Choice and Normative Preference*

Jawwad Noor

Department of Economics,

Boston University,

270 Bay State Road, Boston MA 02215.

Email: jnoor@bu.edu.

October 17, 2005

*I would like to thank Larry Blume, John Duggan, Bart Lipman and particularly Larry Epstein for helpful comments. Any errors are my responsibility.

Abstract

Due to factors such as temptation, choices may not respect normative preference (the agent's own, subjective view of what constitutes his welfare). Nevertheless, the evidence on preference reversals suggests a means of recovering normative preference from choice. A definition of normative preference in terms of choices between sufficiently delayed alternatives is formulated and studied. Mild conditions on behavior are shown to ensure the existence of a normative preference. Two characterizations are provided. It is demonstrated that a notion of welfare may exist inspite of dynamic inconsistency of preferences. An application shows that the evidence on hyperbolic discounting implies that agents' normative discount functions must be exponential.

Keywords: Preference Reversals, Welfare, Temptation, Hyperbolic Discounting, Ethics.

JEL classification number: D11

1. Introduction

We all have the capacity to make normative judgments about our choices – think of an agent who is breaking his diet and is simultaneously saying to himself that he really should not. Such judgments express our personal view of what constitutes our welfare and well being, and are embedded in what we will refer to as the agent's *normative preference* – the ranking of alternatives that reflects the choices he thinks he *should* make.

Since normative preference contains the agent's own, subjective view of what constitutes his welfare, it is the natural guide for welfare policy. Thus, it is worth inquiring: what are the foundations of normative preference? Is normative preference observable? Despite being a cognitive notion, can it be characterized in terms of observables such as choice behavior? The interest in foundations arises from the fact that without them, one cannot form testable hypotheses about what constitutes an agent's welfare.

One approach to providing behavioral foundations is suggested by traditional revealed preference theory: identify normative preference with choice. Thus, an agent would be said to find alternative a normatively preferable to b if and only if, when given the opportunity, he would choose a over b. However, there is no reason to believe that choice necessarily respects normative preference. Choices are influenced by the desires and urges we experience, but normative preference may label some urges as 'bad'; a dieter may desire a burger despite judging it to be normatively inferior to a salad. Therefore it is not clear that revealed preference theory provides appropriate behavioral foundations for normative preference.

We argue that, nevertheless, normative preference may be recoverable from choice. This paper proposes an approach that is motivated by the evidence on *preference reversals*, a well-documented finding in the experimental psychology literature (see Ainslie [2] for a survey of the evidence). An example of a preference reversal is when an agent exhibits the following rankings at some given point in time:

$$\begin{aligned} (\$10,\ now) &\succ (\$15,\ 3\ days) \\ (\$10,\ 7\ days) &\prec (\$15,\ 10\ days). \end{aligned}$$

That is, the agent prefers the smaller earlier reward when it is available immediately, but reverses his preference in favor of the larger later reward when the rewards are pushed into the future by a common number of periods.[1]

Psychologists since Ainslie [1], Rachlin [20] and Rachlin and Green [21] have interpreted preference reversals in terms of a conflict between urges and the agent's "true preference"; see Ainslie [2]. A preference for the larger later reward when the rewards are far suggests that the agent finds the smaller earlier reward inferior, in some sense. However, his preferences switch in favor of this same inferior reward when it is available immediately. This tendency to choose the smaller reward when it becomes imminent is understood as an expression of a temptation.[2]

[1] For an example of a preference reversal that is of a different nature, consider Trope and Liberman [25], who find that subjects prefer watching a noneducational but entertaining movie now to an educational but unentertaining movie now, yet they switch preferences when either movie is to be watched at some given point in the future. Note that there is no temporal gap separating the rewards (that is, the movies), unlike in the example in the text. Also see Read and van Leeuwen [22].

[2] See Rubinstein [23], Fernandez-Villaverde and Mukherji [7] and Halevy [11] for alternative

Taking such an interpretation as given, our main observation is that preference reversals reveal that *delayed temptations are easier to resist than immediate temptations.* This is evident in the fact that although the temptation by the smaller immediate reward could not be resisted when it was available immediately, resisting it became possible when both rewards were (sufficiently) delayed. This suggests a way of deriving an agent's normative preference from his choices: elicit normative preference from choices between "sufficiently delayed" rewards.

This paper formalizes these ideas and presents them in the form of a behavioral definition of normative preference (Section 2). It shows that mild conditions on behavior ensure the existence of normative preference (Section 3). As an application, we inquire how agents would normatively choose to discount the future, given the evidence available on how they actually discount the future (Section 4). The relation with the literature on temptation and hyperbolic discounting is discussed, and it is shown that dynamic inconsistency of preferences need not imply the nonexistence of a notion of welfare (Section 5). All proofs are relegated to the appendices.

2. A Definition

Let $\mathcal{A}$ be a set of alternatives and suppose that the agent's choices at some fixed point in time are summarized by $\{\succsim_t\}_{t=0}^{\infty}$, a set of preference relations defined over $\mathcal{A}$. Each preference $\succsim_t$ ranks alternatives in $\mathcal{A}$ when these alternatives are delayed

explanations for preference reversals. We note, however, that these explanations apply only to cases where there exists a temporal gap between the two rewards. In particular, they cannot account for the kind of reversals mentioned in footnote 1.

by t periods.[3] If, as hypothesized in the introduction, delayed temptations are indeed easier to resist than immediate temptations, then as t grows, the influence of temptation on the agent's ranking $\succsim_t$ of alternatives diminishes. That is, as t grows, the temptation component underlying $\succsim_t$ becomes less significant, and so, each $\succsim_t$ provides an increasingly better approximation of the agent's underlying normative preference $\succsim^*$. For this reason, it is intuitive for normative preference to be identified with the (suitably defined) limit of the sequence $\{\succsim_t\}_{t=0}^{\infty}$:

$$\succsim^* \equiv \lim_{t\to\infty} \succsim_t .$$

This serves as a definition of normative preference. It formalizes the basic idea that normative preference may be revealed by *distancing* the agent from the consequences of his choices. If the distance is sufficiently large, the agent's choice between the rewards is not unduly affected by temptation, and thus, reveals his normative preference. We note in passing that while we focus on temporal distance in this paper, one can imagine other notions of distance. For instance, consider 'availability': an item may be more tempting the more easily available it is, and so, the 'farther' an item is in terms of availability, the less tempting it may be.

The validity of the definition of normative preference rests heavily on the interpretation of preference reversals in terms of temptation. A strong implicit assumption that underlies the interpretation (and that we inherit) is that normative preference is stationary. That is, the interpretation presumes that for all t, the agent finds it normatively better to consume a rather than b after t periods if and only if he finds it normatively better to consume a rather than b today. It

[3] Alternatively, one could take as primitive a single preference relation over the set $\mathcal{A}\times(\mathbb{N}\cup\{0\})$ of delayed rewards. However, this primitive embodies more information than we require.

is only under this assumption that one can make an inference about the agent's normative preference over alternatives to be consumed today by observing his ranking of alternatives to be consumed later. But is stationarity a reasonable assumption?

Stationarity of normative preference is not to be expected in general. An agent may find taking drugs now normatively acceptable if it serves a medicinal purpose, but unacceptable next week when there is no good reason to take drugs. In general, an agent's normative preference over two alternatives may depend on the "situation". But it is possible to deal with non-stationarity without departing from our framework: re-define the set of alternatives so that an alternative includes a situation as part of its description. For instance, 'taking drugs for medicinal purposes' may be such an alternative. With such a modification, the appeal of stationarity is reinstated since there is indeed a sense in which our assessment of what constitutes the 'correct' choice is stable, at least from the perspective of one point in time if not across time. An agent may believe today that consuming drugs is normatively unacceptable for non-medicinal purposes regardless of the date at which the drug is to be consumed.

We conclude with a clarification regarding the sense in which we consider normative preference to be the agent's subjective view of his welfare: what we call normative preference may not coincide with agents' self-reports about what they think is best for them. A smoker may say that abstinence is in his best interest, but we do not regard this as a reflection of his normative preference unless he somehow expresses this *behaviorally*, such as in his choices among relevant delayed alternatives. If an agent truly values his health, then this must reveal itself in *some* choice situation. A lack of any behavioral evidence of health-consciousness implies that he in fact places relatively higher value on things other than his health. Should a smoker who claims that he values his health be distinguished from another who behaves exactly like him in every choice situation, but does not claim that his health is a worthwhile concern?

3. Normative Preference

3.1. Existence

Let C be a compact metric space and let $\mathcal{A} = \Delta(C)$ denote the set of all probability measures on the Borel σ-algebra of C, endowed with the weak convergence topology; $\mathcal{A}$ is compact and metrizable [4]. Generic elements of $\mathcal{A}$ are μ, η, ν. For $\alpha \in [0,1]$ and $\mu, \eta \in \mathcal{A}$, the mixture $\alpha\mu + (1-\alpha)\eta \in \mathcal{A}$ is the measure that assigns $\alpha\mu(A) + (1-\alpha)\eta(A)$ to each A in the Borel σ-algebra of C. The primitive is $\{\succsim_t\}_{t=0}^{\infty}$, a set of preference relations over $\mathcal{A}$ where each preference $\succsim_t$ captures the ranking of alternatives in $\mathcal{A}$ when the alternatives are delayed by t periods. To explore existence and some basic properties of normative preference, consider the following axioms on $\{\succsim_t\}_{t=0}^{\infty}$. The first two are familiar.

Axiom 1 (Order). *$\succsim_t$ is complete and transitive for all t.*

Axiom 2 (Continuity). *The sets $\{\eta : \mu \succsim_t \eta\}$ and $\{\eta : \eta \succsim_t \mu\}$ are closed for all μ and t.*

The next axiom imposes the structure of preference reversals on $\{\succsim_t\}_{t=0}^{\infty}$.

Axiom 3 (Reversal). *If $\mu \prec_t \eta$ (resp. $\mu \precsim_t \eta$) and $\mu \succsim_{t'} \eta$ (resp. $\mu \succ_{t'} \eta$) for some $t' > t$, then $\mu \succsim_{t''} \eta$ (resp. $\mu \succ_{t''} \eta$) for all $t'' > t'$.*

Thus, if pushing a pair of rewards into the future changes its ranking, then the reversed ranking is maintained for all subsequent delays in the rewards. Following the evidence on preference reversals, the axiom imposes the restriction that there can be no more than one reversal for any pair of rewards. Note that, among the axioms considered in this section, Reversal is the only one restriction across the different preferences in $\{\succsim_t\}_{t=0}^{\infty}$.

The next axiom requires that the agent exhibit a preference reversal for at least one pair of rewards.

Axiom 4 (Non-triviality). *$\mu \succ_{t'} \eta$ and $\mu \prec_{t''} \eta$ for some μ,η and $t'' > t'$.*

The final axiom is the familiar independence axiom imposed on each $\succsim_t$.

Axiom 5 (Independence). *For all t,*

$$\mu \succ_t \eta \Longrightarrow \alpha\mu + (1-\alpha)\nu \succ_t \alpha\eta + (1-\alpha)\nu.$$

Theorem 3.1 identifies conditions that ensure the existence of a normative preference $\succsim^* \equiv \lim_{t\to\infty} \succsim_t$. To be precise, we must define a topology for the space of preferences over $\mathcal{A}$. Following Hildenbrand [13], identify any complete, transitive and continuous orders on $\mathcal{A}$ with its graph, a nonempty closed (hence compact) subset of the set $\mathcal{A}\times\mathcal{A}$ endowed with the product topology. Identify the space of such orders on $\mathcal{A}$ with a subset of $\mathcal{P} = \mathcal{K}(\mathcal{A}\times\mathcal{A})$, the space of nonempty compact subsets of $\mathcal{A}\times\mathcal{A}$ endowed with the Hausdorff metric topology. See Appendix A for details.

Say that $\succsim^*$ is *non-trivial* if $\mu \not\sim^* \eta$ for some μ, η, and that $\succsim^*$ satisfies *Independence* if for all μ, η, ν,

$$\mu \succ^* \eta \Longrightarrow \alpha\mu + (1-\alpha)\nu \succ^* \alpha\eta + (1-\alpha)\nu.$$

Theorem 3.1. *Suppose that* $\{\succsim_t\}$ *satisfies Order, Continuity and Reversal. Then normative preference* $\succsim^* \equiv \lim_{t\to\infty} \succsim_t$ *exists. Furthermore:*

(a) $\succsim^*$ *is complete, transitive and continuous.*

(b) $\succsim^*$ *satisfies Independence if* $\{\succsim_t\}$ *satisfies Independence.*

(c) $\succsim^*$ *is non-trivial if* $\{\succsim_t\}$ *satisfies Non-triviality.*

Thus, arguably mild conditions ensure the existence of a normative preference $\succsim^*$. To understand the intuition underlying the existence result, observe that Reversal requires that for every $\mu, \eta \in \mathcal{A}$, post-reversal preferences agree on the ranking of μ and η. This implies that for every $\mu, \eta \in \mathcal{A}$, there exists T such that all preferences $\succsim_t$, $t \geq T$, agree on the ranking of μ and η. In a sense, the difference between $\succsim_t$ and $\succsim_{t+1}$ decreases as t grows, since $\succsim_t$ and $\succsim_{t+1}$ agree on the ranking of more and more pairs of rewards.

3.2. Characterizations

A straightforward implication of Order and Reversal is the existence of a function $\tau : \mathcal{A} \times \mathcal{A} \to \mathbb{R}$ such that for each $(\mu, \eta) \in \mathcal{A} \times \mathcal{A}$, $\tau(\mu, \eta)$ is the number of periods that μ and η need to be delayed before a preference reversal is observed; if no reversal is observed, then $\tau(\mu, \eta) = 0$. For instance, if $\mu \succ_0 \eta$, $\mu \succsim_1 \eta$ and $\mu \prec_t \eta$ for all $t \geq 2$, then $\tau(\mu, \eta) = 2$. For a precise definition of τ, see Appendix B. Given τ, one can define a preference relation $\gtrapprox$ over $\mathcal{A}$ that captures the *post-reversal rankings*:

$$\mu \gtrapprox \eta \iff \mu \succsim_{\tau(\mu,\eta)} \eta. \tag{3.1}$$

The strict preference relation $>$ and the indifference relation $\approx$ are derived from $\gtrapprox$ in the usual way.

One might expect $\gtrapprox$ and $\succsim^*$ to coincide in general. After all, a preference reversal occurs when rewards are sufficiently pushed into the future, thereby causing temptation to become resistible. That is, the post-reversal ranking $\gtrapprox$ is presumably a temptation-free ranking, and thus, should coincide with normative preference $\succsim^*$. However, the following characterizations of normative preference reveal that this is not entirely true.

Let the set $\Omega \subset \mathcal{A} \times \mathcal{A}$ on which τ is upper semicontinuous be given by:

$$\Omega = \{(\mu, \eta) \in \mathcal{A} \times \mathcal{A} : \limsup_{n\to\infty} \tau(\mu_n, \eta_n) \leq \tau(\mu, \eta) \textit{ whenever } (\mu_n, \eta_n) \to (\mu, \eta)\}.$$

Theorem 3.2. *Suppose that $\{\succsim_t\}$ satisfies Order, Continuity and Reversal. Then, for any $\mu, \eta \in \mathcal{A}$, the following statements are equivalent:*

(a) $\mu \succ^* \eta$

(b) $\mu > \eta$ *and* $(\mu, \eta) \in \Omega$

(c) $(\mu_n, \eta_n) \to (\mu, \eta) \Longrightarrow \exists N$ *s.t.* $\forall n \geq N$, $\mu_n > \eta_n$.

The second characterization of $\succsim^*$ ($(a) \Leftrightarrow (c)$) states that $\mu \succ^* \eta$ if and only if there is a neighborhood of (μ, η) in which $\mu' > \eta'$ for each (μ', η') in the neighborhood. Observe that according to this characterization, $\mu \succ^* \eta$ implies $\mu > \eta$. However, the converse is not necessarily true, that is, $\succsim^* \neq \gtrapprox$ in general. The first characterization of $\succsim^*((a) \Leftrightarrow (b))$, which we interpret below, also reflects the non-equivalence of normative preference and post-reversal rankings.

Why may $\succsim^*$ differ from $\gtrapprox$ and what is the relevance of the upper semicontinuity of τ? The answer is best illustrated with an example. Suppose that

$$\mu \succ_t \eta \text{ for all } t,$$

so that $\tau(\mu, \eta) = 0$ and $\mu > \eta$. There are two stories consistent with the absence of a preference reversal. In the first story, there is a 'preference reversal in period 0': the agent's choice is not overwhelmed by temptation, and so, no preference reversal occurs. In such a case, we have both $\mu > \eta$ and $\mu \succ^* \eta$. In the second story, there is a 'preference reversal at infinity': the agent may be normatively indifferent, $\mu \sim^* \eta$, but may have a strict temptation preference for μ regardless of how distant it is. Consequently, no matter how far the rewards are pushed into the future, no preference reversal is observed. In this case, we have $\mu > \eta$ but $\mu \sim^* \eta$. That is, there is a switch from strict preference to indifference only at infinity.

One can distinguish between a preference reversal at zero and a preference reversal at infinity by checking the upper semicontinuity of τ. Upper semicontinuity fails for pairs of rewards at which a preference reversal at infinity occurs. The intuition, in the context of the example, is as follows. If there is a preference reversal at infinity for the pair (μ, η), then, due to normative indifference at (μ, η), temptation is 'infinitely strong' relative to normative preference. Continuity (Axiom 2) suggests that underlying temptation and normative preferences are continuous, and since normative indifference is a knife-edge case, in every neighborhood of (μ, η) one can expect to find a pair of rewards (μ_n, η_n) for which there is normative non-indifference, and in particular, for which temptation is overwhelming but not infinitely so. Since overwhelming temptation expresses itself in the form of a preference reversal, in every neighborhood of (μ, η) one can thus expect to find a pair of rewards (μ_n, η_n) for which $\tau(\mu_n, \eta_n) > 0$. Now, information about the strength of temptation is revealed by the value of $\tau(\mu_n, \eta_n)$; the larger the value of $\tau(\mu_n, \eta_n)$, the stronger the temptation since, in order to induce a preference reversal, the rewards μ_n, η_n have to be delayed by a larger number of periods. Furthermore, the closer (μ_n, η_n) is to (μ, η), the stronger the temptation, and therefore, the higher the value of $\tau(\mu_n, \eta_n)$.[4] But, in our example, $\tau(\mu, \eta) = 0$. Upper semicontinuity of τ is thus violated.

3.3. Elicitation

Our definition of normative preference $\succsim^*$ relies on a large amount of information, namely, $\{\succsim_t\}_{t=0}^{\infty}$. Eliciting the $\succsim^*$-ranking over any two alternatives apparently requires an infinite number of observations. On the other hand, the post-reversal ranking $\gtrapprox$ (defined by 3.1) of any two alternatives could require as few as two observations: if the $\succsim_0$-ranking differs from the $\succsim_t$-ranking for some arbitrarily large t, then we know that the $\gtrapprox$-ranking coincides with the $\succsim_t$-ranking. For instance, if $\mu \succ_0 \eta$ but $\eta \succ_{100} \mu$, then we know that $\eta > \mu$. Given that $\gtrapprox$ is easier to elicit than $\succsim^*$, one would like to know under what conditions the two coincide.

A condition is suggested by the characterization of normative preference $\succsim^*$ in Theorem 3.2. Denote by Λ the set of pairs of rewards for which there is post-reversal non-indifference:

$$\Lambda = \{(\mu, \eta) \in \mathcal{A} \times \mathcal{A} : \mu \not\approx \eta\}.$$

The following corollary of Theorem 3.2 provides a necessary and sufficient condition for $\gtrapprox = \succsim^*$.

Corollary 3.3. *Suppose that $\{\succsim_t\}$ satisfies Order, Continuity and Reversal. Then $\gtrapprox = \succsim^*$ if and only if τ is upper semicontinuous on Λ.*

[4] Appendix E shows formally that for any μ, η such that $\mu > \eta$ and $(\mu, \eta) \notin \Omega$, there exists a sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η) such that $\limsup_{n\to\infty} \tau(\mu_n, \eta_n) = \infty$.

We conclude by noting that a sufficient condition for τ to be upper semicontinuous on Λ is for it to be bounded.[5] That is, boundedness of τ rules out preference reversals at infinity and implies that $\gtrapprox$ coincides with $\succsim^*$.

4. An Application: Normative Discounting

Exponential discounting precludes preference reversals and thus, the evidence on preference reversals contradicts the usual assumption that agents discount future rewards using an exponential discount function $t \longmapsto \delta^t$. Rather, the evidence supports the hypothesis that people use discount functions that satisfy the property of *decreasing impatience* – agents exhibit less impatience when making choices between rewards that are farther in the future (for instance, they prefer \$15 in 10 days to \$10 in 7 days) rather than closer to the present (they prefer \$10 today to \$15 in 3 days). This section asks what such a discount function implies about an agent's *normative discount function*, that is, the discount function that he believes he should use.

In order to elicit the agent's normative view of how he should discount, we require data on how he ranks consumption streams. Let C be the set of lotteries over some compact metric space, endowed with the weak convergence topology, and let the space of $T+1$-period horizon consumption streams be given by$\mathcal{A} = C^T$, which is endowed with the product topology. We impose $T < \infty$ (see footnote

[5] To see this, prove the contrapositive: if τ is not upper semicontinuous on Λ, then there exists μ, η such that $\mu > \eta$ and $(\mu, \eta) \notin \Omega$. But then, as shown in Appendix E, there exists a sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η) such that $\limsup_{n\to\infty} \tau(\mu_n, \eta_n) = \infty$. Thus τ cannot be bounded.

6 for the reason). The primitive is $\{\succsim_n\}_{n=0}^{\infty}$, a set of preferences defined over $\mathcal{A}$ that captures the agent's ranking of delayed $T+1$-period consumption streams. More precisely, each $\succsim_n$ captures, from the perspective of some fixed period 0, the agent's ranking of streams of length $T+1$ that begin *after* n periods. We make three assumptions on $\{\succsim_n\}_{n=0}^{\infty}$:

Assumption 1 *There exist functions $u : C \to \mathbb{R}$ and $\phi : \mathbb{N} \cup \{0\} \to \mathbb{R}$ such that each $\succsim_n$ is represented by the utility function defined by:*

$$U_n(c_0, c_1, .., c_T) = \sum_{t=0}^{T} \phi(t+n)u(c_t), \tag{4.1}$$

for each $(c_0, c_1, .., c_T) \in C^{T+1}$.

That is, $\succsim_n$ is captured by a discounted utility model with discount function $\phi(\cdot)$. Note how n appears in the function U_n: since the consumption stream $(c_0, c_1, .., c_T)$ begins after n periods, the t^{th} element of the stream (that is, c_t) is in fact $t+n$ periods away. Thus, its utility is discounted by $\phi(t+n)$.

Assumption 2 *The instantaneous utility function $u(\cdot)$ is nonconstant, continuous and mixture-linear with $u \geq 0$ and $u(0) = 0$ for some $0 \in C$.*

This assumption requires no discussion.

Assumption 3 *The discount function $\phi(\cdot)$ is weakly decreasing in t, and $\frac{\phi(t+1)}{\phi(t)}$ is weakly increasing in t.*

The first part of the assumption states that a unit of consumption is worth less the farther it is. This has support from studies on time preference (see, for instance, the references in Fredrick, Lowenstein and O'Donoghue [9]) and accords well with introspection. The second part embodies (weakly) decreasing impatience since $\frac{\phi(t+1)}{\phi(t)}$ reflects how, from today's perspective, the agent weighs consumption in period $t+1$ relative to consumption in period t. The literature on preference reversals and decreasing impatience lends empirical support to this assumption. The class of discount functions that satisfy Assumption 3 includes both exponential discount functions δ^t, where $\delta \leq 1$, and generalized hyperbolic discount functions $\left(\frac{1}{1+\alpha t}\right)^{\frac{\lambda}{\alpha}}$, where $\alpha, \lambda > 0$ (Chung and Hernnstein [6], Lowenstein and Prelec [16]).

Though the agent uses the discount function $\phi(\cdot)$ when evaluating consumption streams, he may, for instance, be 'too' impatient in his own opinion. That is, due to a temptation by earlier gratification, the discount function that he uses may be different from his normative discount function, which is the discount function he believes he should use. We elicit the normative discount function from the normative preference over C^{T+1} :

$$\succsim^* \equiv \lim_{n \to \infty} \succsim_n .$$

The normative preference $\succsim^*$ reflects how he would rank streams in the absence of temptation. The normative discount function $\phi^* : \{0, 1, ..., T\} \to \mathbb{R}$ is inferred from the representation of $\succsim^*$.

Theorem 4.1. *Under Assumptions 1-3, the limits* $\succsim^* \equiv \lim_{n\to\infty} \succsim_n$ *and* $\gamma \equiv \lim_{n\to\infty} \frac{\phi(1+n)}{\phi(n)}$ *exist. Furthermore,* $\succsim^*$ *is represented by a utility function* $U^* : C^{T+1} \to R$ *defined by:*

$$U^*(c_0, c_1, .., c_T) = \sum_{t=0}^{T} \phi^*(t) u(c_t), \quad \textit{for any } (c_0, c_1, .., c_T) \in C^{T+1},$$

where $\phi^*(t) = \gamma^t$, *for any* $t \leq T$.

Thus, the assumptions of the Theorem lead to the sharp result that the normative discount function $\phi^*(\cdot)$ is well-defined and has an exponential form. Observe that although the assumption of exponential discounting in models of intertemporal choice has little empirical justification, the result bolsters the case that it nevertheless has normative justification: the result supports prescribing exponential discounting as a guide for choices.

What do commonly studied discount functions imply about the normative discount function? In what follows, we restrict the domain of $\phi(\cdot)$ to $\{0, 1, ..., T\}$ and implicitly assume the quantifiers 'for all δ, β', and 'for all $\alpha, \lambda > 0$' where appropriate.

- $\phi(t) = \delta^t$ implies $\phi^*(t) = \delta^t$.

That is, exponential discounting implies that there is no difference between an agent's actual and normative discount function. Consequently, the standard model precludes any temptation to discount the future in a manner that is different from that prescribed by normative preference.

- $\phi(t) = \begin{cases} 1 & \text{if } t = 0 \\ \beta\delta^t & \text{otherwise.} \end{cases}$ implies $\phi^*(t) = \delta^t$.

Thus, the quasi-hyperbolic discount function implies that the normative discount function sets $\beta = 1$. This is reminiscent of O' Donoghue and Rabin [19].

- $\phi(t) = \left(\frac{1}{1+\alpha t}\right)^{\frac{\lambda}{\alpha}}$ implies $\phi^*(t) = 1$.

This follows from

$$\gamma \equiv \lim_{n\to\infty} \frac{\phi(1+n)}{\phi(n)} = \lim_{n\to\infty} \left(\frac{1+\alpha n}{1+\alpha n+\alpha} \right)^{\frac{\lambda}{\alpha}} = \left(\lim_{n\to\infty} \frac{1+\alpha n}{1+\alpha n+\alpha} \right)^{\frac{\lambda}{\alpha}},$$

and the fact that $\lim_{n\to\infty} \frac{1+\alpha n}{1+\alpha n+\alpha} = 1$ by l'Hopital's rule.[6] The result states that the hyperbolic discount function $\left(\frac{1}{1+\alpha t}\right)^{\frac{\lambda}{\alpha}}$, a special case of which has been used to fit experimental data [17], implies that the agent believes he should weigh the present and the future equally. This holds regardless of the values of the parameters α, λ, that is, regardless of the agent's idiosyncrasies. To the extent that this conclusion may be considered too strong, it would be of interest to inquire whether agents' discount functions are better approximated by a function that is hyperbolic for a range of values of t, but exponential for all sufficiently large t.

5. Remarks

5.1. Dynamic Inconsistency and Welfare

The popular approach to modeling agents with self-control problems, arising from Strotz [24], is the multiple-selves approach (see Laibson [15]). An agent is viewed as consisting of temporal selves (one for each period) that game against each other, and each self t is typically assumed to have quasi-hyperbolic preferences over consumption streams, such as those represented by the utility function:

$$W_t(c_t, c_{t+1}, ...) = u(c_t) + \beta \sum_{\tau=1}^{\infty} \delta^{\tau} u(c_{t+\tau}), \tag{5.1}$$

for all consumption streams $(c_t, c_{t+1}, ...) \in C^{\infty}$, where C is a suitably defined space of alternatives and $\beta, \delta < 1$. The parameter β captures the temptation to

[6] The fact that $\phi^*(\cdot)$ could be 1 for an interesting class of discount functions led us to restrict attention to finite T. For $T = \infty$, U^* is not necessarily real-valued.

'over-discount' the future, and leads to non-exponential discounting in the model.

The model features *dynamic inconsistency*: self t may prefer \$15 in 10 days to \$10 in 7 days, but self $t+7$ may prefer \$10 today to \$15 in 3 days.[7] This feature leads to difficulties in conducting welfare analysis. Is the agent's welfare higher if he is given \$15 in 10 days, or \$10 in 7 days? There is no clear answer to this question since the current, period t self is better off with the larger later reward whereas the period $t+7$ self is better off with the smaller earlier reward. However, this presumes that self t's observed preferences over consumption streams (as represented by W_t) reveal what is best for his welfare, as in revealed preference theory. Indeed, given this assumption, dynamic inconsistency implies that the temporal selves may not agree on a notion of welfare, and that, therefore, there may not exist such a thing as "the agent's welfare".

However, if the model is indeed one that incorporates self-control problems, then it is not valid to assume that self t's observed preferences reflect his welfare. Moreover, rejecting this assumption makes it possible for a notion of welfare to exist despite dynamic inconsistency: even if the observed preferences of each self

[7] To define dynamic consistency precisely, for any stream $s \in C^\infty$ denote by s^{+n} the stream that gives some fixed consumption $\overline{c} \in C$ for the first n periods and the consumption dictated by s in all subsequent periods. For each period t, let $\succsim^t$ be a preference relation over C^∞. The set $\{\succsim^t\}_{t=0}^\infty$ is *dynamically consistent* if for all $t, n \in \mathbb{N} \cup \{0\}$ and streams $s, \underline{s} \in C^\infty$,

$$s^{+n} \succsim^t \underline{s}^{+n} \Longleftrightarrow s \succsim^{t+n} \underline{s}.$$

Viewing each $\succsim^t$ as the period t self, this states that self t prefers facing the stream s over the stream s' after n periods if and only if the self $t+n$ respects this and prefers s over s'. Dynamic inconsistency is the negation of dynamic consistency. See Ainslie and Haendel [3] and Read and van Leeuwen [22] for evidence of dynamic inconsistency disagree, it may nevertheless be the case that *the normative preferences of each self agree*. Take, for instance, quasi-hyperbolic preferences (5.1). One can verify that the normative preference of self t is represented by the utility function:[8]

$$U_t^*(c_t, c_{t+1}, c_{t+2}...) = u(c_t) + \sum_{\tau=1}^{\infty} \delta^\tau u(c_{t+\tau}).$$

Furthermore, the normative preferences of the selves are *dynamically consistent*: that is, self t has a normative preference for \$15 in 10 days over \$10 in 7 days if and only if self $t+7$ normatively prefers \$15 in 3 days over \$10 today. Thus, receiving \$15 in 10 days may be better than receiving \$10 in 7 days from the perspective of *both* self t and self $t+7$. Given dynamically consistent normative preferences, the normative preference of self 0 (as represented by U_0^*) serves as a guide for welfare policy. In particular, a notion of welfare exists for the agent inspite of dynamic inconsistency.[9]

[8] Keep with the notation in the earlier footnote and for each n, define $\succsim_n^t$ over C^∞ by $s \succsim_n^t \underline{s} \iff W_t(s^{+n}) \geq W_t(\underline{s}^{+n})$, for all $s, \underline{s} \in C^\infty$. Thus, $\succsim_n^t$ captures self t's ranking of streams delayed by n periods. Verify that $\succsim_n^t = \succsim_1^t$ for all $n \geq 1$, so that $\succsim_1^t = \lim_{n\to\infty} \succsim_n^t$. That is, self t's normative preference is $\succsim_1^t$.

[9] O' Donoghue and Rabin [19] propose a different way of deriving a welfare criterion for the quasi-hyperbolic model: they identify welfare with the preferences of a self residing in a fictitious ex-ante period where no choices are to be made and the undue bias for the present is absent [19, pg 113]. When each self's preferences are given by (5.1), their approach also delivers U_0^* as a welfare criterion.

5.2. Other Definitions

Gul and Pesendorfer [10] study a notion of normative preference (they refer to it as 'commitment preference') defined in terms of an ex-ante preference under commitment. We describe their definition and compare it to ours.

Gul and Pesendorfer focus on an agent's ranking of choice problems (menus). For some space of alternatives $\mathcal{A}$, the space of menus is given by the set of non-empty subsets of $\mathcal{A}$. Their model presumes a two period time-line: in period 0 the agent chooses a menu, and in period 1 he makes a choice from this menu. Gul and Pesendorfer identify normative preference over $\mathcal{A}$ with the agent's ranking of singleton menus, that is, his commitment preference. Thus, the agent has a normative preference for a vegetarian meal v in period 1 rather than a burger b if and only if he has a period 0 preference for going to the restaurant $\{v\}$ that serves only v rather than the restaurant $\{b\}$ that serves only burgers b. That is, the ex-ante (period 0) preference to commit to consuming v in period 1 rather than b reveals the normative preference for v over b.

Some differences between our definition of normative preference and this 'commitment definition' include the following.

- While our definition presumes stationarity of normative preference (Section 2), the commitment definition does not. To demonstrate, suppose $\mathcal{A} = C \times C$ and let $(v, b) \in \mathcal{A}$ represent an alternative that gives v in period 1 and b in some period 2. The commitment definition delivers a normative preference over $\mathcal{A}$, and it is possible for the agent to have a normative preference for (v, c) over (b, c), for any $c \in C$, and also a normative preference for (c, b) over (c, v), for any $c \in C$.

That is, normative preference over v and b can depend on the date of consumption. Note that stationarity is a testable hypothesis in this formulation, whereas it is a maintained hypothesis in ours. See, however, the discussion in the third bullet below.

- While our definition does not presume dynamic consistency of normative preference, the commitment definition does – period 1 normative preference is *assumed* to coincide with the agent's period 0 normative view of what he should have for dinner in period 1. Indeed, there is not even any mention of period 1 choice in the definition, which is based purely on period 0 choice. Thus, dynamic consistency of normative preference is not a testable hypothesis in this formulation, while it is in our formulation (see the previous subsection).

- The commitment definition also presumes that the agent's ranking of menus is not subject to temptation. It does not allow for the possibility that the agent may be tempted today by the *opportunity* of having a burger tomorrow, that is, it precludes the story in which agents may be tempted by the restaurant $\{b\}$. Such temptation may lead the agent to choose $\{b\}$ over $\{v\}$, in contrast to his normative preference. Thus, in order to apply the definition, an outside observer must first obtain an agent's ranking of menus in the absence of temptation. The data required is special, and it is not altogether obvious that it is available: to the extent that an agent may be tempted by restaurants, it is not clear that we can deduce how he would rank restaurants in the absence of temptation. In contrast, our definition is formulated in terms of choices that may well be subject to temptation, and thus the observability of our primitive is not in question.[10]

[10] In Noor [18] we show how foundations for Gul and Pesendorfer type models can nevertheless

5.3. Other Perspectives

An agent's view of what constitutes the best course of action may not be determined solely by a consideration of hedonic pleasure and pain, but also by his convictions regarding what is right or wrong. That is, ethics may be a part of normative preference.

In suitable settings, our definition of normative preference may be taken to be a definition of the agent's ethics. Consider, for instance, an Ultimatum game where a Proposer has a sense of distributional fairness, which is captured by an 'ethical preference' over the set of possible divisions of a certain sum of money between himself and the Decider. Let this set be denoted by $\mathcal{A}$ and imagine that, despite his ethics, the Proposer is tempted to act selfishly. Due to this temptation, his ethics may not be reflected in his choices between elements of $\mathcal{A}$. But, to the extent that distant temptations are easier to resist, his ethics may be revealed by how he ranks these elements when the outcome of the game is to be enforced in the future. Thus, our definition of normative preference can serve as definition of ethical preferences in this setting. In particular, ethical preferences can be given behavioral foundations.

A. Appendix: Topology on $\mathcal{P}$

Since is $\mathcal{A}$ is compact and metrizable, $\mathcal{A} \times \mathcal{A}$ is compact and metrizable under the product topology. Let d be a metric that generates the topology on $\mathcal{A} \times \mathcal{A}$. Denote the space of nonempty

be provided by making use of our definition of normative preference.

compact subsets of $\mathcal{A} \times \mathcal{A}$ by $\mathcal{P}$. For any $A, B \in \mathcal{P}$, let $d(a, B) = \inf_{b \in B} d(a, b)$ and $d(b, A) = \inf_{a \in A} d(b, a)$. The Hausdorff metric h_d induced by d is defined by

$$h_d(A, B) = \max\{\sup d(a, B), \sup d(b, A)\},$$

for all $A, B \in \mathcal{P}$. An ε-ball centered at A is defined by

$$B(A, \varepsilon) = \{B : h_d(A, B) < \varepsilon\}.$$

The Hausdorff metric topology on $\mathcal{P}$ is the topology for which the collection of balls $\{B(A, \varepsilon)\}_{A \in \mathcal{P}, \varepsilon \in (0, \infty)}$ is a base.

View the elements of $\mathcal{P}$ as binary relations on $\mathcal{A}$ by identifying a binary relation B on $\mathcal{A}$ with $\Gamma(B)$, the graph of B:

$$\Gamma(B) = \{(\mu, \eta) \in \mathcal{A} \times \mathcal{A} : \mu B \eta\}.$$

If B is a weak order (complete and transitive binary relation) then $\Gamma(B)$ is nonempty. If B is also continuous then $\Gamma(B)$ is closed, and hence compact.[11] Thus, the set of continuous weak orders on $\mathcal{A}$ is a subset of $\mathcal{P}$.

By [4, Thm 3.71(3)], compactness of $\mathcal{A} \times \mathcal{A}$ implies that $\mathcal{P}$ is compact. Also, under compactness of $\mathcal{A} \times \mathcal{A}$, $\Gamma(B)$ is the Hausdorff metric limit of a sequence $\{\Gamma(B_n)\} \subset \mathcal{P}$ if and only if $\Gamma(B)$ is the 'closed limit' of $\{\Gamma(B_n)\}$ [4, Thm 3.79]. To define the closed limit of a sequence $\{\Gamma(B_n)\}$, first define the topological limit superior $Ls\Gamma(B_n)$ and topological limit inferior $Li\Gamma(B_n)$ of the sequence $\{\Gamma(B_n)\}$:

$Ls\Gamma(B_n) = \{a \in \mathcal{A} \times \mathcal{A} :$ for every neighborhood V of a,

$$V \cap \Gamma(B_n) \neq \phi \text{ for infinitely many } n\}$$

[11] To show that $\Gamma(B)$ is closed if B is a continuous weak order, use [8, Lemma 5.1 and Exercise 3.16]. Note that the space of lotteries $\mathcal{A}$ is connected, and moreover, it is separable since it is compact metric.

$$Li\Gamma(B_n) = \{a \in \mathcal{A} \times \mathcal{A} : \text{for every neighborhood } V \text{ of } a,$$

$$V \cap \Gamma(B_n) \neq \phi \text{ for all but a finite number of } n\}.$$

The sequence $\{\Gamma(B_n)\}$ converges to a closed limit $\Gamma(B)$ if $\Gamma(B) = Ls\Gamma(B_n) = Li\Gamma(B_n)$.

B. Appendix: Definition of τ

For any $\{\succsim_t\}_{t=0}^{\infty}$ that satisfies Order (in fact, completeness of each $\succsim_t$ is all we need), define the function $\tau : \mathcal{A} \times \mathcal{A} \to \mathbb{R}$ that captures the time at which a reversal takes place for each (μ, η) in the following way. First consider any $(\mu, \eta) \in \mathcal{A} \times \mathcal{A}$ such that $\mu \succsim_0 \eta$. If $\mu \sim_t \eta$ for all t or $\mu \succ_t \eta$ for all t, then define $\tau(\mu, \eta) = 0$, and if there exists T such that $\mu \prec_T \eta$, then define

$$\tau(\mu, \eta) = \min\{t : \mu \prec_t \eta\}.$$

If $\mu \succ_0 \eta$, and there exists t such that $\mu \sim_t \eta$ and there is no t' such that $\mu \prec_{t'} \eta$, then define

$$\tau(\mu, \eta) = \min\{t : \mu \sim_t \eta\}.$$

Finally, let $\tau(\mu, \eta) = \tau(\eta, \mu)$ for all μ, η.

Lemma B.1. *Suppose $\{\succsim_t\}_{t=0}^{\infty}$ satisfies Order and Reversal and take any μ, η such that $\mu \succsim_0 \eta$. If $\tau(\mu, \eta) = 0$ then $\mu \sim_t \eta$ for all t or $\mu \succ_t \eta$ for all t. If $\tau(\mu, \eta) > 0$ then only one of the following holds:*

(a) $\mu \succ_t \eta$ for $t < \tau(\mu, \eta)$ and $\mu \prec_t \eta$ for all $t \geq \tau(\mu, \eta)$;

(b) $\mu \succ_t \eta$ for $t < \tau(\mu, \eta)$ and $\mu \sim_t \eta$ for all $t \geq \tau(\mu, \eta)$;

(c) There is $0 \leq T < \tau(\mu, \eta)$ such that $\mu \succ_t \eta$ for all $t < T$, $\mu \sim_t \eta$ for all $T \leq t < \tau(\mu, \eta)$, and $\mu \prec_t \eta$ for all $t \geq \tau(\mu, \eta)$.

Proof. The case with $\tau(\mu, \eta) = 0$ follows from the definition of $\tau(\cdot)$. For the second part, first consider the case where $\mu \succ_0 \eta$ and suppose $\tau(\mu, \eta) > 0$, so that there is t such that $\mu \precsim_t \eta$. Let

T^* be the first integer for which preferences reverse; thus, $\mu \succ_t \eta$ for $t < T^*$ and $\mu \precsim_{T^*} \eta$. By Reversal, we must have $\mu \precsim_t \eta$ for all $t \geq T^*$. If $\mu \prec_t \eta$ for all $t \geq T^*$, then given the definition of $\tau(\cdot)$, we have $\tau(\mu, \eta) = T^*$ and we are in case (a) in the statement of the Lemma. Similarly, if $\mu \sim_t \eta$ for all $t \geq T^*$ then we are in case (b).

If we are in neither case, then given that we must have $\mu \precsim_t \eta$ for all $t \geq T^*$, there exist $t', t'' \geq T^*$ such that $\mu \sim_{t'} \eta$ and $\mu \prec_{t''} \eta$. By Reversal, $\mu \succ_0 \eta$ and $\mu \prec_{t''} \eta$ implies $\mu \prec_t \eta$ for all $t \geq t''$, and so $t' < t''$. For the same reason, it must also be that $\mu \sim_{T^*} \eta$. Let T^{**} be the first integer larger than T^* for which indifference turns into strict preference, that is $\mu \sim_t \eta$ for $T^* \leq t < T^{**}$ and $\mu \prec_{T^{**}} \eta$. Reversal ensures $\mu \prec_t \eta$ for all $t \geq T^{**}$. Given the definition of $\tau(\cdot)$, we have $\tau(\mu, \eta) = T^{**}$ and we are in case (c).

The above established the result for μ, η such that $\mu \succ_0 \eta$ and $\tau(\mu, \eta) > 0$. Now consider μ, η such that $\mu \sim_0 \eta$ and $\tau(\mu, \eta) > 0$. Let T^* be the first integer for which preferences reverse, that is, $\mu \sim_t \eta$ for $t < T^*$ and wlog, $\mu \prec_{T^*} \eta$. By Reversal, $\mu \prec_t \eta$ for all $t \geq T^*$. By definition of of $\tau(\cdot)$, $\tau(\mu, \eta) = T^*$ and we are in case (c) in the Lemma. This completes the proof. ■

C. Appendix: Proof of Theorem 3.1

Assume that $\{\succsim_t\}_{t=0}^{\infty}$ satisfies Order, Continuity and Reversal, and define the time-of-reversal function $\tau(\cdot)$ as in Appendix B. Since each $\succsim_t$ is a continuous weak order, $\{\Gamma(\succsim_t)\}$ is a sequence in $\mathcal{P}$. We show that the closed limit of $\succsim_t$ as t goes to infinity is the preference $\succsim^*$ defined as follows: first define the preference relation $\gtrapprox$ over $\mathcal{A}$ that captures the post-reversal rankings by

$$\mu \gtrapprox \eta \Longleftrightarrow \mu \succsim_{\tau(\mu,\eta)} \eta,$$

and then let the preference order $\succsim^*$ over $\mathcal{A}$ be defined by

$$\mu \succsim^* \eta \Longleftrightarrow \exists \text{ sequence } \{(\mu_n, \eta_n)\} \text{ converging to } (\mu, \eta) \text{ s.t. } \forall n,\ \mu_n \gtrapprox \eta_n. \qquad \text{(C.1)}$$

Note that

$$\Gamma(\succsim^*) = \overline{\Gamma(\gtrapprox)},$$

where $\overline{\Gamma(\gtrapprox)}$ denotes the closure of the graph of $\gtrapprox$. Note also that the definition of $\succsim^*$ directly implies:

$$\mu \gtrapprox \eta \Longrightarrow \mu \succsim^* \eta \tag{C.2}$$

C.1. Proof of Existence

To establish the existence of a closed limit, it suffices to show that $Ls\Gamma(\succsim_t) \subset Li\Gamma(\succsim_t)$, since $Li\Gamma(\succsim_t) \subset Ls\Gamma(\succsim_t)$ always holds.

Step 1: $Ls\Gamma(\succsim_t) \subset \Gamma(\succsim^*)$.

Suppose $(\mu^*, \eta^*) \notin \Gamma(\succsim^*)$. Closedness of $\Gamma(\succsim^*)$ implies that there exists a neighborhood U_1 of (μ^*, η^*) such that $U_1 \cap \Gamma(\succsim^*) = \phi$. By the contrapositive of (C.2),

$$U_1 \cap \Gamma(\gtrapprox) = \phi. \tag{C.3}$$

In particular, $(\mu^*, \eta^*) \notin \Gamma(\succsim_{\tau^*})$ where $\tau^* \equiv \tau(\mu^*, \eta^*)$. However, $\Gamma(\succsim_{\tau^*})$ is closed and so there exists a neighborhood U_2 of (μ^*, η^*) such that

$$U_2 \cap \Gamma(\succsim_{\tau^*}) = \phi. \tag{C.4}$$

Let $V = U_1 \cap U_2$ and observe that V is a neighborhood of (μ^*, η^*).

Given (C.3), the post-reversal preference for any $(\mu, \eta) \in V$ ranks η strictly higher than μ, that is,

$$\mu \prec_{\tau(\mu,\eta)} \eta \text{ for all } (\mu, \eta) \in V. \tag{C.5}$$

But by (C.4) it is also true that $\mu \prec_{\tau^*} \eta$ for all $(\mu, \eta) \in V$. It follows from Reversal that

$$\tau(\mu, \eta) \leq \tau^* \text{ for all } (\mu, \eta) \in V,$$

and hence (C.5) implies $\mu \prec_t \eta$ for all $(\mu, \eta) \in V$ and $t \geq \tau^*$. That is, $V \cap \Gamma(\succsim_t) = \phi$ for all $t \geq \tau^*$. Conclude that $(\mu^*, \eta^*) \notin Ls\Gamma(\succsim_t)$ since V is a neighborhood of (μ^*, η^*) that does not intersect with infinitely many $\succsim_t$.

Step 2: $\Gamma(\succsim^*) \subset Li\Gamma(\succsim_t)$.

First show that $\Gamma(\gtrapprox) \subset Li\Gamma(\succsim_t)$. Observe that if $(\mu, \eta) \in \Gamma(\gtrapprox)$, then $(\mu, \eta) \in \Gamma(\succsim_t)$ for all $t \geq \tau(\mu, \eta)$. Hence for every neighborhood V of (μ, η), $V \cap \Gamma(\succsim_t) \neq \phi$ for all but a finite number of t. It follows that $(\mu, \eta) \in Li\Gamma(\succsim_t)$, thus establishing that $\Gamma(\gtrapprox) \subset Li\Gamma(\succsim_t)$. To complete the proof of Step 2, note that from the closedness of $Li\Gamma(\succsim_t)$ (see [4, Lemma 3.67]), it follows that $\overline{\Gamma(\gtrapprox)} \subset Li\Gamma(\succsim_t)$. But $\Gamma(\succsim^*) = \overline{\Gamma(\gtrapprox)}$. The assertion follows.

By Steps 1 and 2, $Ls\Gamma(\succsim_t) \subset \Gamma(\succsim^*) \subset Li\Gamma(\succsim_t)$. Hence,

$$Li\Gamma(\succsim_t) = Ls\Gamma(\succsim_t) = \Gamma(\succsim^*).$$

This completes the proof.

C.2. Proof of (a)

To establish completeness, take any μ, η and suppose $\eta \not\succsim^* \mu$. By (C.2), $\eta \not\gtrapprox \mu$, and by completeness of $\gtrapprox$, $\mu \gtrapprox \eta$.[12] Then, again by (C.2), $\mu \succsim^* \eta$. To establish transitivity, take any take any μ, η, ν and suppose $\mu \succsim^* \eta \succsim^* \nu$. Then by definition of $\succsim^*$, there exist sequences $\{(\mu_n, \eta_n)\}, \{(\eta_n, \nu_n)\}$ such that $(\mu_n, \eta_n) \to (\mu, \eta)$ and $(\eta_n, \nu_n) \to (\eta, \nu)$, and for each n, $\mu_n \gtrapprox \eta_n$ and $\eta_n \gtrapprox \nu_n$. By Lemma B.1, $\mu_n \succsim_t \eta_n \succsim_t \nu_n$ for all $t \geq T \equiv \max\{\tau(\mu_n, \eta_n), \tau(\eta_n, \nu_n)\}$ and so by transitivity of $\succsim_t$, $\mu_n \succsim_t \nu_n$ for all $t \geq T$, implying that for each n, $\mu_n \gtrapprox \nu_n$. It follows that $\{(\mu_n, \nu_n)\}$ is a sequence that converges to (μ, ν) and $\mu_n \gtrapprox \nu_n$ for all n. By definition of $\succsim^*$, $\mu \succsim^* v$, thus establishing transitivity of $\succsim^*$.

To establish continuity, we show that $\{\eta : \eta \succsim^* \mu\}$ is closed; the other case holds by an analogous argument. Take a sequence $\{\nu_n\}$ such that $\nu_n \succsim^* \mu$ for all n and $\nu_n \to \nu$. Also, for each i let $V_i \subset \mathcal{A} \times \mathcal{A}$ be a ball of radius 2^{-i} that contains (ν, μ). Because $\nu_n \to \nu$, for every i there exists n such that $(\nu_n, \mu) \in V_i$. Furthermore, $\nu_n \succsim^* \mu$ and the definition of $\succsim^*$ imply

[12] Completeness of $\gtrapprox$ follows from Order.

the existence a sequence $\{(\nu'_m, \mu'_m)\}$ such that $(\nu'_m, \mu'_m) \to (\nu_n, \mu)$ and $\nu'_m \gtrapprox \mu'_m$ for all m. Since V_i is also a neighborhood of (ν_n, μ), for each i there exists m_i such that $(\nu'_{m_i}, \mu'_{m_i}) \in V_i$. By construction, $\nu'_{m_i} \gtrapprox \mu'_{m_i}$ for each i and furthermore, $(\nu'_{m_i}, \mu'_{m_i}) \to (\nu, \mu)$ as $i \to \infty$. Thus $\nu \succsim^* \mu$, as desired.

C.3. Proof of (b)

This is proved in 5 steps, and will make use of Lemma D.1 below. The set of points on which τ is upper semicontinuous is denoted by Ω and is defined in the next Appendix.

Step 1: $\mu \succ_t \eta \Longleftrightarrow \mu\alpha\nu \succ_t \eta\alpha\nu$, for all t.

Axioms 1, 2 and 4 together imply this stronger version of Independence*.

Step 2: $\tau(\mu, \eta) = \tau(\mu\alpha\nu, \eta\alpha\nu)$ and $\mu \succ_{\tau(\mu,\eta)} \eta \Longleftrightarrow \mu\alpha\nu \succ_{\tau(\mu\alpha\nu,\eta\alpha\nu)} \eta\alpha\nu$.

This follows from Step 1.

Step 3: $(\mu, \eta) \notin \Omega \Longrightarrow (\mu\alpha\nu, \eta\alpha\nu) \notin \Omega$.

If $\{(\mu_n, \eta_n)\}$ is a sequence that converges to (μ, η) and

$$\limsup_{n\to\infty} \tau(\mu_n, \eta_n) > \tau(\mu, \eta),$$

then $\{(\mu_n\alpha\nu, \eta_n\alpha\nu)\}$ is a sequence that converges to $(\mu\alpha\nu, \eta\alpha\nu)$ and, by the first assertion in Step 2,

$$\limsup_{n\to\infty} \tau(\mu_n\alpha\nu, \eta_n\alpha\nu) > \tau(\mu\alpha\nu, \eta\alpha\nu).$$

Thus, $(\mu\alpha\nu, \eta\alpha\nu) \notin \Omega$.

Step 4: $\mu \sim^* \eta \Longrightarrow \mu\alpha\nu \sim^* \eta\alpha\nu$.

Suppose $\mu \sim^* \eta$. Then

$\mu \sim^* \eta$

$\Longrightarrow \mu \sim_{\tau(\mu,\eta)} \eta$ or $(\mu, \eta) \notin \Omega$ by Lemma D.1 below

$\Longrightarrow \mu\alpha\nu \sim_{\tau(\mu\alpha\nu,\eta\alpha\nu)} \eta\alpha\nu$ or $(\mu\alpha\nu, \eta\alpha\nu) \notin \Omega$ by Steps 2 and 3

$\Longrightarrow \mu\alpha\nu \sim^* \eta\alpha\nu, \qquad$ as desired.

Step 5: $\mu \succ^* \eta \Longrightarrow \mu\alpha\nu \succ^* \eta\alpha\nu$.

By the main theorem in Herstein and Milnor [12], under order and continuity of $\succsim^*$, Step 4 implies the result.

C.4. Proof of (c)

As in the proof of (b), we make use of Lemma D.1 and the set of points Ω on which τ is upper semicontinuous; see the next Appendix. We show that $\mu \succ_{t'} \eta$ and $\mu \prec_{t''} \eta$ for $t'' > t'$ implies $\mu < \eta$ and $(\mu, \eta) \in \Omega$. It then follows from Lemma D.1 below that $\mu \prec^* \eta$, as desired. Given $\mu \succ_{t'} \eta$ and $\mu \prec_{t''} \eta$ for $t'' > t'$, Lemma B.1 implies $\mu \prec_{\tau(\mu,\eta)} \eta$, which in turn implies the first assertion $\mu < \eta$. To show that $(\mu, \eta) \in \Omega$, first observe that by Reversal, $\mu \succ_{t'} \eta$ and $\mu \prec_{t''} \eta$ for $t'' > t'$ implies $\mu \succ_0 \eta$. Now take any sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η). By Continuity, there exists N such that for all $n \geq N$, $\mu_n \succ_0 \eta_n$ and $\mu_n \prec_{\tau(\mu,\eta)} \eta_n$. It follows by Reversal that for all $n \geq N$, $\tau(\mu_n, \eta_n) \leq \tau(\mu, \eta)$, and so, $\limsup_{n\to\infty} \tau(\mu_n, \eta_n) \leq \tau(\mu, \eta)$. That is, $(\mu, \eta) \in \Omega$, as was to be shown.

D. Appendix: Proof of Theorem 3.2

In Appendix C.1 we showed the equivalence between (a) and (c). Here we establish the equivalence of (b) and (c). Define $\gtrapprox$ and $\succsim^*$ as in Appendix C.1. The set Ω of points in $\mathcal{A} \times \mathcal{A}$ on which τ is upper semicontinuous will be important. Formally,

$$\Omega = \{(\mu, \eta) \in \mathcal{A} \times \mathcal{A} : (\mu_n, \eta_n) \to (\mu, \eta) \Longrightarrow \limsup_{n\to\infty} \tau(\mu_n, \eta_n) \leq \tau(\mu, \eta)\}.$$

For later, note that since $\tau(\mu, \eta) = \tau(\eta, \mu)$ for all μ, η by definition of $\tau(\cdot)$, it follows that $(\mu, \eta) \in \Omega$ implies $(\eta, \mu) \in \Omega$ as well.

Lemma D.1. $\mu \succ^* \eta \Longleftrightarrow [\mu > \eta$ *and* $(\mu, \eta) \in \Omega]$.

Proof. $\Longleftarrow$: Take μ and η such that $\mu > \eta$ and $(\mu, \eta) \in \Omega$. Given Lemma B.1, $\mu > \eta$ implies $\mu \succ_{\tau(\mu,\eta)+1} \eta$. Since $\succsim_{\tau(\mu,\eta)+1}$ is continuous, for every sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η) there exists N such that

$$\mu_n \succ_{\tau(\mu,\eta)+1} \eta_n, \text{ for all } n \geq N.$$

By hypothesis, $\limsup_{n\to\infty} \tau(\mu_n, \eta_n) \leq \tau(\mu, \eta)$. Therefore, there exists N' such that

$$\tau(\mu, \eta) + 1 > \tau(\mu_n, \eta_n), \text{ for all } n \geq N'.$$

It follows that for all $n \geq N'$, the ranking of (μ_n, η_n) by $\succsim_{\tau(\mu_n,\eta_n)}$ must agree with that by $\succsim_{\tau(\mu,\eta)+1}$, and so,

$$\mu_n \succ_{\tau(\mu_n,\eta_n)} \eta_n, \quad \text{for all } n \geq \max\{N, N'\}.$$

But $\mu_n \succ_{\tau(\mu_n,\eta_n)} \eta_n$ is equivalent to $\mu_n > \eta_n$. This establishes that for any sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η), there exists M such that $\mu_n > \eta_n$ for all $n \geq M$. In particular, there is no sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η) such that $\eta_n \gtrapprox \mu_n$ for all n. Thus $\eta \not\succsim^* \mu$, as desired.

$\Longrightarrow$: Take μ, η such that $\mu \succ^* \eta$. Then (C.2) yields

$$\mu > \eta \tag{D.1}$$

thus establishing the first assertion in the implication. To establish the second assertion, take any sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η). Since $\mu \succ^* \eta$ and since $\succsim^*$ is continuous (Theorem 3.1), there exists N such that

$$\mu_n \succ^* \eta_n, \quad \text{for all } n \geq N.$$

By (C.2),

$$\mu_n > \eta_n, \quad \text{for all } n \geq N. \tag{D.2}$$

Without loss of generality, let $N = 1$. Suppose by way of contradiction that

$$\limsup_{n\to\infty} \tau(\mu_n, \eta_n) > \tau(\mu, \eta).$$

Then, there exists a subsequence $\{(\mu_{n_m}, \eta_{n_m})\} \subset \{(\mu_n, \eta_n)\}$ where for all m,

$$\tau(\mu_{n_m}, \eta_{n_m}) > \tau(\mu, \eta) \geq 0. \tag{D.3}$$

By construction, $\mu_{n_m} > \eta_{n_m}$ for all m, that is, $\mu_{n_m} \succ_{\tau(\mu_{n_m}, \eta_{n_m})} \eta_{n_m}$ for all m. Thus, by Lemma B.1 and (D.3),

$$\eta_{n_m} \succsim_{\tau(\mu,\eta)} \mu_{n_m}, \quad \text{for all } m.$$

However, since $\succsim_{\tau(\mu,\eta)}$ is continuous and $(\mu_{n_m}, \eta_{n_m}) \to (\mu, \eta)$, we have $\eta \succsim_{\tau(\mu,\eta)} \mu$. This is equivalent to $\eta \gtrapprox \mu$, which contradicts (D.1). ■

E. Appendix: Reversal at Infinity

We show that $\mu > \eta$ and $(\mu, \eta) \notin \Omega$ implies that there exists a sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η) such that $\limsup_{n\to\infty} \tau(\mu_n, \eta_n) = \infty$:

By definition, $(\mu, \eta) \notin \Omega$ implies that there exists a sequence $\{(\mu_n, \eta_n)\}$ that converges to (μ, η) and $\limsup_{n\to\infty} \tau(\mu_n, \eta_n) > \tau(\mu, \eta)$. Without loss of generality, $\tau(\mu_n, \eta_n) > \tau(\mu, \eta)$ for all n. Suppose by way of contradiction that $\limsup_{n\to\infty} \tau(\mu_n, \eta_n) = T < \infty$. Thus, there exists N such that for all $n \geq N$, $T+1 > \tau(\mu_n, \eta_n)$. Also, for large enough n, $\mu_n \succ_{\tau(\mu,\eta)} \eta_n$, and since

$$T + 1 > \tau(\mu_n, \eta_n) > \tau(\mu, \eta),$$

it follows that for all large enough n, $\eta_n \succsim_{T+1} \mu_n$. By continuity of $\succsim_{T+1}$, $\eta \succsim_{T+1} \mu$. But since $T + 1 > \tau(\mu, \eta)$, this contradicts the hypothesis that $\mu \succ_{\tau(\mu,\eta)} \eta$. Therefore, $\limsup_{n\to\infty} \tau(\mu_n, \eta_n) = \infty$.

F. Appendix: Proof of Corollary 3.3

We show that $[\mu > \eta \iff \mu \succ^* \eta]$ if and only if τ is upper semicontinuous on Λ. If τ is not upper semicontinuous on Λ, then it follows from the characterization of normative preference in Lemma

D.1 that $[\mu > \eta \not\Longleftarrow \mu \succ^* \eta]$. If τ is upper semicontinuous on Λ, then the characterization implies $[\mu > \eta \Longrightarrow \mu \succ^* \eta]$. Furthermore, by (C.2), $[\mu \succ^* \eta \Longrightarrow \mu > \eta]$ holds for all μ, η, and this completes the proof.

G. Appendix: Proof of Theorem 4.1

To derive the representation of normative preference, note that each $\succsim_n$ is also represented by the utility function

$$\widehat{U}_n(c_0, c_1, .., c_T) = \sum_{t=0}^{T} \frac{\phi(t+n)}{\phi(n)} u(c_t),$$

which is obtained by dividing $U_n(\cdot)$ by the positive constant, $\phi(n)$. We prove the Theorem in a series of steps.

Step 1: $\frac{\phi(t+n)}{\phi(n)}$ is bounded above by 1.

By assumption, $\phi(\cdot)$ is decreasing. Therefore, for all n and t, $\phi(t+n) \leq \phi(n)$.

Step 2: $\frac{\phi(t+n)}{\phi(n)}$ is weakly increasing in n.

Note that

$$\frac{\phi(t+n)}{\phi(n)} = \frac{\phi(t+n)}{\phi(t+n-1)} \cdot \frac{\phi(t+n-1)}{\phi(t+n-2)} \cdots \frac{\phi(n+2)}{\phi(n+1)} \cdot \frac{\phi(n+1)}{\phi(n)},$$

$$\frac{\phi(t+n+1)}{\phi(n+1)} = \frac{\phi(t+n+1)}{\phi(t+n)} \frac{\phi(t+n)}{\phi(t+n-1)} \cdot \frac{\phi(t+n-1)}{\phi(t+n-2)} \cdots \frac{\phi(n+2)}{\phi(n+1)},$$

and therefore,

$$\frac{\frac{\phi(t+n+1)}{\phi(n+1)}}{\frac{\phi(t+n)}{\phi(n)}} = \frac{\frac{\phi(t+n+1)}{\phi(t+n)}}{\frac{\phi(n+1)}{\phi(n)}}.$$

But by assumption, $\frac{\phi(r+2)}{\phi(r+1)} \geq \frac{\phi(r+1)}{\phi(r)}$ for any integer r. Thus, $\frac{\frac{\phi(t+n+1)}{\phi(t+n)}}{\frac{\phi(n+1)}{\phi(n)}} \geq 1$ and in particular, $\frac{\frac{\phi(t+n+1)}{\phi(n+1)}}{\frac{\phi(t+n)}{\phi(n)}} \geq 1$ for all t and n.

Step 3: The sequence $\{\widehat{U}_n\}$ converges uniformly to U^*.

By Steps 1 and 2, for each t, the sequence $\{\frac{\phi(t+n)}{\phi(n)}\}_{n=0}^{\infty}$ is a bounded, increasing sequence. Thus, $\phi^*(t) \equiv \lim_{n\to\infty} \frac{\phi(t+n)}{\phi(n)}$ is well-defined. Observe further that $\{\widehat{U}_n\}$ is a sequence of continuous real functions defined on a compact space C^{T+1}, and that it is monotone increasing and converges pointwise to the function $U^* : C^{T+1} \to R$ defined by:

$$U^*(c_0, c_1, .., c_T) = \sum_{t=0}^{T} \phi^*(t) u(c_t),$$

for all $(c_0, c_1, .., c_T) \in C^{T+1}$. By Dini's Theorem [4, Theorem 2.62], the convergence is uniform.

Step 4: U^* represents $\succsim^*$.

Since u is nonconstant, $U^*(\rho) > U^*(\nu)$ for some $\rho, \nu \in C^{T+1}$. By linearity of u, for every $\mu, \eta \in C^{T+1}$,

$$U^*(\mu) \geq U^*(\eta) \Longrightarrow U^*(\mu\alpha\rho) > U^*(\eta\alpha\nu), \text{ for all } \alpha \in (0,1)\,. \tag{G.1}$$

This observation will be used below. Let $\succsim^\infty$ be the preference relation represented by U^*. As in Appendix A, identify any binary relation B on $\mathcal{A}$ with its graph $\Gamma(B) \subset \mathcal{A} \times \mathcal{A}$. We show that $\Gamma(\succsim^\infty) = \lim_{n\to\infty} \Gamma(\succsim_n)$, and it follows that $\succsim^\infty = \succsim^*$, that is, U^* is a representation of normative preference $\succsim^*$, as desired for the step.

Claim $\quad \Gamma(\succsim^\infty) = \lim_{n\to\infty} \Gamma(\succsim_n)$.

Proof. First establish $Ls\Gamma(\succsim_n) \subset \Gamma(\succsim^\infty)$. If $(\mu, \eta) \in Ls\Gamma(\succsim_n)$ then there is a subsequence $\{\Gamma(\succsim_{n(m)})\}$ and a sequence $\{(\mu_m, \eta_m)\}$ that converges to (μ, η) such that $(\mu_m, \eta_m) \in \Gamma(\succsim_{n(m)})$ for each m. Therefore, for each m,

$$U_{n(m)}(\mu_m) \geq U_{n(m)}(\eta_m).$$

Since $U_{n(m)}$ converges to U^* uniformly, it follows that $U^*(\mu) \geq U^*(\eta)$. Hence $(\mu, \eta) \in \Gamma(\succsim^\infty)$, as desired.

Next establish $\Gamma(\succsim^\infty) \subset Li\Gamma(\succsim_n)$. Let $(\mu, \eta) \in \Gamma(\succsim^\infty)$ and take any neighborhood O of (μ, η). By (G.1), there exists $\alpha \in (0,1]$ s.t $(\mu\alpha\rho, \eta\alpha\nu) \in O$ and $U^*(\mu\alpha\rho) > U^*(\eta\alpha\nu)$. By Step

3, there exists $T < \infty$ such that $U_n(\mu\alpha\rho) > U_n(\eta\alpha\nu)$ for all $n \geq T$, that is, $(\mu\alpha\rho, \eta\alpha\nu) \in \Gamma(\succsim_n)$ for all $t \geq T$. Hence,

$$V \cap \Gamma(\succsim_n) \neq \phi \text{ for all but a finite number of } n,$$

that is, $(\mu, \eta) \in Li\Gamma(\succsim_n)$. ■

Step 5: $\phi^*(t) = \gamma^t$, where $\gamma = \lim_{n\to\infty} \frac{\phi(1+n)}{\phi(n)}$.

For any k, $\{\frac{\phi(1+n+k)}{\phi(n+k)}\}_{n=0}^{\infty}$ is a subsequence of the sequence $\{\frac{\phi(1+n)}{\phi(n)}\}_{n=0}^{\infty}$ that we showed to be convergent in Step 3. Hence, for all k,

$$\lim_{n\to\infty} \frac{\phi(1+n)}{\phi(n)} = \lim_{k\to\infty} \frac{\phi(1+n+k)}{\phi(n+k)}$$

Define $\gamma \equiv \lim_{n\to\infty} \frac{\phi(1+n)}{\phi(n)}$ and recall that we defined $\phi^*(t) \equiv \lim_{n\to\infty} \frac{\phi(t+n)}{\phi(n)}$ above. For any t,

$$\frac{\phi(t+n)}{\phi(n)} = \Pi_{r=0}^{t-1} \frac{\phi(t+n-r)}{\phi(t+n-r-1)}.$$

Therefore,

$\phi^*(t)$

$= \lim_{n\to\infty} \frac{\phi(t+n)}{\phi(n)}$

$= \lim_{n\to\infty} \Pi_{r=0}^{t-1} \frac{\phi(t+n-r)}{\phi(t+n-r-1)}$

$= \Pi_{r=0}^{t-1} \lim_{n\to\infty} \frac{\phi(t+n-r)}{\phi(t+n-r-1)}$

$= \Pi_{r=0}^{t-1} \lim_{n\to\infty} \frac{\phi(1+n)}{\phi(n)}$

$= \Pi_{r=0}^{t-1} \gamma$

$= \gamma^t$, as desired.

References

[1] Ainslie, G. (1975): 'Specious Reward: A Behavioral Theory of Impulsiveness and Impulse Control', *Psychological Bulletin* 82, pp 463-496.

[2] Ainslie, G. (1992): *Picoeconomics*, Cambridge University Press.

[3] Ainslie, G. and V Haendel (1983): 'The Motives of the Will', in Gotteheil et al (Eds), *Etiologic Aspects of Alcohol and Drug Abuse* , Springfield, Il: Thomas.

[4] Aliprantis, C. and K. Border (1994): *Infinite Dimensional Analysis: a Hitchhiker's Guide*, 2nd Edition, Springer Verlag.

[5] Brown, A. and C. Pearcy (1995): *An Introduction to Analysis*, Springer, New York.

[6] Chung, S. and R. Hernnstein (1961): 'Absolute and Relative Strength of Response as a Function of Frequency of Reinforcement', *Journal of the Experimental Analysis of Behavior* 4, pp 267-272.

[7] Fernandez-Villaverde, J. and A. Mukherji (2002): 'Can we Really Observe Hyperbolic Discounting?', Penn Institute for Economic Research working paper 02-008.

[8] Fishburn, P. (1970), *Utility Theory for Decision Making*, John Wiley and Sons, New York.

[9] Fredrick, S., G. Lowenstein and T. O'Donoghue (2002): 'Time Discounting and Time Preference: A Critical Review', *Journal of Economic Literature* 40(2), pp 351-401.

[10] Gul, F. and W. Pesendorfer (2001): 'Temptation and Self-Control', *Econometrica* 69, pp 1403-1435.

[11] Halevy, Y. (2004): 'Diminishing Impatience and Non-Expected Utility: A Unified Approach', mimeo.

[12] Herstein, I. N. and J. Milnor (1953): 'An Axiomatic Approach to Measurable Utility', *Econometrica* 21, pp 291-297.

[13] Hildenbrand, W. (1974): *Core and Equilibria of a Large Economy*, Princeton University Press.

[14] Kirby, K., and R. Hernnstein (1995): 'Preference Reversals due to Myopic Discounting of Delayed Reward', *Psychological Science*, pp 83-89.

[15] Laibson, D. (1997): 'Golden Eggs and Hyperbolic Discounting', *Quarterly Journal of Economics* 112, pp 443-77.

[16] Lowenstein, G. and D. Prelec (1992): 'Anomalies in Intertemporal Choice: Evidence and an Interpretation', *Quarterly Journal of Economics* 57(2), pp 573-598.

[17] Mazur, J. E. (1987): 'An Adjustment Procedure for Studying Delayed Reinforcement' in M. L. Commons, J. E. Mazur, J. A. Nevin and H. Rachlin (Eds), *Quantitative Analyses of Behavior: Vol. 5. The Effect of Delay and Intervening Events on Reinforcement Value*, pp 55-73, Hillsdale, NJ: Erlbaum.

[18] Noor, J. (2005): 'Temptation, Welfare and Revealed Preference', University of Rochester Working Paper.

[19] O'Donoghue, T., and M. Rabin (1999): 'Doing it Now or Later', *American Economic Review*, 89, pp 103-24.

[20] Rachlin, H. (1970): *Introduction to Modern Behaviorism*, San Francisco: W. H. Freeman.

[21] Rachlin, H. and L. Green (1972): 'Commitment, Choice and Self-Control', *Journal of the Experimental Analysis of Behavior* 17, pp 15-22.

[22] Read, D. and B. van Leeuwen (1998), "Predicting Hunger: The Effects of Appetite and Delay on Choice," *Organizational Behavior and Human Decision Processes* 76(2), pp 189-205.

[23] Rubinstein, A. (2003), "'Economics and Psychology'? The Case of Hyperbolic Discounting," *International Economic Review* 44(4), pp 1207-1216.

[24] Strotz R. (1955): 'Myopia and Inconsistency in Dynamic Utility Maximization', *Review of Economic Studies* XXIII, pp 165-180.

[25] Trope, Y., and N. Liberman (2000), "Temporal Construal and Time-dependent changes in Preference," *Journal of Personality and Social Psychology* 79, pp 876-889.