Do Stock Price Bubbles Influence Corporate Investment?[1]

Simon Gilchrist
Boston University and and NBER

Charles P. Himmelberg
Federal Reserve Bank of New York and Columbia University

Gur Huberman
Columbia University

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Abstract: Building on recent developments in behavioral asset pricing, we develop a model in which an increase in the dispersion of investor beliefs under short-selling constraints predicts a rise in stock price above its fundamental value, or bubble. The model predicts managers respond to bubbles by issuing new equity and increasing capital expenditures. We test these predictions (among others) using the variance of analysts' earnings forecasts – a proxy for the dispersion of investor beliefs – to identify the "bubble" component in Tobin's Q. We document the dynamic response to bubble shocks using a panel data VAR. Using recursive orderings of the VAR for additional identification, we find that orthogonalized bubble shocks have positive and statistically significant effects on Tobin's Q, net equity issuance, and real investment, consistent with the predictions of the model.

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1 Introduction

To date, most of the effort of behavioral finance has focused on decisions of individuals and on asset prices. Regarding asset prices, the behavioral approach has progressed beyond the statement that asset prices need not equal fundamentals. Starting from models of the limits of arbitrage, it has produced variables capable of predicting excess risk-adjusted returns.

This paper focuses on corporate decisions, especially investment and stock issuance. Starting with the dispersion of analysts' earnings forecasts, which is known to predict stock returns,[2] this paper traces the correlation of that dispersion with future values of Tobin's Q, corporate investment and stock issuance. If managers take their firms' stock prices as signals of investment opportunities, they may do so even if the equity prices are wrong (relative to fundamentals) and may predictably change in the future. From the perspective of traditional finance theory, there is no obvious reason why dispersion of analysts' forecasts should predict issuance or investment. If, however, high dispersion is associated with temporarily inflated stock prices, the firm should take advantage of the situation and sell more equity, which is the motivation for this work.

This study first explores the theoretical predictions of a simple model that seeks to capture the above intuition, and then employs a vector autoregression (VAR) estimated on firm-level panel data to explore the time series relation among of annual investment, net equity issuance, Tobin's Q, marginal product of capital, and dispersion of analysts' earnings forecasts. Combined with a "structural" ordering of the residuals, the VAR-based impulse response analysis traces the impact of a shock to a variable on the whole set of variables in subsequent periods, and assesses the relative weight of the shock in explaining variation of the variables in the subsequent periods. We focus on the response of the system to dispersion shocks.

[2]Diether, Malloy and Scherbina (2002).

Among previous papers that have attempted to document bubbles and their effect on real investment, Polk and Sapienza (2002) is probably closest to ours in spirit. They provide theoretical reasons to expect bubbles to be related to discretionary earnings accruals, equity issuance, and lagged returns, and use these as bubble proxies in an investment equation which uses Tobin's Q to control for investment opportunities. Consistent with their predictions, they find that all three variables are statistically significant in a regression for investment.[3] While these tests are generally valid, they are difficult to interpret. As we argue in the model section (section 2), the predicted signs on bubble proxies can be counter-intuitive because measured Q is itself a function of the bubble. Both the choice of our proxy variable and the recursive orderings of our VAR are motivated by these concerns.

Other related research includes Blanchard, Rhee and Summers (1993) and Mϕrck, Shleifer and Vishny (1991), both of which compare the responsiveness of investment to Tobin's Q and fundamentals and broadly conclude that investment is driven primarily by fundamentals. Chirinko and Schaller (1996, 2001) test for bubbles by including both fundamental and market q measures, and conclude that the evidence favors the existence of bubbles. Erickson and Whited (1999) argue that measurement error distorts estimates of the classic relation between investment and Tobin's Q, and suggest that bubbles in stock prices represent one possible source of error. Similarly, Bond and Cummins (2000) show that measurement error in Q is serially correlated, and they, too, speculate that stock price bubbles are a likely source. Baker, Stein, and Wurgler (2002) argue that certain firms are intrinsically more dependent on equity for their external financing and show that such firms display more sensitivity to stock prices.[4] Our research builds on these efforts as well as recent insights from the finance

[3] Polk and Sapienza (2002) also point out that abnormally high investment levels may be caused in part by stock bubbles, in which case they should predict low subsequent returns. This is indeed what they find.

[4] A related literature examines the behavioral biases of executives and its potential impact on corporate investment decisions. Heaton (1999) develops a model in which CEOs are both overconfident and overoptimistic. Malmendier and Tate (2002) use the timing of stock option exercise to measure

literature and new data to develop and test new theoretical predictions.

1.1 Behavioral Asset Pricing: Arguments and Implications for Investment and Equity Financing

Prices are major determinants of resource allocation. Asset prices guide corporate executives in their investment and equity flotation choices. How managers should and do use prices in these decisions depends on their beliefs regarding the relation between prices and fundamentals and the anticipated market's reaction to investment and equity issuance.

The traditional and the behavioral approaches to asset pricing offer radically different views on the relation between prices and fundamentals. To understand their different implications for corporate investment, it is worthwhile to review these different approaches.

According to traditional financial economics, markets are efficient and therefore stock prices equal their fundamental values, given the available information. Moreover, since traders can buy and sell unlimited quantities of an asset at the same price, in equilibrium, they agree on an asset's valuation, and prices reflect all their views. Otherwise, a dissenter will pursue further expected profits (in his eyes) by trading more, thereby upsetting the equilibrium.

Behavioral finance takes issue with the position that the "price is right," espousing Adam Smith's view that "[t]he value of a share in a joint stock is always the price which it will bring in the market; and this may be either greater or less, in any proportion, than the sum which its owner stands credited for in the stock of the company."

Numerous studies of individual decision making underlie the behavioral approach. They document systematic deviations of decisions from the norm of optimality that

overconfidence. Bertrand and Schoar (2002) report evidence that CEOs appear to have managerial styles that they carry with them when they change jobs.

is routinely invoked in economic models. Recent surveys of behavioral finance include Barberis and Thaler (2002), Hirshleifer (2001) and Shleifer (2000).

Acknowledgement that individuals systematically err does not automatically imply that market prices can be away from fundamentals. Decisions of market participants may differ from those of typical people because market participants form a self-selected group with motives and incentives different from those of typical individuals. Moreover, errors of market participants can offset each other and in the aggregate have no trace in prices. Finally, pricing anomalies are likely to attract arbitrageurs whose profit-motivated trades will undo the anomalies.

The two ingredients of behavioral models that lead to asset pricing anomalies are disagreements among market participants and constraints on portfolio choice. Prices may fail to reflect all available information if the opinion of those who disagree with the prices cannot be translated into trades because such trades are excessively costly due to missing markets.

In the ideal world of an arbitrageur, he can set up a position as soon as he spots a pricing anomaly, and the anomaly is corrected as soon as he establishes the position, thereby delivering a sure and immediate profit. Real-life situations are considerably more complex. Some markets are missing, and therefore certain positions can be established only at a cost, to the extent that arbitrageurs may prefer to set up proxies for these positions, leaving themselves exposed to some risks. Shleifer and Vishny (1997) apply this insight to elaborate on the limits of arbitrage.

When looking for a specific cross-firm pricing anomaly, the behavioral approach considers a statistic related to the firm, argues why and how the statistic or variables correlated with it may lead market participants to assign the wrong value to the firms under consideration, and proceeds to examine the prediction empirically. Prime examples are models of value investing (DeBondt and Thaler, 1985, Lakonishok, Shleifer and Vishny, 1994.) Difficult identification challenges plague attempts to relate asset mispricing to corporate actions such as investment and stock issuance, because the firm-related statistic that indicates mispricing may be correlated with the firm's investment opportunities.

Rather than starting with variables that may lead investors to misprice assets, one can start with the institutional feature most glaringly in the way of market efficiency: the difficulty of short selling.

The asymmetry between the ease of buying and the difficulty of selling a stock short is the standard and prominent example of a missing market, or at least a market in which it is costly to trade. This asymmetry is important when traders who have sold all the stock they owned still believe a stock is overpriced and would like to sell it short.

Anticipating the current attention given to the behavioral approach, Miller (1977) suggests that there are assets that traders would like to short, but cannot. These assets' prices fail to reflect the opinion of these traders and therefore are too high. Recently, Hong and Stein (2002) formalize this argument.

Short selling costs are important when short selling is desired. Focusing on a period in the 1920s and 30s when the market for stocks to short was at the NYSE and the costs of shorting were made public and reported in the Wall Street Journal, Lamont and Jones (2002) document that stocks that were expensive to short did indeed deliver lower returns than other stocks. On average, shorting them was profitable even after accounting for the cost of shorting.

Proxies must replace records of the explicit cost of short selling when these costs are unavailable. Two proxies have been proposed: Chen, Hong and Stein (2002) consider breadth – the extent to which a stock is held by mutual funds – and Diether, Malloy and Scherbina (2002) and Park (2001) consider dispersion of analysts' earnings forecasts. Both papers report that the proxies they choose indeed help forecast the cross section of returns. Breadth captures the extent to which fund managers want to hold the stock; when it is small for a giver stock relative to its historical average, many fund managers would have liked to short it, but are legally prevented from shorting. Therefore breadth is positively correlated with subsequent returns.

The intuition of Diether, Malloy and Scherbina (2002) and Park (2001) is that the dispersion of analysts' earnings forecasts reflects disagreements among investors, and the wider the dispersion, the stronger the disagreements. When disagreements are strong, more market participants would like to short the stock, and some of them are probably prevented from doing so either because shorting that stock is costly or because they are legally bound not to short stocks.

Chen, Hong and Stein (2002) report that "using data on mutual fund holdings, we find that stocks whose change in breadth in the prior quarter is in the lowest decile of the sample outperform those in the top decile by 6.38% in the twelve months after formation. Adjusting for size, book-to-market, and momentum, the figure is 4.95%." Diether, Malloy, and Scherbina (2002) report that "a portfolio of stocks in the highest quintile of dispersion underperforms a portfolio of stocks in the lowest quintile of dispersion by 9.48% percent per year."

Behavior-based distortion of asset prices can affect corporate decisions through two channels. One, firms may dismiss the possibility that asset prices are distorted and use them as the correct signals for investment and equity flotation. Firms will invest too much (or too little) even if firms are not betting that prices are different from fundamentals, but they are. Two, firms realize that asset prices are distorted, anticipate changes in them, and exploit the perceived distortions in financial markets. A firm's capital market activity supports its real investment activities but is also a speculation on future price moves. The market's reaction will constrain a firm's issuance (repurchase) of stock when price is too high (low). If investors interpret a firm's stock issuance (or repurchase) as indication of the management's disagreement with the market price, it will adjust, thereby closing the opportunity to exploit the perceived mispricing. To mask a purely speculative stock issuance (repurchase) the firm may also increase (decrease) investment, especially in marginal projects.

Thus, whether through the first or second channel – or a combination thereof

– deviations of asset prices from fundamentals will lead to deviations of corporate investment from following a pure net present value rule. This paper examines this proposition empirically.

Empirically, detecting systematic deviations of corporate investments from their optimal levels due to mispricing of the equity is a challenge for a few reasons. One, detecting mispricing is hard. Two, establishing optimal investment, or deviation from it, is hard. Three, both pricing and investment are jointly determined along with a few other relevant variables. Four, the pertinent variables are observed at different frequencies, announced and recorded after market participants are at least partially aware of their realizations.

As empirical indicators of possible bubbles, breadth and dispersion have the advantage that they are exogenous to the firm, in contrast with variables such as earnings to price ratio, recent returns, sales growth, stock issuance, quality of earnings etc. Breadth and dispersion may reflect information about the firm, but they should not affect its action; to the extent they contain information about the firm, that information should be incorporated in the price of the firm's equity. For our empirical work, we chose to use the dispersion of analysts' earnings forecasts because it seemed more directly related to its theoretical analog, and therefore more promising.

In the next section, we explore a simple equilibrium model of heterogeneous investor beliefs under short-selling constraints. This delivers a pricing function (inverse demand function) for new share issues which depends on both the dispersion of beliefs and the size of the share issue. We consider the optimal share issuance and real investment decisions for a rational, bubble-aware manager who seeks to maximize the objective value of the firm. We characterize the equilibrium relationship among dispersion, equity issuance, investment and Tobin's Q, especially with regard to empirical implications. In section 3, we briefly describe the data and econometric approach, followed by a description of our empirical results in section 4. Section 5 concludes.

2 A Model of Real Investment, Equity Issuance, and Bubbles

The behavior of a firm in the presence of bubbles can be derived from the following simple model. In breif, we assume the manager of a firm controls a technology that generates future cash flows as a function of an endogenous capital stock and an exogenous value shock. The manager can rationally assess the objective distribution of future cash flows, and seeks to maximize this payout net of capital costs. The market price of equity in our model may not reflect fundamental value. Managers recognize mispricing when it occurs, and in the model below, we show they exploit it by issuing new shares and increasing investment. We begin by deriving the market demand and equilibrium price for shares when investors have heterogeneous beliefs and face short-selling constraints, and then derive the equilibrium response of equity issuance, bubbles, real investment, cost of capital, and Tobin's Q in response to exogenous dispersion shocks.

2.1 The Demand for New Share Issues

We begin by assuming investor valuations are represented as vV, where V is the true value of the firm, and $v \in [0, \infty]$ is a random variable representing the cross-sectional distribution of beliefs. Let P denote the market value (price) of the firm.[5] We assume the investor's portfolio demand for a firm's shares (i.e., the fraction of the investor's wealth invested in the firm) is given by[6]

$$\omega_v = \gamma \left(vV - P \right). \tag{1}$$

[5] Investors and managers disagree about the value of the firm, but for simplicity, we assume they all use the same discount rate.

[6] This portfolio demand can be derived from a model in which investors have CARA utility.

Multiplying ω_v by investor wealth and dividing by the market value of the firm translates the investor's demand from a portfolio share to a proportional claim on the firm's equity value, $n_v = \omega_v W/P$. Without loss of generality, we assume investor wealth and the total mass of investors both equal one. Thus, $n_v \in [-1, 1]$ is the investor's demand for shares, given by $n_v = \gamma\left(vB^{-1} - 1\right)$, where $B = P/V$.

Under short-selling constraints, the only investors who take positions in the stock are those for whom $vV \geq P$, or $v \geq B$. Hence, assuming v has the distribution function $F(v)$, the aggregate demand for shares is

$$n^d(B;\sigma) = \gamma \int_B^\infty \left(vB^{-1} - 1\right) dF(v). \tag{2}$$

Denote the fraction of total shares supplied to the public by n. By inverting the function $n = n^d(B;\sigma)$ for B, we define the inverse demand function, $B(n;\sigma)$, which gives the level of the price above fundamentals as a function of dispersion and the number of shares issued.

To characterize the inverse demand function we assume that v is log-normally distributed with $\ln v \sim N\left(-0.5\sigma^2, \sigma^2\right)$, so that $E(v) = 1$. This normalization says that average beliefs are unbiased, and also implies that net demand for shares is zero when $P = V$ (or $B = 1$) and short-sale constraints are not binding. It will also be convenient to work with a normalized log transformation of B, namely:

$$b \equiv \frac{\ln B + 0.5\sigma^2}{\sigma}. \tag{3}$$

Then, using properties of the log-normal distribution, equation 2 can be expressed as

$$n^d(B;\sigma) = \gamma\left[\left(1 - \Phi(b - \sigma)\right)B^{-1} - \left(1 - \Phi(b)\right)\right]. \tag{4}$$

where Φ denotes the c.d.f of the standard normal distribution.

It turns out to be convenient to express the above demand function in terms of

b only. Let ϕ denote the pdf of the standard normal. Using the definition of b in equation in equation 3, it is readily verified that $\phi(b)\, B = \phi(b - \sigma)$. Solving this for B and inserting into equation 4 yields the alternative expression:

$$n^d(B;\sigma) = \gamma(1 - \Phi(b)) \left[\frac{h(b)}{h(b-\sigma)} - 1 \right]. \tag{5}$$

where $h(b)$ denotes the hazard rate for the standard normal distribution, defined as

$$h(b) \equiv \frac{\phi(b)}{1 - \Phi(b)}.$$

Because the hazard rate is strictly increasing, the ratio $h(b)/h(b-\sigma)$ is greater than one, hence market demand is strictly positive for $B > 0$. We formally characterize these and additional properties of the demand for shares in the following proposition:

Proposition 1 *Assume the distribution of investor beliefs in equation (2) is lognormally distributed according to* $\ln v \sim N(\mu, \sigma^2)$, *with* $\mu = -\sigma^2/2$ *so that average beliefs are unbiased (i.e.,* $E(v) = 1$*). Then for every* $n > 0$ *and* $\sigma > 0$ *there exists a unique* $B > 0$ *that satisfies equation 4. This implicitly defines the inverse demand curve,* $B(n;\sigma)$*. Moreover, the inverse demand curve has the properties*

$$B_n = \frac{-B^2}{\gamma(1 - \Phi(b-\sigma))} < 0 \tag{6}$$

and

$$B_\sigma = Bh(b-\sigma) > 0, \tag{7}$$

where $B_n \equiv \frac{\partial B(n;\sigma)}{\partial n}$ *and* $B_\sigma \equiv \frac{\partial B(n;\sigma)}{\partial \sigma}$ *denote the partial derivatives.*

Proof: *See appendix.*

The above proposition says that the inverse demand curve is downward sloping in the size of the equity issue, and shifts outward in response to an increase in dispersion. The derivatives in equations 6 and 7 lead to simple expressions for the respective demand elasticities. In particular, the inverse-price elasticity of demand $\eta_n \equiv -\frac{\partial \ln B}{\partial \ln n}$ is

$$\eta_n = 1 - \frac{h(b-\sigma)}{h(b)}. \tag{8}$$

The ratio $h(b-\sigma)/h(b)$ is bounded between zero and one. Hence, the demand curve is elastic (the inverse-demand curve is inelastic) over its entire range – a finding which is relevant when we consider the firm problem below. Computing the semi-elasticity of the price (bubble) to dispersion, $\eta_\sigma \equiv \frac{\partial \ln B}{\partial \sigma}$, we have

$$\eta_\sigma = h(b-\sigma).$$

The shift in demand caused by an increase in dispersion depends on the degree of truncation, and hence the hazard rate of the normal distribution evaluated at the bubble. To understand the implications of such a demand shift for investment, we now turn to the firm problem.

2.2 Equity Issuance and the Equilibrium Price Bubble

Given the demand curve for shares implied by the inverse-demand function $B(n;\sigma)$, we can now formally consider the firm's problem. Suppose the expected value of installed capital, K, is given by $V(K) = E[\Pi(K,\theta)] + (1-\delta)K$. Assume the installation of new capital incurs an adjustment cost $\frac{1}{2}\psi K^2$. We assume managers recognize mispricing and choose K to maximize the true value of the firm from the perspective of old shareholders.[7] They can finance this investment using risk-free debt at the rate r, or they can issue new equity by selling a fraction n of the firm's equity. They can invest the proceeds in K, or pay them out as a dividend to the old shareholders.[8]

[7] For example, managers might own a stake in the firm for incentive reasons, in which case their incentives are to act on behalf of old rather than new shareholders.

[8] The firm's ability and desire to issue equity when faced with a bubble (and the market response to this issuance) distinguishes our model setting from the one used by Chirinko and Schaller (1996) to study the test for bubbles using investment behavior.

The market value of equity is given by $B(n;\sigma)V(K)$, so proceeds from new equity issues are given by the discounted value of the new shareholders' claim, or

$$X = \frac{1}{1+r} nB(n;\sigma)V(K). \tag{9}$$

Thus the firm's optimization problem is:

$$\max_{I,X,n} -K - \frac{1}{2}\psi K^2 + X + (1-n)\frac{1}{1+r}V(K) \tag{10}$$

subject to equation (4). Note that the future value of the firm in equation (10) is multiplied by $1-n$ to reflect the dilution of old shareholders.

The firm is a monopolist in the supply of its own shares, so issue size depends on the elasticity of the demand curve as well as the marginal (dilution) cost of issuing. As shown above, the demand curve is strictly elastic over its entire range, allowing the firm ample scope to exploit its monopoly position. Just as a profit-maximizing monopolist never sets price equal to marginal cost, the firm never issues enough shares to drive the bubble down to its fundamental value. That is, equilibrium features $B > 1$. To see this logic formally, the first-order condition for equity issuance derived from equation (10) is:

$$B(n;\sigma) - 1 + nB_n(n;\sigma) = 0. \tag{11}$$

Under the assumption that the inverse demand for new shares curve slopes down (which we showed in Proposition 1), it follows that the bubble satisfies $B > 1$ when the firm is issuing new shares ($n > 0$), and $B < 1$ when the firm is repurchasing ($n < 0$).

To determine the equilibrium values of equity issuance and the bubble (n and B), and their response to an increase in dispersion, σ, we combine the market's demand (equation 4) with the firm's first-order condition for equity issuance (equation 11). The existence and and functional form characterization of equilibrium is given in the following proposition:

Proposition 2 *There exists a unique equilibrium bubble as a function of dispersion,* $B(\sigma) > 1$, *that satisfies*

$$B = \frac{h(b)}{h(b-\sigma)}. \tag{12}$$

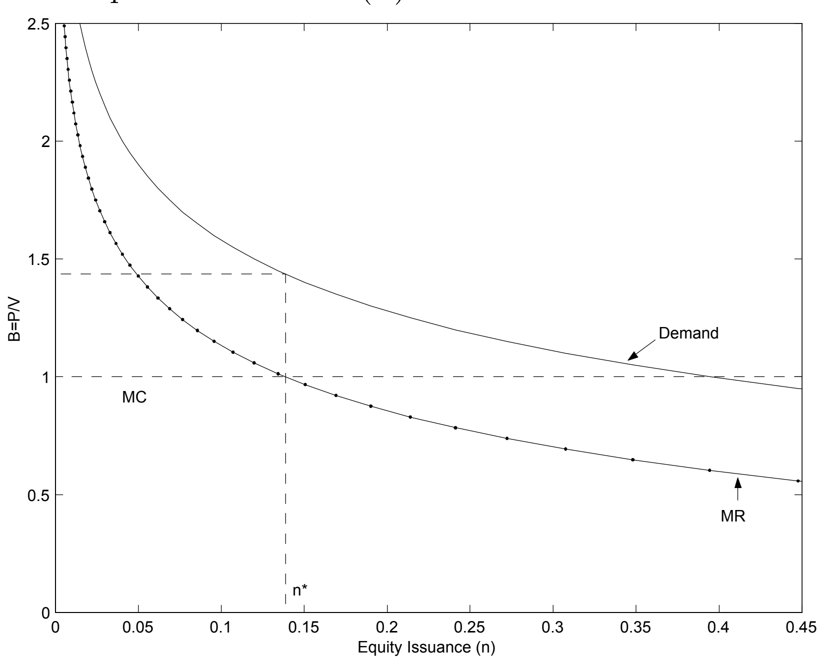
Given the equilibrium mapping $B(\sigma)$, *the equilibrium value of equity issuance is uniquely determined by the equation*

$$n(\sigma) = \gamma (1 - \Phi(b)) (B(\sigma) - 1). \tag{13}$$

Proof: *See appendix.*

It is worth noting that the equilibrium value of the bubble B can be expressed solely a function of σ and hence does not depend on other model parameters, notably the demand parameter γ. In contrast, the quantity of shares issued is increasing in the demand parameter γ. This result implies that the cost of capital , which depends inversely on $n(B-1)$ is decreasing in γ.

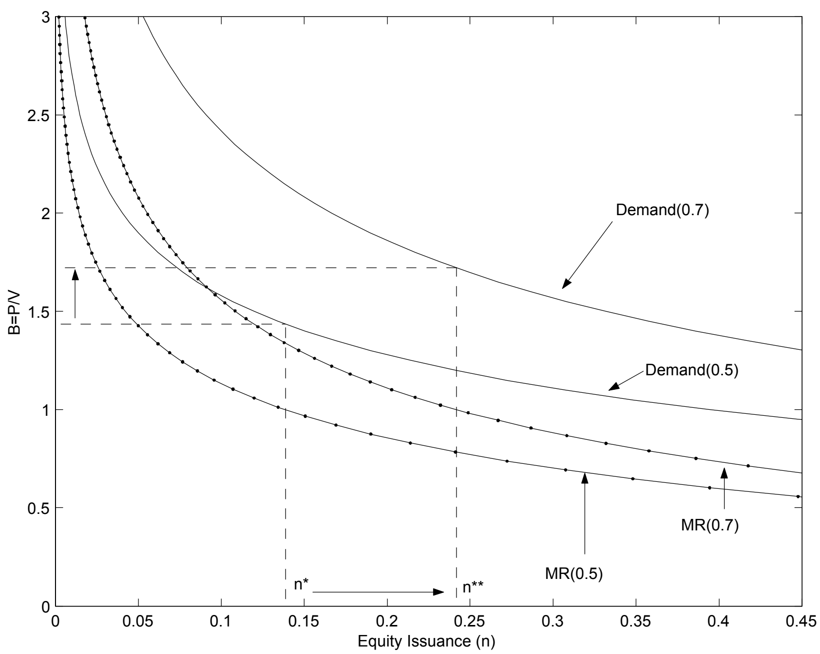
Figure 1: Equilibrium Bubble (B) and Fraction of Shares Issued (n).



For $\sigma = 0.5$, $\gamma = 1$, the equilibrium is depicted in figure 1. Figure 1 plots the market demand curve and the marginal revenue curve for new equity issuance. Equilibrium equity issuance is denoted by n^*. For these parameter values the bubble is sizable – on the order of 50%.

Now consider the effect of an increase in dispersion on the equilibrium bubble B and equity issuance n. As depicted in figure 2, an increase in dispersion represents an outward shift in the market demand for shares and an increase in the equilibrium value of the bubble. In equilibrium, equity issuance depends on both the average net-revenue per share $(B - 1)$ and the percentage of market participants $(1 - \Phi(b))$. In general, a dispersion-driven increase in the share bubble B could cause either an increase or decrease in share issuance, depending on whether the revenue per participant increases enough to offset the drop in market participation. In our case, as shown in the previous proposition, an increase in dispersion causes an increase in share issuance.

Figure 2: The Effect of an Increase in Dispersion



In Figure 2, we depict the effect of an increase in σ from 0.5 to 0.7. An increase in dispersion causes both a rise in the stock price bubble and an increase in fraction of equity issued (from n^* to n^{**}). That this is a general property of this model (assuming a log-normal distribution for investor valuations) is stated and proven in the following proposition.

Proposition 3 *An exogenous increase in dispersion causes the following changes in equilibrium:*

1. *Share issuance, n, increases;*

2. *The stock price bubble, B, increases.*

Proof: *See appendix.*

While the derivatives describing the equilibrium response of equity issuance and price to an increase in dispersion are considerably more complicated than the simple demand elasticities derived above would suggest, the proof of proposition 3 also shows that the equilibrium response of the bubble to an increase in dispersion satisfies

$$\frac{d\ln(B)}{d\sigma} < h(b-\sigma) = \eta_\sigma.$$

In other words, the positive equilibrium response of the bubble to an increase in dispersion is less than the partial-equilibrium analysis (holding issuance fixed) would suggest because the equilibrium response of firms is to increase the supply of shares, and partially offsets the increase in the bubble.

2.3 Dispersion, Investment and Tobin's Q

A convenient feature of our model formulation is that it has allowed thus far to consider the equilibrium behavior of share issuance and stock pricing without jointly considering the choice of investment. As we now show, however, it does not follow from this that investment decisions are independent of dispersion and the stock price bubble.

Tthe first-order condition with respect to capital (for the firm's problem in equations 10 and 9) implies the (modified) q equation

$$1 + \psi K = \frac{1 + n\,(B - 1)}{1 + r} V_k. \tag{14}$$

For the case where there is no bubble ($B \equiv 1$), this reduces to $1 + \psi K = \frac{1}{1+r} V_k$. This is the usual first-order condition for investment, which says that the firm invests up to the point where the marginal cost of investment, $1 + \psi K$, equals the discounted expected marginal value of capital, $\frac{1}{1+r} V_k$ (or marginal Q). It forms the basis for econometric specifications based on Tobin's Q (or *average* Q), about which we will have more to say more below.

The first-order condition in equation (14) makes it clear that the investment choice depends on the bubble. The precise effect is summarized in the next proposition:

Proposition 4 *In response to an exogenous increase in dispersion, investment increases.*

Proof: *See appendix.*

The most intuitive way to see the effect of the bubble on investment is to substitute for V_k and, assuming no adjustment costs ($\psi = 0$), approximate the resulting equation (14) as

$$E\left[\Pi_k\right] \simeq r + \delta - n\,(B - 1)\,, \tag{15}$$

where the approximation used is valid when $n\,(B - 1)$ is small.[9] This expression allows us to interpret the effect of bubbles in terms of a modification to the Jorgensonian cost of capital, which is defined as the right side of equation (15). When $n\,(B - 1)$ is zero, that is, when there is no bubble, or when there is a bubble but the firm does not issue, the approximation is exact and we have the familiar optimality condition

[9] This logic still holds without the approximation, but a linear approximation simplifies the intuition.

for capital which sets the marginal profitability of capital equal to its user cost. That is, $E[\Pi_k] = r + \delta$.

If, however, the bubble is positive *and* the firm actively exploits the bubble by issuing shares, then this has the effective of reducing the cost of capital by $n(B-1)$. In particular, from Proposition 3 we know that an increase in dispersion increases the size of the bubble and increases new share issuance. Hence, in equation (15), this clearly causes the cost of capital to fall, which should increase investment. Just as in equation (14), equation (15) shows that the amount by which a positive stock price bubble reduces the cost of capital depends on the extent to which new equity is issued. That is, it depends not only on the size of the bubble, but also on the elasticity of share demand.

Finally, we do not estimate traditional Q regressions in our empirical work, but we do present evidence on the response of Q to shocks in dispersion, hence it is of interest to define and characterize *measured* Q in equilibrium. Moreover, our model generates an equilibrium relation among investment, Tobin's Q, bubbles, and share issuance which has potentially important implications for alternative testing strategies based on estimates of modified Q equations.[10]

Proposition 5 *Define Tobin's Q as the ratio of the market value of equity to replacement value of capital:*

$$Q \equiv \frac{1}{1+r} E\left[\frac{BV}{K}\right]. \tag{16}$$

If $\Pi(K,\theta)$ *is linearly homogeneous in* K*, then the relationship between in investment* (K) *and* ***measured*** *Tobin's Q is given by*

$$1 + \psi K = \frac{1 + n(B-1)}{B} Q. \tag{17}$$

Moreover, in response to an exogenous increase in dispersion, the equilibrium value

[10] We are grateful to Andy Abel for bringing these points to our attention and for particularly lucid comments.

of Tobin's Q increases.

Proof: *See appendix.*

We can solve the investment-Q equation for measured (Tobin's) Q as

$$Q = \left(\frac{B}{1 + n\,(B-1)} \right) (1 + \psi K)\,. \tag{18}$$

If adjustment costs and the stock price bubble were both zero, then we'd have $Q = 1$. Otherwise, since the first term on the right side of equation (18) is increasing in B, either bubbles *or* adjustment costs (or both) are sufficient to imply $Q > 1$.

The investment-Q relation in equation (18) is often used to estimate the adjustment cost parameter ψ.[11] For estimation, it is more common to normalize by solving for K instead of Q. The resulting expression can be approximated as:[12,13]

$$K \simeq \frac{1}{\psi} \left[Q - 1 - (B-1)(1-n) \right]. \tag{19}$$

Equation (19) suggests that a regression of investment on Tobin's Q and a proxy for the bubble term $(B-1)(1-n) > 0$ would yield a *negative* coefficient on the latter. This appears to contradict the predicted positive relationship between bubbles and investment implied by a response to dispersion in proposition 4. The apparant paradox stems from having conditioned on measured Q, which itself depends on the bubble. Conditioning on "true" Q, if it were observable, would generate more intuitive results. To see this, note that measured Q is approximately $Q \simeq V_k + B - 1$,

[11] In more elaborate derivations of the Q model, the marginal cost-of-adjustment term is usually a function of the rate of investment rather than the level. The static environment here is too simple to accomodate growth rates.

[12] Investment, Q, bubbles, and equity issuance are all endogenous in this equation, so the choice of variable appearing on the left side of the equation is strictly arbitrary. Though common, this normalization often tempts users to assign (incorrectly) a causal relationship from Q to investment.

[13] The "epsilon approximations" we have used to obtain this approximation of the Q equation are accurate in a neighborhood of $Q = 1$, $B = 1$, and $n = 0$.

so equation (19) can be written in terms of "true" Q as

$$K \simeq \frac{1}{\psi}\left[V_k - 1 + n\left(B - 1\right)\right]. \tag{20}$$

In contrast to a regression based on equation (19), a regression based on equation (20) would estimate a positive coefficient on a proxy for the bubble term, thus resolving the paradox. Of course, equation (19) is the relevent empirical specification since it is based on observable data.

To summarize the results in this section, heterogeneous beliefs and short-selling constraints can generate bubbles. From the perspective of rational managers who maximize true value, these bubbles reduce the marginal cost of capital, and thus stimulate real investment. But the link is more subtle than it first appears. The reduction in the cost of capital depends not only on the size of the bubble but also on the *elasticity* of the bubble with respect to new share issues. For example, if even an enormous bubble could be eliminated by a very small share issue, the financing benefit of such a bubble (and hence the effect on the cost of capital) would be negligible. Firms would have no incentive to exploit such a highly elastic bubbles. Large bubbles could theoretically persist in equilibrium yet have only a trivial impact on investment. It depends on the elasticity.

We've shown that proxies for the dispersion of investor beliefs can be used to construct a test for the real effects of bubbles. When the distribution of investor valuations is lognormal, increases in dispersion increase both the size of the bubble and the amount of new equity issued. This lowers the cost of capital and therefore stimulates investment. We've also shown that the elasticity of the bubble is implicitly revealed by the size of the equity issue, hence new equity issues provide an additional (or alternative) indicator of stock price bubbles. In our model, changes in the dispersion of investor beliefs are the only shocks that affect the new issue decision. In a more general model, we would expect shocks to investment opportunities to affect issuance, too, and our empirical specification allows for this. Finally, we showed that the equilibrium value of Tobin's Q (that is, average *measured* Q as opposed to average *true* Q) is increasing in not only the rate of investment but also in the size of the bubble. Thus, our results provide a measure of support for the common practice of using Tobin's Q (or market-to-book ratios) as indirect measures of stock price bubbles.

3 Data and Econometric Framework

Polk and Sapienza (2002) is the first and only paper of which we are aware that attempts to measure the impact of bubbles on investment using endogenous indicators to identify stock price bubbles. Their empirical results are clearly consistent with the hypothesis that bubbles exist and that firms exploit them. As such, it makes an important contribution. At the same time, the model in the previous section highlights difficult identification issues in the traditional investment-Q framework. The problem with using variables like equity issuance and lagged returns to identify bubbles is that they are obviously good indicators of unobserved investment opportunities, too. Moreover, Tobin's Q is *itself* a function of the bubble. It is therefore not a very suitable control variable.[14]

Our empirical strategy is motivated in part by such identification problems. Our startegy is to use a bubble indicator which is plausibly orthogonal to investment opportunities. We pursue two ideas. First, we use the variance of analysts' earnings forecast as a bubble indicator. In contrast to variables like equity issuance and lagged stock returns, there is no obvious reason why this measure would be correlated with investment opportunities (it isn't even obvious why earnings analysts should disagree in the first place when they have access to the same public information, including one another's' forecasts). Second, we use recursively ordered VARs to

[14]More generally, the problem with Tobin's Q as a control variable arises because the relation between investment and Tobin's Q is not causal. Rather, it is an equilibrium condition, and only under certain restrictive conditions. Under more general conditions, Tobin's Q is not a sufficient statistic for the marginal value of capital, and it is potentially misleading to treat it as such.

further isolate and identify the exogenous component of this variable. This approach is a (minimally) structural attempt to improve identification. In addition to aiding identification, recursively ordered VARs provide a parsimonious description of the information contained in the conditional first and second moments of the data, and the system approach explicitly recognizes and accommodates the joint endogeneity of pricing, investment, and equity financing. Plots of the impulse response functions provide an easy visual inspection of an otherwise complex system, and variance decompositions provide convenient summary statistics for quantifying the importance of the respective shocks to the system.[15]

We estimate three basic systems, each nested within the former. The first VAR includes just three variables: the logarithms of the investment-to-capital ratio, Tobin's Q and our dispersion measure (described in more detail below). In the second VAR we include the above three variables plus the logarithm of the marginal product of capital (hereafter "MPK"). This variable is added to improve our ability to distinguish bubbles from "fundamental" information about discounted marginal returns to capital. Finally, to the third VAR, we add the value of net equity issuance as a fraction of the market value of equity. This variable is included for two reasons – first, so that we may examine its endogenous response to various shocks in the VAR system, and second, so that we can examine the system response to orthogonalized shocks to equity issuance.

We assemble annual, firm-level data from two sources. Data on sales, capital expenditures, net cash flows from equity issuance, (and the book values of) total assets, total liabilities, preferred equity, and property, plant and equipment are obtained from Standard & Poors Compustat. These variables are merged with data from the IBES dataset on analysts' earnings forecasts, which were kindly provided by Anna Scherbina. These data are used in Deither, Malloy and Scherbina (2002). Their paper

[15] We do not exploit identification strategies based on the imposition of structural cross-equation restrictions.

explains the importance of using their data rather than the split-adjusted forecasts which are provided in the standard-issue dataset.

Variable definitions are as follows.

- *Investment* $(i_t - k_t)$ is the logarithm of the ratio of capital expenditures to the net book value of property, plant and equipment at the beginning of the fiscal year.

- *Net equity issuance* (n_t) is cash from net share issues during the fiscal year divided by the market value of equity at the beginning of the year.

- *Tobin's Q*, (q_t) is the logarithm of the following ratio: market value of equity plus the book value of preferred equity plus the book value of total liabilities divided by the book value of total assets (end-of-fiscal-year values).

- *Dispersion* (d_t) is the logarithm of the fiscal year average of the monthly standard deviation of analysts' earnings forecasts (per share), times the number of shares, then divided by the book value of total assets. That is,

$$d_t = \log \left(\frac{\sum_{j=1}^{12} N_{t-j} \sigma_{t-j} / 12}{Total\ Assets} \right),$$

  where N_{t-j} is the number of shares outstanding, and σ_{t-j} is the variance of earnings forecasts for all analysts making verified forecasts for the month (where "verification" means that IBES has confirmed the forecast is not stale).

- *Marginal profit of capital* (mpk_t, or "MPK") is the logarithm of sales divided by lagged book value of property, plant and equipment (end-of-fiscal-year values).

The lower and upper tails for each of these variables are truncated to their respective values at the first and 99th percentiles. This is a simple if somewhat crude method for reducing the impact of extreme ratio outliers which are common for ratios in firm panels drawn from accounting data. The use of logs (where possible) also mitigates these problems. We drop observations for which the lag between consecutive fiscal-year-ends is not exactly 12 months. (The month in which the fiscal years sometimes changes for such reasons as mergers or restructurings.)

We start with a three variable VAR system that includes Tobin's Q, investment, and dispersion. This VAR represents the smallest system we can analyze and still hope to trace out the links between investment, stock prices and dispersion in analysts' forecasts. In this system, the dispersion measure represents our indicator of disagreement among investors. Tobin's Q plays two potential roles: under the Q-theory of investment, Tobin's Q provides a statistical summary of all future investment opportunities to which current investment should respond; it also captures the stock market response to increased investor disagreement in a behavioral-based model of stock market bubbles. A full description of the role of behavioral finance mechanisms in stock pricing and investment decisions will have to distinguish between these two roles for Q – i.e. provide a clean separation between stock price movements that are attributable to fundamentals versus those that are attributable to bubbles. In what follows below, we consider additional VAR specifications that aid in this task.

The error structure of each equation in the VAR allows for a firm-specific and time-specific components. Let $\mathbf{f}_i = \left\{f_i^q, f_i^d, f_i^i\right\}'$ denote a vector of firm-specific component that varies across firms but is constant over time, and let $\mathbf{e}_t = \left\{e_t^q, e_t^d, e_t^i\right\}'$ denote a vector of time-specific (aggregate) shocks that are common to all firms but vary over time. Finally, denote the vector of idiosyncratic shocks by $\mathbf{v}_{it} = \left\{v_{it}^q, v_{it}^d, v_{it}^i\right\}'$. Letting $\mathbf{y}_{it} = \{q_{it}, d_{it}, i_{it} - k_{it}\}'$ we can express the VAR compactly as

$$(\mathbf{I} - \mathbf{A}(L))\mathbf{y}_{it} = \mathbf{f}_i + \mathbf{e}_t + \mathbf{v}_{it} \tag{21}$$

where $\mathbf{A}(L)$ is a vector lag operator. We assume $E\left(\mathbf{v}_{it}\mathbf{v}_{it}'\right) = \Sigma$ and $E\left(\mathbf{v}_{it}\mathbf{v}_{it-s}'\right) = 0,\ s \neq t$, so that, conditional on $\mathbf{f}_j$ and $\mathbf{e}_t$, lagged values of $\mathbf{y}_{it}$ are valid instruments for estimating $\mathbf{A}(L)$:

$$E\left(\mathbf{v}_{it}\mathbf{y}_{it-s}\right) = 0,\ s > 0. \tag{22}$$

To estimate the coefficients of $\mathbf{A}(L)$ we transform our structure to remove fixed time and firm effects. To remove time (fiscal year) effects, we first transform all variables to deviations from their fiscal-year-specific means. This effectively removes 12 time dummies per year, because firms have fiscal years ending in all 12 months. More fiscal years end in December than in any other month, but enough end on each of the other months of the year to permit a full set of time dummies.

Fixed firm effects are removed using forward-mean differencing (Arellano and Bover, 1995). This is a forward-filter which subtracts the forward mean and rescales to preserve homoskedasticity. This transformation removes firm fixed effects while preserving orthogonality between the lagged values in the VAR and the transformed residuals. Formally, assuming that the data $\mathbf{y}_{it}$ have already been time-differenced, the filter can be written

$$\tilde{\mathbf{y}}_{it} = c_t \left(\mathbf{y}_{it} - \frac{1}{T-t} \sum_{s=t+1}^{T} \mathbf{y}_{is} \right)$$

where $c_t^2 = (T-t)\,/\,(T-t+1)$. Equation 21 can therefore be expressed as[16]

$$(1 - \mathbf{A}(L))\tilde{\mathbf{y}}_{it} = \tilde{\mathbf{v}}_{it}. \tag{23}$$

We can estimate the coefficients of $\mathbf{A}(L)$ in equation (23) using standard GMM methods and including time dummies to control for $\tilde{\mathbf{e}}_t$, the aggregate component of the transformed error structure. Estimates of Σ are obtained directly from our empirical estimates of the covariance matrix of the transformed error $\tilde{\mathbf{v}}_{it}$.

[16] As explained above, this transformation has two desirable properties. First, it preserves instruments, so that

$$E(\widetilde{v}_{jt} y_{jt-s}) = 0, \quad s \geq 0,$$

and second, it does not induce heterokedasticity or serial correlation, so that

$$\begin{aligned} E(\widetilde{v}_{jt}\widetilde{v}_{jt}) &= \Sigma, \\ E(\widetilde{v}_{jt}\widetilde{v}_{jt-s}) &= 0, \quad s \neq 0. \end{aligned}$$

4 Empirical Results

4.1 Baseline Results

Table 1 reports the coefficient values of a two-lag VAR system for $\mathbf{y}_{it} = \{i_{it}, q_{it}, d_{it}\}'$. Table 1 also reports the t-statistics for the coefficients along with the R-squared of the transformed regression.[17] Consistent with behavioral finance theories that emphasize a positive link between disagreement among investors and stock prices, our baseline empirical estimates indicate that dispersion in analysts' earnings forecasts has a statistically significant, positive effect on Tobin's Q. This finding is also consistent with the existing empirical findings of Diether, Malloy and Scherbina (2001) and other researchers who document a negative relation between dispersion and future asset returns. In addition to a positive effect of dispersion on Q, we also find a statistically significant, positive effect of Tobin's Q on dispersion. This result suggests that an increase in stock prices predicts increased disagreement among analysts (and conversely). In contrast, lagged investment has no explanatory power for Tobin's Q and only a marginal effect on dispersion.

The last column of table 1 reports the regression of investment on lags of Tobin's Q, lags of dispersion in analysts' forecasts and lags of investment. The Tobin's Q coefficient for investment at the first lag is positive and highly statistically significant. Tobin's Q has a great deal of explanatory power for investment – a result that is to be expected from Q-theory.[18] The last column of table 1 also indicates that dispersion has statistically significant, positive effect on investment even when controlling for investment opportunities through Tobin's Q.

To gain some understanding of the quantitative implications of the parameter

[17] These equations are estimated using instrumental variables, so these R^2 statistics are not particularly informative.

[18] The empirical literature also finds a statistically significant role for Q in explaining investment, but often finds implausibly low parameter values. As discussed below, the elasticities of investment with respect to Q obtained here are much more plausible. We believe this reflects our logarithmic specification.

Table 1: VAR Coefficient Estimates: 3 Variable System

	q_t	d_t	$i_t - k_t$
q_{t-1}	0.636	0.118	0.595
	(40.81)	(5.717)	(28.873)
q_{t-2}	0.023	0.141	-0.116
	(1.98)	(8.517)	(6.580)
d_{t-1}	0.072	0.499	0.049
	(7.07)	(28.578)	(2.877)
d_{t-2}	0.041	0.105	0.051
	(7.29)	(10.486)	(5.552)
$i_{t-1} - k_{t-1}$	0.005	-0.002	0.440
	(0.546)	(0.155)	(31.134)
$i_{t-2} - k_{t-2}$	-0.000	-0.031	0.011
	(0.068)	(3.396)	(1.106)
Nobs	22945	22945	22945

estimates in Table 2, it is interesting to compute the implied long-run elasticities associated with an increase in dispersion on Tobin's Q and investment. The long-run elasticity of Tobin's Q with respect to dispersion may be interpreted as the elasticity with respect to a permanent increase in dispersion holding other variables fixed (for the full dynamic effect, we use the impulse response function, discussed below). The long-run elasticity of Tobin's Q, for example, is computed by dividing the coefficient on lagged dispersion, a_{qd}, by one minus the coefficient on its own lag, a_{qq}, or $\frac{a_{qd}}{1-a_{qq}}$, where a_{qd} and a_{qq} are estimated elements of the matrix $\mathbf{A}(L)$. Table 2 provides the long-run elasticity estimates implied by the VAR coefficients for all three variables.

These estimates imply that dispersion has a quantitatively significant effect on both Tobin's Q and investment – a permanent 1% increase in dispersion leads to a 0.33% increase in Tobin's Q and an 0.18% increase in investment. For comparison's sake, a 1% increase in Tobin's Q leads to an 0.87% increase in investment. In terms of long-run elasticities, investment has very little effect on either Tobin's Q or dispersion.

Table 2: Long-Run Elasticity Estimates: 3 Variable System

	Response		
Shock	q	d	$i-k$
q	–	0.268	0.872
d	0.331	–	0.182
$i-k$	0.001	-0.055	–

The lack of feedback from investment to the other two variables implies that we can approximate the total effect of a permanent shock to dispersion on investment by combining the direct effect of dispersion on investment (0.182) and the indirect effect working through Tobin's Q (0.872*0.331) to obtain a total long-run elasticity of 0.47.[19]

To characterize the full dynamic effect of dispersion on Tobin's Q and investment we now consider the impulse response functions implied by the VAR. This requires imposing more structure on the error vector v_{jt} to identify the underlying source of (independent) shocks. We assume that $\mathbf{v}_{it}$ is related to a set of mutually orthogonal fundamental shocks $\boldsymbol{\eta}_{it} = \left\{\eta^q_{it}, \eta^d_{it}, \eta^i_{it}\right\}'$ through the following recursive structure:

$$\begin{aligned} v^q_{it} &= \eta^q_{it} \\ v^i_{it} &= \rho_{iq}\eta^q_{it} + \eta^i_{it} \\ v^d_{it} &= \rho_{dq}\eta^q_{it} + \rho_{di}\eta^i_{jt} + \eta^d_{jt} \end{aligned} \tag{24}$$

The coefficients ρ_{iq}, ρ_{dq}, and ρ_{di} may be interpreted as projection coefficients obtained from the regression of v_{it} on to η_{it}. These projection coefficients are obtained by applying a Cholesky decomposition to the covariance matrix of the v_{it}'s.

Although it is not necessary from a statistical point of view, it is worthwhile considering the sources of uncertainty that lead to innovations in Tobin's Q, asset

[19] This estimate is based on the counterfactual exercise of considering an exogenous permanent increase on dispersion, holding Q fixed. The full dynamic effect implied by the impulse response function is discussed below.

prices and investment. Because Tobin's Q reflects both fundamentals and bubbles, orthogonal innovations to the Q equation (η_{it}^{q}) represent changes in expected profits or changes in future discount rates, as well as changes in the bubble component of stock prices. Because stock prices are forward looking, this increase may or may not be reflected in current profits. Given the possibility of significant planning horizons for investment, such a shock may or may not be reflected in current investment. Hence, our preferred ordering of the VAR allows for this possibility. In the adjustment cost framework, orthogonal investment shocks capture exogenous chances in the marginal cost of investing and changes in the price of new capital goods that are not reflected in Tobin's Q. Orthogonal dispersion shock (η_{it}^{d}) reflect changes in the dispersion of analysts' earnings forecasts that are not associated with either changes in fundamentals or changes in current stock prices, but which may cause changes in next period's bubble component of Q (plus any direct effects on investment behavior). Since dispersion is measured as an average over the year, and Tobin's Q represents end-of-year stock prices, it may be more natural to order dispersion before Tobin's Q rather than afterwards. By constructing shocks to dispersion that are orthogonal to shocks to Q, we stack the deck against finding much evidence in favor of an independent mechanism working through dispersion. Our results are robust to alternative orderings, so we restrict our discussion to the above ordering.

Figure 1 plots the impulse response of Tobin's Q, investment and dispersion to orthogonal dispersion shocks with a magnitude of one-standard deviation. The first row shows the effect of a shock to η_{it}^{q} on Tobin's Q, investment and dispersion in that order. A one standard deviation increase in η_{it}^{q} implies a 0.28 percent increase in Tobin's Q. The effect dissipates over time and Tobin's Q returns to baseline after ten years. The rise in η_{it}^{q} leads to an immediate 0.08 percent increase in investment. The investment response peaks one year later at 0.2 percent – a value that is roughly in line with the Tobin's Q response at that horizon. Following year 2, investment tracks Q along the return to baseline values. In response to predictable movements in Q, the quadratic adjustment cost model of investment implies that investment and Tobin's Q should co-move with unit elasticity. In line with our long-run estimates documented in Table 3, the VAR impulse response functions imply that in percent terms, predictable movements in investment are only slightly less than the predictable movements in Tobin's Q. Thus, except for the first period response, movements in Q and investment are broadly consistent with an adjustment cost model. The immediate reaction of Tobin's Q in the first period combined with the delayed response of investment could be easily justified within an adjustment cost framework that is augmented to include lags in the investment planning horizon.

Dispersion of analysts' forecasts also increases in response to a positive shock to η_{it}^{q}. Again, we see a delay in the peak response – this time at the two-year horizon. We do not offer a structural interpretation of this result, except to observe that in the data, rising stock prices (Granger) cause an increase in disagreement among analysts.

The shock to investment, η_{it}^{i} has very little effect on Tobin's Q, a small transitory effect on dispersion and a sustained effect on investment itself. Although we do not have a strict structural interpretation, the response to such shocks appears to be consistent with our interpretation that these shocks represent changes in the marginal cost of adjusting investment or the price of capital goods that are not well captured by Tobin's Q.

We now consider a one standard deviation shock to the dispersion of analysts' forecasts, η_{it}^{d}. Dispersion rises 0.45 percent in response to this shock and returns to baseline fairly quickly thereafter – fifty percent of the rise in dispersion has disappeared after one year. By construction, this shock is orthogonal to both Tobin's Q and investment, and, therefore, has zero effect on either variable upon impact. Tobin's Q rises by nearly 0.06 percent over a two year period before returning to baseline. Under our bubble interpretation, this implies that the stock market is slow to incorporate such disagreement into the stock price. Or put differently, dispersion is a leading indicator of the bubble. It is also consistent with the notion that bubbles tend to be persistent and are difficult to arbitrage. The slow rise in Q in response to an increase in dispersion is not an artifact of the ordering. If we order dispersion first, so that Q and investment can respond immediately to dispersion shocks, we obtain a 0.01 percent rise in Q and a 0.03 percent rise in investment upon impact. Both variables exhibit strong hump-shaped dynamics with the peak effects on the order of 0.06-0.08 percent occurring at the two- to three-year horizon.

The shock to dispersion also causes a slow, sustained rise in investment. The peak response of investment is 0.08 percent and occurs at the two-to-three year horizon. Looking at the relation between Q and investment along the dynamic path, we again see a strong co-movement between the two variables that is consistent with an adjustment cost model with close to unit elasticity between Q and investment. Most of the investment dynamics are transmitted through movements in Tobin's Q rather than the direct effect of dispersion on investment. If we interpret orthogonal shocks to dispersion as proxies for exogenous changes in disagreement among investors (which causes prices to depart from fundamentals), these results suggest that a stock price bubbles are transmitted to investment via Tobin's Q.

Variance decompositions for this three variable VAR are reported in table 3. The variance decomposition reports the percent of the explainable variation in each variable that can be attributed to each of the fundamental shocks η_{it}^{q}, η_{it}^{d}, and η_{it}^{i} at various horizons. We consider the one-year, five-year and ten-year horizon. The variance decomposition computed at the ten-year horizon is, for all practical purposes, equivalent to the unconditional variance decomposition. The variance decomposition is conditional on the ordering. We report the decomposition for the ordering discussed above, but the results are insensitive to the ordering.

According to the variance decomposition reported in Table 3, over 97 percent of the variation in both dispersion and Tobin's Q are explained by their own shocks η_{it}^{q} and η_{it}^{d}. This is true at all horizons. The variance of investment is explained by a combination of η_{it}^{q}, the shocks to Q, and η_{it}^{i}, the shocks to investment itself. At the one

Table 3: Variance Decomposition: 3 Variable System

	Explained Variance (fraction of total)		
Shock	q	d	$i-k$
	1-Year Horizon		
q	0.990	0.006	0.180
d	0.010	0.990	0.000
$i-k$	0.000	0.004	0.820
	5-Year Horizon		
q	0.976	0.016	0.369
d	0.023	0.980	0.005
$i-k$	0.001	0.004	0.626
	10-Year Horizon		
q	0.972	0.020	0.435
d	0.027	0.976	0.006
$i-k$	0.001	0.004	0.559

year horizon, 82 percent of the variance of investment is explained by the investment shock while 18% is explained by the shock to Tobin's Q. Over time, the share of investment variation attributed to Q shocks increases. At the ten year horizon, the share is 44 percent. Shocks to dispersion account for virtually none of the variation in investment at any horizon. Beyond the one year horizon, they explain approximately 2 percent of the variance in Tobin's Q. If stock market bubbles were driven only by dispersion in analysts' forecasts, these findings would imply that bubbles are a quantitatively insignificant phenomena for either Q or investment. Placing dispersion higher in the ordering increases the fraction of investment variance it can explain, but only to about one percent.

An alternative view is that dispersion in analysts' forecasts helps identify bubble-like behavior but that most of the variation in Q that is due to bubbles is not well explained by the dispersion in analysts' forecasts.[20] The VAR leaves 30 percent of

[20]In the VAR money literature, Sims (200?) argues for a similar instrumental-variables interpretation of identified monetary policy shocks to rationalize why alternative identifying schemes produce monetary policy innovations that have very similar implications for output and inflation, despite

the variation in (log) Tobin's Q unexplained at the 10-year horizon. This number represents an upper bound on the investment variation explained by bubbles.

4.2 The VAR Augmented with Fundamentals

We now consider an expanded version of the VAR that includes a measure of investment opportunities that does not rely on the stock market. Our goal is to control for fundamentals as best we can, before examining the effect of dispersion on Tobin's Q and investment. To the extent that Tobin's Q does not adequately measure expected changes in the future *marginal* profitability of capital (MPK), movements in dispersion of analysts' forecasts that are orthogonal to Q may still contain some news about investment opportunities. This is especially likely when new investment projects have higher fundamental uncertainty than the capital stock as a whole. In this case, we might well expect to see an increase in the degree of disagreement among analysts regarding how such investment opportunities should be incorporated into earnings forecasts. By considering shocks to dispersion that are orthogonal to our fundamentals based-measure of investment opportunities, we guard against this possibility. Adding MPK to the VAR also allows us to consider its endogenous response to various shocks, and lets us decompose the variance of Tobin's Q into shocks that can be easily traced to MPK vs. those that cannot, and therefore provides an upper bound on the fraction of the variance of Q that is potentially due to stock price bubbles.

Various assumptions regarding production and market structure justify alternative measures of MPK. Investment in a standard adjustment cost model is a function of the present discounted value of future MPK. Under quite general assumptions about technology and demand (Cobb-Douglas production and isoelastic demand), the log

the fact that the various measures of policy innovations have virtually uncorrelated with each other and a low correlation with Fed-funds futures market-based measures of the innovation in the Federal Funds rate.

Table 4: VAR Coefficient Estimates : 4 Variable System

	mpk_t	q_t	d_t	$i_t - k_t$
mpk_{t-1}	0.870	0.175	0.360	0.289
	(34.601)	(7.441)	(10.303)	(7.666)
mpk_{t-2}	-0.065	-0.170	-0.210	-0.239
	(3.661)	(9.491)	(8.161)	(9.115)
q_{t-1}	0.196	0.625	0.085	0.579
	(17.080)	(39.133)	(4.048)	(27.356)
q_{t-2}	-0.088	0.024	0.145	-0.112
	(8.781)	(2.096)	(8.836)	(6.399)
d_{t-1}	0.037	0.078	0.512	0.054
	(4.111)	(7.832)	(29.549)	(3.197)
d_{t-2}	0.015	0.042	0.105	0.048
	(3.354)	(7.459)	10.765	5.377
$i_{t-1} - k_{t-1}$	-0.190	-0.033	-0.103	0.371
	(19.124)	(3.152)	(6.128)	(20.867)
$i_{t-2} - k_{t-2}$	0.048	0.062	0.052	0.097
	(5.836)	(7.187)	(3.972)	(6.879)
Nobs	23334	23334	23334	23334

of MPK is linear in the log of the sales to capital ratio. Accordingly, we augment the VAR to include mpk, which is constructed as the log of the sales to capital ratio. Table 4 reports the coefficient estimates for this augmented VAR, while Tables 5 reports the implied long run elasticities. Figure 2 and Table 6 provide the impulse response functions and variance decompositions, using the ordering $mpk, q, i - k, d$.

Adding MPK has very little effect on either the coefficient values or the predictive value of dispersion for Tobin's Q and investment. Dispersion in analysts' forecasts adds a significant degree of explanatory power to both of these variables even with MPK in the regression, suggesting that dispersion forecasts Q and investment for

Table 5: Long-run Elasticities: 4 Variable System

	Response			
Shock	mpk	q	d	$i-k$
mpk	–	0.014	0.392	0.098
q	0.554	–	0.601	0.912
d	0.267	0.342	–	0.199
$i-k$	-0.728	0.083	-0.133	–

reasons other than omitted variable bias due to incorrectly measured MPK. Dispersion also has significant explanatory power for MPK itself, perhaps because dispersion is correlated with fundamental uncertainty, which in turn leads to higher levels of expected MPK in equilibrium. It follows that properly controlling for fundamentals is important when assessing the role of disagreement for investment behavior.

As table 5 indicates, adding our measure of fundamentals to the VAR leaves the long-run elasticities documented in table 2 essentially unchanged. Dispersion continues to have an economically important effect on Tobin's Q, and an economically important effect on investment, both directly and indirectly through the Q equation. Tobin's Q and MPK both have a positive effect on dispersion. Interestingly, the long-run elasticity of investment with respect to MPK is fairly low, this finding reflects the fact that lags 1 and 2 of MPK enter the investment equation with equal and opposite signs. Thus, Tobin's Q does a good job of capturing the long-run link between investment and profit opportunities while MPK appears to do relatively better at capturing some of the short-run dynamics.

Figure 2 displays the impulse response functions for the four variable VAR provide. We order the variables $\{mpk, q,\ i-k, d\}$. The impulse response functions for Tobin's Q and investment in response to an orthogonalize dispersion shock are essentially unchanged when MPK is added to the VAR. There is again a delayed pricing response through Tobin's Q, accompanied by strong co-movement between Tobin's Q and investment along the dynamic path. Adding fundamentals to the VAR changes the relation between investment and Q only in response to a shock to fundamentals itself. The MPK shock, η_{it}^{mpk}, when ordered first, leads to a 0.2 percent increase in MPK and a 0.2 percent increase in investment after one period. The Q response at 0.04 is substantially lower than the investment response in this situation. Thus, a positive shock to MPK elicits a sharp rise in MPK, a modest increase in Q and, after some delay, a sharp rise in investment. A positive shock to Tobin's Q (orthogonal to MPK) elicits a sharp rise in Tobin's Q, a modest rise in MPK, and a delayed response of investment. What is interesting here is that the latter response is very much like the response of Tobin's Q and investment to an increase in disagreement as measured by the dispersion in analysts' forecasts.

In summary, fundamental shocks and non-fundamental shocks elicit dynamic responses from investment and Tobin's Q that are quite different. Investment moves four times more than Tobin's' Q in response to fundamental shocks than non-fundamental shocks.[21] This finding suggests the following interpretation: managers rationally respond to fundamental shocks by increasing investment to maximize value. In response to non-fundamental shocks, they partially respond by increasing investment, and partially respond in some other manner, either by ignoring or effectively discounting some of the signal provided by the stock market in response to non-fundamental shocks, or by issuing shares but not channelling funds into new investment opportunities. At this point, the argument is speculative. In the next section, we consider a VAR that includes equity issuance to provide additional insight into this argument. Before doing so, we consider the implications of adding fundamentals to the variance decomposition.

Table 6 provides the variance decomposition of fundamentals, Tobin's Q, invest-

[21] This finding is consistent with previous empirical results documented by Gilchrist and Himmelberg (1995) who argue that the relationship between investment and Tobin's Q is much stronger when instrumented using only fundamentals based information such as lags of sales to capital ratios and earnings to capital ratios. Similar evidence is reported by Bond and Cummins (2000), who instrument with the mean of analysts earnings, and by Erickson and Whited (2001), who derive clever instruments from higher moment conditions.

Table 6: Variance Decomposition: 4 Variable System

	Explained Variance (fraction of total)			
	mpk	q	$i-k$	d
Shock	1-Year Horizon			
mpk	0.908	0.043	0.273	0.017
q	0.026	0.944	0.125	0.002
$i-k$	0.062	0.001	0.600	0.005
d	0.004	0.011	0.003	0.976
	5-Year Horizon			
mpk	0.824	0.071	0.311	0.042
q	0.050	0.900	0.272	0.004
d	0.119	0.003	0.410	0.011
$i-k$	0.008	0.026	0.006	0.942
	10-Year Horizon			
mpk	0.801	0.081	0.325	0.053
q	0.056	0.884	0.323	0.005
d	0.137	0.004	0.345	0.014
$i-k$	0.009	0.031	0.007	0.929

ment and dispersion using that ordering. Analogous to table 3, we consider the 1, 5, and 10 year horizons. At the one year horizon, 90 percent or more of the variation in fundamentals, Tobin's Q and dispersion is caused by own shocks. Although these numbers fall as we extend the horizon, it's still the case that 80 percent or more of the variation can be ascribed to each variable's own shock. Dispersion continues to explain only a small fraction of the other variables in the system, including investment. It is worth noting however that shocks to the dispersion of analysts' forecasts does explain 2.5 to 3 percent of the variance in Tobin's Q at the five and ten year horizon, while fundamental shocks explain 7 to 8 percent of the variance in Tobin's Q. We obtain this finding despite having ordered fundamentals first and dispersion last.

The last column of table 6 provides the variance decomposition of investment. Fundamental shocks explain roughly 30 percent of the variance of investment at all horizons. Shocks to Tobin's Q that are orthogonal to fundamentals explain an increasing fraction of the variance at longer horizons. At the one year horizon η_{it}^q explains 12 percent of investment variance while at the ten year horizon, η_{it}^q explains 30 percent of the variance. Shocks to investment that are orthogonal to fundamentals and Q explain the rest – 60 percent at the one year horizon and 34 percent at the 10 year horizon, while shocks to dispersion explain at most 1 percent of the variation in investment at any horizon. In summary, although shocks to dispersion explain only a small fraction of investment, they account for a larger share of the variation in Tobin's Q. More importantly, somewhere between 12 and 30 percent of the variance in investment is explained by shocks to Tobin's Q that are orthogonal to shocks to MPK. We take these numbers as an upper bound on how much of the variation in investment one can plausibly ascribe to behavioral mechanisms working through stock prices.

4.3 Bubbles, Dispersion and Equity Issuance

We now consider a VAR that is augmented to include new equity issuance. We include in the VAR two lags of our measure of fundamentals, Tobin's Q, investment, equity issuance and dispersion. To compute impulse response functions and variance decompositions, we consider an orthogonalization of shocks to the variables in that order. The impulse response functions are plotted in figure 3 and the variance decompositions are provided in Table 7.

Adding equity issuance to the VAR leaves the impulse response functions of the other variables to their own shocks essentially unchanged. We therefore focus on the response of equity issuance to the various shocks in the system, and the response of all variables to the shock to equity issuance. Shocks to MPK, Tobin's Q and investment all lead to an increase in equity issuance. The size of the increase depends on the shock. One way to gauge the relative magnitude of equity issuance is to compare it to the response of investment for any given shock. We expect that as investment

Table 7: Variance Decomposition: 5 Variable System

	Explained Variance (fraction of total)				
	mpk	q	$i-k$	n	d
Shock	1-Year Horizon				
mpk	0.909	0.043	0.277	0.004	0.016
q	0.026	0.937	0.120	0.048	0.000
$i-k$	0.063	0.002	0.602	0.001	0.006
n	0.000	0.007	0.000	0.940	0.002
d	0.002	0.011	0.001	0.007	0.976
	5-Year Horizon				
mpk	0.828	0.075	0.326	0.017	0.042
q	0.049	0.880	0.261	0.063	0.001
$i-k$	0.119	0.004	0.411	0.005	0.016
n	0.000	0.017	0.000	0.884	0.003
d	0.004	0.024	0.001	0.031	0.939
	10-Year Horizon				
mpk	0.807	0.086	0.342	0.030	0.053
q	0.056	0.861	0.310	0.079	0.001
$i-k$	0.133	0.005	0.345	0.097	0.020
n	0.000	0.020	0.001	0.825	0.003
d	0.004	0.029	0.002	0.056	0.923

opportunities increase, firms will resort to equity issuance to finance new investment. In response to an increase in MPK, the relative amount financed appears to be fairly small – on the order of 1/40 of the investment response.[22] In response to shocks to Tobin's Q that are orthogonal to fundamentals, this ratio is much larger – on the order of 1/8 of the investment response. Shocks to investment produce virtually no response of equity issuance while shocks to dispersion again produce a number on the order of 1/8. Consistent with our interpretation of orthogonalized shocks to Tobin's Q and dispersion as representing a response to bubbles in the stock market, these results suggest that the amount of equity issuance relative to the amount of new investment that needs to be financed is an order of magnitude higher in response to shocks that can be linked to stock market bubbles relative to other shocks that are more closely tied to fundamentals. This evidences suggests that firms systematically take advantage of mispricing by issuance relatively large amounts of equity when stock prices are high relative to fundamentals.

In addition to studying the quantity of equity issued, it is also worth discussing the timing. In response to shocks to both Tobin's Q and dispersion, new equity issuance tracks Tobin's Q. In particular, an orthogonalized shock to dispersion produces a delayed hump-shaped response of Tobin's Q that peaks several years out. Equity issuance exhibits a similar hump-shaped response to this shock, again peaking several years out. These findings support the idea that firms are actively timing the market. Rather than dump stock on the market immediately, managers wait for the increased dispersion to be reflected in stock prices before issuing new equity. A shock to MPK generates a similarly persistent, hump-shaped response from Tobin's Q, but in sharp contrast to the effect of Q or dispersion shocks, the response of equity issuance to an

[22] These calculations are based on comparing the peak investment response to the peak response of equity issuance. For example, in response to a shock to fundamental Q, investment rise 0.2 percent while equity issueace rise by 0.005 percent. The ratio is 1/40. These calculations are admittedly rough, though hopefully informative. In future versions, we plan to translate the responses into more interpretable values (e.g. dollar amount issued and invested relative to the market value).

MPK shock is immediate and transitory.[23]

The shock to equity issuance also produces dynamic responses that are consistent with the notion that stock market bubbles drive corporate policy. Innovations to equity issuance that are orthogonal to MPK, Tobin's Q, and investment are essentially transitory. These shocks have virtually no predictive power for MPK or investment. By contrast, shocks to equity issuance produce a sharp drop in Tobin's Q (and also reduces future dispersion). This result is consistent with the view that equity issuance is a good measure of mispricing in the stock market. Equity issuance tells the market that the stock is overvalued, and this reduces valuation ratios (e.g., Tobin's Q). That such a clear public signal of overvaluation would reduce the future dispersion of opinion among investors is also consistent with the theory.

Table 7 reports the variance decompositions for the VAR that includes equity issuance. The explanatory power of orthogonal shocks to equity issuance is similar to the explanatory power of shocks to dispersion – neither shock can explain a significant percentage of the variation in fundamentals, measured by mpk, or investment. Both shocks explain a small but nontrivial fraction of the variation in Tobin's Q. At the ten year horizon, the combined explanatory power of these two shocks for Tobin's Q is on the order of 5 percent. In comparison, the fundamentals shock explains 8.6 percent of the variance in Tobin's Q at this horizon. As with dispersion, most of the variation in equity issuance is explained by its own shock – 82 percent at the 10 year horizon. The fact that equity issuance is largely driven by factors unrelated to either fundamentals, investment or Tobin's Q suggest that it is a good candidate to consider as a direct measure of stock market mispricing owing to bubbles.

In summary, adding equity issuance to the VAR corroborates the previous results. The magnitude of the equity response relative to the investment response appears to

[23] The investment shock, which, if interpreted as as variation in the adjustment cost technology or price of capital goods, may also be viewed as a shock to fundamentals, produces even less comovement between equity issuance and Tobin's Q: orthogonalizes shocks to investment cause equity issuance to rise but cause Tobin's Q to fall.

be much larger for non-fundamental shocks than fundamental shocks, the degree of co-movement between equity issuance and Tobin's Q is higher in response to non-fundamental shocks, and Tobin's Q falls following new equity issuance even though there is no evidence of a decline in fundamentals. These findings are broadly consistent with the notion that mispricing in the stock market produces corporate investment and financing responses that are consistent with managers systematically timing the market to issue new equity, and channeling a non-trivial component of the proceeds into physical investment.

5 Conclusion

We have developed a model which predicts managers respond to bubbles by issuing new equity and increasing capital expenditures. We test these predictions (among others) using the variance of analysts' earnings forecasts – a proxy for the dispersion of investor beliefs – to identify the "bubble" component in Tobin's Q. We document the dynamic response to bubble shocks using a panel data VAR. Using recursive orderings of the VAR for additional identification, we find that orthogonalized bubble shocks have positive and statistically significant effects on Tobin's Q, net equity issuance, and real investment, consistent with the predictions of the model. The percentage of the time-series variation (or "within" variation) in Tobin's Q, real investment and equity issuance that can be explained by shocks to our dispersion measure is relatively small. But dispersion is an imperfect proxy for bubbles, so these estimates should be viewed as lower bounds on the variation in Tobin's Q attributable to a bubble component.

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Figure 1
Impulse response functions for the ordering $\{q, i-k, d\}$
(i.e., $\{LTQ1, LCXK, LDSPRA\}$)

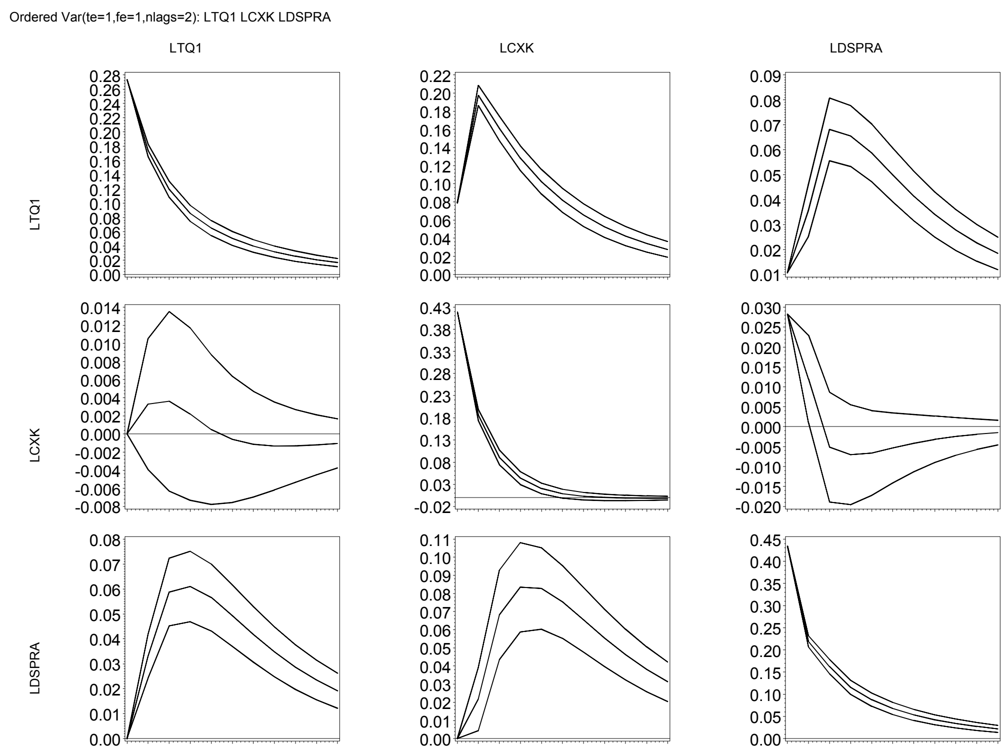


Figure 2
Impulse response functions for the ordering $\{mpk, q, i-k, d\}$
(i.e., $\{LMPK, LTQ1, LCXK, LDSPRA\}$)

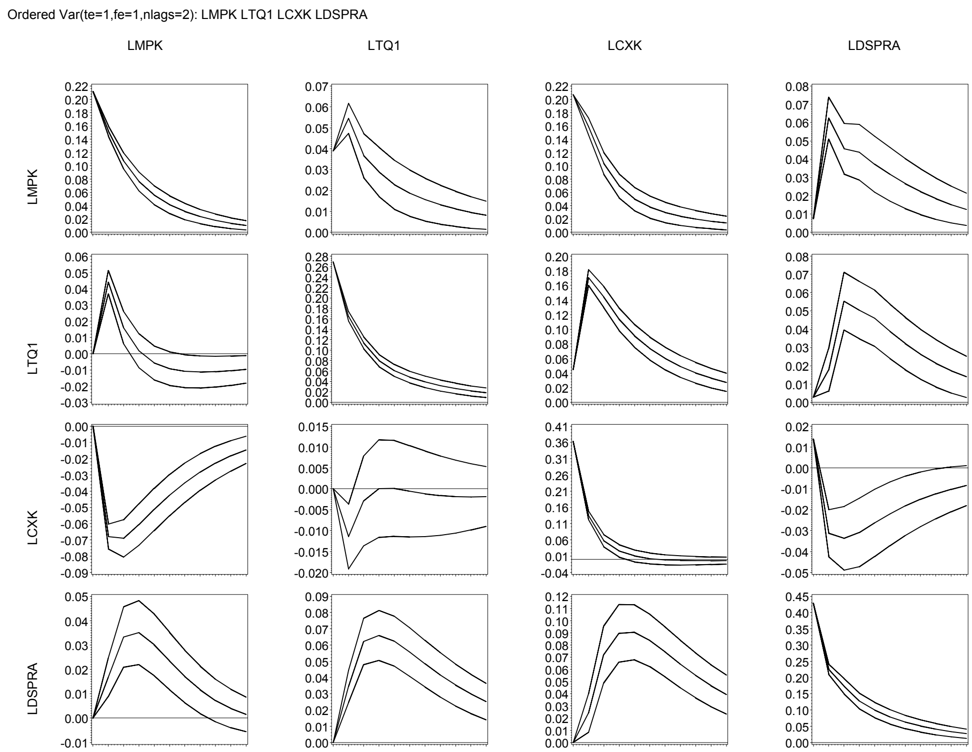
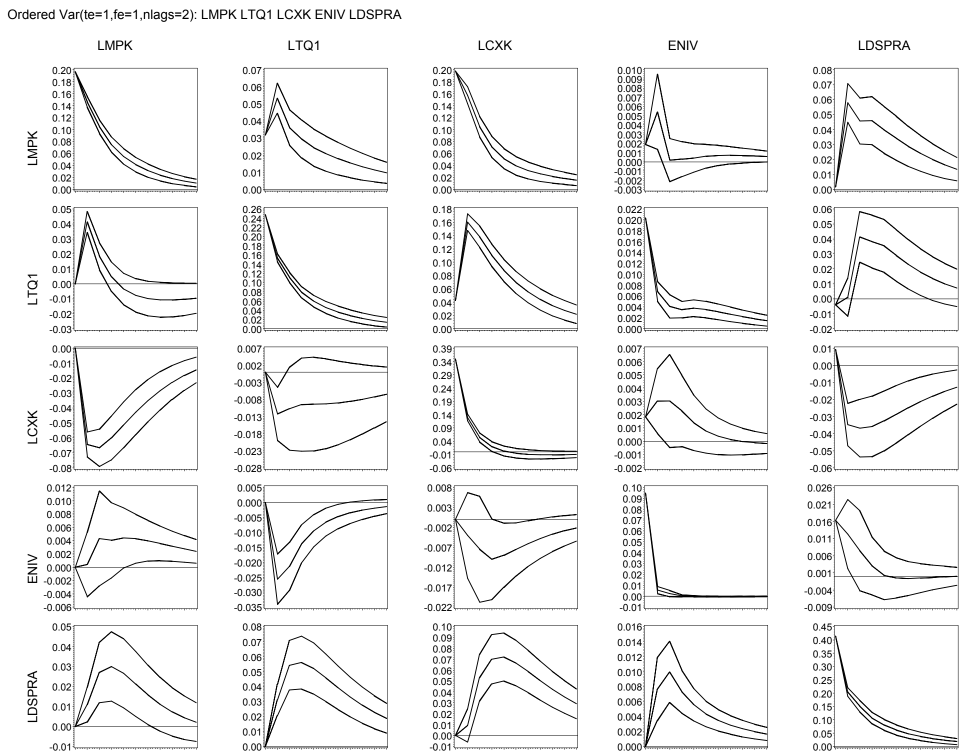


Figure 3
Impulse response functions for the ordering $\{mpk, q, i-k, n, d\}$
(i.e., $\{LMPK, LTQ1, LCXK, ENIV, LDSPRA\}$)



7 Appendix

Proof of Proposition 1: We first use equation 4 combined with equation 3 to establish the limiting behavior of the demand curve. We then compute and sign the derivatives B_n and B_σ. Since $\lim_{B\to 0} b = -\infty$ and $\lim_{B\to\infty} b = \infty$

$$\lim_{B\to 0} \gamma \left[(1-\Phi(b-\sigma)) B^{-1} - (1-\Phi(b)) \right] = \lim_{B\to 0} \gamma B^{-1} = \infty \tag{25}$$

To compute the upper limit, we note that $\frac{\partial b}{\partial B} = \frac{1}{\sigma B}$ and apply l'Hopital's rule to obtain

$$\lim_{B\to\infty} \gamma \left[(1-\Phi(b-\sigma)) B^{-1} - (1-\Phi(b)) \right] = \lim_{B\to\infty} -\gamma \frac{\phi(b-\sigma)}{\sigma B} = 0. \tag{26}$$

We now compute the partial derivative of n with respect to B, recognizing that b is a function of B:

$$\frac{\partial n}{\partial B} = \gamma \left[-\left[\phi(b-\sigma)B^{-1} - \phi(b) \right] \frac{\partial b}{\partial B} - \left[1-\Phi(b-\sigma) \right] \frac{1}{B^2} \right].$$

Equation **??** implies that the first term in this expression is zero, so that

$$\frac{\partial n}{\partial B} = -\gamma \left[1-\Phi(b-\sigma) \right] B^{-2} < 0. \tag{27}$$

The limiting conditions in equations **??** and **??** and the derivative in equation **??** establish that the market demand curve in equation 4 is invertible with

$$B_n = \frac{-B^2}{\gamma \left[1-\Phi(b-\sigma) \right]} < 0.$$

To compute $B_\sigma = \frac{\partial B}{\partial \sigma}$, we totally differentiate equation (4), holding n fixed:

$$\begin{aligned} 0 &= \gamma \left[-\left(\phi(b-\sigma)B^{-1} - \phi(b) \right) \frac{\partial b}{\partial \sigma} + \phi(b-\sigma))\frac{1}{B} \right] \partial\sigma + \frac{\partial n}{\partial B}\partial B \\ &= \gamma \frac{\phi(b-\sigma)}{B}\partial\sigma + \frac{\partial n}{\partial B}\partial B. \end{aligned}$$

Solving for $\frac{\partial B}{\partial \sigma}$ we obtain:

$$B_\sigma = -\gamma\phi(b)B_n.$$

Substituting in our expression for B_n, we obtain

$$B_\sigma = Bh(b-\sigma) > 0.$$

Proof of Proposition 2: We first prove that a unique solution to equation 12 exists. Using equation 3 we can express equation 12 as an equation in b:

$$\exp(\sigma b - 0.5\sigma^2) = \frac{h(b)}{h(b-\sigma)}.$$

The left-hand-side of this equation is strictly positive and monotonically increasing in b. The hazard rate for the standard normal $h(b)$ is monotonically increasing so that $\frac{h(b)}{h(b-\sigma)} > 1$ and

$$\begin{aligned}
\lim_{b\to-\infty} \frac{h(b)}{h(b-\sigma)} &= \infty \\
\lim_{b\to-\infty} \frac{h(b)}{h(b-\sigma)} &= 0
\end{aligned}$$

The derivative of the right-hand-side of equation 12 satisfies

$$\begin{aligned}
\frac{\partial}{\partial b}\left(\frac{h(b)}{h(b-\sigma)}\right) &= \frac{h(b)}{h(b-\sigma)}\left(\frac{h'(b)}{h(b)} - \frac{h'(b-\sigma)}{h(b-\sigma)}\right) \\
&= \frac{h(b)}{h(b-\sigma)}\left(h\left(b\right) - h\left(b-\sigma\right) - \sigma\right) < 0,
\end{aligned}$$

where the inequality follows from the log-concavity of $h\left(x\right)$ (see the appendix of Gilchrist and Williams, 2001). These results are sufficient to guarantee a unique equilibrium value of b and hence B for equation 12. The uniqueness of the equilibrium value $n(\sigma,\gamma)$ for equity issuance, equation 13, follows directly from these results.

Proof of Proposition 3: Before analyzing the equilibrium response of B and n to an increase in dispersion, we first establish that equilibrium occurs at $b > \sigma$. To do so we note that at $b = \sigma$ we have

$$\begin{aligned}
\frac{h(b)}{h(b-\sigma)} - B &= \exp(-0.5\sigma^2)\left[\frac{\exp(0.5\sigma^2)}{2(1-\Phi(\sigma))} - 1\right] \\
&> 0 \;\; for \;\; \sigma > 0.
\end{aligned}$$

To obtain the inequality, we note that at $\sigma = 0$, the term in brackets on the right-hand-side of this expression is identically zero. It is also strictly increasing in σ and therefore positive for $\sigma > 0$. Since $h(b)/h(b-\sigma)$ is decreasing in b and $B = \exp(\frac{b-0.5\sigma^2}{\sigma})$ is increasing in b, the equilibrium must occur at $b > \sigma$.

We now wish to totally differentiate equation (12) to obtain $\frac{dB}{d\sigma}$. For short-hand notation, let $g(B,\sigma) = h(b)/h(b-\sigma)$ so that equilibrium implies $B = g$. It is straightforward to show that

$$\frac{dB}{d\sigma} = \frac{\left[g_b \frac{\partial b}{\partial \sigma} + g_\sigma\right]}{\left[1 - g_b \frac{\partial b}{\partial B}\right]}.$$

where

$$\begin{aligned} g_b &= g\left[h(b) - h(b-\sigma) - \sigma\right] < 0 \\ g_\sigma &= g[h(b-\sigma) - (b-\sigma)] > 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial b}{\partial \sigma} &= \frac{-(b-\sigma)}{\sigma} \\ \frac{\partial b}{\partial B} &= \frac{1}{B\sigma}. \end{aligned}$$

Inserting these expressions and simplifying, we obtain

$$\begin{aligned} \frac{dB}{d\sigma} &= \frac{\left[\frac{-(b-\sigma)}{\sigma} g\left[h(b) - h(b-\sigma) - \sigma\right] + g[h(b-\sigma) - (b-\sigma)]\right]}{\left[1 - g\left[h(b) - h(b-\sigma) - \sigma\right]\frac{1}{B\sigma}\right]} \\ &= \frac{B\left[-(b-\sigma)\left[h(b) - b - (h(b-\sigma) - (b-\sigma)\right] + \sigma[h(b-\sigma) - (b-\sigma)]\right]}{\left[\sigma - \left[h(b) - h(b-\sigma) - \sigma\right]\right]} \end{aligned}$$

Again, log-concavity of the hazard rate implies that

$$h(b-\sigma) - (b-\sigma) - (h(b) - b) > 0$$

Simplifying we have:

$$\frac{dB}{d\sigma} = \frac{B\left(b\left[h(b-\sigma) + \sigma - h(b)\right)\right] + \sigma\left[h(b) - b\right])}{(\sigma + \left[h(b-\sigma) + \sigma - h(b)\right])} > 0. \tag{28}$$

All the terms in square brackets in equation 28 are positive which establishes the inequality.

Equation 4 and equation 12 provide an alternative expression for equity issuance in equilibrium:

$$n = \gamma\left[\left(1 - \Phi(b-\sigma)\right)B^{-1} - \left(1 - \Phi(b)\right)\right]. \tag{29}$$

Totally differentiating equation 29 with respect to σ we obtain

$$\frac{dn}{d\sigma} = -\gamma\left[\phi(b-\sigma)B^{-1} - \phi(b)\right]\frac{db}{d\sigma} + \gamma\left(1 - \Phi(b-\sigma)\right)B^{-2}\frac{dB}{d\sigma} + \frac{\gamma\phi(b-\sigma)}{B}$$

The term in brackets is identically zero. Rearranging:

$$\frac{dn}{d\sigma} = \gamma \frac{[(1 - \Phi(b - \sigma))]}{B} \left(h(b - \sigma) - \frac{dB/B}{d\sigma} \right). \tag{30}$$

To sign this we note that $\left[1 - g_b \frac{\partial b}{\partial B}\right] > 1$ so that

$$\begin{aligned} \frac{dB/B}{d\sigma} &= \frac{\left[\frac{g_b}{g}\frac{\partial b}{\partial \sigma} + \frac{g_\sigma}{g}\right]}{\left[1 - g_b \frac{\partial b}{\partial B}\right]} \\ &< \frac{g_b}{g}\frac{\partial b}{\partial \sigma} + \frac{g_\sigma}{g} \end{aligned}$$

where

$$\begin{aligned} \frac{g_b}{g}\frac{\partial b}{\partial \sigma} + \frac{g_\sigma}{g} &= [h(b - \sigma) - h(b) + \sigma]\left(\frac{b - \sigma}{\sigma}\right) + [h(b - \sigma) - (b - \sigma)] \\ &= h(b - \sigma) - [h(b) - h(b - \sigma)]\left(\frac{b - \sigma}{\sigma}\right) \end{aligned}$$

As established above, equilibrium requires $b > \sigma$. The term in brackets is positive implying,

$$\frac{g_b}{g}\frac{\partial b}{\partial \sigma} + \frac{g_\sigma}{g} < h(b - \sigma)$$

therefore

$$\frac{dB/B}{d\sigma} < h(b - \sigma)$$

and

$$\frac{dn}{d\sigma} > 0.$$

Hence, increases in dispersion raise the equilibrium bubble and cause an increase in equity issuance.

To summarize results, we have established that there are unique equilibrium values $B = B(\sigma), n = n(\sigma; \gamma)$ which satisfy

$$B = \frac{h(b)}{h(b - \sigma)} > 1$$

with

$$\frac{dB}{d\sigma} > 0$$

and

$$n = \gamma(1 - \Phi(b))\,(B - 1) > 0$$

with

$$\frac{dn}{d\sigma} > 0$$

Hence, in equilibrium, both the bubble and issuance are positive, as long as $\sigma > 0$. Furthermore, the above proof also establishes that

$$\frac{d\ln(B)}{d\sigma} < h(b-\sigma),$$

owing to equity issuance, the equilibrium semi-elasticity of B to an increase in σ is strictly less than the semi-demand elasticity $\eta_\sigma = h(b-\sigma)$. Finally, it is straightforward to see that as $\sigma \to 0$, and heterogeneity of beliefs dissapears, we have $n \to 0$, and $B \to 1$ in which case assets are correctly priced.

Proof of Proposition 4: Substituting for the definition of V_k, we can write the first-order condition for capital in equation (14) as

$$\frac{1+r}{1+n(B-1)} = \frac{\Pi_k + 1 - \delta}{1+\psi K}. \tag{31}$$

Assuming that the marginal profit of capital (Π_k) weakly decreasing in K, it follows immediately that the right side of equation (31) is monotonically decreasing in K. By Proposition 3, an increase in dispersion causes equilibrium values of B and n to increase, so the left side of equation (31) is decreasing in dispersion, so the equilibrium choice of K must be increasing in dispersion.

Proof of Proposition 5: If $\Pi(K)$ is homogeneous of degree one, then $V_k = V/K$. From equation (16), the definition of measured Tobin's Q is

$$Q \equiv \frac{1}{1+r} E\left[\frac{BV}{K}\right]. \tag{32}$$

Plugging the above results into the first-order condition for investment in equation (14) yields equation (17) in the Proposition:

$$1+\psi K = \frac{1+n(B-1)}{B} Q.$$

To see that Q is increasing in σ, it suffices to note that because V is homogeneous of degree one in K, the ratio V/K is independent of K. Hence, the ratio is independent of K, so Q depends on σ only through B, which is increasing in σ.