The three main goals of these modules: “Why should I take another math class – in grad school?!" you might ask. We do not intend to make this course as a remedial math tour. Instead, we would like to focus our attention on revisiting 4 areas of engineering math, with the intention of addressing 3 questions:

a) How are these branches of mathematics connected to each other? (Answer: Linear algebra!)

b) Where will these math concepts come up in your grad career in biomedical engineering?

c) How do I deal with advanced math topics that I’ll need for future research that wasn’t covered in these courses?

Enrollment: All graduate students or senior undergraduates (need department approval) are welcome!

2 math tracks tailored to your own interest... with a la carte registration!

For the 2017 - 2018 academic year, we are offering 4 math modules under the BE 601-604 course heading. Out of these 4 modules, you are free to choose (a minimum of 2) topics that suits your interest. The completion of 4 total credits from the BE 601-604 series will satisfy the math requirements for the BME graduate curriculum.

Note: These classes were designed such that the topics within a given module cascade on top of previous ones. Please check out the flowchart on the next page to see the preferred pre-requisites for each course!
BE 601 – 604: Cascaded module design / preferred pre-requisites

Prerequisites for Linear algebra (BE 601):
• Basic vector calculus
• Basic undergraduate physics

Prerequisites for Stats / Numerical (BE 604):
• Basic probability and statistics exposure is helpful!
• Previous exposure to linear algebra is essential
• Working knowledge of a programming language

Prerequisites for ODEs (BE 602)
• Previous exposure to undergraduate ODEs is helpful
• Linear algebra (finding eigenvalues, eigenvectors, and diagonalizing a matrix)
• Previous exposure to Fourier series
• Working knowledge of a programming language

Prerequisites for PDEs (BE 603):
• Previous exposure to at least 1 of these 2 orthogonal functions commonly used in the generalized Fourier series approximations of periodic functions:
  - Legendre polynomials
  - Bessel functions
• Have seen Fourier series and can find Fourier coefficients using inner products
• Previous exposure to finite-difference schemes for ODEs or PDEs will help!
• Previous exposure to Laplace and Fourier Transforms is helpful!

** For people choosing to skip BE 601 or 602 and dive straight into PDEs (BE 603)

If you need a refresher on Sturm-Liouville problems, Legendre polynomials, Bessel functions, and Laplace / Fourier transforms, or finite-difference approximations to differential equations, I will have pre-taped video lectures that you can watch at your leisure. This allow us to concentrate on solving PDE-centric problems during lectures, thus avoiding the class being bogged down on rehashing math topics covered in either BE 601, 602, and undergraduate signals & systems courses.
**Homework and Friday recitations:** Problem sets will be handed out usually on **Wednesdays** and will be due on the **following Friday** during recitation. During Friday recitations, we will discuss any questions regarding both the previous and next problem sets.

**Matlab:** Since matrix-dependent elements will be in no short supply in this class, we will adopt Matlab as the standard software from which all course materials, homeworks, and take-home tests will be analyzed with. Programming tips in Matlab will be provided throughout the course during recitations, office hours, or in announcements with template scripts uploaded onto Blackboard Learn.

**Textbooks:** There are no required textbooks for BE 601. Please see pages 5 – 7 for more explanations!

**Exams:** **Take-home** exam at the end of each module  
Duration = 1 week

**Grading:** The breakdown per module is:  
80% homeworks (~6 per module) + participation during recitation  
20% from take-home exam (1 per module)
### Linear algebra syllabus: (1st half of Fall 2018)

<table>
<thead>
<tr>
<th>Lectures (theme)</th>
<th>Topics</th>
<th>Key concepts</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4 (Ax = b)</td>
<td>Column multiplication</td>
<td>Alternative to $A^{-1}$ when $A$ is large</td>
<td>Intro to <strong>lumped element modeling</strong> of PDEs</td>
</tr>
<tr>
<td></td>
<td>Gaussian elimination / LU factorization</td>
<td>Exposure to matrix manipulations, “numerical linear algebra” style</td>
<td>Intro to discrete convolutions + manipulation of data</td>
</tr>
<tr>
<td></td>
<td>Finite difference eqs</td>
<td>Gentle intro into matlab if students are unfamiliar with it</td>
<td></td>
</tr>
<tr>
<td>5 (Ax = b)</td>
<td>Least squares $(A^TAx = A^Tb)$</td>
<td>How to approximate the solution to $Ax = b$ when matrix $A$ is rectangular</td>
<td>Curve fitting</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Exposure to minimization problems</td>
<td>Multi-variable linear regression</td>
</tr>
<tr>
<td>6-8 (Qc = b)</td>
<td>Orthogonal matrices, inner products and Fourier series</td>
<td><strong>Introduction to orthogonality</strong> and basis sets</td>
<td>Solid deformations (continuum mechanics)</td>
</tr>
<tr>
<td></td>
<td>Isometry / similarity / change of basis</td>
<td>Generalized Fourier series</td>
<td>Signals &amp; systems (Fourier series)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Understanding the relationship between change of basis and similarity in the context of geometrical transformations</td>
<td>Image compression (Haar wavelets / biorthogonal matrices)</td>
</tr>
<tr>
<td>9-10a (Ax = $\lambda x$)</td>
<td>Eigenvalues / eigenvectors</td>
<td>Symmetric matrices leads to <strong>orthogonal eigenvectors</strong> (important in ODES and PDEs)</td>
<td>Transformations in solid mechanics</td>
</tr>
<tr>
<td></td>
<td>Diagonalization</td>
<td><strong>Diagonalization = decoupling</strong> of systems of equations</td>
<td>The covariance matrix (intro to statistics)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Solving ODE systems</td>
</tr>
<tr>
<td>10b-11 (Ax = $\lambda x$)</td>
<td>Quadratic forms, $p$-norms</td>
<td><strong>Real-life applications of diagonalization and decompositions</strong></td>
<td>Principal component analysis / clustering</td>
</tr>
<tr>
<td></td>
<td>Inverse shifted power iterations</td>
<td>Euclidean vs. statistical “distances”</td>
<td>Google PageRank (Markov chains)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Using the Rayleigh quotient as a vehicle to iteratively solve for eigenvectors and eigenvalues</td>
<td></td>
</tr>
<tr>
<td>12-14 (Ax = b) (Ax = $\lambda x$)</td>
<td>Eigenvalue and Singular value decompositions (SVD)</td>
<td><strong>Relationship between Fourier series, orthogonality, and eigenvectors</strong></td>
<td>Image processing / data compression using SVD</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reducing the complexity of a giant matrix $A$ by only retaining its “dominant” characteristics</td>
<td>Polar decompositions (continuum mechanics)</td>
</tr>
</tbody>
</table>
Recommended “all-purpose” reference textbook for engineering math: One of the most comprehensive all-purpose engineering math textbooks out on the market is the one written by Greenberg. Since it is quite expensive on Amazon, I will have selected scanned chapters on Blackboard for you to read (mostly in the ODE sections). We will supplement what Greenberg lacks (linear algebra!) with additional online book chapters that students can access for free via Blackboard Learn.


Other “all purpose” text(s) that supplement the BE 601 – 604 modules: Aside from Greenberg’s book, there are numerous “compendium” math texts that span the standard elements of linear algebra, ODE, PDEs, complex variables, and numerical methods. I personally like Riley’s book because its contents were written at a higher level than Greenberg, but I also like Mary Boas’ book for her simple and concise explanations. For more advanced students, and for people considering taking the numerical analysis module, I highly recommend purchasing Strang’s (1986) book because it correlates well with his MIT Open Courseware video lectures for 18.085 and 18.086.

** Note: The number of “✓” refers of quality and extent of coverage (I own all 5 books and they’re my opinions only!), whereas “♦” denotes the level of mathematical intuition the author expects the reader to have for full comprehension of the material (ie. Not just knowing how to plug + chug basic problems!).

<table>
<thead>
<tr>
<th>Author</th>
<th>Linear Algebra</th>
<th>ODEs</th>
<th>PDEs</th>
<th>Calculus of variations</th>
<th>Complex Variables</th>
<th>Tensors</th>
<th>Numerical methods / optimization</th>
<th>Prob + Stats</th>
<th>Price (new)</th>
<th>Level of math maturity required</th>
<th>Book focus</th>
</tr>
</thead>
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<tr>
<td>Riley</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>$</td>
<td>♦♦♦</td>
<td>Chemistry Physics (symmetry)</td>
</tr>
<tr>
<td>Boas</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<td>$$$</td>
<td>♦</td>
<td>Physics</td>
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<tr>
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<td>✓</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>$$$$</td>
<td>♦</td>
<td>Engineering</td>
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<tr>
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<td>$$$</td>
<td>♦</td>
<td>Engineering</td>
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<tr>
<td>Strang (1986)</td>
<td>✓✓✓</td>
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<td>✓</td>
<td>✓</td>
<td>$</td>
<td>♦♦♦</td>
<td>Computational Physics</td>
</tr>
</tbody>
</table>

Additional “All-purpose” math textbooks:

Reading assignments: I will frequently assign readings from this list (especially the red ones) via Blackboard Learn!! Selected PDFs will be available for you to download on the class website.

Linear algebra and tensors:

Heat transfer:

Numerical methods:
Statistical methods (for life scientists); statistical analysis


EK 100 dates:

9/7 (Fri)

9/21 (Fri)

11/2 (Fri)

11/16 (Fri)