# Quantum Mechanics

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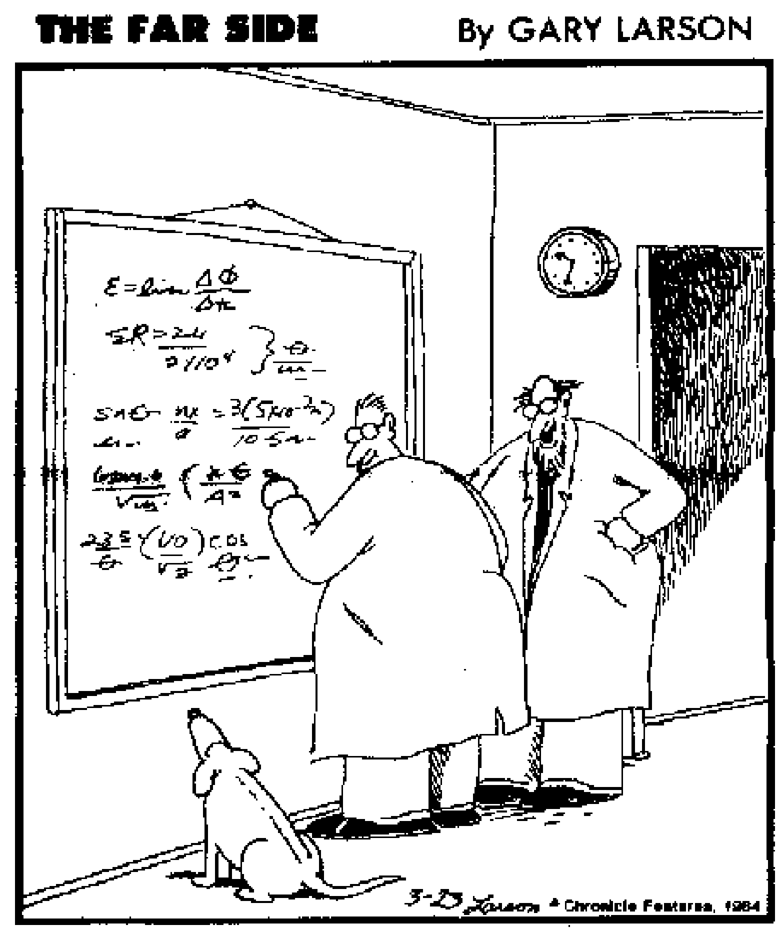
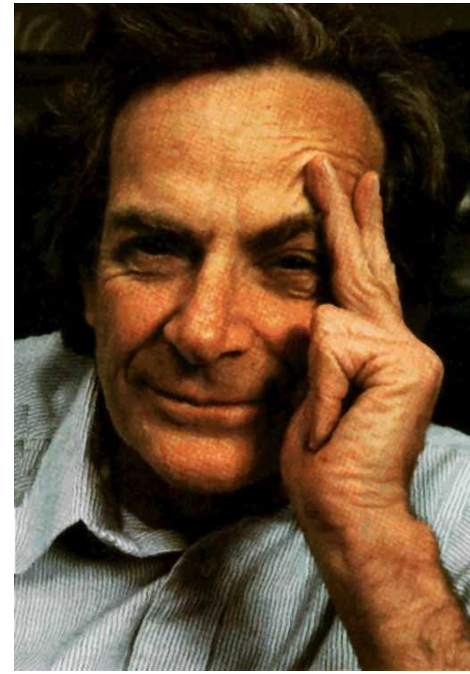



"Ohhhhhhh . . . Look at that, Schuster . . . Dogs are so cute when they try to comprehend quantum mechanics."

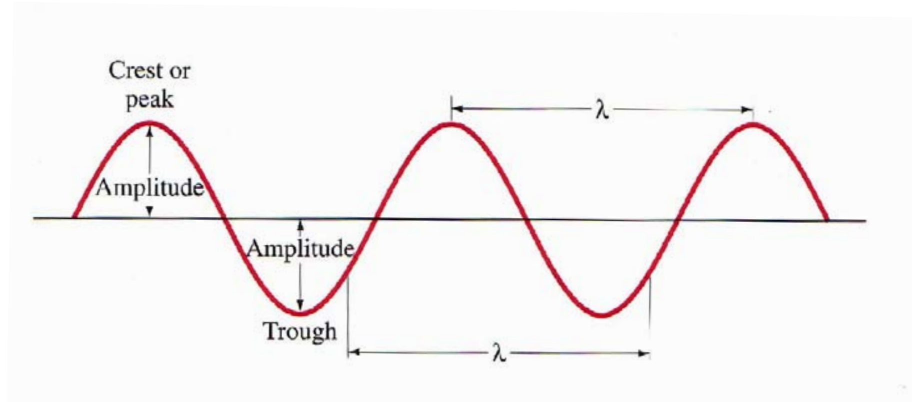
# Introduction



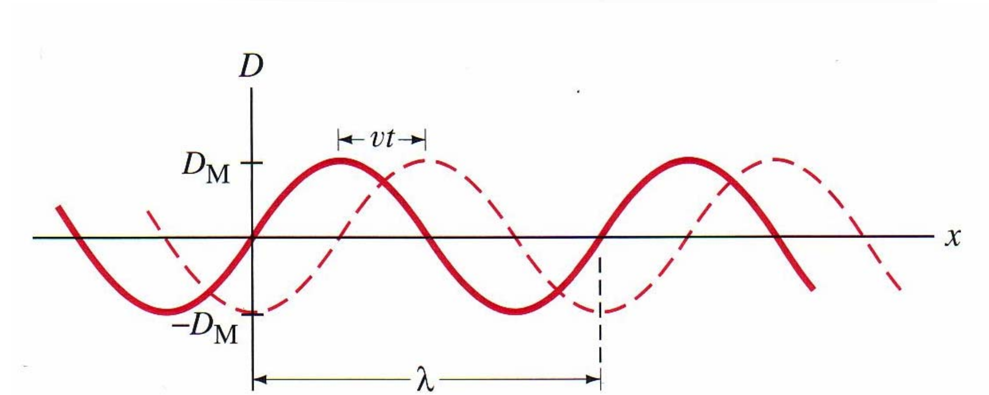
- When things are:
  - Fast – Special Relativity
  - Big – General Relativity
  - Small – Quantum Mechanics
- "I think I can safely say that nobody understand quantum mechanics." – Richard Feynman[1]
- Scientists can make extremely accurate predictions with quantum mechanics without understanding exactly how it works.

# Waves

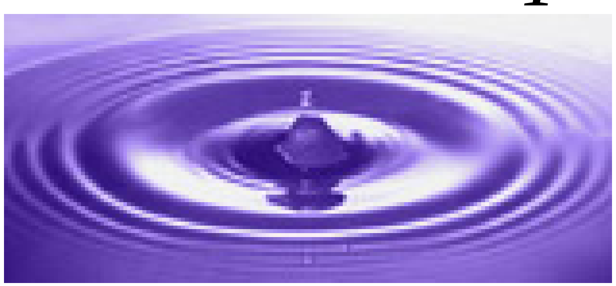
**Frequency** – the number of complete cycles that pass a given point per unit time




**Period** – the time required for one complete cycle of a wave to pass a given point along the line of travel




$$v = f\lambda \qquad f = \frac{1}{T}$$



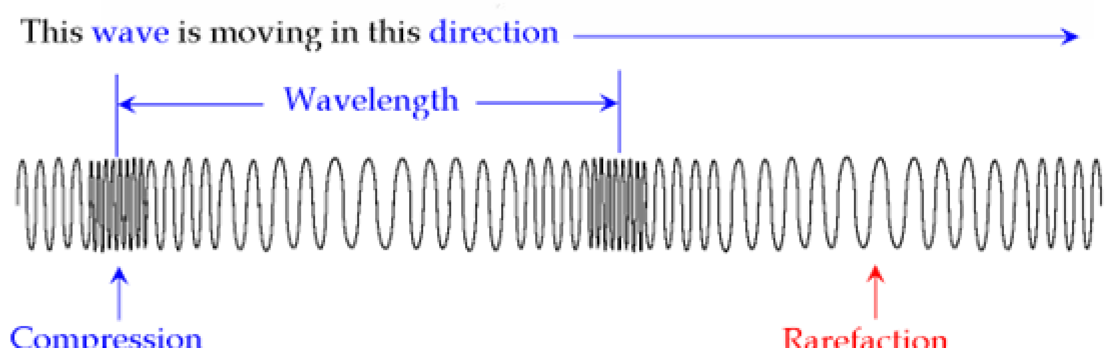
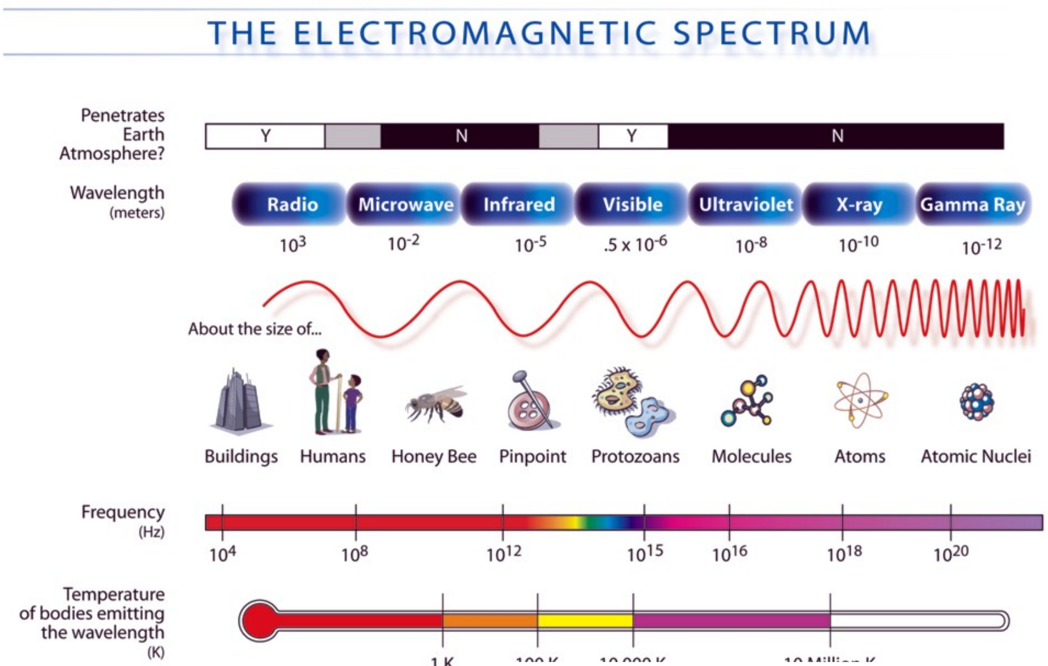
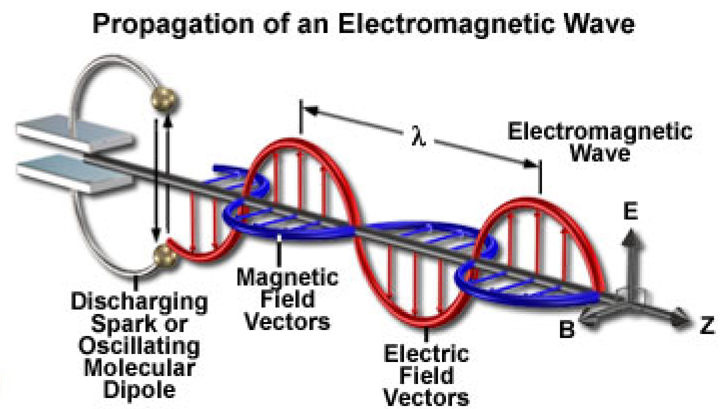



*Diagram Source (upper right): D. C. Giancoli, Physics for Scientists and Engineers, Volume I (Prentice Hall, Upper Saddle River, 2000).*

# Electromagnetic Waves



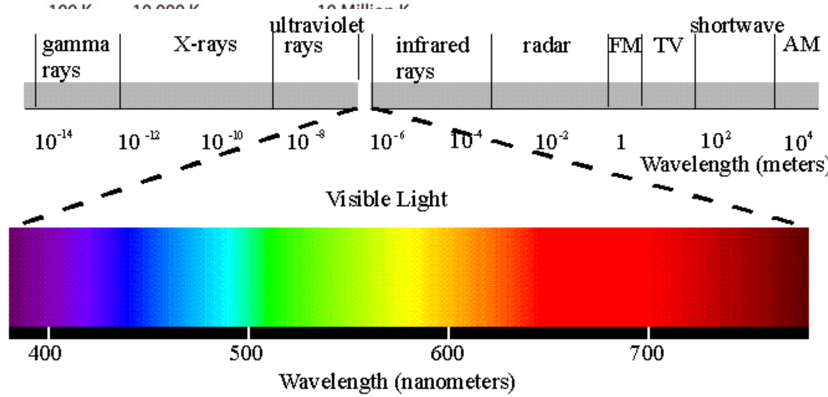




*Diagram Source (upper right): Molecular Expressions Optical Microscopy Primer, http://micro.magnet.fsu.edu/primer/index.html*

# Standing Waves

- Definition: The resultant of two wave trains of the same wavelength, frequency, and amplitude traveling in opposite directions through the same medium
- The wave doesn't appear to travel but it has points that oscillate in a fixed pattern

$$L = \frac{\lambda}{2} n \qquad \lambda = \frac{2L}{n}$$

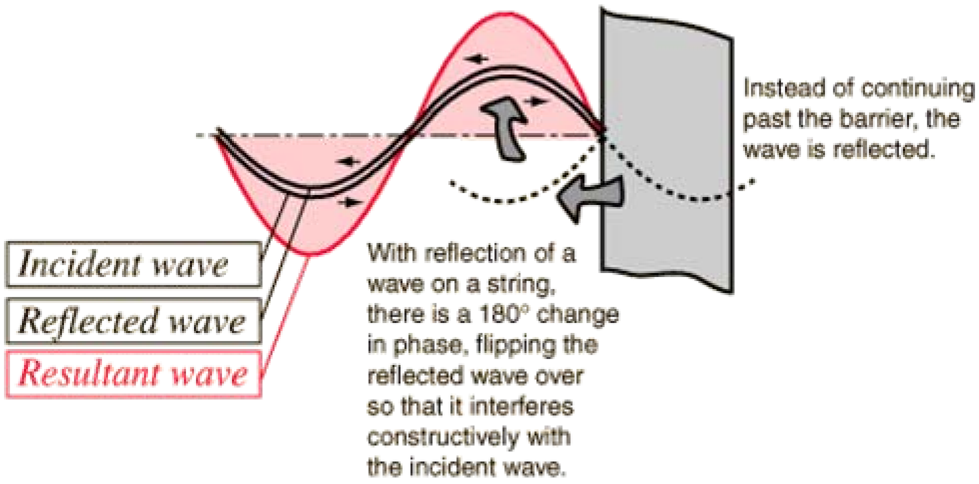
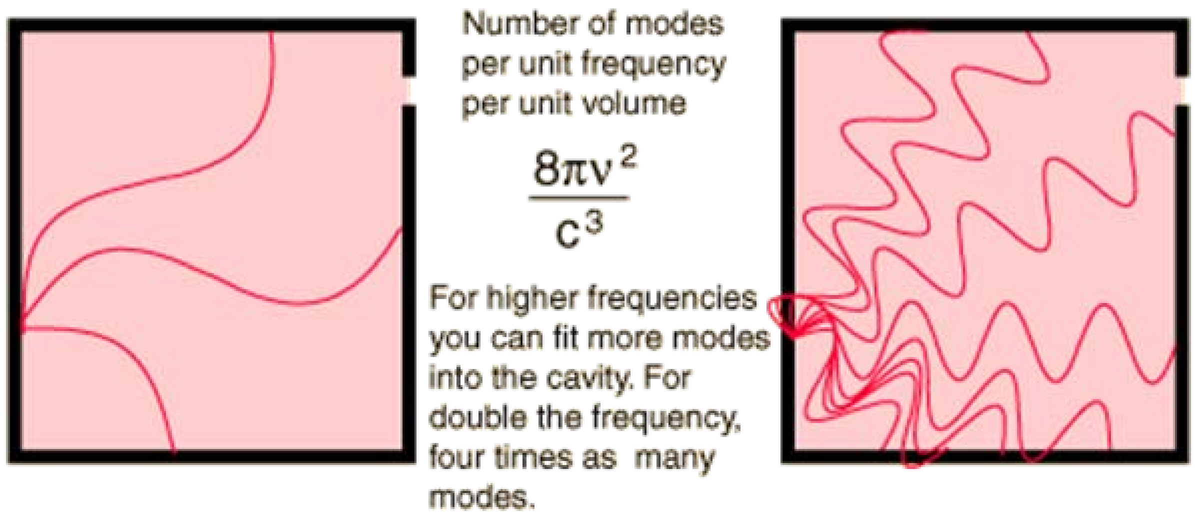



Diagram Source: HyperPhysics Concepts, http://hyperphysics.phy-astr.gsu.edu/Hbase/hph.html

# Ultraviolet Catastrophe



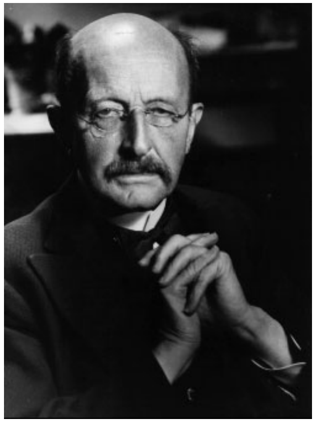

Standing waves exist inside the oven and so there are some integral number of peaks and troughs inside.

Electromagnetic waves carry energy and if you heat up the walls of an oven there will be EM waves in the oven.

Classically each mode has the same average energy. As the frequency increases the number of modes increases and the total energy inside the oven approaches infinity!

*Diagram Source (upper right): HyperPhysics Concepts, http://hyperphysics.phy-astr.gsu.edu/Hbase/hph.html*

# Ultraviolet Catastrophe



Max Planck

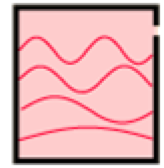
In 1900 Max Planck solves the problem!

But…

He assumed that the walls could only emit and absorb energy in units of **hf** (**h** is Planck's constant)

$$h = 6.6261 \times 10^{-34}\ J \cdot s$$

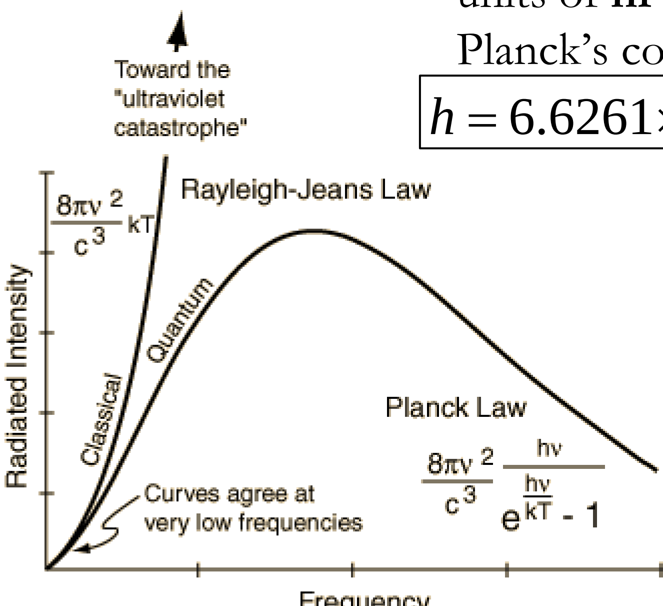
Radiation modes in a hot cavity provide a test of quantum theory

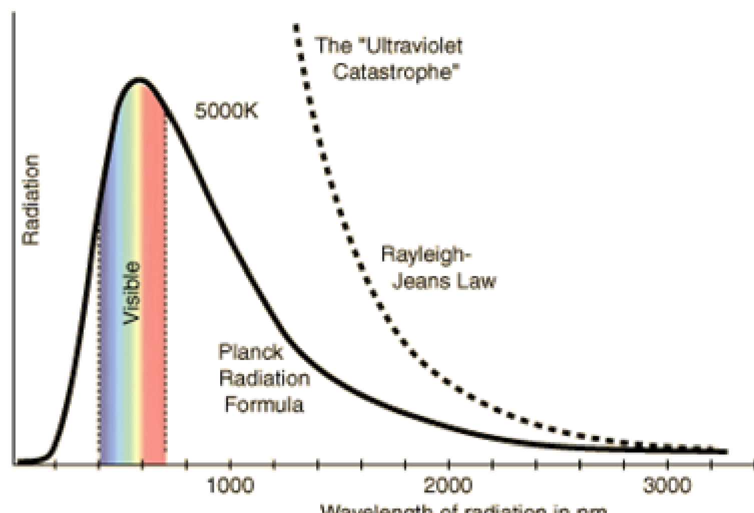


| | #Modes per unit frequency per unit volume | Probability of occupying modes | Average energy per mode |
|---|---|---|---|
| CLASSICAL | $\frac{8\pi\nu^2}{c^3}$ | Equal for all modes | $kT$ |
| QUANTUM | $\frac{8\pi\nu^2}{c^3}$ | Quantized modes: require $h\nu$ energy to excite upper modes, less probable | $\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$ |

Planck's calculation agrees with experimental results and avoids the ultraviolet catastrophe.

Planck wins the Nobel Prize in 1918.



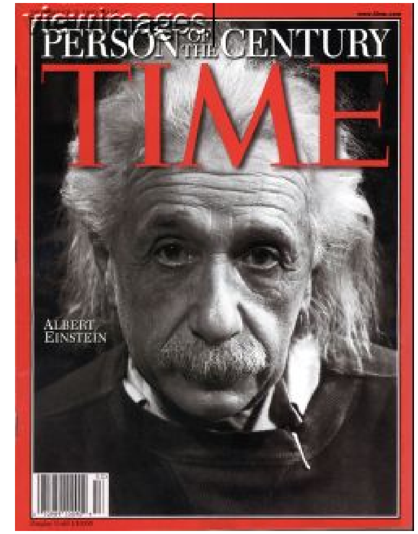




Diagrams Source: HyperPhysics Concepts, http://hyperphysics.phy-astr.gsu.edu/Hbase/hph.html

# Photoelectric Effect

1887 – Heinrich Hertz notices that when light shines on certain metals, electrons are emitted.

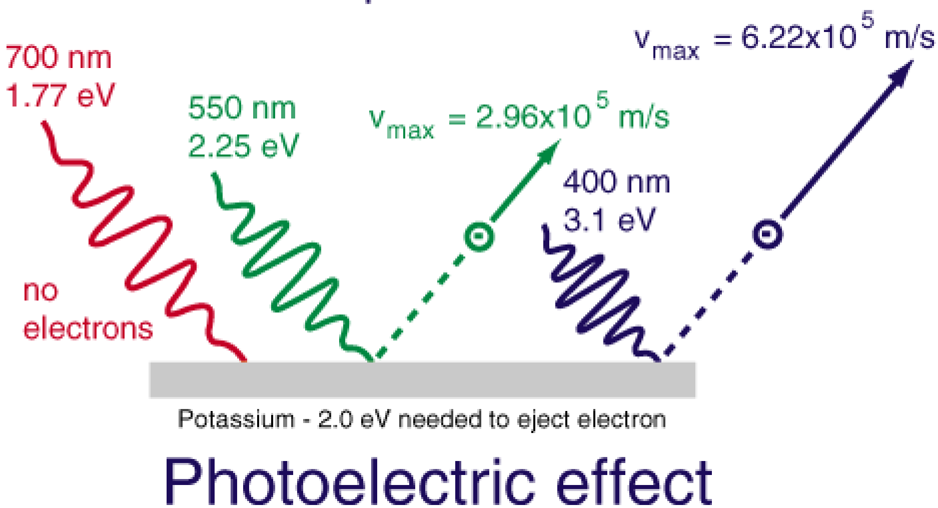
$$\frac{1}{2}m_e v^2 = hf - W$$




As the intensity of the incident energy is increased more electrons are emitted.

As the frequency of the incident light is increased the electrons emitted have a greater velocity.

$$E_{photon} = h\nu$$



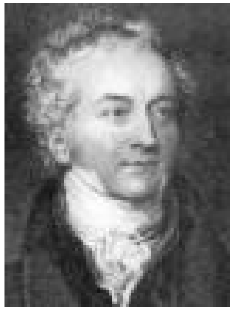

A certain minimum amount of energy is necessary to free an electron from the metal.

In 1905 Einstein explains the photoelectric effect using the idea that light is composed of many tiny packets (now called photons).

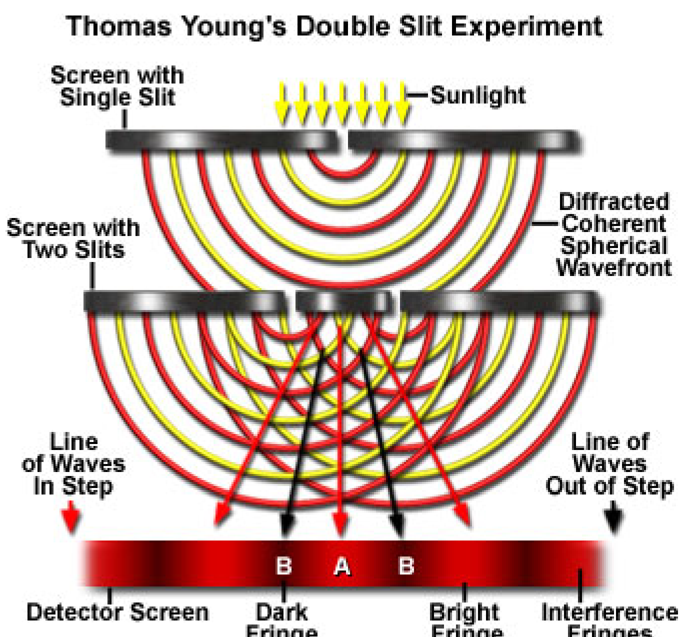
Einstein wins the Nobel Prize in 1921.

*Diagram Source (lower left): HyperPhysics Concepts, http://hyperphysics.phy-astr.gsu.edu/Hbase/hph.html*

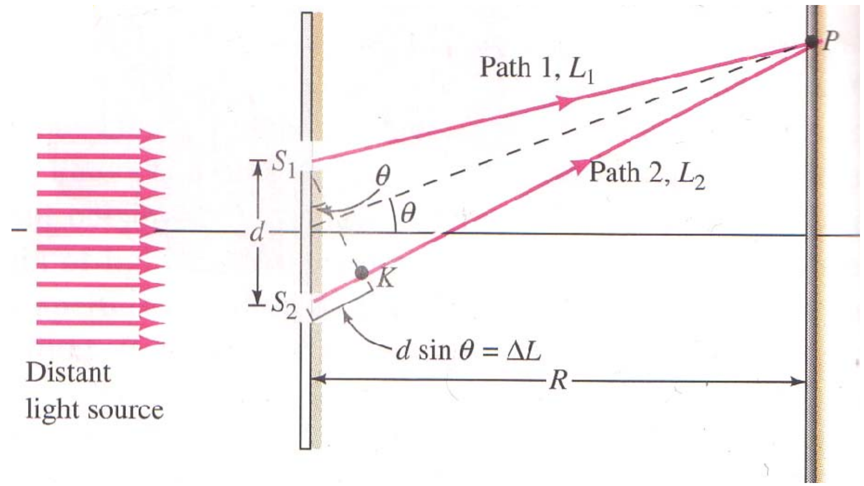
# Double Slit Experiment



Thomas Young




**The geometry used to find the interference pattern conditions for the light that reaches point P**




**Constructive interference**

$$\Delta L = m\lambda$$

$$d\sin\theta = m\lambda$$

**Destructive interference**

$$\Delta L = d\sin\theta$$

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

*Diagram Source (left): Molecular Expressions Optical Microscopy Primer, http://micro.magnet.fsu.edu/primer/index.html*

*Diagram Source (right): P. M. Fishbane, S. Gasiorowicz, and S.T. Thornton, Physics for Scientists and Engineers, Volume I (Prentice Hall, Upper Saddle River, 1996).*

# Double Slit Experiment

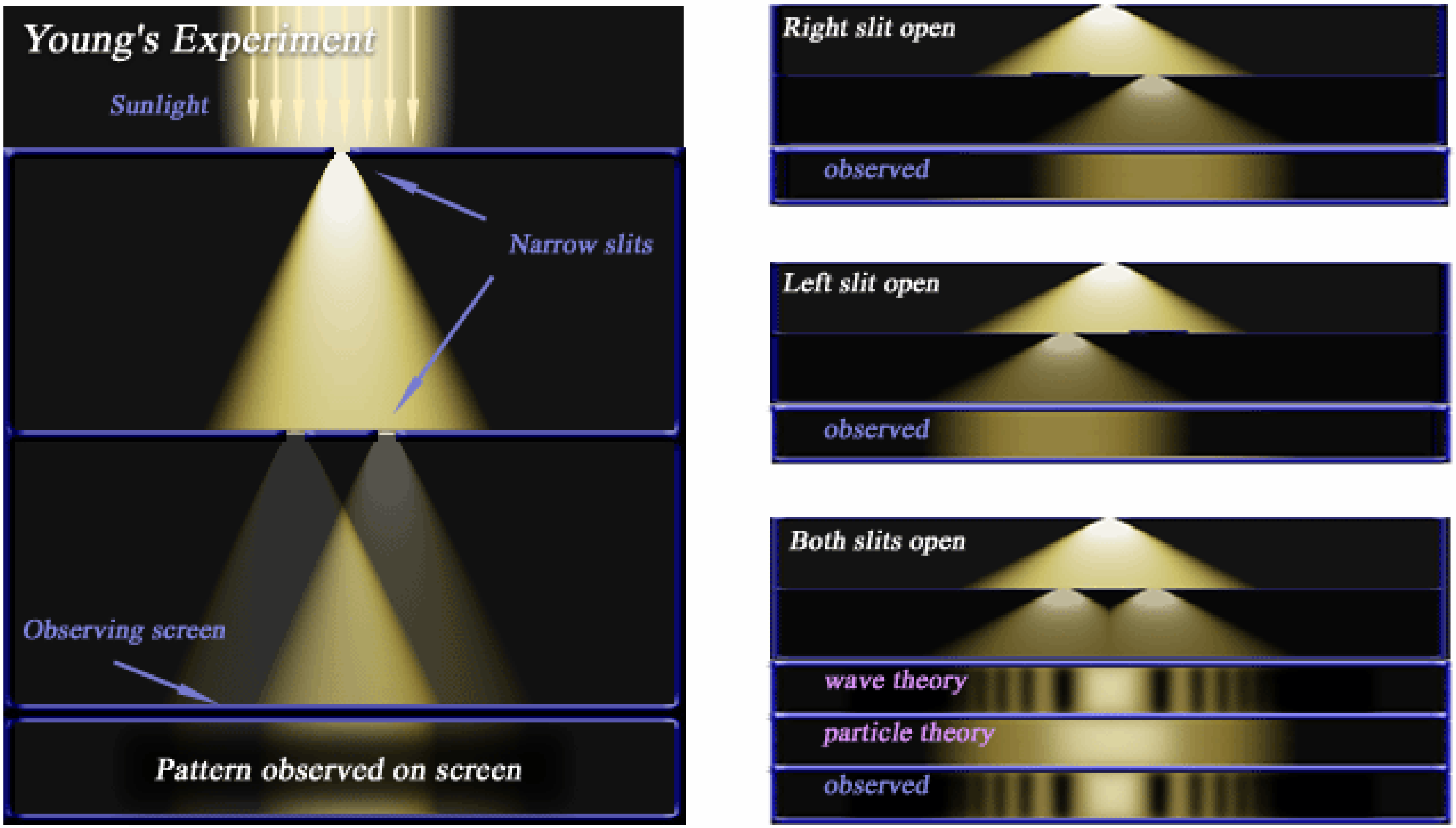



*Diagram Source: Wikipedia, www.wikipedia.com*

# De Broglie Wavelength

Prince Louis de Broglie

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

$$p = \frac{E}{c} = \frac{hf}{c}$$

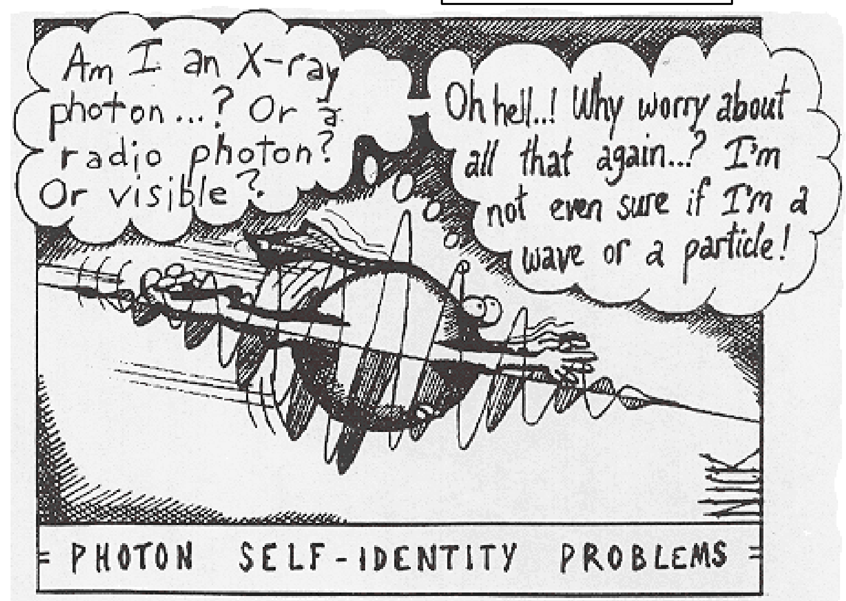
$$c = f\lambda$$

$$p = \frac{h}{\lambda}$$

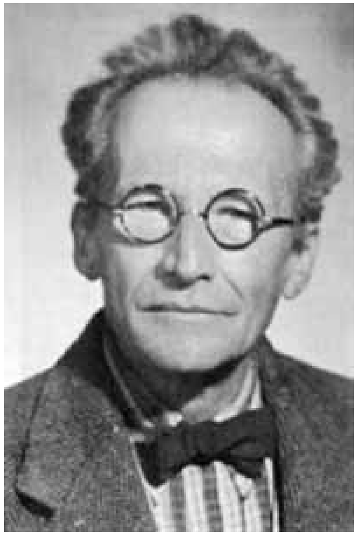
$$\boxed{\lambda = \frac{h}{p}}$$

In 1923 Prince Louis de Broglie suggests that matter has wave properties.

Louis de Broglie wins the Nobel Prize in physics in 1929.



# Wave functions



Erwin Schrödinger

The probability of finding an electron at time t, in a volume element is:

$$\left|\psi\left(\vec{r},t\right)\right|^2 d^3\vec{r}$$

When the system is measured the wave function collapses to a value.

$$\hbar = \frac{h}{2\pi}$$

**Schrödinger Equation**

$$-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)+V(x)\psi(x,t)=i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

$$-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}u(x)+V(x)u(x)=Eu(x)$$

**Schrödinger's Cat**


*Diagram Source (lower right): Wikipedia, www.wikipedia.com*

# Uncertainty Relations

Werner Heisenberg

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\Delta E = \frac{2p\Delta p}{2m} = \frac{p\Delta p}{m}$$

$$\Delta E = v\Delta p \geq v\left(\frac{\hbar}{2\Delta x}\right)$$

$$\Delta E \geq \left(\frac{1}{\Delta t}\right)\left(\frac{\hbar}{2}\right)$$

One can try to determine the position of an electron with light.

Light with a short wavelength reduces the uncertainty in position

But…

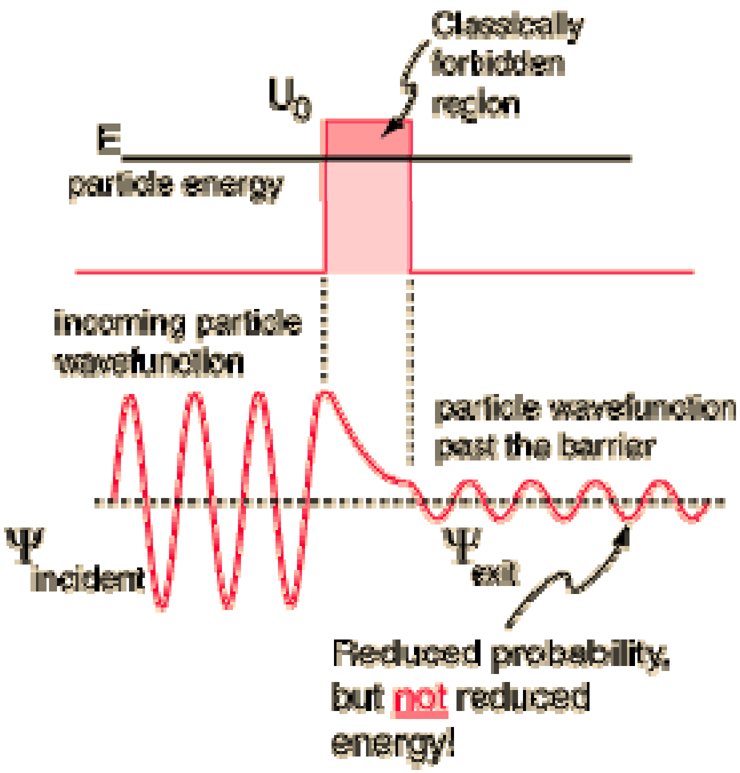
Short wavelength light has a high frequency and therefore more energy. This light will increases the uncertainty in the electron's momentum.

$$\boxed{\Delta x \Delta p_x \geq \frac{\hbar}{2}}$$

$$\boxed{\Delta E \Delta t \geq \frac{\hbar}{2}}$$

# Quantum Tunneling

The quantum mechanical wave function is able to "tunnel" through a potential energy barrier.



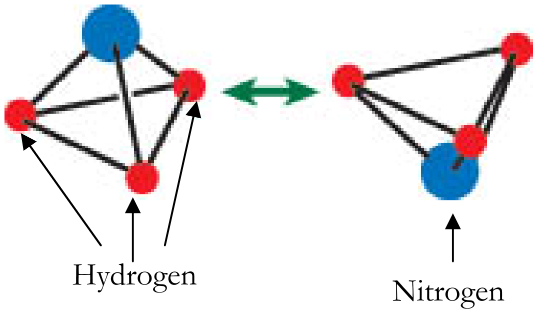


$$|T|^2 \cong e^{-\frac{2}{\hbar}\int\sqrt{2mV(x)}dx}$$

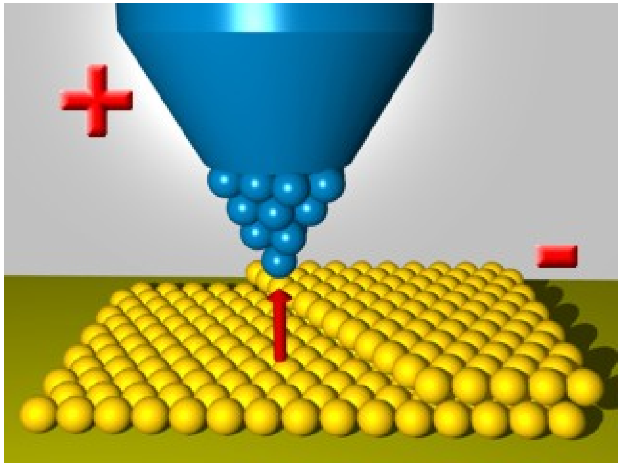
Diagram Source (upper right): HyperPhysics Concepts, http://hyperphysics.phy-astr.gsu.edu/Hbase/hph.html

# Applications

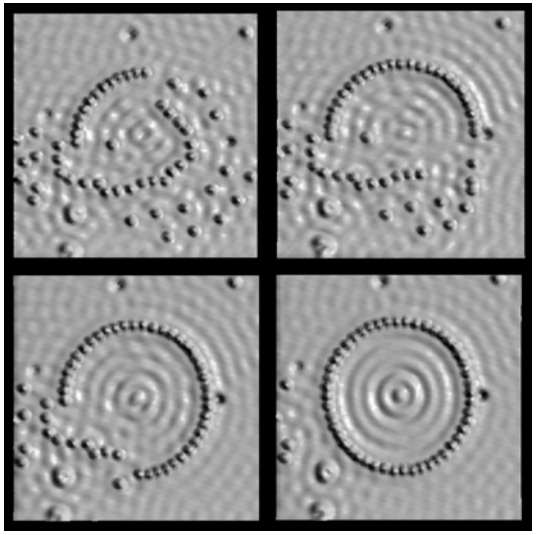
## Ammonia Molecule



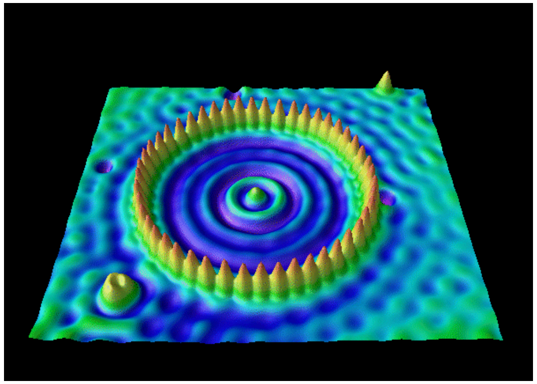

## Scanning Tunneling Microscope (STM)



## The Making of the Circular Corral



## Quantum Corral



A scanning tunneling microscope detects the tunneling current between a positively charged tip and surface. The microscope can use this information to construct an image of the surface or the microscope can be used to move individual atoms.

*Diagram Source (lower left): IBM STM Image Gallery* http://www.almaden.ibm.com/vis/stm/gallery.html

# Conclusion

The Microscopic World is an Interesting Place!

Physics is Fun!

Have a Great Summer!

Good Luck Next Year!

# References


1. B. Greene, The Elegant Universe (Vintage Books, New York, 1999). (In this reference the quote is cited from Richard Feynman, *The Character of Physical Law* (Cambridge, Mass.: MIT Press, 1965), p. 129. )
2. D. C. Giancoli, Physics for Scientists and Engineers, Volume I (Prentice Hall, Upper Saddle River, 2000).
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5. J. Bernstein, P. M. Fishbane, and S Gasiorowicz, Modern Physics (Prentice Hall, Upper Saddle River, 2000).
6. P. M. Fishbane, S. Gasiorowicz, and S.T. Thornton, Physics for Scientists and Engineers, Volume I (Prentice Hall, Upper Saddle River, 1996).
7. Wikipedia, www.wikipedia.com
8. IBM STM Image Gallery http://www.almaden.ibm.com/vis/stm/gallery.html