An Alternative Trend-Cycle Decomposition using a State Space Model with Mixtures of Normals: Specifications and Applications to International Data

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Abstract

This paper first generalizes the trend-cycle decomposition framework of Perron and Wada (2005) based on an unobserved components models with innovations having a mixtures of Normals distribution, which is able to handle sudden level and slope changes to the trend function as well as outliers. We investigate how important are the differences in the implied trend and cycle compared to the popular decomposition based on the Hodrick and Prescott (HP) (1997) filter. Our results show important qualitative and quantitative differences in the implied cycles for both real GDP and consumption series for the G7 countries. Most of the differences can be ascribed to the fact that the HP filter does not handle well slope changes, level shifts and outliers, while our method does so. Third, we assess how such different cycles affect some so-called "stylized facts" about the relative variability of consumption and output across countries. Our results show again important differences. In particular, the cross-country consumption correlations are generally higher than the output correlations, except for the period from 1975 to 1985, provided Canada is excluded. Our results therefore provide a partial solution to this puzzle. The evidence is particularly strong for the most recent period.

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Key Words: Trend-Cycle Decomposition, Unobserved Components Model, International Business Cycle, Non Gaussian Filter.

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1 Introduction

Interest in the business cycle has a long standing history in both theoretical investigations and empirical applications. The important contribution of Burns and Mitchell (1946) paved the way for methods to measure it. The literature has, however, departed from their methods due to its complexity and the need for subjective evaluations. Instead, much of the work has concentrated on easily applicable mechanical and non-subjective methods. In the last few decades, many alternative procedures have been suggested. The need for a quantitative measure of the business cycle arises in great part because most macroeconomic models deliver implications that pertain to the non-trending component of series. In order to confront these models with the data, there is, accordingly, a need to separate the trend and the cycle.

Popular methods to extract the cyclical component include the following, among others: the Beveridge-Nelson (1981) decomposition based on unconstrained ARIMA models (Campbell and Mankiw, 1987, Watson, 1986, Cochrane, 1986), Unobserved Components models (Clark, 1987, Morley et al., 2003), the Hodrick-Prescott (1997) (HP) filter, and the Band-Pass (BP) filter (Baxter and King, 1999). Note that the latter is not per se a trend-cycle decomposition since it also eliminates high-frequency components but for most macroeconomic variables it yields a cycle that is close to that obtained with the HP filter.

Recently, Perron and Wada (2005) showed the importance of accounting for structural changes in the trend function of a time series when performing a trend-cycle decomposition, especially when using fully parameterized unobserved components models. They considered the US real GDP series and argued that once a change in the slope of the trend function is allowed in 1973:1, standard unobserved components models and the Beveridge-Nelson decomposition deliver the same trend and cycle, the trend being a simple piecewise deterministic linear function. They also proposed a generalized unobserved components model where the errors affecting the slope of the trend function are drawn from a mixtures of Normals distribution [1]. This permits sudden changes in the slope occurring occasionally at dates that need not be pre-specified but which are the outcome of the smoothed trend estimate.

In general allowing for the possibility of changes in only the slope of the trend function is insufficient. As discussed in Section 2, when dealing with real GDP series for the G7 countries, one is also faced with the problems of level shifts and severe outliers. Our aim is, therefore, first to generalize the trend-cycle decomposition framework of Perron and Wada

[1] They also consider such a distribution for the shock to the cyclical component to allow different variances in expansions and recessions.

(2005) and extend their algorithm to estimate the resulting structural models. Secondly, we wish to investigate how important are the differences in the implied trend and cycle for the various countries compared to other methods. Since, in empirical macroeconomic analyses, the most frequently used detrending procedure is the HP filter, we shall restrict our comparative analysis to our detrending procedure and the HP filter. Our results will show important qualitative and quantitative differences in the implied cycles for both real GDP and consumption series for the G7 countries. Most of the differences can be ascribed to the fact that the HP filter does not handle well slope changes, level shifts and outliers, while our method does so. Hence, our results first lead to a different picture of the cyclical component of important macroeconomic time series. Third, we assess how such different cycles affect some so called "stylized facts" about the relative variability of consumption and output across countries. Our results show again some important differences. In particular, if one excludes Canada, whose consumption series is atypical, the most recent period shows that for the vast majority of countries the consumption correlation is higher than the output correlation (and for those countries for which this does not hold the differences are very small). The period prior to 1975 also shows strong support for a similar conclusion. The period in which the so-called consumption-output correlation puzzle is in effect is mostly from 1975 to 1985. Hence, our results can be viewed as a partial resolution to this puzzle, in particular for the last decade for which the evidence is strong.

The plan of the paper is the following. Section 2 motivates the subsequent analyses by looking at the real GDP series for the G7 countries. Section 3 establishes the theoretical framework for the trend-cycle decompositions and Section 4 discusses how to estimate the models. Section 5 presents the results for the trend-cycle decomposition of the real GDP series for the G7 countries and compare the results to those obtained with an HP filter. Section 6 reassesses the findings about important measures of cyclical movements in output and consumption across the G7 countries using our trend-cycle decomposition, with emphasis on the relative volatilities of the cyclical components of output (real GDP) and consumption, and the cross-country correlations in these components. Section 7 offers brief concluding comments.

2 Motivation

Figure 1 presents the seasonally adjusted (log) real GDP series for the G7 countries using postwar quarterly series that extend as far back as was available to us [2]. These graphs reveal a number of interesting features. First, most countries show a decline in the rate of growth occurring near 1973. This is a feature that has received a lot of attention. For example, Perron (1989) argues that once one allows for a change in the slope of the trend function in 1973:1, one can reject the hypothesis that the US real GDP series contains a unit root (see Perron, 1997, for evidence pertaining to real GDP series for the G7 countries). Also, Bai, Lumsdaine and Stock (1998) estimate a multivariate model of the growth rates of real GDP for the G7 countries imposing a common break. They find statistical evidence for a change in mean with a 90% confidence interval that covers the period 1972:2-1975:2. More evidence is presented in Perron and Yabu (2005) who find a statistically significant change in the slope of the trend function for all countries allowing the noise component to be stationary or to have an autoregressive unit root. In all such studies the change is modelled as being sudden (i.e., a structural change at some date). This is important since standard detrending methods such as the HP filter are able to allow for a decrease in the rate of growth through time but not in a sudden fashion.

Another feature that is present in many series are more or less sudden level shifts. This is especially evident for the case of Germany which shows a dramatic increase in 1991 at the time of the re-unification. Such level shifts also occur for Canada and the UK in the early 1980s. Another type of aberrant pattern occurs in the case of France for which an extreme outlier occurs in 1968 in the form of a sudden temporary drop due to the general strike in May 1968.

What we wish to highlight from this visual inspection of the series are the following facts: structural changes in the slope and level of the trend function and outliers seem to be features affecting all real GDP series for the G7 countries (though not all features are present for all series). Also, large occurrences of such features are relatively rare, mostly once and at most a few times in the postwar sample.

This first suggest the distinct possibility that standard methods of detrending such as the

[2] The sample period and data sources for the various countries are as follows. For Canada: 1947:1-1997:2, CANSIM, Statistics Canada; for France: 1963:1-2004:2, OECD Economic Outlook (FRA.GDPV); for Germany: 1960:1-2004:4, OECD Economic Outlook (DEU.GDPV); for Italy: 1960:1-2004:4, OECD Economic Outlook (ITA.GDPV); for Japan: 1955:2-2001:1, Statistics Bureau (Gross Domestic Expenditure); for the United Kingdom: 1955:1-2005:1, National Statistics (ABMI); for the U.S.: 1947:1-2004:2, Bureau of Economic Analysis).

HP filter will provide a distorted picture of the cyclical component. Take for instance the case of Germany. As we shall see the HP detrending procedure is unable to account for the sudden increase at the time of the re-unification. Instead a smooth trend is fitted with the implication that the period a few quarters before 1991 was a deep recession and the period a few quarters after was a drastic short-lived expansion. Perron and Wada (2005) documents extensively, for the case of the US, how a single change in slope can affect the outcome of various detrending procedures and that accounting for such a change can drastically alter the resulting cyclical component. So, clearly, slope and level changes, as well as major outliers, can have a substantial impact on what a detrending procedure delivers as the cycle.

In many instances researchers aware of such problems will use ad hoc methods to provide a remedial; e.g., avoiding the period contaminated by such effects, interpolating (c.f., Stock and Watson, 2005) or using sub-samples. But such methods involve a substantial loss of information. One would therefore like to have a detrending method that is able to account for such features in an endogenous fashion, i.e., without having to specify a break date, a type or a number of changes, and deliver a cyclical component that is uncontaminated by such aberrant events. On this front little has been done [3] and it is our aim to suggest a procedure for doing so.

3 The trend-cycle decomposition framework

The most general specification of the class of models considered is the following,

$$
\begin{aligned}
y_t &= \tau_t + c_t + \omega_t && (1)\\
\tau_t &= \tau_{t-1} + \beta_t + \eta_t && (2)\\
\beta_t &= \beta_{t-1} + \upsilon_t && (3)\\
\phi(L)\, c_t &= \epsilon_t, && (4)
\end{aligned}
$$

where y_t is the observed series, τ_t is the trend function, c_t is the cyclical component and ω_t the measurement errors. The shocks ω_t, η_t, υ_t and ϵ_t are assumed to be mutually uncorrelated as well as serially uncorrelated. If the errors were Normally distributed, this would be a standard unobserved components model which has already been used extensively in the literature under various levels of generality (see, among others, Clark, 1987, Morley et al, 2003, Harvey and Jaeger, 1993). Our departure from the basic specification is to model the

[3] A recent exception is Giordani, Kohm aamd van Dijk (2005) who use a Bayesian methodology applied to an extended State Space model.

errors as having a mixtures of Normal distributions. Let x_t represent either ω_t, η_t, υ_t or ϵ_t, the distribution of x_t is specified to be

$$x_t = \lambda_t \gamma_{1t} + (1 - \lambda_t)\, \gamma_{2t}$$

where

$$\gamma_{it} \sim i.i.d.N(0, \sigma_i^2)$$

and $\lambda_t \sim i.i.d.Bernoulli\,(\alpha)$. Hence, with probability α, the error at time t is drawn from a $N\,(0, \sigma_1^2)$ and with probability $(1 - \alpha)$ it is drawn from a $N\,(0, \sigma_2^2)$. This will permit sudden changes if α is close to 1 and σ_2^2 is much larger than σ_1^2. In this case, most of the time the errors are drawn from a low variance distribution which characterizes "normal periods"; but occasionally a large shock occurs, which characterizes "atypical events".

Consider the implications of such a specification for the various error terms in the model. First, if this scenario applies to the measurement errors ω_t, this would imply small or zero measurement errors in "normal periods" and occasional outliers. Second, for the error η_t affecting the trend function, this would allow a random walk component (or a deterministic trend if $\sigma_1^2 = 0$) with occasional level shifts. Third, for the error υ_t affecting the slope of the trend function, this allows small or null changes in the slope in "normal periods" with occasional large changes. Finally, a mixtures of Normal distributions for the error ϵ_t can allow for different variances in recessions and expansions (though here α would be the probability of being in an expansion and one would expect σ_2^2 to be larger than σ_1^2 but not by a large factor, see Perron and Wada, 2005).

Hence, the use of mixtures of Normal distributions for the errors is potentially a powerful tool to permit structural changes in the slope and level of the trend function as well as outliers. It is important to note that the probabilities that the errors be drawn from one regime or the other are independent of past realizations. This is in contrast to the popular Markov switching type model (e.g., Hamilton, 1989). Here the different regimes affect the magnitude of the shocks and not per se what regime is in effect. Our goal is to have a framework which can allow special events such as a one-time productivity slowdown. Hence, the probability that we draw from the high variance distribution should be independent of whether past draws came from the small or large variance distributions. Accordingly, it is more appropriate to make the probabilities of being in one regime or the other as independent of past realizations.

The introduction of such mixtures of Normals distributions for the errors introduces considerable additional complexities for the estimation of the model. In particular, allowing such mixtures distributions for all components leads to an unstable algorithm. Hence, we need to restrict the model somewhat to obtain sensible outcomes. Our choice of restrictions follows from our discussion of the main features of the GDP series for the various G7 countries. In all cases, except the US, the errors ϵ_t for the cyclical component are specified as Normally distributed. The main reason is that we want to highlight the importance of slope and level shifts to the trend function in influencing the resulting extracted cycle. We allow it for the US to relate our results to those of Perron and Wada (2005). In all cases, the errors υ_t affecting the slope of the trend function is modelled with a mixtures of Normal distributions given that slope changes are likely to be present for all countries. We allow a mixtures of Normals distribution for the error η_t affecting the level of the trend function for Canada, Germany, Italy and UK (for France, Japan and the US, large sudden level shifts are not present). Finally, we allow a mixtures of Normals for the measurement error ω_t only for France since it seems the only country to have been subject to a large one-time decrease in GDP caused by the May 1968 strike. Finally, in all cases, the cyclical component is modelled as an $AR(2)$, i.e., $\phi(L) = 1 - \phi_1 L - \phi_2 L^2$; diagnostic tests revealed this to be an appopriate specification.

4 Estimation method

For concreteness, we discuss the method for the case where the shocks η_t (to the level of the trend) and υ_t (to the slope of the trend) have a mixture of Normal distributions and the measurement errors ω_t and shock to the cyclical component ϵ_t are Normally distributed (for the other cases, only minor modifications are needed). As a matter of notation we let α_1 (resp., α_2) be the probabiity that a draw for η_t (resp., υ_t) come from the low variance regime denote $\sigma^2_{\eta 1}$ and $\sigma^2_{\upsilon 1}$ (while the higher variances are denoted $\sigma^2_{\eta 2}$ and $\sigma^2_{\upsilon 2}$). The State Space model is of the form

$$\begin{aligned} y_t &= Hx_t + \omega_t \\ x_t &= Fx_{t-1} + Gu_t \end{aligned}$$

where $x_t = [\tau_t, c_t, c_{t-1}, \beta_t]'$, $H = [1, 1, 0, 0]$

$$F = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \phi_1 & \phi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $u_t = [\eta_t, \epsilon_t, v_t]'$. What is different from the usual State Space model is that the distribution of u_t is not Normal. However, we can view the specification as a State Space model with Normal errors but with four possible states. These states are defined by the combined values of the Bernoulli random variables and imply four possible covariance matrices for the vector of errors u_t, namely

$$Q = \left\{ \begin{bmatrix} \sigma_{\eta 1}^2 & 0 & 0 \\ 0 & \sigma_{\epsilon_t}^2 & 0 \\ 0 & 0 & \sigma_{v1}^2 \end{bmatrix}, \begin{bmatrix} \sigma_{\eta 1}^2 & 0 & 0 \\ 0 & \sigma_{\epsilon}^2 & 0 \\ 0 & 0 & \sigma_{v2}^2 \end{bmatrix}, \begin{bmatrix} \sigma_{\eta 2}^2 & 0 & 0 \\ 0 & \sigma_{\epsilon_t}^2 & 0 \\ 0 & 0 & \sigma_{v1}^2 \end{bmatrix}, \begin{bmatrix} \sigma_{\eta 2}^2 & 0 & 0 \\ 0 & \sigma_{\epsilon_t}^2 & 0 \\ 0 & 0 & \sigma_{v2}^2 \end{bmatrix} \right\}$$

where each component occurs with probabilities $\alpha_1\alpha_2$, $\alpha_1 (1 - \alpha_2)$, $(1 - \alpha_1)\alpha_2$, and $(1 - \alpha_1)(1 - \alpha_2)$, respectively. This interpretation is helpful in constructing an algorithm for estimation.

Our generalization complicates the estimation procedure considerably. The basic principles are, however, the same as for the estimation of the usual State Space model with Normal errors. The likelihood function is estimated using a variant of the Kalman filter and a by-product is an estimate of the conditional expectation of the State vector x_t using information available up to time t. These are denoted $x_{t|t}$ and are called filtered estimates. One can also construct estimates using the full sample, i.e., $x_{t|T}$ which are obtained using a smoothing algorithm and are, accordingly, called smoothed estimates. The main goal here is to obtain smoothed estimates of the trend function τ_t and of the cyclical component c_t. We describe the main steps below.

Since this estimation and the filtering procedure are similar to the ones for Markov switching models, the basis for the construction of our computer codes was the GAUSS program written by Chang-Jin Kim (KIM_JE1.OPT) as discussed in Kim and Nelson (1999). The code is available from the book's website. Let $Y_t = (y_1, ..., y_t)$ be the vector of data available up to time t. The objective function to be maximized is

$$\begin{aligned} \ln(L) &= \ln\left[\sum_{t=1}^{T} p\left(y_t|Y_{t-1}\right)\right] \\ p\left(y_t|Y_{t-1}\right) &= \sum_{s_t=1}^{4}\sum_{s_{t-1}=1}^{4} p\left(y_t|s_{t-1}, s_{t,}Y_{t-1}\right)\Pr\left(s_{t-1} = i, s_t = j|Y_{t-1}\right) \end{aligned}$$

Also, let the prediction errors be

$$\nu_{t|t-1}^{ij} = y_t - E[y_t|Y_{t-1}, s_{t-1} = i, s_t = j] = y_t - Hx_{t|t-1}^{ij}.$$

Here, and throughout, the superscripts (ij) refers to the value of the variable conditional on the process being in state i at time $t-1$ and state j at time t. Conditional on the states at periods t and $t-1$ taking values i and j, respectively, and the value of Y_{t-1}, the prediction errors are such that

$$(\nu_{t|t-1}^{ij}|s_{t-1}=i,s_t=j,Y_{t-1})\sim N\left(0,f_{t|t-1}^{ij}\right) \tag{5}$$

with

$$f_{t|t-1}^{ij}=E\left(\nu_{t|t-1}^{ij}\nu_{t-1}^{ij\prime}\right)=HP_{t|t-1}^{ij}H'$$

so that

$$p\left(y_t|s_{t-1},s_{t,}Y_{t-1}\right)=\frac{1}{\sqrt{2\pi}}\left|f_{t|t-1}^{ij}\right|^{-1/2}\exp\left\{-\frac{\nu_{t|t-1}^{ij\prime}(f_{t|t-1}^{ij})^{-1}\nu_{t-1}^{ij}}{2}\right\}$$

Also,

$$\begin{aligned}
\Pr\left(s_{t-1}=i,s_t=j|Y_{t-1}\right) &= \Pr\left(s_t=j|s_{t-1}=i\right)\Pr\left(s_{t-1}=i|Y_{t-1}\right)\\
&= \Pr\left(s_t=j\right)\Pr\left(s_{t-1}=i|Y_{t-1}\right)\\
\Pr\left(s_{t-1}=i,s_t=j|Y_t\right) &= \Pr\left(s_{t-1}=i,s_t=j|y_t,Y_{t-1}\right)=\frac{p\left(y_t,s_t,s_{t-1}|Y_{t-1}\right)}{p\left(y_t\mid Y_{t-1}\right)}\\
&= \frac{p\left(y_t|s_t,s_{t-1,}Y_{t-1}\right)\Pr\left(s_{t-1}=i,s_t=j|Y_{t-1}\right)}{p\left(y_t|Y_{t-1}\right)}\\
\Pr\left(s_t=j|Y_t\right) &= \sum_{i=1}^{4}\Pr\left(s_{t-1}=i,s_t=j|Y_t\right).
\end{aligned}$$

The basic inputs are therefore the best estimates of the sate vector and their mean squared errors, namely

$$\begin{aligned}
x_{t|t-1}^{ij} &= Fx_{t-1|t-1}^{i}\\
P_{t|t-1}^{ij} &= FP_{t-1|t-1}^{i}F'+GQ^jG'
\end{aligned}$$

where

$$\begin{aligned}
x_{t|t-1}^{ij} &= E\left[x_t|Y_{t-1},s_{t-1}=i,s_t=j\right]\\
x_{t-1|t-1}^{i} &= E\left[x_{t-1}|Y_{t-1},s_{t-1}=i\right]\\
P_{t|t-1}^{ij} &= E\left[\left(x_t-x_{t|t-1}\right)\left(x_t-x_{t|t-1}\right)'|Y_{t-1},s_{t-1}=i,s_t=j\right]\\
P_{t-1|t-1}^{i} &= E\left[\left(x_{t-1}-x_{t-1|t-1}\right)\left(x_{t-1}-x_{t-1|t-1}\right)'|Y_{t-1},s_{t-1}=i\right]
\end{aligned}$$

for $i,j=1,2,3,4$. The problem that arises with four possible states is that the number of estimates for the state vector and their mean square error matrices grows exponentially with time. Indeed, at a given time t, we have t^4 estimates of the state vector to compute. The solution we adopt is to use the re-collapsing procedure suggested by Harrison and Steven (1976) which effectively provides re-approximations at each time t. These are given by:

$$
\begin{aligned}
x^j_{t|t} &= \frac{\sum_{i=1}^4 \Pr\left(s_{t-1}=i, s_t=j|Y_t\right) x^{ij}_{t|t}}{\Pr\left(s_t=j|Y_t\right)} \\
P^j_{t|t} &= \frac{\sum_{i=1}^4 \Pr\left(s_{t-1}=i, s_t=j|Y_t\right) \left\{P^{ij}_{t|t} + \left(x^j_{t|t} - x^{ij}_{t|t}\right)\left(x^j_{t|t} - x^{ij}_{t|t}\right)'\right\}}{\Pr\left(s_t=j|Y_t\right)}
\end{aligned}
$$

where now a single superscript j refers to the value of the variable conditional on the process being in state j at period t. The filtered estimate of the state vector is then obtained as:

$$x_{t|t} = \sum_{j=1}^4 \Pr\left(s_t=j|Y_t\right) x^j_{t|t}.$$

4.1 Initial Values

Since one component of the state vector is non-stationary, we cannot initialize all components of the state vector and its covariance matrix to their unconditional expected values. The initial values we used are:

$$x_{0|0} \; = [y_1, 0, 0, \Delta y_2]'$$

and

$$P_{0|0} = \begin{bmatrix} 1e+08 & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & 1e+08 \end{bmatrix}$$

where the submatrix P is given by

$$vec\left(P\right) = \left[I_2 - F1 \otimes F1\right]^{-1} vec\left(Q1\right)$$

with

$$F1 = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix} \quad Q1 = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & 0 \end{bmatrix}$$

The initial value of the trend τ_t is set to the first observation of the series, and the initial value of the slope β_t is set to the difference between the second and first observations. We set their variances to a very large number to reflect a diffuse prior on their values, following Harvey and Phillips (1979). Note that the results are not sensitive to these particular specifications.

The other components of the state vector are stationary and we use their steady state values as initial conditions.

4.2 Restrictions, Initial conditions and computations

A practical difficulty in the estimation of such Gaussian mixture models is the so called "label-switching problem" (see, e.g., Hamilton, Waggoner and Zha, 2004). This problem is due to the fact that the likelihood function $p\left(y_t|Y_{t-1}\right)$ does not change if the individual components of $p\left(y_t|s_{t-1}, s_{t,}Y_{t-1}\right)\Pr\left(s_{t-1}=i, s_t=j|Y_{t-1}\right)$ are interchanged, and likewise for $p\left(y_t|s_{t-1}, s_{t,}Y_{t-1}\right)\Pr\left(s_{t-1}=i, s_t=j|Y_{t-1}\right)$, so that

$$\begin{aligned} & p(y_t|s_{t-1}, s_{t,}Y_{t-1})\Pr\left(s_{t-1}=i, s_t=j|Y_{t-1}\right)+p\left(y_t|s_{t-1}, s_{t,}Y_{t-1}\right)\Pr\left(s_{t-1}=i^*, s_t=j|Y_{t-1}\right) \\ = \; & p\left(y_t|s_{t-1}, s_{t,}Y_{t-1}\right)\Pr\left(s_{t-1}=i^*, s_t=j|Y_{t-1}\right)+p\left(y_t|s_{t-1}, s_{t,}Y_{t-1}\right)\Pr\left(s_{t-1}=i, s_t=j|Y_{t-1}\right)\end{aligned}$$

Hence, we cannot identify the sates i and i^* without some normalization. To overcome this problem, we impose some restrictions discussed next.

It is important to note that, as stated, not all parameters are identified, though the trend-cycle decomposition is. To get parameter estimates we impose the following restrictions pertaining to the parameters in the mixtures of Normals distributrions. The value under the column "α" refers to the minimal value of the probability of being in the low variance regime, while the value under the column "σ_1" denotes the largest value of the standard deviation in the low variance part of the mixtures.

Country	η_t (level)		υ_t (slope)		ϵ_t (cycle)		ω_t (errors)	
	α	σ_1	α	σ_1	α	σ_1	α	σ_1
Canada	0.8	none	0.5	0.1	NA	NA	NA	NA
France	NA	NA	0.8	0.1	NA	NA	0.9	none
Germany	0.8	none	0.5	0.1	NA	NA	NA	NA
Italy	0.8	none	0.5	0.1	NA	NA	NA	NA
Japan	NA	NA	0.5	0.001	NA	NA	NA	NA
UK	0.5	none	0.5	0.1	NA	NA	NA	NA
US	NA	NA	0.9	0.01	none	none	NA	NA

Note that in many cases the probability of having a draw from the small variance distribution is simply set to be greater than 0.5, which amounts to a simple normalization. Note also that most of these restrictions are non-binding, with some exceptions (e.g., Canada). Though such restrictions are needed to get parameter estimates, the implied trend-cycle decomposition is not sensitive to them.

All estimations are implemented using the programming language GAUSS, version 6.0. To maximize the chances of obtaining parameter estimates that correspond to the global maximum of the likelihood function, we re-estimate the model 300 times with different initial values for the parameters that are drawn from a $N(0,3)$. The convergence criterion is set at $1e-5$ in the GAUSS command 'optmum'. Finally, we compute the likelihood function for observations $t = 10$ onwards because of potential nonstationarity.

4.3 Quasi-Smoothing

The smoothing algorithm used is that suggested by Kitagawa (1994). We outline the main steps here. Let $Y^t = \{y_t, y_{t+1}, \cdots, y_T\}$, the smoothed density is then

$$p(x_t|Y_T) = p(x_t|Y_{t-1})\, p\left(Y^t|x_t\right) p\left(Y^t|Y_{t-1}\right).$$

Note that $p(Y^t|Y_{t-1})$ does not depend on t, and the smoothed density is obtained by the one step ahead projection density $p(x_t|Y_{t-1})$ and the backward filtering density, $p(Y^t|x_t)$. The latter is given by

$$p\left(Y^{t+1}|x_t\right) = \int_{-\infty}^{\infty} p\left(Y^{t+1}|x_{t+1}\right) p(x_{t+1}|x_t)\, dx_{t+1}$$

and

$$p\left(Y^t|x_t\right) = p\left(Y^{t+1}|x_t\right) p(y_t|x_t),$$

given the initial condition $p\left(Y^T|x_T\right) = p(y_T|x_T)$. This involves the same algorithm as for the forward filtering procedure and we use the same collapsing method. Once the backward filtering step is performed, the smoothed estimates are obtained using the following recursions:

$$\begin{aligned} x_{t|T} &= x_{t|t-1} + J_t\left(z_{t|t} - x_{t|t-1}\right) \\ J_t &= P_{t|t-1}\left(P_{t|t-1} + U_{t|t}\right)^{-1}. \end{aligned}$$

In practice, the collapsing method is implemented using

$$x_{t|T} = \sum_i \sum_j \Pr\left(s_t^{ij}\right) x_{t|T}^{ij}$$

where

$$\begin{aligned}
x_{t|T}^{ij} &= x_{t|t-1}^{i} + J_t^{ij}\left(x_{t|t}^{i} - x_{t|t-1}^{j}\right) \\
J_t^{ij} &= P_{t|t-1}^{i}\left(P_{t|t-1}^{i} + U_{t|t}^{j}\right)^{-1}
\end{aligned}$$

and

$$\begin{aligned}
\Pr\left(s_t^{ij}\right) &\equiv \Pr\left(s_t = i|Y_{t-1}\right)\Pr\left(s_t = j|Y^t\right) \\
&= \left(\sum_j \Pr\left(s_t = i, s_{t-1} = j|Y_{t-1}\right)\right)\left(\sum_k \Pr\left(s_t = j, s_{t+1} = k|Y^t\right)\right).
\end{aligned}$$

5 Results for the trend-cycle decompositions.

We now present the trend-cycle decompositions obtained for the Real GDP series of the G7 countries (denoted MN for Mixtures of Normals). These will also be compared to decompositions obtained using the Hodrick-Prescott filter (denoted HP) (we set the smoothed parameter λ to be 1600, as usual). The results are presented in Figures 2 to 5.

Consider first the case of Canada. The MN decomposition picks up a few minor level shifts in the mid-50's and early 60's and also show a decrease in the rate of growth after the mid 70's with the decrease continuing gradually until the end of the sample. Overall, the fitted trend is quite smooth contrary to that using the HP filter, which follows the series more closely. Hence, the implied cycle using the MN decomposition is more important (higher variance) than that obtained with the HP filter. Otherwise the movements are similar. For France, the MN decomposition easily accounts for the outlier in 1968, which the HP filter assigns to the cyclical component. Otherwise, the trend function is a straight line with a change in slope near 1973 and a temporary decrease in level at the time of the break. Since the trend obtained with the HP filter follows the actual series more closely and thereby ascribes less movements to the cycle, the variance of the cycle with the MN decomposition is again larger.

The case with the most drastic differences is Germany. The trend function with the MN decomposition is a smooth trend with two major changes: a decrease in slope near 1973 and a large increase in level in 1991 associated with the re-unification. The trend function obtained from the HP filter again follows the series closely until the early 80s (so that the cycle is again less variable) but it misses the level shift. This gives a completely different characterization of the cyclical component after the early 80's. The HP cycle shows a mild expansion for much of the 80s while the MN cycle shows an important recession. From the late 80s to the early 90s, the HP cycle shows a decrease in activity while the MN cycle shows an increase. The period a few quarters before the re-unification is characterized by a sharp recession with the HP cycle and by an expansion with the MN decomposition. The period a few quarters after the re-unification is characterized by an impressive boom with the HP cycle and by a more reasonable expansion with the MN cycle. The HP cycle shows much of the later part of the 90s to be below trend activity while the MN cycle shows a performance roughly on par with the trend level. Hence, it is clear that the failure to account for the sudden upward level shift at the time of the re-unification leads to a very different picture of the cyclical component, and as we shall see below this also has implications for cross-country correlation analyses.

The case of Italy is one for which the HP and MN decompositions yield basically the same results. Again the MN trend is smoother than the HP trend so that the cycle is more variable but the differences are not large. For Japan, the trend function implied by the MN decomposition is a very smooth function. It consists in roughly three parts: a linear trend with high growth until 1973, followed by a linear trend with much reduced slope until the early 90s after which it exhibits a further gradual decline. The HP trend is similar, with again the exception that it is less smooth and follows the series more closely so that the cycle is somewhat less variable.

For the United Kingdom, the MN and HP trends and cycles are very different. The MN trend shows important level shifts prior to 1980 after which it is simply a stable straight line. The difference can most easily be seen by looking at the implied cycle. According to the MN decomposition almost all the period from 1960 to the mid-70 is characterized by above trend activity, while the HP cycle shows four sustained periods of below trend activity. After 1980, the HP trend follows the actual series more closely than the straight line of the MN trend. Accordingly, the MN decomposition shows the recessions of the early 80s and early 90s to be much more severe (compared to the HP cycle) and the expansion of the late 80s to be more important. For the US, the MN trend and cycle is as documented in Perron and Wada (2005). The trend is simply a deterministic function with a change in slope occurring mostly in 1973. The HP filter again follows the actual data more closely so that the implied cycle is much less volatile than the MN cycle. One important difference is the period at the end of the 50s which is characterized by an expansion with the HP cycle but by a recession with the MN cycle.

In summary, the main qualitative differences between the MN and HP trend cycle decompositions are: 1) the MN trend function is (absent sudden level or slope changes and outliers) much smoother than the HP trend. In statistical terms, the random walk component is much less important, if at all present, for the majority of the countries. The HP trend follows the actual series much more closely with the implication that more of the movements in the series are characterized as being of a permanent nature, i.e., the random walk component is much more important; 2) As a consequence, the HP filter ascribes much less movements to the cyclical component compared to the MN cycle, which shows higher variability; 3) The MN decomposition is able to capture sudden occasional changes in the level and slope of the trend function, while the HP filter does so in a much more gradual fashion. In such cases, especially with large level shifts, the implied cycle can be drastically different. The failure to account for such shifts in the trend function by the HP filter is largely responsible for the fact that HP trend contains a larger random walk component and ascribes less movements to the cycle. This follows from the results in Perron (1989, 1990) to the effect that unaccounted changes in a trend function biases usual statistical procedures towards accounting for such changes via a random walk or autoregressive unit root characterization. We shall see in the next section that these features have implications for the cross-countries comparisons of cyclical volatility and correlations.

6 Cross-country comparisons of cyclical movements in output and consumption

In this section, we wish to reassess the findings about important measures of cyclical movements in output and consumption across the G7 countries using our trend-cycle decomposition. The issues to be addressed are: the volatility of the cyclical components of output (real GDP) and consumption, and the cross-country correlations in these components. In order to do so it is important to have an homogenous data set to minimize potential discrepancies in the construction of the output and consumption measures across countries. Accordingly, we shall use a slightly different data set from a unique source, namely the OECD Economic Outlook database [4].

For these series, we again computed the MN and HP trend cycle decompositions [5]. For

[4] The consumption data used is "private final consumption expenditure", hence durable, non durables and services are included. The series for all countries end in 2004:4. The starting date is 1960:1 for all countries, except Canada for which it is 1961:1 and France for which it is 1963:1. The mnemonic for the series are GDPV for real GDP and CPV for consumption.

[5] We also performed all results that follow with the Band Pass filter (see, Baxter and King, 1999). We used the modified filter described in Christiano and Fitzgerald (2003) with the extracted cycle composed of movements with periods between 6 and 32 quarters. The results obtained are very similar to those using the HP filter and, hence, not reported.

the real GDP series the trend-cycle decompositions show no appreciable differences and, hence, we shall not repeat the results. The trend-cycle decompositions for the consumption series are presented in Figures 6 to 9. Consider first the case of Canada, which stands as an extreme case. The actual series is quite smooth but exhibits sudden changes in level and slope (early 70s, mid-70s and especially the early 80s and early 90s). Such shifts are ascribed to the trend function by the MN decomposition, with the implication that barely anything is left for the cyclical component. A way to interpret this result is to note that the actual series is affected by important shocks that are large enough to be viewed as having a permanent effect and hence are part of the trend. The fact that little is left to the cyclical component implies that the Canadian economy adapts quickly to such permanent shocks. The HP cycle is much more volatile but it is interesting to note that most of the movements occur near these periods of sudden changes in level and slope. This case of extreme smoothing is certainly worthy of further investigations.

The cases for France and Italy are similar. The HP and MN trends have roughly the same characteristics, though unlike the case for output, the MN cycle has slightly higher variability than the HP cycle. For Japan the opposite occurs and the MN cycle is much less variable than the HP cycle. This is again due to the presence of sudden changes which are ascribed to the trend function by the MN decomposition. For the United Kingdom, the trend-cycle decomposition for consumption is similar to that for output. The trend is variable prior to the early 80s after which it becomes a straight line and the cyclical component is accordingly more variable with the NM than with the HP filter. For the US, the trend function is again very smooth (basically a straight line with a blip in the early 80s) and, since the HP trend follows more closely the series, the cycle is accordingly more variable with MN than with HP. Finally, the case of Germany shows important differences between the MN and HP cycles. This is again due to the sudden level shift at the time of the re-unification, which has a profound impact on the HP cycle, which cannot account for it.

Table 1 presents summary measures of the volatility in the cyclical components of output and consumption for two sub-periods: a) the full period 1965:1-2004:3 [6]; b) 1970:1-1990:2, the period used by Backus et al. (1992), which allows a comparison with their results. For the full period, the volatility of the output-cycle is greater using the MN decomposition than with the HP filter for all series except Germany, which is expected given our discussion above.

[6] We start in 1965:1 to avoid the use of the decomposition at the very beginning of the sample, which can be imprecise due to startup effects; and also to have a common period for all countries. But the cyclical components used to construct the volatility measures is estimated from the full sample.

For the sample 1970:1-1990:2, the same comparative results hold except that Germany is no longer an exception since the re-unification is no longer part of the sample and has no impact on the HP cycle. The volatility of the consumption cycle is also larger using the MN decomposition, except for Canada and Japan, and this for both samples.

Consider now the relative volatility of the cyclical components of output and consumption. Standard economic theory suggests that the volatility of consumption should be smaller than the volatility of output since agents wish to smooth out their consumption over time. Using the MN decomposition with the full sample, this is the case for US (barely), Canada (to an extreme extent), France and Japan. With the HP filter, the only difference is that consumption is less variable than output for Italy as well. With the MN decomposition, the same holds using the sample 1970:1-1990:3 except for the US. With the HP filter, consumption is now more volatile for France (barely) and less volatile for Germany.

Figures 10 and 11 present rolling estimates of these volatilities over the period 1970:1-2000:1, for output and consumption, respectively. These show, for both the MN and HP decompositions that there is a clear tendency for a decline in the volatility of output, especially after the mid-80s. This is consistent with the finding of Stock and Watson (2002). The same is true for consumption, except perhaps for France whose time profile for the volatility is quite stable, and for Canada whose volatility is constant at a near zero level.

6.1 Consumption Correlation Puzzle

Since the influential work by Backus et al. (1992), one of the well known features or "stylized fact" of the international business cycle is that cross-country consumption correlations are lower than cross-country output correlations (from an empirical analysis using an HP filter to extract the trend component). However, a general equilibrium model with risk averse agents implies that the consumption correlation should be larger than the output correlation, due to the consumption smoothing behavior of agents (see, e.g., Baxter, 1995, and Backus et al., 1995). To try to disentangle this puzzle, two avenues have been followed. The first is to introduce market frictions to the basic model which imply lower consumption correlation than with complete market. Baxter and Crucini (1995) show that if the productivity shock is a random walk without international spillover, incomplete market economies would have smaller consumption correlation than output correlation. Another example is Kehoe and Perri (2002), who suppose that international loans are imperfectly enforceable. In that case, also due to the market frictions, the consumption correlation is not necessarily higher than that of output.

A second approach is to assess the reliability of the "stylized facts" using alternative econometric techniques. Ambler et al. (2004) consider the reliability of the results to using different sub-samples with a wide variety of countries. They use a standard HP filter to extract the cyclical components and their results show that the consumption-output correlation puzzles is even more pronounced, i.e., the cross-country consumption correlations are much smaller than the output correlations than previously reported. Pakko (2004) shows that the consumption correlation is not always smaller than the output correlation, depending on the frequency bands used to extract the cyclical component using a band pass type filtering procedure. It is, nevertheless, smaller when using frequency bands usually associated with the business cycle. We here pursue this sensitivity analysis to see if the detrending procedure used can yield different results or provide additional evidence for the "stylized facts".

The results of the cross-country correlations for output and consumption are presented in Table 2 for the full sample. The results with the HP filter confirm the stylized fact that output correlations are higher than consumption correlations. Of the 21 cases, only three pairs of countries have consumption correlation higher than output correlation, and only by a rather small margin. The results with the MN decomposition are more "encouraging" in that 8 pairs of countries have consumption correlation higher than output correlation. Still, the evidence is not entirely convincing.

To assess the stability of these correlations over time, Figures 12 to 15 present rolling 10 years moving-average estimates of the output (top part) and consumption (middle part) correlations, as well as the difference between the two (bottom part, so that a value above the zero line indicates consumption correlation higher than output correlation). Figure 12 presents the results for pairs not involving Japan, Germany and Canada. Figure 13 presents the results for pairs of countries involving Japan, Figure 11 for pairs involving Germany and Figure 12 for pairs involving Canada. Several features stand out from these results.

Consider first the output correlations. First, for the pairs of countries involving the Euro-zone or English speaking countries, the output correlations are high and generally increasing through time, which accords with the results of Stock and Watson (2005). Note also that the estimates obtained using the MN decomposition are more stable than those obtained using the HP filter, especially when Germany is involved. For pairs of countries not involving Germany and Japan, overall the correlations are not as high but again there is a tendency for an increase over time, especially with estimates obtained using the MN decomposition. For pairs of countries involving Japan, the results are markedly different. The correlations fluctuate more and show a clear decrease to negative levels as we reach the most recent periods. For other pairs of countries involving Germany (with Canada, US and UK), both the MN and HP estimates show a very big decrease at the time of the re-unification, but the correlations have gradually increased since then. In summary, the correlations have increased and reached a high level for most pairs, with the estimates in recent periods being higher than reported by Ambler et al. (2004). Two notable special cases stand out: the fact that the correlations of Japanese output with any other country have been decreasing to negative levels, and the fact the German re-unification have temporary altered the estimates for pairs involving Germany and Canada, the US and UK.

Consider now the consumption correlations. A broad summary of the main features follows. The estimates obtained with the MN decomposition are generally higher than those obtained with the HP filter. For most pairs the correlation has been increasing throughout the sample period. Again, pairs involving Japan show a decrease in the correlation (similar to that of output). The German re-unification again had a substantial temporary effect on the estimates. But the most striking feature is the fact that the correlations for any pairs involving Canada are near zero throughout the sample with no tendency to increase, and a decline for some countries (e.g., France). This is certainly due to the fact that the consumption correlation for Canada is so low, indeed atypical.

A summary of the results for the output and consumptions correlation for different periods across the sample is presented in Table 3. It gives the correlations estimates for the following sub-samples: 1965:3-1974:4, 1975:1-1984:4, 1985:1-1994:4 and 1995:1-2004:3. Since Canada is such an atypical case, let us consider the results for all pairs not involving Canada, which leaves 15 pairs. The results show interesting differences across sub-samples and across methods of detrending. The number of pairs of countries showing consumption correlation higher than output correlation are as follows: 11 with MN for 1965:3-1974:4 (5 with HP), 2 with MN for 1975:1-1984:4 (1 with HP), 7 with MN for 1985:1-1994:4 (4 with HP), 12 with MN for 1995:1-2004:3 (6 with HP). Hence, if one considers our trend-cycle decomposition (and one excludes Canada), the consumption-output correlation puzzle has disappeared in the last decade. In fact the evidence is even stronger if one looks at the point estimates for the 3 pairs which have consumption correlation smaller than output correlation for that period. These are .73 versus .78 for Italy-France, .71 versus .80 for Italy-Germany, and .79 versus .86 for UK-France. Hence, the differences are quite small and likely not significant. The puzzle is quite weak also for the period 1965-1975 for which the consumption correlation is higher than the output correlation for many countries.

The bottom panels in Figures 12 to 15 show rolling 10 years moving-average estimates of the difference between consumption and output correlations. A first feature of interest is that the estimates are generally higher when using the MN decomposition compared to using the HP filter. For most countries they show substantial variations over time. Only three countries have differences that are consistently above 0, namely Italy-UK, Japan-UK, and US-France. All pairs involving Canada (with the exception of Japan before 1975 and after 1985, and Canada-France before 1975) show little evidence for a difference above zero. For most of the other countries, the differences are near or above zero in the post 1985 period (with the exception of US-Italy and Germany-Japan for which the differences are greater than zero only after 1995). Some pairs of countries also show estimates of the differences above zero in the early part of the sample (e.g., France-UK, US-UK until 1975).

Hence, our results provide considerable evidence that the cross-country consumption correlation is greater than the output correlation. If one excludes Canada, 7 of the 15 pairs have full sample estimates of the difference above 0 (with Canada, only Japan has a full sample estimate above 0 and for the other countries there is little evidence to that effect throughout the sample). For the remaining 8 countries, the estimates are close to or above 0 after 1985 (1995 for US-Italy and Germany-Japan). What seems to be responsible for the full-sample result that output correlation is higher than consumption correlation for these countries is the 1975-1985 period, which shows a quite different pattern compared to more recent and earlier periods. Of course, generally, the method of detrending matters to a great extent.

7 Concluding Remarks

Our paper presented an alternative detrending procedure that allows for trend functions having level and slope shifts and series affected by important outliers. When applied to postwar quarterly real GDP and consumption series, we have shown that it performs well, is able to account for sudden changes and yields reasonable estimates of the cyclical component. When compared with the HP filter, a major difference is the fact that our cyclical component is more variable. This is due to the fact that the HP filter does not account well for the level and slope shifts that are present and, accordingly, ascribes more movements to the trend leaving little to the cyclical component. Some exceptions to this rule occurs, for example with Germany for which the sudden increase in level at the time of the re-unification substantially contaminates the cyclical component obtained with the HP filter so that the latter show huge spikes near the time of the level shift.

When analyzing the cyclical components obtained and performing cross-country comparisons, our main findings are the following. First, the output correlations have increased from 1965 to 2004 with the exception of pairs involving Japan. Second the consumption correlations have also shown an increase, with the exception of pairs involving Canada. Third, our results show that for the most recent period (md to some extent the pre-1975 periods as well), consumption correlations are higher than output correlations if one excludes pairs involving Canada. The consumption-correlation output puzzle is no longer present but two puzzling features have replaced it, namely why Japanese output and Canadian consumption are so different from other countries?

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Table 1: Standard Deviations of the Cyclical Component of Real GDP and Consumption

a) 1965:1-2004.3		US	CA	FR	GE	IT	JA	UK
Output	**MN**	**2.52**	**2.03**	**1.58**	**1.53**	**1.66**	**1.89**	**1.73**
	HP	1.55	1.44	1.16	1.93	1.32	1.45	1.43
Consumption	**MN**	**2.44**	**0.13**	**1.03**	**2.40**	**1.92**	**0.58**	**2.62**
	HP	1.23	1.20	0.96	2.29	1.25	1.19	1.69

b) 1970:1-1990:2		US	CA	FR	GE	IT	JA	UK
Output	**MN**	**2.61**	**2.00**	**1.58**	**1.63**	**1.83**	**1.99**	**2.03**
	HP	1.86	1.46	0.91	1.43	1.50	1.47	1.64
Consumption	**MN**	**2.76**	**0.14**	**1.07**	**2.83**	**1.98**	**0.68**	**3.22**
	HP	1.52	1.23	0.95	1.36	1.27	1.46	1.88

Table 2: Cross-country Output and Consumption Correlations; 1965:1-2004:3.
(Upper triangle: Consumption; Lower triangle: Output)
(Underlined numbers indicate consumption correlation greater than output correlation)

a) MN Decomposition

	US	CA	FR	GE	IT	JA	UK
US		0.28	<u>0.29</u>	<u>0.34</u>	0.14	<u>0.15</u>	0.38
CA	0.61		0.03	0.13	0.07	<u>-0.01</u>	0.16
FR	0.28	0.41		0.62	0.54	0.42	<u>0.45</u>
GE	0.29	0.20	0.63		0.24	0.17	<u>0.15</u>
IT	0.35	0.40	0.63	0.46		0.40	<u>0.49</u>
JA	0.13	-0.04	0.46	0.62	0.47		<u>0.28</u>
UK	0.63	0.77	0.33	0.12	0.26	-0.01	

b) HP Filter

	US	CA	FR	GE	IT	JA	UK
US		0.53	<u>0.31</u>	0.00	0.09	0.30	0.46
CA	0.71		0.13	-0.26	0.20	0.08	<u>0.49</u>
FR	0.23	0.27		0.23	0.30	0.35	0.30
GE	0.13	-0.05	0.36		0.25	0.04	-0.15
IT	0.30	0.42	0.49	0.26		0.21	0.14
JA	0.34	0.15	0.36	0.36	0.35		<u>0.42</u>
UK	0.63	0.56	0.39	0.00	0.33	0.34	

Table 3: Cross-Country Output and Consumption Correlations; Sub-samples.
(Upper triangle: Consumption, Lower Triangle: Output)
(Underlined numbers indicate consumption correlation greater than output correlation)

a) Mixtures of Normals Decomposition

1965:3-1974:4

	US	CA	FR	GE	IT	JA	UK
US		$\underline{0.51}$	$\underline{0.61}$	$\underline{0.10}$	$\underline{0.13}$	$\underline{0.28}$	$\underline{0.37}$
CA	0.41		$\underline{0.45}$	0.08	-0.14	$\underline{0.01}$	0.04
FR	-0.47	0.31		$\underline{0.65}$	0.15	0.10	$\underline{0.58}$
GE	-0.12	0.34	0.41		-0.18	-0.25	$\underline{0.54}$
IT	0.03	0.07	0.35	-0.03		$\underline{0.42}$	$\underline{0.10}$
JA	-0.08	-0.06	0.21	0.56	0.32		$\underline{0.09}$
UK	0.27	0.08	-0.25	0.01	-0.74	-0.20	

1975:1-1984:4

	US	CA	FR	GE	IT	JA	UK
US		0.33	0.41	0.76	-0.39	0.26	-0.22
CA	0.88		-0.01	0.34	0.16	-0.14	0.23
FR	0.42	0.29		0.68	0.20	0.58	-0.03
GE	0.82	0.82	0.75		0.17	0.50	0.12
IT	0.48	0.55	0.75	0.82		0.15	$\underline{0.50}$
JA	0.35	0.34	0.85	0.74	0.77		$\underline{0.51}$
UK	0.84	0.77	0.30	0.68	0.26	0.11	

1985:1-1994:4

	US	CA	FR	GE	IT	JA	UK
US		0.38	0.06	$\underline{-0.43}$	0.00	-0.18	0.80
CA	0.97		-0.15	-0.12	-0.11	0.02	0.24
FR	0.23	0.22		$\underline{0.73}$	0.89	0.81	$\underline{0.56}$
GE	-0.45	-0.47	0.73		$\underline{0.61}$	0.68	$\underline{0.03}$
IT	0.45	0.49	0.91	0.49		$\underline{0.74}$	0.49
JA	-0.14	-0.14	0.88	0.89	0.69		$\underline{0.32}$
UK	0.93	0.97	0.28	-0.39	0.55	-0.08	

1995:1-2004:3

	US	CA	FR	GE	IT	JA	UK
US		0.15	$\underline{0.80}$	$\underline{0.72}$	$\underline{0.95}$	$\underline{0.13}$	$\underline{0.92}$
CA	0.64		-0.15	-0.03	0.25	$\underline{0.13}$	0.14
FR	0.53	0.94		$\underline{0.82}$	0.73	$\underline{0.00}$	$\underline{0.79}$
GE	0.49	0.46	0.59		0.71	$\underline{0.02}$	$\underline{0.59}$
IT	0.36	0.62	0.78	0.80		$\underline{0.05}$	$\underline{0.85}$
JA	-0.03	-0.49	-0.50	-0.16	-0.25		$\underline{0.12}$
UK	0.80	0.89	0.86	0.46	0.60	-0.23	

b) HP Filter

1965:3-1974:4

	US	CA	FR	GE	IT	JA	UK
US		0.53	$\underline{0.44}$	0.17	-0.07	$\underline{0.67}$	0.57
CA	0.68		$\underline{0.26}$	-0.19	-0.13	$\underline{0.24}$	0.45
FR	0.06	0.17		0.43	$\underline{0.30}$	$\underline{0.45}$	0.25
GE	0.18	0.22	0.45		-0.07	0.07	0.03
IT	0.12	0.22	0.25	0.00		$\underline{0.36}$	-0.04
JA	0.46	0.20	0.21	0.48	0.12		0.48
UK	0.69	0.51	0.29	0.39	0.14	0.52	

1975:1-1984:4

	US	CA	FR	GE	IT	JA	UK
US		0.47	0.34	0.61	-0.17	0.25	0.32
CA	0.78		-0.03	0.54	0.39	-0.06	0.27
FR	0.44	0.17		0.41	0.09	0.46	0.29
GE	0.73	0.65	0.74		0.44	0.51	0.62
IT	0.46	0.58	0.60	0.77		0.20	0.23
JA	0.45	0.42	0.71	0.70	0.63		$\underline{0.72}$
UK	0.62	0.35	0.67	0.65	0.38	0.41	

1985:1-1994:4

	US	CA	FR	GE	IT	JA	UK
US		0.76	-0.23	$\underline{-0.73}$	-0.42	-0.18	0.54
CA	0.87		0.25	-0.77	0.03	$\underline{0.14}$	0.81
FR	0.15	0.27		-0.07	0.64	0.56	$\underline{0.37}$
GE	-0.74	-0.72	0.07		0.28	-0.09	$\underline{-0.62}$
IT	0.26	0.40	0.83	0.07		0.50	0.02
JA	-0.05	0.05	0.76	0.27	0.68		$\underline{0.19}$
UK	0.73	0.89	0.30	-0.66	0.47	0.07	

1995:1-2004:3

	US	CA	FR	GE	IT	JA	UK
US		0.41	$\underline{0.63}$	$\underline{0.54}$	$\underline{0.73}$	-0.34	$\underline{0.61}$
CA	0.47		-0.05	0.10	$\underline{0.66}$	$\underline{-0.22}$	0.46
FR	0.35	0.75		0.76	0.38	-0.33	0.45
GE	0.43	0.74	0.87		0.43	-0.31	$\underline{0.57}$
IT	0.10	0.53	0.73	0.61		-0.50	$\underline{0.62}$
JA	0.31	-0.23	-0.18	-0.04	-0.04		-0.36
UK	0.57	0.48	0.54	0.55	0.59	0.40	

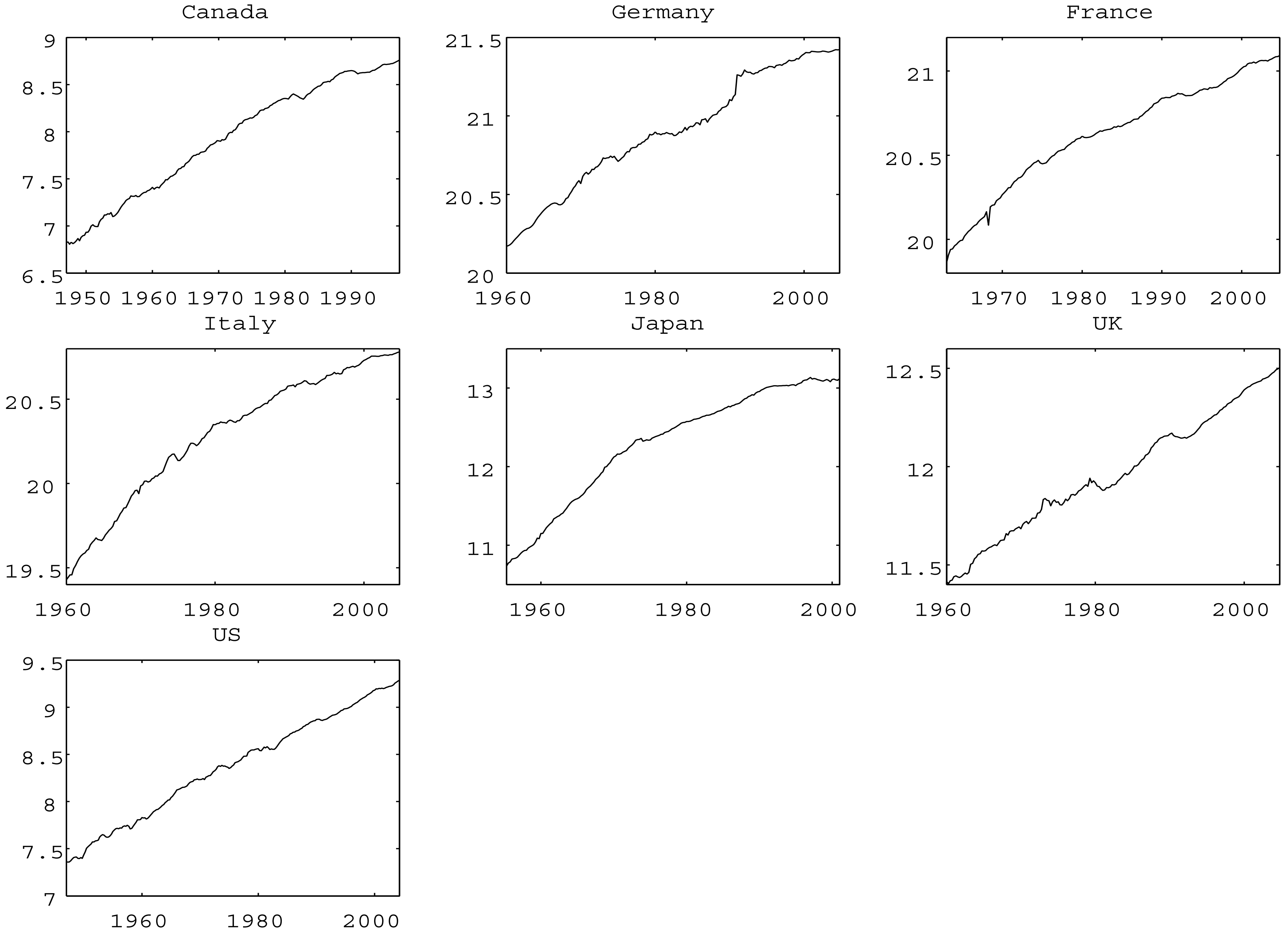

Figure 1: Postwar Quaterly (log) Real GDP Series for the G7 Countries.

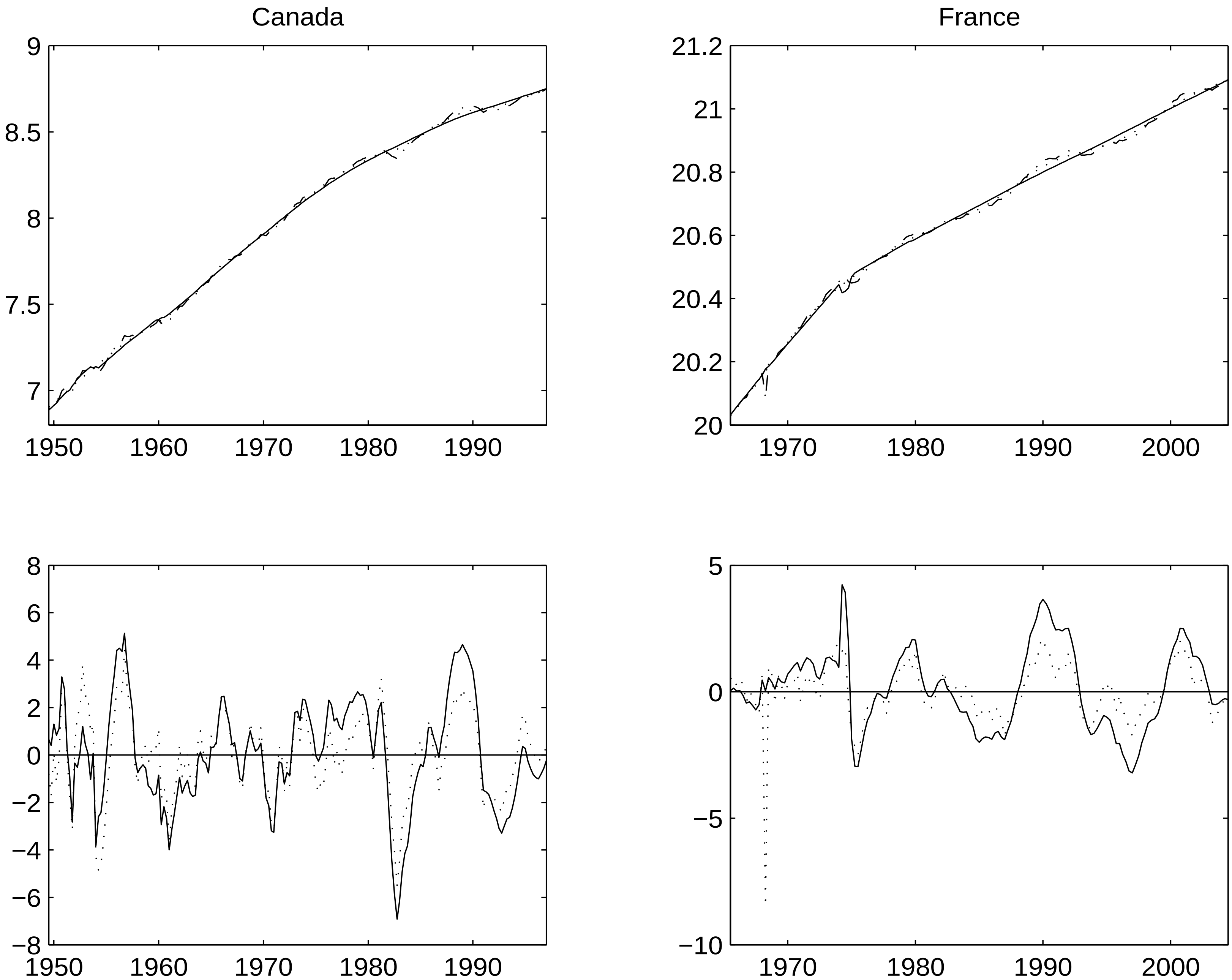

Figure 2: GDP of Canada and France: Mixture (solid), HP (dash), and data (broken line)

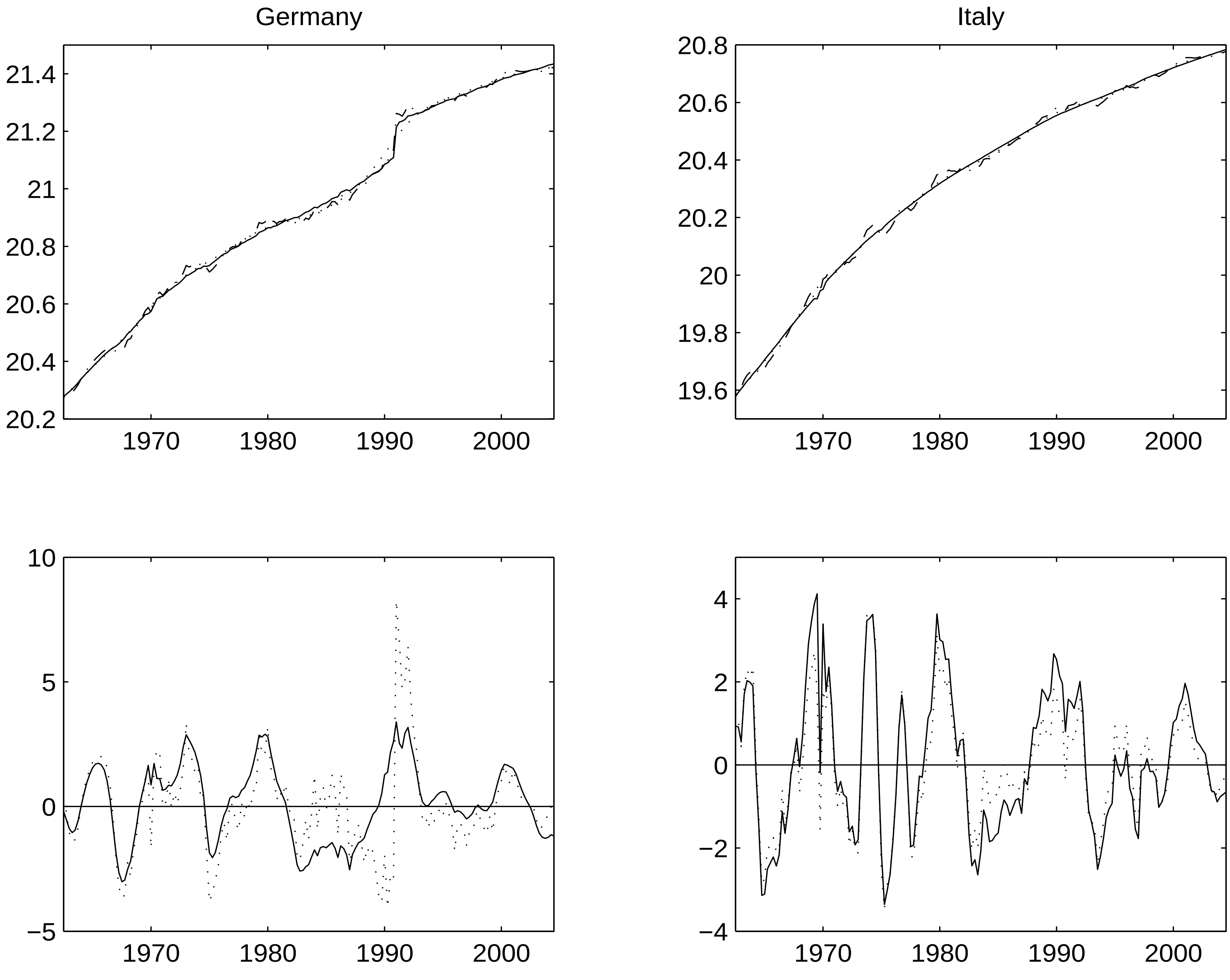

Figure 3: GDP of Germany and Italy: Mixture (solid), HP (dash), and data (broken line)

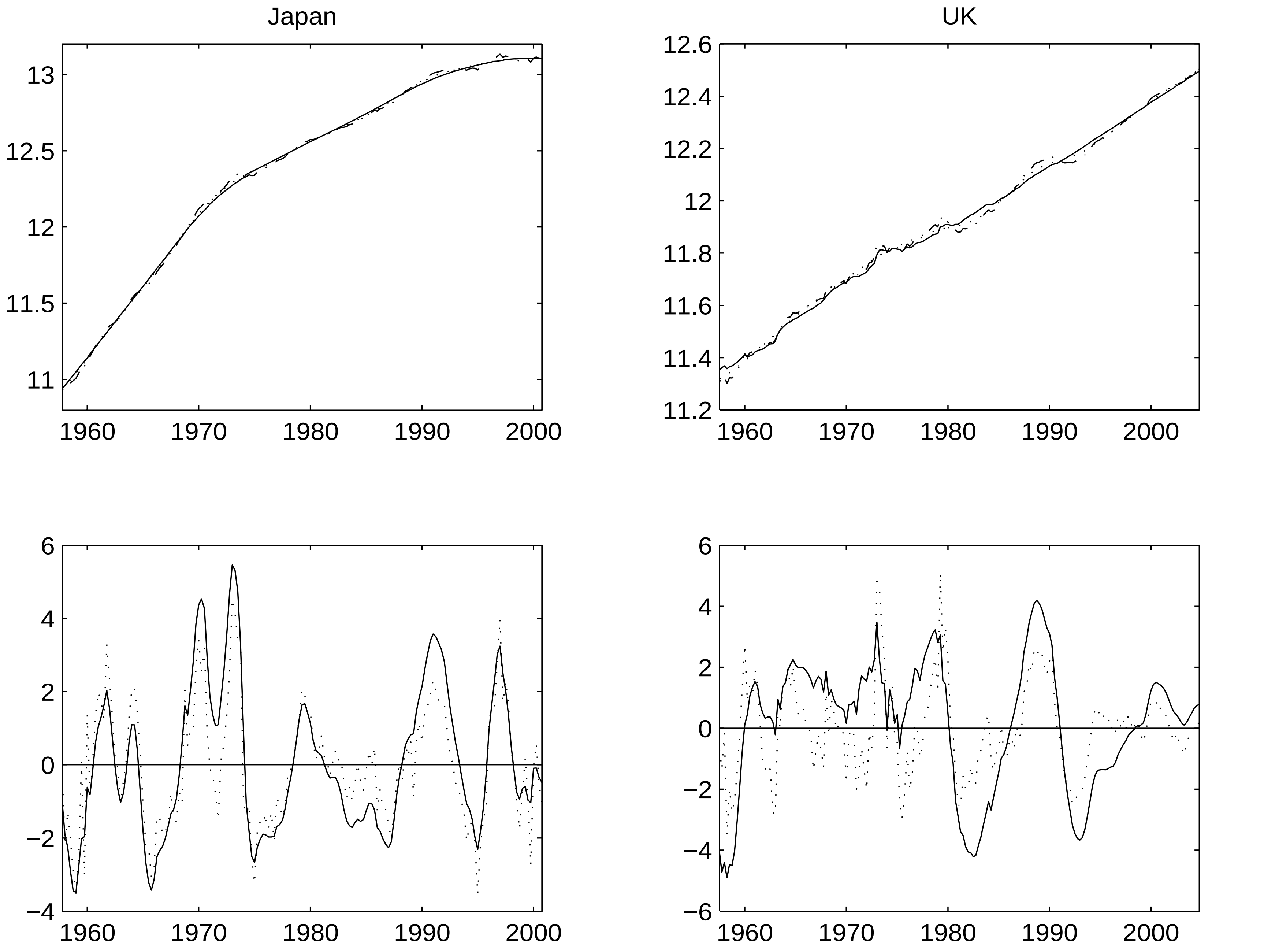

Figure 4: GDP of Japan and UK (solid), HP (dash), and data (broken line)

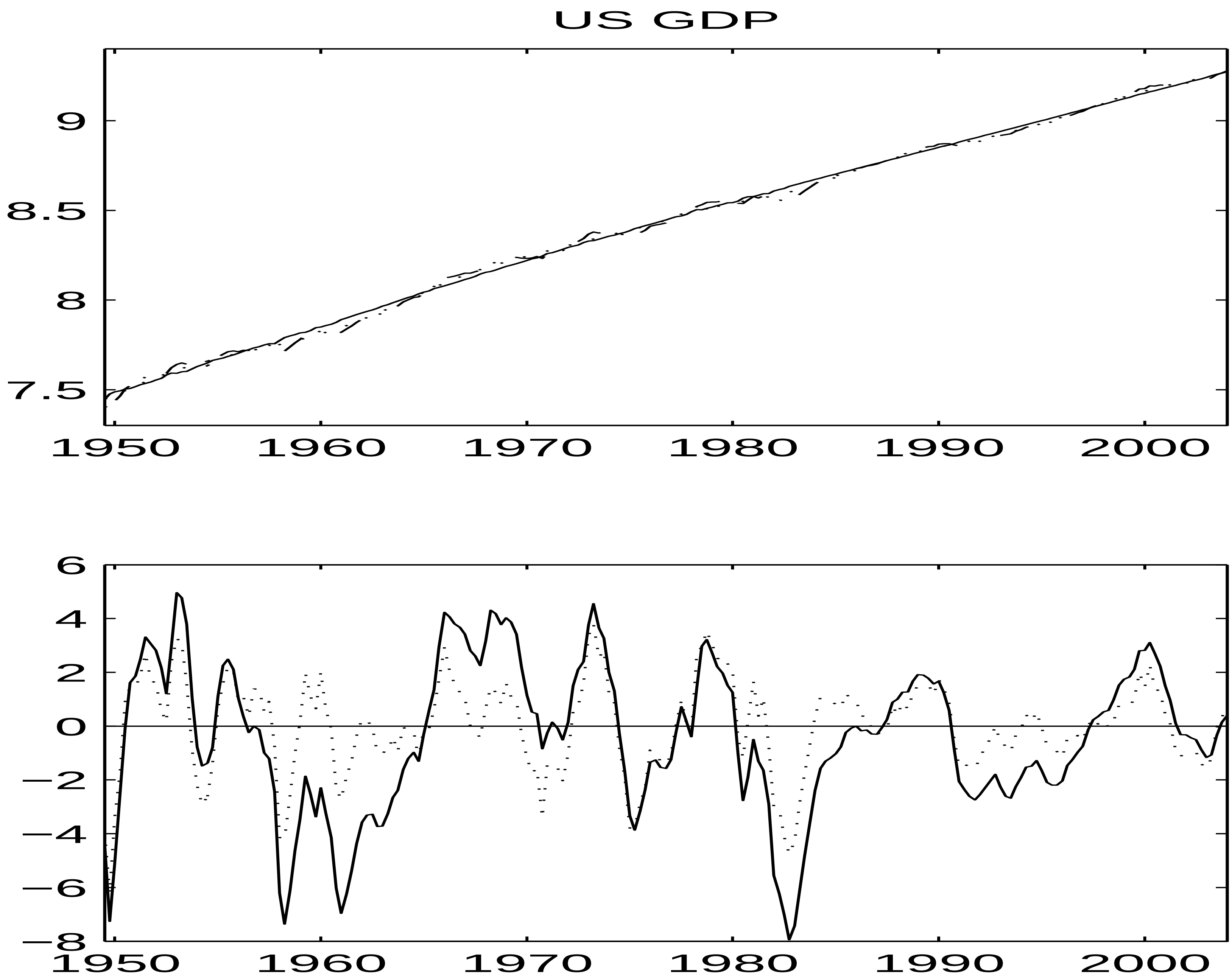

Figure 5: GDP of US: Mixture (solid), HP (dash), and data (broken line)

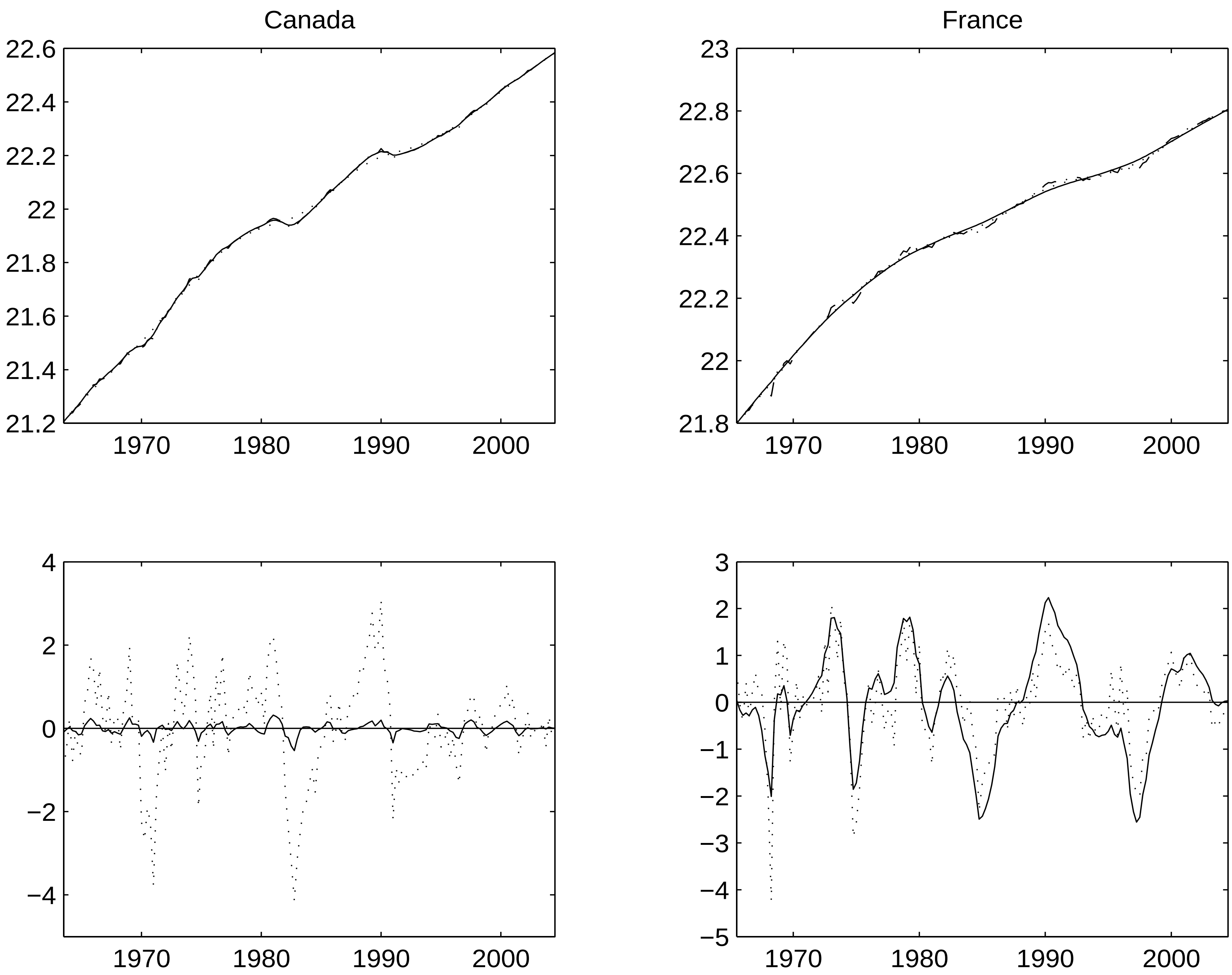

Figure 6: Consumption of Canada and France: Mixture (solid), HP (dash), and data (broken line)

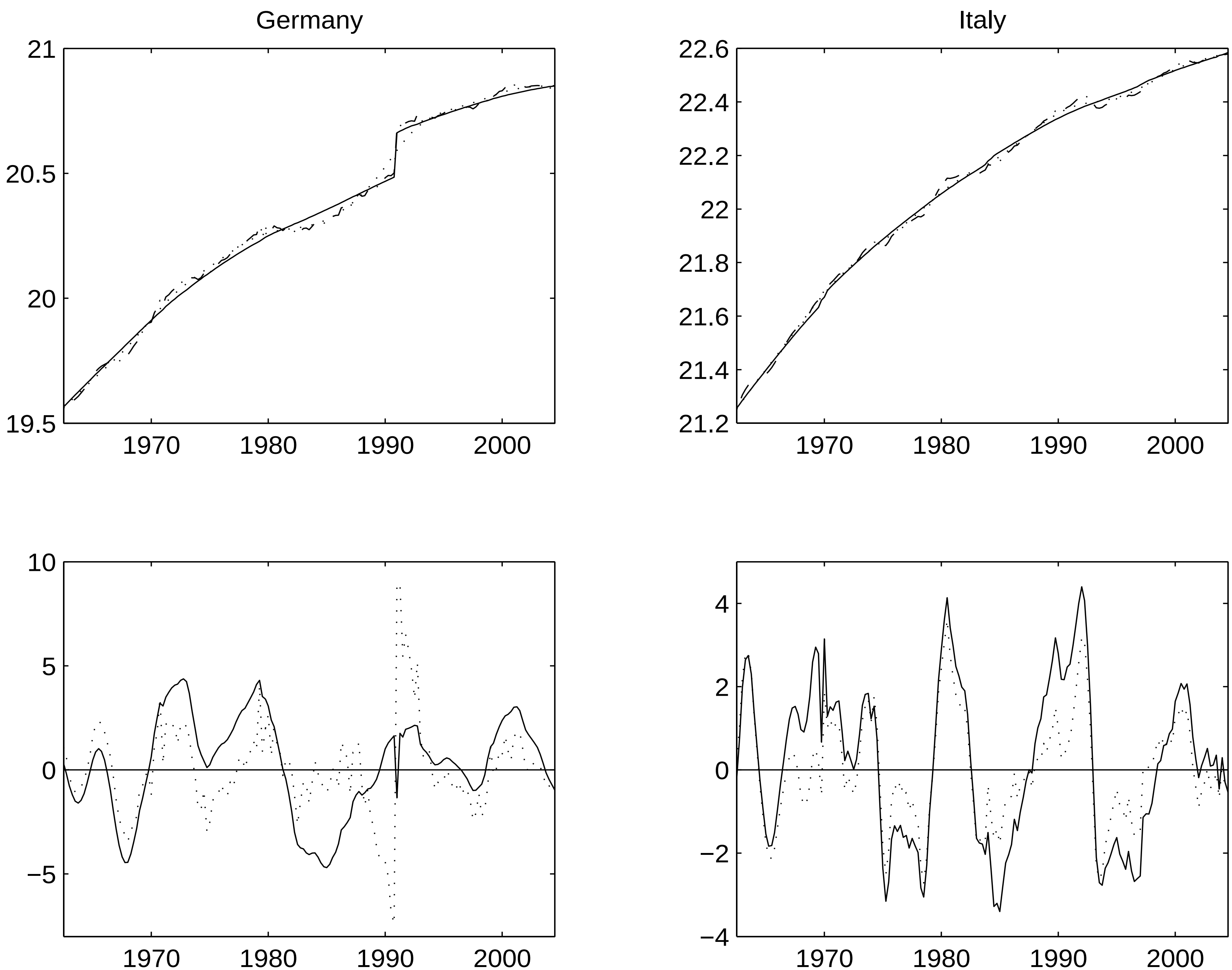

Figure 7: Consumption of Germany and Italy: Mixture (solid), HP (dash), and data (broken line)

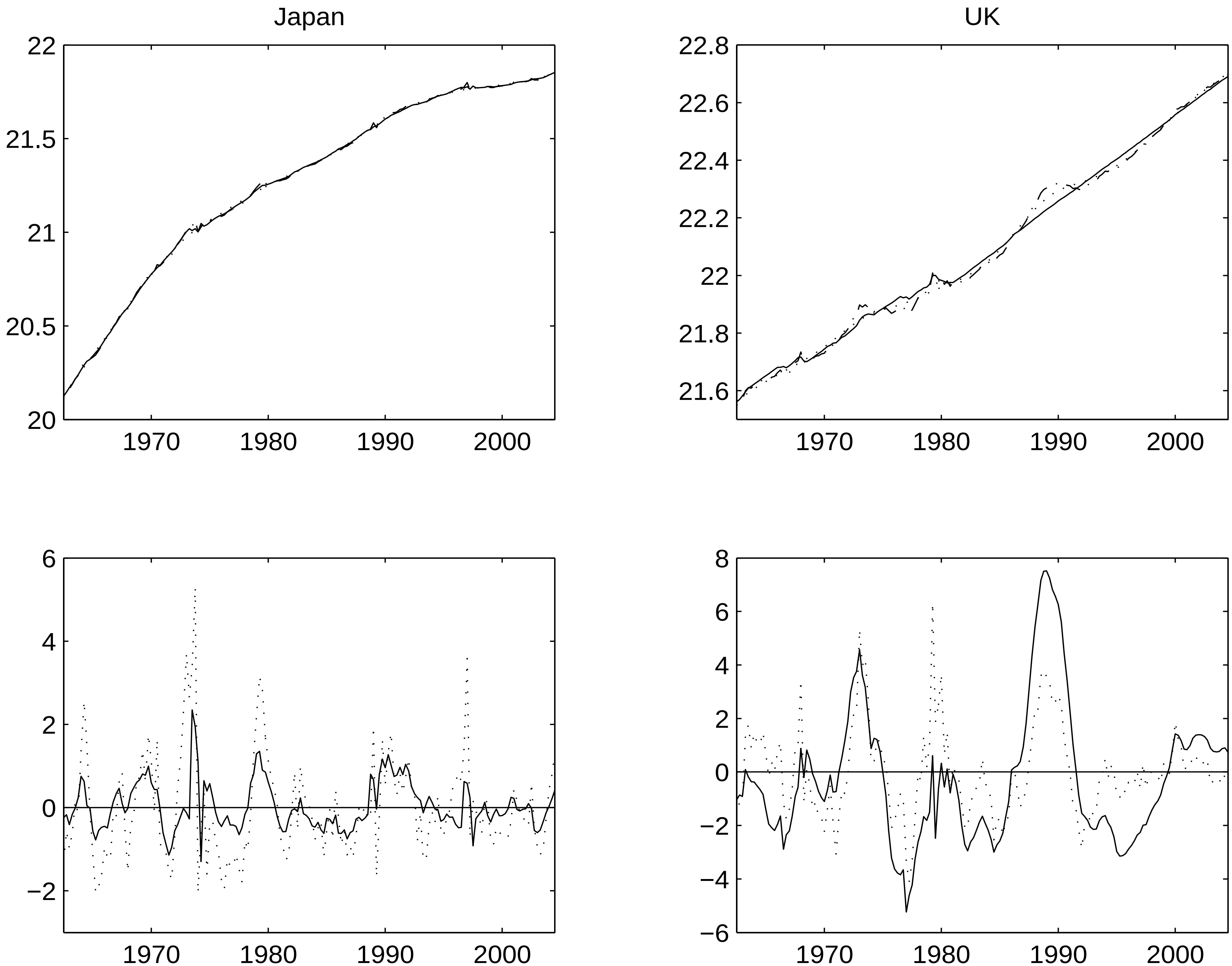

Figure 8: Consumption of Japan and UK: Mixture (solid), HP (dash), and data (broken line)

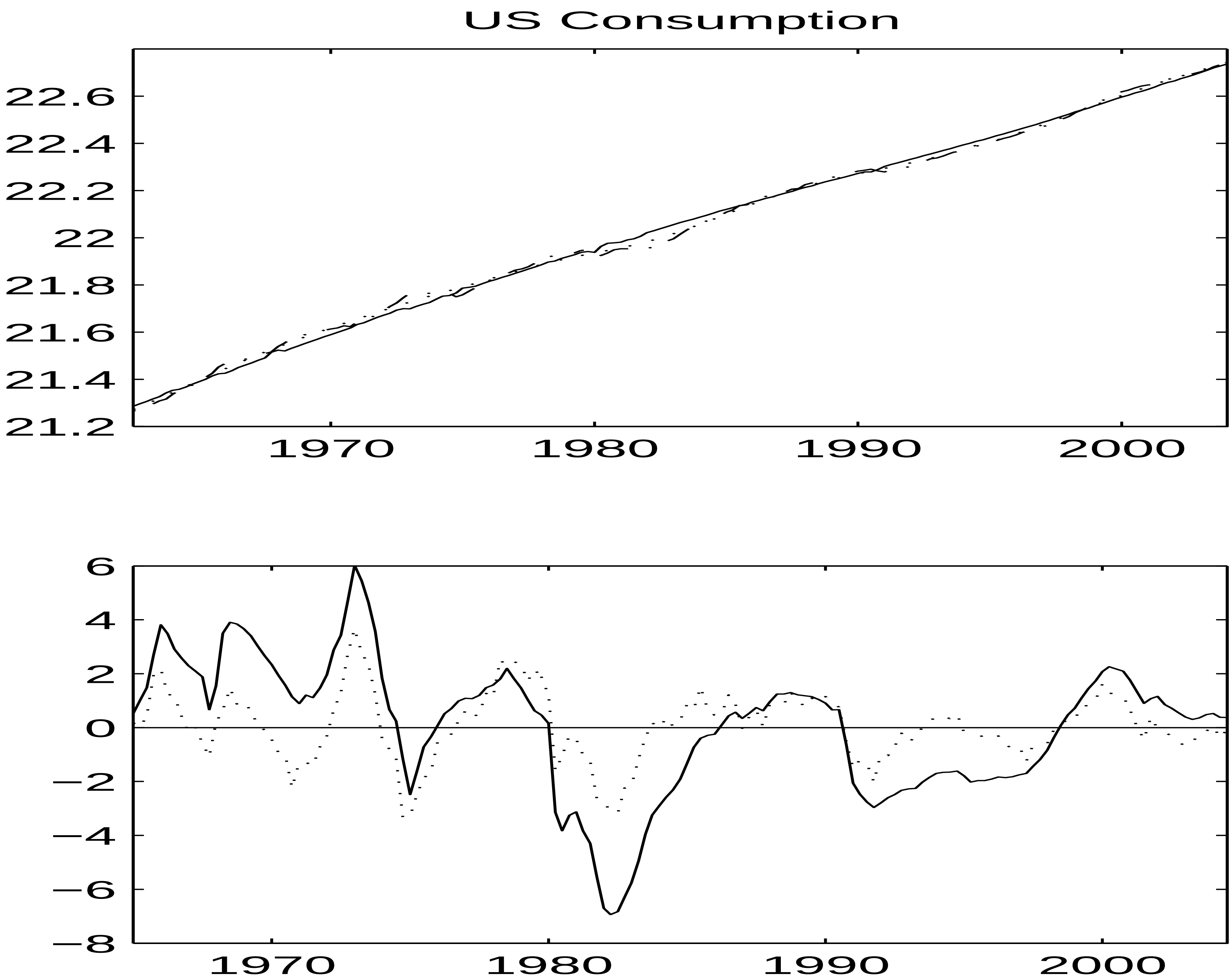

Figure 9: Consumption of US: Mixture (solid), HP (dash), and data (broken line)

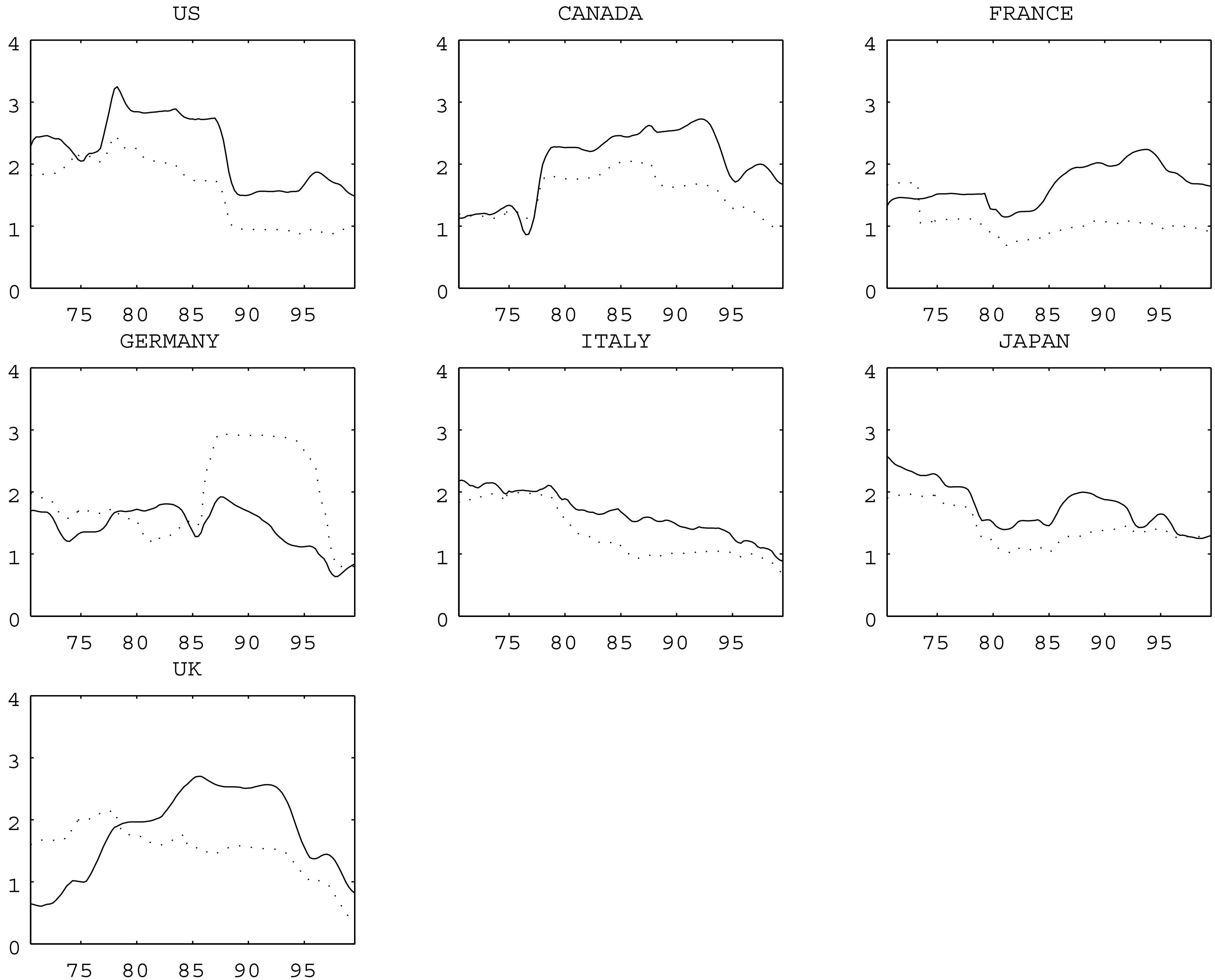

Figure 10: Standard Deviation of teh Cyclical Component of GDP: MN (solid) and HP (dash).

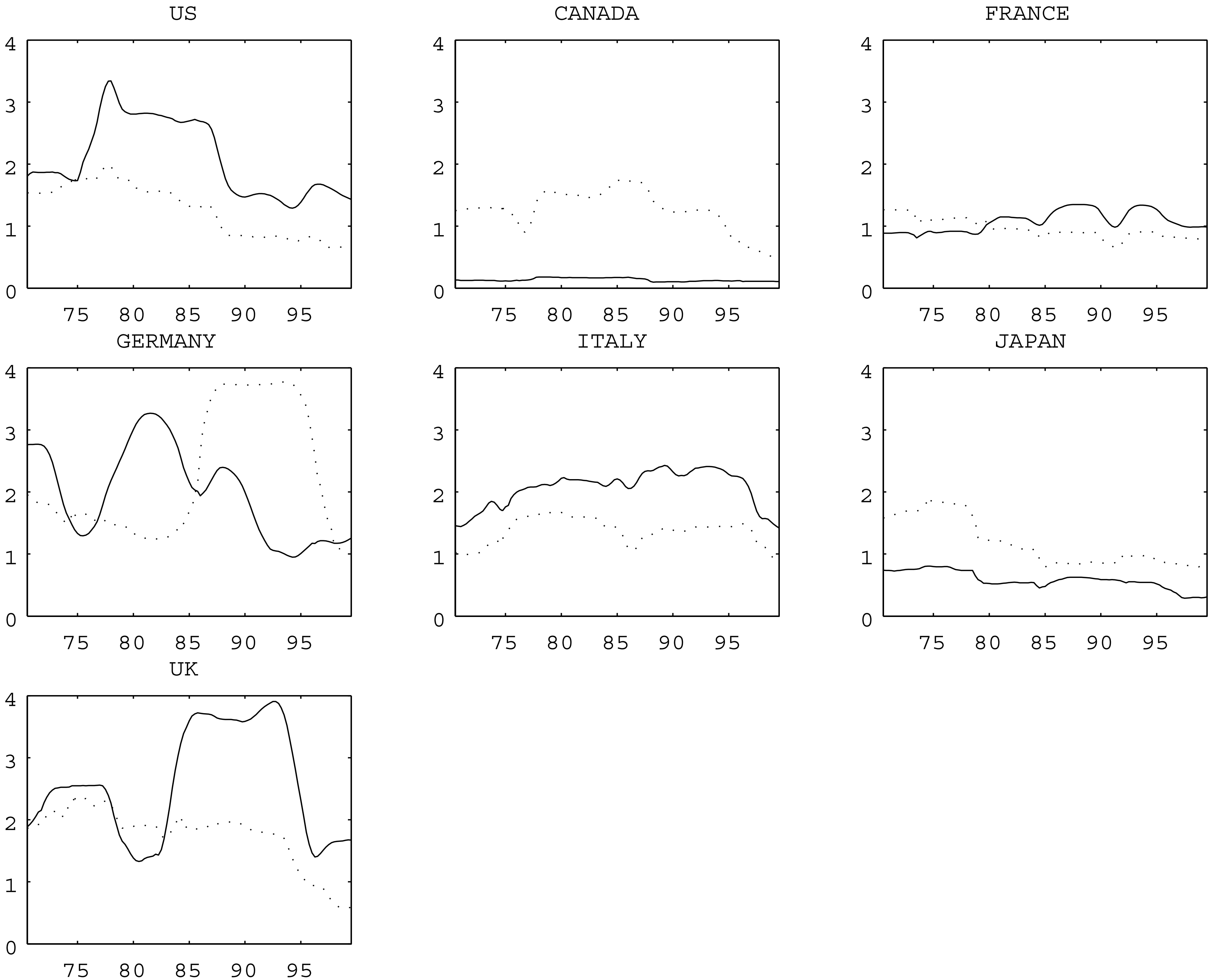

Figure 11: Standard Deviation of the Cyclical Component of Consumption: MN (solid) and HP (dash).

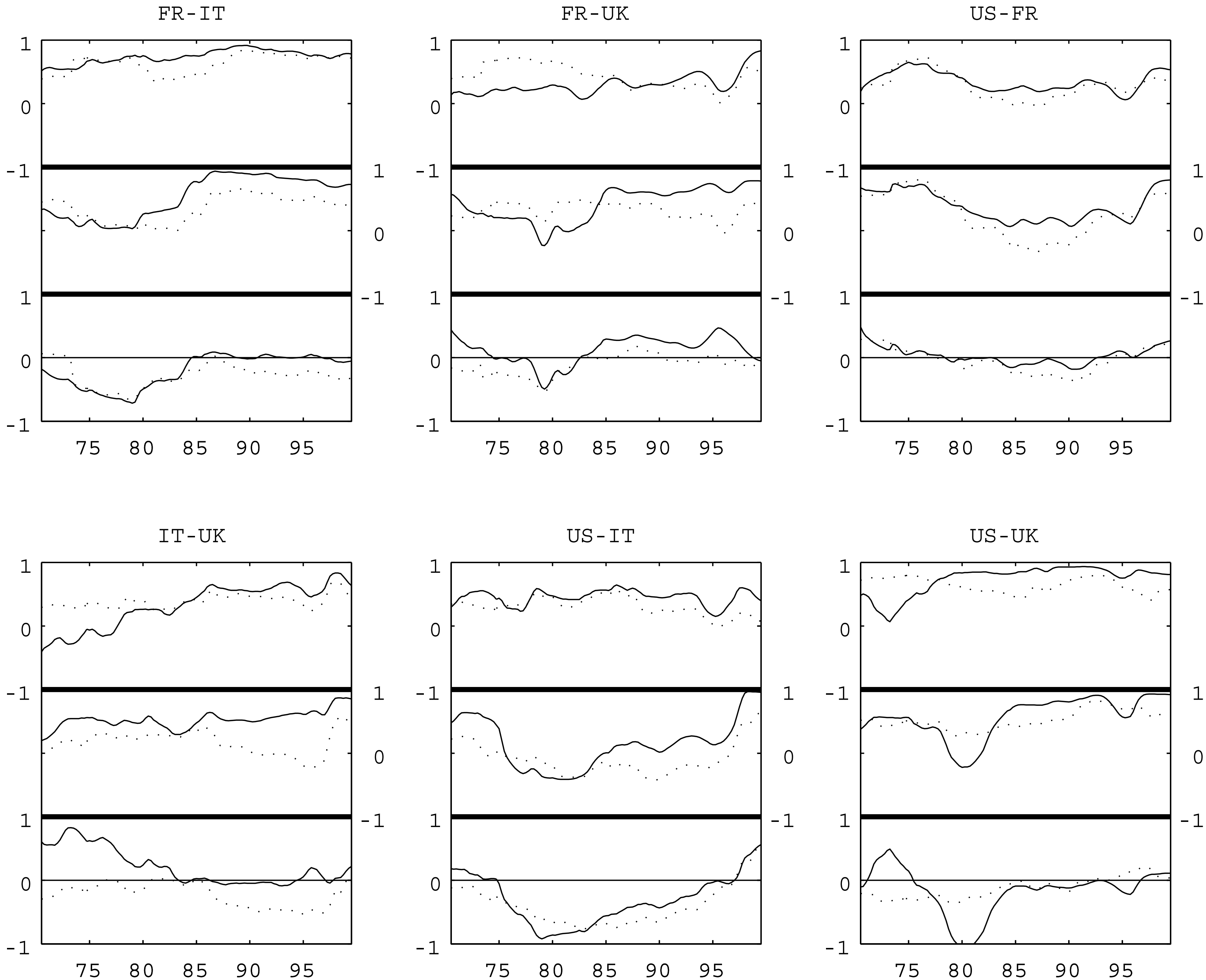

Figure 12: Correlations. Panels: Top (Output), Middle (Consumption), Bottom (difference, C-O). MN (solid) and HP (dash)

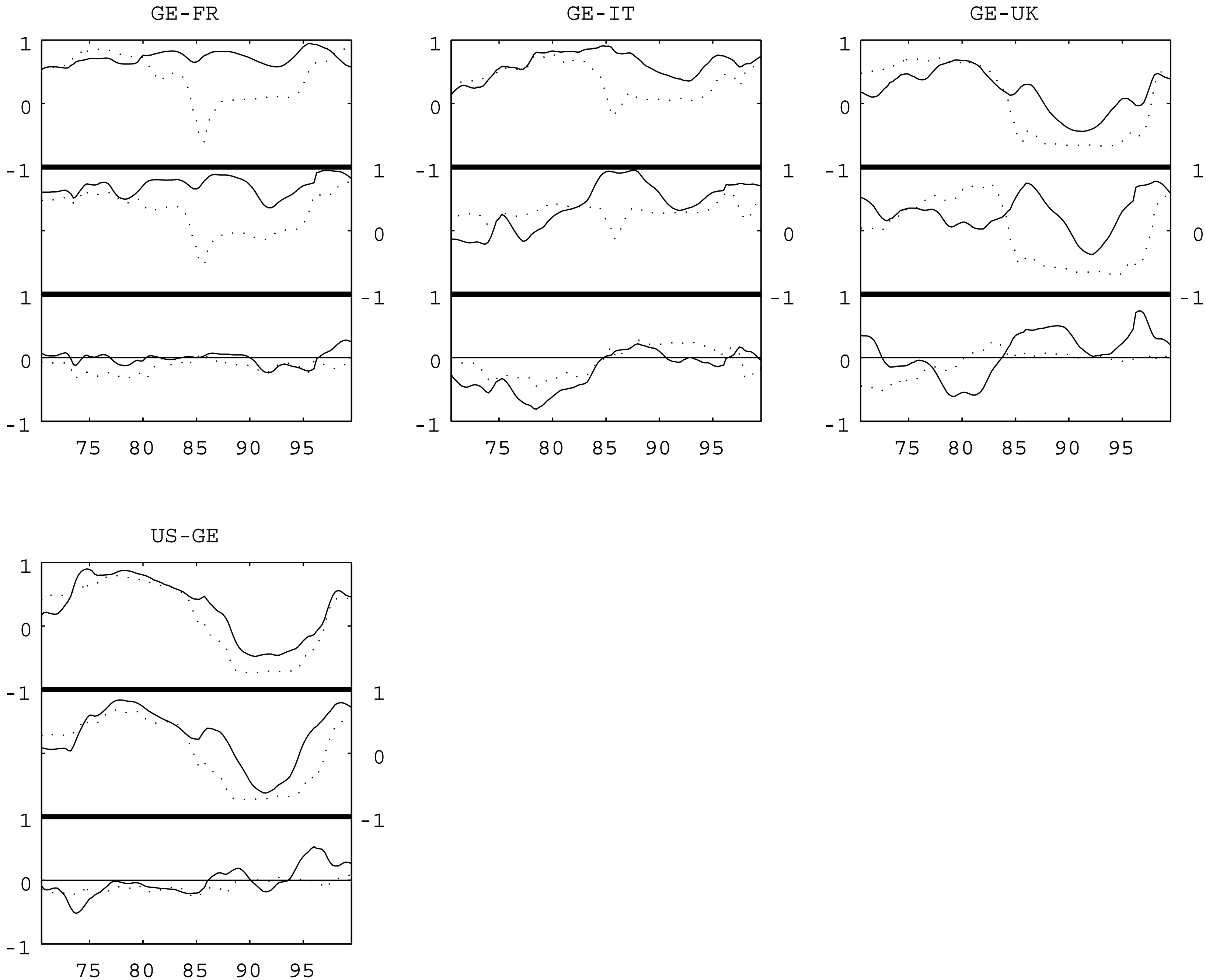

Figure 13: Correlations. Panels: Top (Output), Middle (Consumption), Bottom (difference, C-O). MN (solid) and HP (dash).

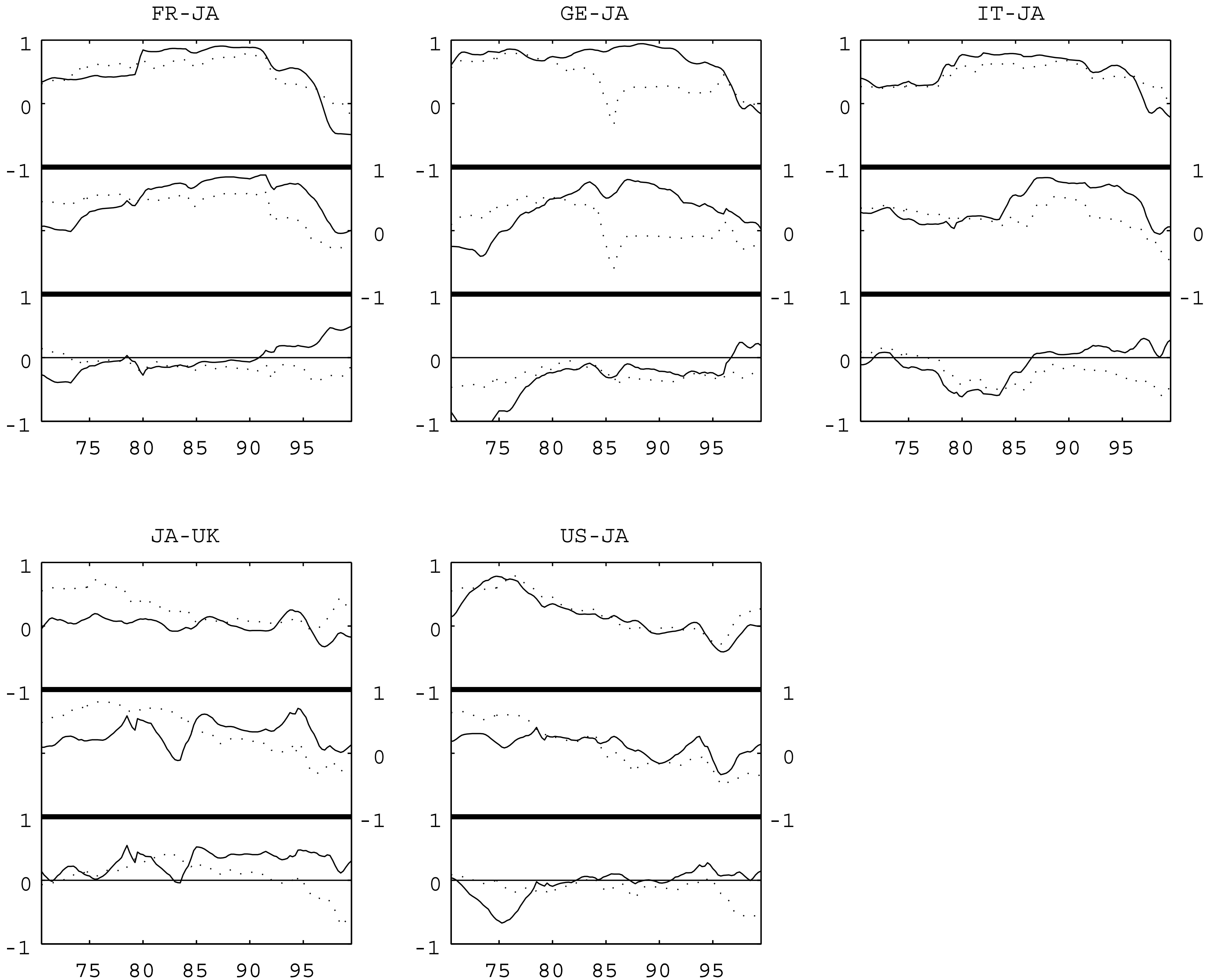

Figure 14: Correlations. Panels: Top (Output), Middle (Consumption), Bottom (difference, C-O). MN (solid) and HP (dash).

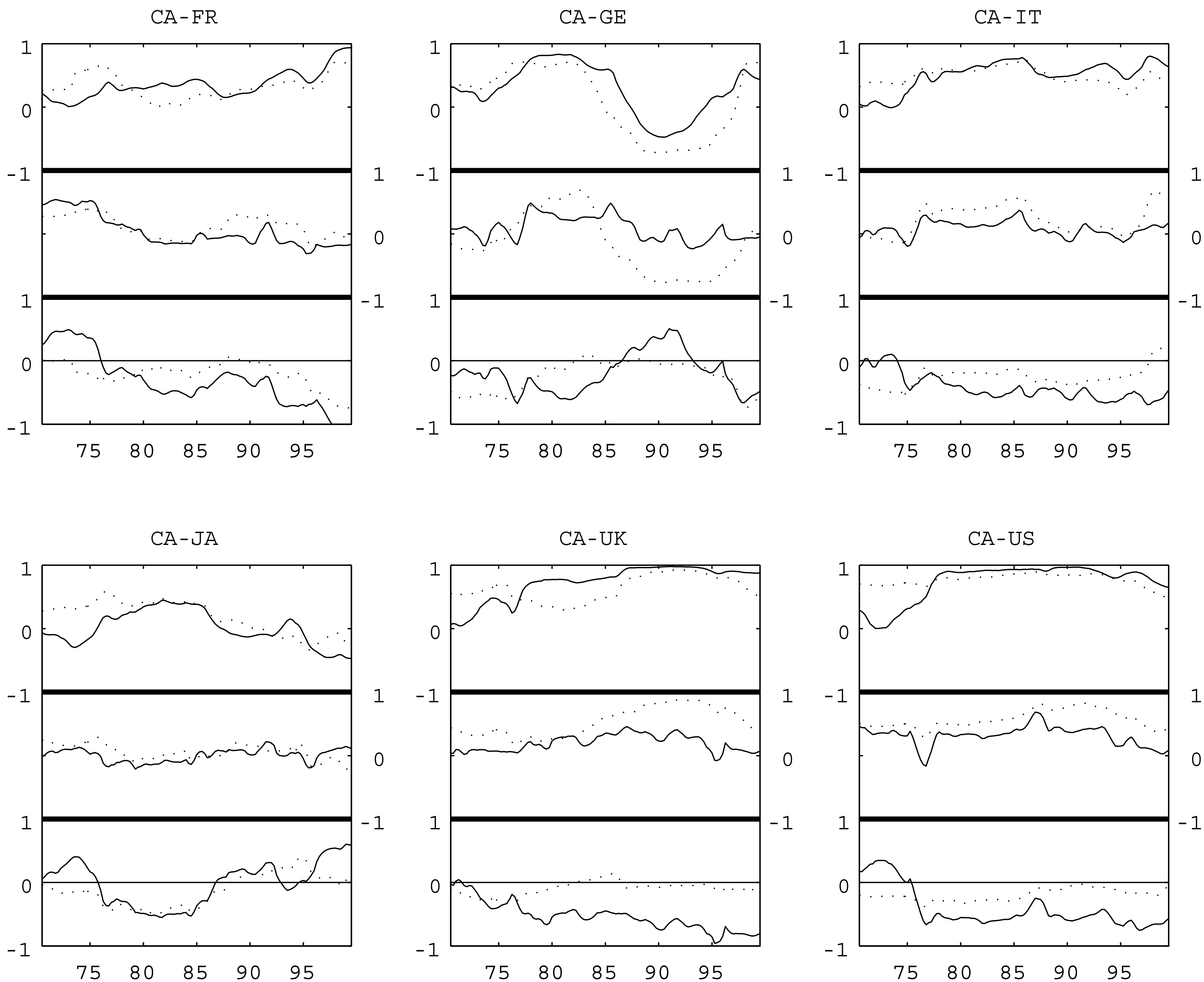

Figure 15: Correlations. Panels: Top (Output), Middle (Consumption), Bottom (difference, C-O). MN (solid) and HP (dash).