Calderón's inverse conductivity problem and quasiconformal maps

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In 1980 A. P. Calderón posed the following problem: Suppose $\Omega \subset \mathbb{R}^n$ is smooth and bounded, $\sigma \in L^\infty(\Omega)$ is bounded away from zero, and that $u \in H1(\Omega)$ is the weak solution of

$$div(\sigma \nabla u) = 0$$

$$u|_{\partial\Omega} = f$$

where $f \in H^{1/2}(\partial\Omega)$. Define the Dirichlet - to - Neumann map $\Lambda_\sigma : H^{1/2}(\partial\Omega) \to H^{-1/2}(\partial\phi)$ by

$$\Lambda_\sigma(f) = \sigma \frac{\partial u}{\partial \nu} |_{\partial\Omega}$$

where ν is the unit outer normal to $\partial\Omega$. Calderón's problem now reads: Does Λ_σ iniquely determine σ? In physical terms this can be refrased as: Can one determine the conductivity of a body by measuring voltages and currents on it's surface.

The inverse problem to determine σ from Λ_σ is also known as *Electrical Impedance Tomography.* It has been proposed as a valuable diagnostic tool especially for detecting breast cancer and for monitoring heart and lungs.

In the talk we discus the history of the problem, it's connections to scattering theory for the Schrödinger equation and finally give a constructive complete solution in two dimensions. It was found by combining the theory of quasiconformal maps to the Beals-Coifman theory of complex geometric optics. The work is a joint study with K. Astala from Helsinki.