textbooks.

In their *International Pendulum Project* paper, Galili and Sela (2004) pointed out that students could be successful in physics assessments if they knew the technique of solving problems. The theory might have had a secondary role, if any. All this concerns pendulum problems. They also add, however: "In the existing orientation on problem solving, students abandon regular physics textbooks, since they can manage without mastering the theoretical content of physics courses" (Galili and Sela 2004, p. 469). According to Kalman (2011b), "Students do not conceive of science in terms of a coherent theoretical framework. The student's paradigm, in the Kuhnian sense, is that the subject consists of solving problems using a tool kit of assorted practices" (Kalman 2011b, pp. 159–160). It turns out in both studies that there is a gap between theory and problem-solving. In order to bridge this gap, it could be useful to show that problem-solving strategies depend on the foundations of the theory (Sects. 2, 3.2, 4.2, 5.5).

Science teaching experts agree that it is important to develop logical and critical thinking.[17] The variety of problem-solving strategies stemming from the history of science (Sects. 3.1, 4.1, 5.1, 5.4) and the connection between foundations of mechanics and problem-solving (Sects. 3.3, 5.2, 5.6) provide us with a *plurality* of lines of thought.[18] This could strongly support logical thinking, since it concerns the very important, perhaps the most important, subject from the point of view of a student: problem solving.

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[17] See for example Hodson (1992), Bailin (2002), Kalman (2002, 2011), Zemplén (2007), Doménech et al. (2007), Galili (2009), Malamitsa et al. (2009).

[18] In a very recent paper, Ha et al. (2012) pointed out the problem: "Most science senior-level teaching consists of transmitting de-contextualized abstract knowledge along with a ''template'' (solely numerical) problem-solving approach. Only one dominant interpretation of nature is presented, which comes from the instructor and the textbook" (Ha et al. 2012, p. 2).

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