Course Description
The course content divides essentially into four parts. Derivation of the field equations (essential to understand their applicability for modeling); fundamental theorems which gives results we can state for all materials and/or problems; basic elastostatics which covers classical solution methods to classical canonical problems; advanced elastostatics which covers a selection of advanced topics. The selection of the advanced topics depends on class interest and my current interests. An emphasis will be placed on concepts (e.g. modeling, linearity, superposition, scaling) and fundamental phenomena (e.g. boundary layers, timescales, energy) exhibited through a sampling of model problems.

Prerequisites
Two semesters of undergraduate mechanics of solids and/or fluids. Advanced calculus, and vectors. Familiarity with linear algebra is very useful.

Reference Materials
- Mal, Ajit K. 2015 Course reader for linear and nonlinear elasticity. NB: This book is available electronically at our course website, made available by courtesy of the author. It will be the primary reference used for the lecture materials.
- Sadd, Martin H. 2011 Elasticity: Theory, Applications, and Numerics, Academic Press, Third Ed. NB: This book is available electronically through the BU Library. It’s been the main text for the course in some past years.
- Love, A.E.H. 1892 A Treatise on the Mathematical Theory of Elasticity, Cambridge University Press. NB: The year of publication of this book is not a misprint. It provides excellent historical content and context, and it covers many of the classical solutions. It also has a lot of material that is not generally covered in modern texts (e.g. the elastic), and so the only and best comprehensive coverage of those topics. The notation is difficult for a modern audience.

Grading
The final grade for the course is based on written homework solutions, written and oral reports from small research problems, and participation in homework discussions. Discussions will be held during which students will be expected to present the solutions of homework problems.

Topics
Part I: Field Equations (3 weeks)
1. Vector and tensor algebra: Vector and tensor calculus.
2. Analysis of deformation: Definition of strain, linearized strain, principal strains, compatibility equations.
4. Linear elastic material behavior: Hooke’s law, anisotropy, isotropy, thermoelastic constitutive equations. Strain energy, energetic bounds on material constants.

Part II: Basic theorems and elementary 3D solutions (2 weeks)
7. Tension, torsion, and bending of bars by St. Venant’s semi-inverse method.
8. Spherically symmetric stress distributions.
9. Reciprocity and integral theorems.
Part III: Basic elastostatics (3 weeks)

   (a) Plane stress & strain as special cases of 3D problems.
   (b) Airy stress functions and separation of variables:
       • Wedge and Flamant problem (dimensional analysis).
       • Stress concentrations around a circular hole (topology).
       • Crack tip stress fields (asymptotic behavior of stress fields).

11. Three-dimensional Problems
    • Displacement potential functions.
    • Kelvin’s problem.

Part IV: Advanced Topics (4 weeks)

12. Waves in elastic solids.
    (a) Shear and compressional waves; Helmholtz decomposition.
    (b) Reflection, transmission, and mode conversion at an interface.
    (c) Surface waves (Rayleigh waves).
    (d) Guided waves (SH waves in a layer); end effects.

13. Poroelasticity.
    (a) Biot poroelasticity: confined vs. unconfined compression
    (b) Multiscale poroelasticity: “vascularized” poroelastic solids.

14. Variational Methods: (2-3 lectures)
    (a) Principle of minimum potential energy.
    (b) Principle of minimum complementary energy.
    (c) Principle of virtual work.
    (d) Rayleigh-Ritz and finite element methods.

15. Effective properties of materials with microstructures.
    (a) Basic (Hashin-Shtrikman) variational bounds on material properties.
    (b) Advanced (Willis) variational bounds on material properties.
    (c) Asymptotic homogenization methods.

    (a) Slight eccentricity.
    (b) Small hole.
    (c) Thin walled tube.

17. Rods, plates, and shells as asymptotic limit of 3D theory.
    (a) Asymptotic derivation of elementary equations.
    (b) Boundary layer edge effects: matched asymptotic expansions.

    (a) Asymptotic method of multiple scales; derivation of effective properties.
    (b) Exotic continuum behavior of materials with extreme microstructures.

Instructor

Paul E. Barbone, EMA 221, (617)353-6063, barbone@bu.edu
Office hours: Wednesdays 4-5pm, or by appointment (email me).