Solutions for Vector Homeworks

All of these assigned problems come from:

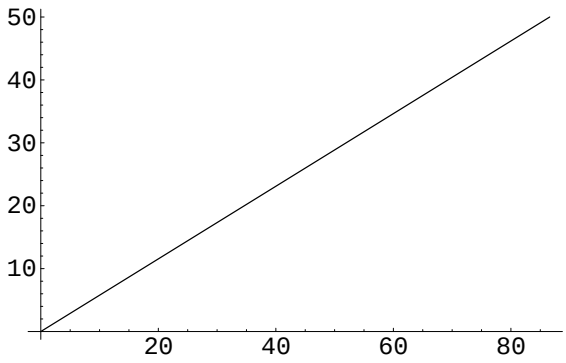
CPO Physics by Tom Hsu on pages 140-142

Assignment 1

25 points

1. Draw a vector to show a displacement of 100 meters at an angle of 30 degrees. Use a scale for your drawing.

   Ans:



   The scale is given from the axes.

2. You walk 1 kilometer north to the store, turn around and return home. What is your displacement?

   Ans: $d =< 1, N > + < 1, S >=< 0, 1 > + < 0, -1 >=< 0, 0 >= \vec{0}$

3. What is the maximum resultant of a 1 centimeter vector and a 4 centimeter vector? What is the minimum?

   Ans: The resultant vector is $< x_0 + x_1, y_0 + y_1 >$

   For use, $< x_0, y_0 >=< 4\cos\theta, 4\sin\theta >$ and $< x_1, y_1 >=< 1\cos\alpha, 1\sin\alpha >$

   Lets have $\theta = 0$ for convenience. What is our resultant vector as a function of α

   $r(\alpha) =< 4 + \cos\alpha, 0 + \sin\alpha >$

   This is maximum when they are pointing in the same direction:

   $r(0) =< 4 + 1, 0 + 0 >=< 5, 0 >$ cm

Minimum when pointing in opposite directions:

$r(180) =< 4-1, 0 >=< 3, 0 >$ cm

4. If a vector is at 45 degrees, what do you know about the magnitude of its components?

   Ans: The components of a vector $< r, 45$ degrees $>$ are $< r\cos 45, r\sin 45 >=< r\frac{1}{\sqrt{2}}, r\frac{1}{\sqrt{2}} >$ and so the components are of equal magnitude.

Assignment 2

30 points

1. Add the following set of vectors:

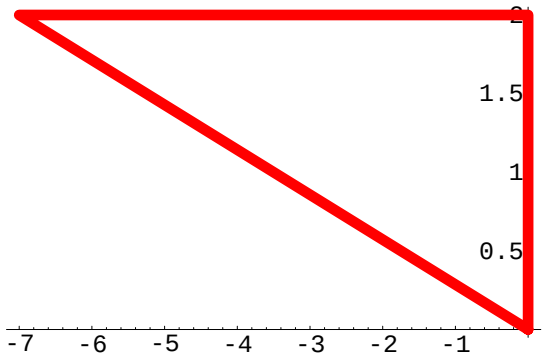
   (a) 2 cm N + 7 cm W

   Ans: < 2 cm, 90 degrees$> + <$ 7 cm, 180 degrees$>$

   $=< 2\cos 90, 2\sin 90 > + < 7\cos 180, 7\sin 180 >$

   $=< 0, 2 > + < -7, 0 >$

   $=< -7, 2 >$ cm



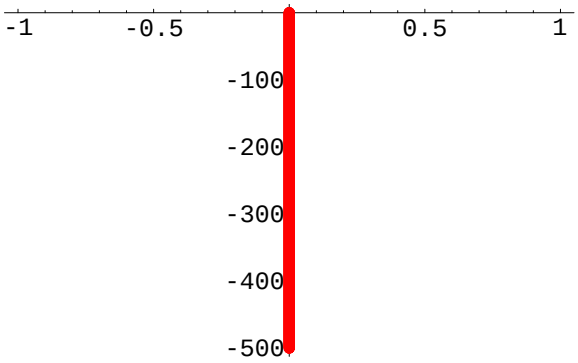
   (b) 5 m S + 8 cm N

   Ans: < 5 m, 270 degrees$> + <$ 8 cm, 90 degrees$>$

   $=< 500\cos 270, 500\sin 270 > + < 8\cos 90, 8\sin 90 >$

   $=< 0, -500 > + < 0, -8 >$

   $=< 0, -492 >$ cm



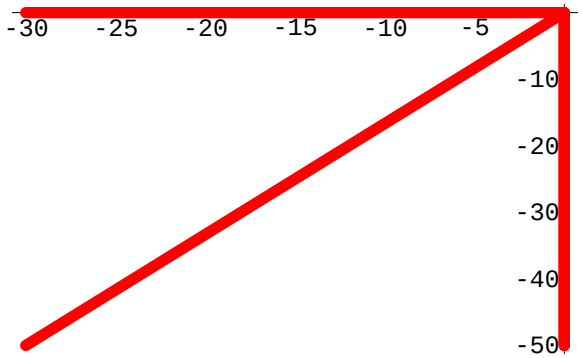
   (c) 30 m/s W + 50 m/s S

   Ans: $<$ 30 m/s, 180 degrees$> + <$ 50 m/s, 270 degrees$>$

   $=< 30\cos 180, 30\sin 180 > + < 50\cos 270, 50\sin 270 >$

$=< -30, 0 > + < 0, -50 >$

$=< -30, -50 >$ m/s



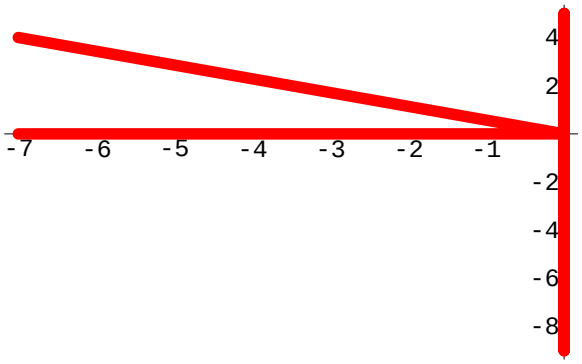
(d) 5 cm N + 7 cm W + 9 cm S

Ans:$<$ 5 cm, 90 degrees$>$ + $<$ 7 cm, 180 degrees$>$ + $<$ 9 cm, 270 degrees$>$

$=< 5\cos 90, 5\sin 90 > + < 7\cos 180, 7\sin 180 > + < 9\cos 270, 9\sin 270 >$

$=< 0, 5 > + < -7, 0 > + < 0, -9 >$

$=< -7, -4 >$ cm



2. Resolve the vector $<$ 6 cm, 25 degrees $>$ into Cartesian Coordinates (x-y components).

   Ans: $< 6\,cm,$ 25 degrees $>=< 6\cos 25, 6\sin 25 > \approx < 5.4, 2.5 >$ cm

3. Calculate the components of the vector representing velocity of 40 meters per second at an angle of 55 degrees.

   Ans: $<$ 40 m/s, 55 degrees $>=< 40\cos 55, 40\sin 55 > \approx < 22.9, 32.8 >$

4. A pilot wants to fly directly west. The engine pushes the plane at 100 m/s and there is a crosswind blowing to the south at 30 m/s. Determine the exact angle at which the pilot should fly.

Ans: Since the pilot wants to go west, the resultant vector of the planes velocity and the cross wind should point westward, i.e. $v =< -x, 0 >$.

$v_{plane} =< 100\cos\theta, 100\sin\theta > m/s$

$v_{wind} =< 0, -30 > m/s$

$v =< -x, 0 >=< 0, -30 > m/s+ < 100\cos\theta, 100\sin\theta > m/s$

$\Rightarrow 0 = -30 + 100\sin\theta$

$\Rightarrow \theta = \sin^{-1}\frac{30}{100} \approx 17.5$ degrees northernly of west

$= 287.5$ degree bearing on the compass.

5. You and a friend are rowing a boat. You aim it directly toward a dock directly across the 100 meter-wide river and paddle at a speed of 1 m/s. You both are concentrating on rowing and do not notice that there is a 2 m/s current pushing you downstream. How far from the dock will you be when you reach the shore? Will the time it takes to cross the river be affected by the current?

   Ans: The resultant velocity vector is $v = v_{rowing} + v_{current}$

   $=< 1, 0 > + < 0, -2 >$

   $=< 1, -2 > m/s$

   since x-y components are independent, the it will take 100 seconds to cross the river.

   Recall: $d = vt =< 1, -2 > *100s$

   $=< 100, 200 > m$

   So the boat lands 200 m downstream from the dock and the time it took to cross was not affected by the current.

Assignment 3

5 points

Match the following words with the provided definitions:

1. Velocity (e)
2. Projectile (f)
3. Displacement (c)
4. Components (d)
5. Trajectory (b)

(a) A scalar quantity

(b) The path a projectile follows

(c) The distance and direction an object is from its starting point

(d) Vectors at right angles that combine to make another vector

(e) An object moving through the air that is only affected by the force due to gravity

Assignment 4

20 points

1. You take a running jump off the end of a diving board at a speed of 7 m/s and splash into the water 1.5 seconds later.

   (a) How far horizontally do you land from your takeoff point?

   (b) How high is the diving board?

2. A model rocket is launched into the air so that its initial horizontal speed is 20 m/s and its initial vertical speed is 39.2 m/s. Complete a table of data that gives the horizontal speed and vertical speed every second for 8 seconds.