Marginal vs. Average Bundle and the Intertemporal Elasticity of Substitution

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Abstract

Saving an extra dollar means foregoing consumption today - but the consumption that you forego is not the average consumption good, rather it is a luxury that you trade off for a luxury later on. Hence, the price that affects the trade-off is not the price of the average consumption bundle (the CPI) but an index that overweighs luxury goods. Heterogeneity in goods leads thus to the presence of an additional term in the standard Euler equation. I compute this additional term and reestimate a simple Euler equation. In preliminary work, the correction appears minor: estimates of the IES are increased, but the absolute change in the estimated IES is small.

1 Introduction

The estimation of consumption Euler equations has been at the forefront of macroeconomics and finance since Hall (1978) and Hansen and Singleton (1983). Estimating the intertemporal elasticity of substitution of consumption is important: this parameter determines, for instance, the speed of convergence of economies towards their steady-states, and the desirability of capital taxation. Most of the work in this literature has made the standard simplification that there is one good. (On top of the standard representative agent assumption.) However, conditions under which this aggregation *over goods* is possible are stringent: as will be demonstrated below, one needs to assume either homotheticity of preferences, i.e. that all income elasticities are unity, or that relative prices are constant. In this paper, I examine the consequences of relaxing both assumptions. I show that this results in the presence of an additional term in the usual log-linearized first-order condition. (This term reduces to zero either when all income elasticities

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equal one, or when all the relative prices are constant.) I proceed to estimate the intertemporal elasticity of substitution, taking this extra term into account. I follow Hall (1988) and run simple regressions of consumption growth on interest rates, with my correction term added.

The intuition for the presence of the additional term when income elasticities differ from one is straightforward. When saving an extra dollar, people trade off consumption today for consumption tomorrow. But a marginal decrease in consumption is achieved by cutting back the consumption of today's luxuries, not by cutting back on necessities (almost by definition). Symmetrically, the extra future consumption will lead to an increase in the consumption of the household's luxuries, not their necessities. It follows that the correct price to measure this intertemporal margin is not the price of the average bundle (the consumer price index), but the price of the marginal bundle. The intuition suggests that luxuries should be overweighed.. This intuition is confirmed by the algebra.

The results given here are preliminary. Overall, they suggest that the additional correction has little importance. The empirical fact driving this result is that measured relative prices do not move much, and they move in a way that is not strongly correlated with asset returns or consumption growth. However, it could be that better data or more refined econometric procedures allow to uncover a more substantial effect. I discuss some possibilities in the conclusion.

The remark made here might also be important for attempts at estimating the intertemporal elasticity of substitution in labor. I offer some preliminary computations.

Related Literature

This work is related to several strands of the literature. Obviously, the paper is related to the studies by Hall (1978, 1988), Hansen and Singleton (1983), Mulligan (2003,2004) and many others who estimate the IES (Intertemporal elasticity of substitution in consumption). Second, recent work in asset pricing emphasizes that non-separabilities can help explain the behavior of asset prices (see e.g. Pako (2004), Piazzesi, Schneider and Tuzel (2004) and Yogo (2004), and Cochrane (2001, chap. 21) for a discussion). Whereas these papers parametrize utility functions and use quantity data to back out marginal utilities, I show that one can use data on relative prices to avoid the tricky part of specifying a utility function over different goods.[1] My method also allows me to consider the whole CPI basket instead of emphasizing one particular good (but I do not consider leisure). Thirdly, this paper also relates to studies of how the relative prices of

[1]However, the relevant price for durable goods like houses or cars is a rental price, which is typically not observable. In my current empirical implementation, I use capital prices.

goods, in particular durable goods, investment goods or luxuries, change over the business cycle (Murphy, Shleifer and Vishny (1989) and Bils and Klenow (1998)).

Organization

Section 2 derives the correct index and analyzes some examples to illustrate the effect at work. Section 3 proceeds with a rough empirical implementation. Section 4 concludes.

2 Deriving the correct measure of the marginal rate of substitution with multiple goods

Assume a representative agent with flow utility over N goods $u(X_1, X_2, ..., X_N)$. Let $X^t = (X_1^t, ..., X_N^t)$ be the bundle of goods consumed at time t. The lifetime utility is the expected discounted sum of these flows,

$$U_0 = \mathbb{E}_0 \sum_{t \geq 0} \beta^t u(X^t).$$

Let $V(P_1, ..., P_N, M)$ be the indirect utility in a period, i.e.

$$\begin{aligned} V(P_1, ..., P_N, M) &= \max_{X_1,...,X_N} u(X_1, X_2, ..., X_N) \\ s.t. &: \sum_{i=1}^{N} P_i X_i \leq M. \end{aligned} \tag{2.1}$$

Let $R_{t,t+1}^{nom}$ be a nominal gross return between periods t and $t+1$. The first-order condition for an interior solution is

$$V_M(P^t, M^t) = \mathbb{E}_t \left[\beta R_{t,t+1}^{nom} V_M(P^{t+1}, M^{t+1}) \right],$$

where P^t denotes the price vector at time t and M^t is nominal consumption expenditure at t.

First, note that the ratio of marginal utilities of nominal wealth

$$m_{t,t+1}^{nom} = \frac{\beta V_M(P^{t+1}, M^{t+1})}{V_M(P^t, M^t)},$$

is related to the usual discount factor for real return as follows. Let $\overline{P}_t$ be the CPI price level, then

$$\begin{aligned} 1 &= \mathbb{E}_t \left[R_{t,t+1}^{nom} \frac{\beta V_M(P^{t+1}, M^{t+1})}{V_M(P^t, M^t)} \right] \\ 1 &= \mathbb{E}_t \left[R_{t,t+1}^{nom} \frac{\overline{P}_t}{\overline{P}_{t+1}} \frac{\beta V_M(P^{t+1}/\overline{P}_{t+1}, M^{t+1}/\overline{P}_{t+1})}{V_M(P^t/\overline{P}_t, M^t/\overline{P}_t)} \right] \\ 1 &= \mathbb{E}_t \left[R_{t,t+1}^{\text{real}} \frac{\beta V_M(P^{t+1}/\overline{P}_{t+1}, M^{t+1}/\overline{P}_{t+1})}{V_M(P^t/\overline{P}_t, M^t/\overline{P}_t)} \right] \end{aligned} \tag{2.2}$$

(The second line used the fact that since V is homogeneous of degree 0 in (P, M), so that V_M is homogeneous of degree -1 in (P, M).) $M^t/\overline{P}_t$ is real consumption, so that we have a standard function of real consumption, but it now includes marginal utility shifters: the relative prices $P^t/\overline{P}_t$:

$$m_{t,t+1}^{\text{real}} = \frac{\beta V_M(P^{t+1}/\overline{P_{t+1}}, M^{t+1}/\overline{P_{t+1}})}{V_M(P^t/\overline{P_t}, M^t/\overline{P_t})}.$$

One could specify a utility function to obtain a form for the marginal utility V_M. A more tractable approach is to approximate the ratio of marginal utilities instead to obtain the equivalent to the standard log-linear approximation to this Euler equation. The following paragraphs show how to obtain the approximation. The busy reader may want to go directly to the result 1 stated below.

Derivation of the approximation

Using a first-order Taylor approximation:

$$V_M(P^{t+1}, M^{t+1}) \simeq V_M(P^t, M^t) + \frac{\partial V_M(P^t, M^t)}{\partial M}\left(M^{t+1} - M^t\right) + \sum_{i=1}^{N} \frac{\partial V_M(P^t, M^t)}{\partial P_i}\left(P_i^{t+1} - P_i^t\right)$$

$$\begin{aligned}\frac{V_M(P^{t+1}, M^{t+1})}{V_M(P^t, M^t)} &\simeq 1 - \left(\frac{-V_{MM}(P^t, M^t)M^t}{V_M(P^t, M^t)}\right)\left(\frac{M^{t+1} - M^t}{M^t}\right) \\ &\quad + \sum_{i=1}^{N}\left(\frac{V_{MP_i}(P^t, M^t)P_i^t}{V_M(P^t, M^t)}\right)\left(\frac{P_i^{t+1} - P_i^t}{P_i^t}\right)\end{aligned}$$

$$\simeq 1 - \sigma_t\left(\frac{M^{t+1} - M^t}{M^t}\right) + \sum_{i=1}^{N}\left(\frac{V_{MP_i}(P^t, M^t)M^t}{V_M(P^t, M^t)X_i^t}\right)\left(\frac{P_i^t X_i^t}{M^t}\right)\left(\frac{P_i^{t+1} - P_i^t}{P_i^t}\right) \tag{2.3}$$

where I denoted $\sigma_t = \left(\frac{-V_{MM}(P^t,M^t)M^t}{V_M(P^t,M^t)}\right)$. Note that $\sigma_t = \frac{-V_{MM}(P^t,M^t)M^t}{V_M(P^t,M^t)} = \frac{-V_{MM}(P^t/\overline{P_t},M^t/\overline{P_t})M^t/\overline{P_t}}{V_M(P^t/\overline{P_t},M^t/\overline{P_t})}$ is the usual measure of the curvature of the value function, expressed in real terms. (I again used the homogeneity of degree -1 of V_M.) Now use Roy's lemma:

$$X_i^t = X_i\left(P^t, M^t\right) = -\frac{V_{P_i}(P^t, M^t)}{V_M(P^t, M^t)}$$

where $X_i(P, M)$ is the solution of the problem (2.1). Differentiating with respect to M, I obtain the income elasticity of X_i :

$$\eta_{it} \overset{def}{=} \frac{M}{X_i}\frac{\partial X_i}{\partial M} = \frac{M}{X_i}\left(-\frac{V_{P_iM}}{V_M} + \frac{V_{P_i}V_{MM}}{V_M^2}\right)$$

Hence,

$$V_{P_iM} = -\frac{\eta_i X_i}{M}V_M + \frac{V_{P_i}V_{MM}}{V_M}$$

and substituting into (2.3) yields, using Roy's lemma again:

$$\frac{V_M(P^{t+1}, M^{t+1})}{V_M(P^t, M^t)}$$

$$\simeq \quad 1 - \sigma_t \widehat{M}^{t+1} + \sum_{i=1}^{N} \left(\left(-\frac{\eta_{it} X_i^t}{M^t} V_M(P^t, M^t) + \frac{V_{P_i}(P^t, M^t) V_{MM}(P^t, M^t)}{V_M(P^t, M^t)} \right) \frac{M^t}{V_M(P^t, M^t) X_i^t} \right) s_{it} \widehat{P}_i^{t+1}$$

$$\frac{V_M(P^{t+1}, M^{t+1})}{V_M(P^t, M^t)} \simeq 1 - \sigma_t \widehat{M} + \sum_{i=1}^{N} \left(-\eta_{it} - \sigma_t \right) s_{it} \widehat{P}_i^{t+1}$$

where $s_{it} = P_i^t X_i^t / M^t$ is the expenditure share of good i, η_{it} is the income elasticity of good i, and hats denote % changes. This expression can be further simplified:

$$\begin{aligned} \frac{V_M(P^{t+1}, M^{t+1})}{V_M(P^t, M^t)} - 1 \quad &\simeq \quad -\sigma_t \left(\widehat{M}^{t+1} - \widehat{P}^{t+1} \right) - \sum_{i=1}^{N} \eta_{it} s_{it} \widehat{P}_i^{t+1} \\ &\simeq \quad -\sigma_t \left(\widehat{M}^{t+1} - \widehat{P}^{t+1} \right) - \sum_{i=1}^{N} \eta_{it} s_{it} \left(\widehat{P}_i^{t+1} - \widehat{P}^{t+1} \right) - \widehat{P}^{t+1} \end{aligned}$$

The first line uses that the change in the overall price level (as computed by the Consumer Price Index) is

$$\widehat{P}^{t+1} \stackrel{def}{=} \frac{\overline{P_{t+1}} - \overline{P_t}}{\overline{P_t}} \simeq \sum_{i=1}^{N} s_{it} \widehat{P}_i^{t+1},$$

and the second line uses the adding-up identity

$$\sum_{i=1}^{N} s_{it} \eta_{it} = 1,$$

i.e. income elasticities must be one on average across goods, by budget constraint logic. The last inflation term is taken out if we consider a real marginal rate of substitution (i.e. a s.d.f. that prices real returns), as we noted in equation (2.2). We can now state the

Result 1: the real marginal rate of substitution (aka stochastic discount factor) is, to a first-order approximation:

$$\log m_{t,t+1}^{\text{real}} \simeq \log \beta - \sigma_t \left(\widehat{M}^{t+1} - \widehat{P}^{t+1} \right) - \sum_{i=1}^{N} \eta_{it} s_{it} \left(\widehat{P}_i^{t+1} - \widehat{P}^{t+1} \right),$$

where σ_t is the local curvature of the value function in terms of aggregate real consumption; η_{it} are the (local) income elasticities of the intratemporal program (2.1); s_{it} are the expenditure shares; $\widehat{M}^{t+1} - \widehat{P}^{t+1}$ is the increase in aggregate real consumption (as computed by the NIPA); and $\widehat{P}_i^{t+1} - \widehat{P}^{t+1}$ are the relative prices changes.

I now examine a series of simple cases to shed light on this formula.

Case (i): One Good.

Assume $N = 1$. Then $\widehat{P}^t = \widehat{P}_1^t$ obviously, and as a result $\log m_{t,t+1} \simeq \log \beta - \sigma_t \left(\widehat{M}^{t+1} - \widehat{P}^{t+1} \right)$. We obtain the usual formula $\log m_{t,t+1} \simeq \log \beta - \sigma_t \widehat{c}_{t+1}$. With CRRA utility, $\sigma_t = \sigma$.

Case (ii): Relative prices are constant.

In this case we also obtain $\log m_{t,t+1} \simeq \log \beta - \sigma_t \widehat{c}_{t+1}$. This is the Hicksian aggregation case (See Deaton and Muellbauer p. xxx).

Case (iii): Homothetic preferences

In this case $\eta_{it} = 1$ for all i, so that

$$\begin{aligned} \sum_{i=1}^{N} \eta_{it} s_{it} \left(\widehat{P}_i^{t+1} - \widehat{P}^{t+1} \right) &= \sum_{i=1}^{N} s_{it} \left(\widehat{P}_i^{t+1} - \widehat{P}^{t+1} \right) \\ &= \sum_{i=1}^{N} s_{it} \widehat{P}_i^{t+1} - \widehat{P}^{t+1} \\ &= 0, \end{aligned}$$

and the additional term is again zero. This is the second case in which preferences over goods can be aggregated.

Case (iv): Two goods with different income elasticities.

In this case we have a luxury, say 1, and a necessity, 2: $\eta_1 > 1 > \eta_2$. Omitting time subscripts, the correction term is

$$\begin{aligned} & s_1 \eta_1 \left(\widehat{P}_1 - \widehat{P} \right) + s_2 \eta_2 \left(\widehat{P}_2 - \widehat{P} \right) \\ = \; & \widehat{P}_2 - \widehat{P} + s_1 \eta_1 \left(\widehat{P}_1 - \widehat{P}_2 \right) \\ = \; & s_1 \left(1 - \eta_1 \right) \left(\widehat{P}_2 - \widehat{P}_1 \right) \end{aligned}$$

where I used that $\widehat{P} = s_1 \widehat{P}_1 + s_2 \widehat{P}_2$ and $s_1 \eta_1 + s_2 \eta_2 = s_1 + s_2 = 1$. Hence, we see immediately that this term will be big only if (i) income elasticities differ substantially from 1, and (ii) the relative price moves significantly.

Case (v): Three goods, one has an income elasticity equal to 1.

Let 3 be the good for which $\eta_3 = 1$. By the same logic, the correction is

$$\begin{aligned} & s_1 \eta_1 \left(\widehat{P}_1 - \widehat{P} \right) + s_2 \eta_2 \left(\widehat{P}_2 - \widehat{P} \right) + s_3 \eta_3 \left(\widehat{P}_3 - \widehat{P} \right) \\ = \; & s_1 \left(\eta_1 - 1 \right) \left(\widehat{P}_1 - \widehat{P} \right) + s_2 \left(\eta_2 - 1 \right) \left(\widehat{P}_2 - \widehat{P} \right) + s_3 \left(\eta_3 - 1 \right) \left(\widehat{P}_3 - \widehat{P} \right) \\ = \; & s_1 \left(\eta_1 - 1 \right) \left(\widehat{P}_1 - \widehat{P} \right) + s_2 \left(\eta_2 - 1 \right) \left(\widehat{P}_2 - \widehat{P} \right) \end{aligned}$$

where again I used the adding-up constraint $\sum s_i \eta_i = 1$ in line 2, and line 3 exploits that $\eta_3 = 1$. It follows that the effect will be interesting only if, as in case (iv), the relative prices move and income elasticities differ from unity, but also only if the expenditure shares of the goods are large enough.

Empirical Implementation

The basic equation that people run following equation (2.2) is, for a risk-free asset:

$$\log m_{t,t+1}^{\text{real}} = -\log R_{t,t+1}^{\text{real}}$$

and economists such as Hall and Mulligan run regressions of log consumption growth on the real interest rate:

$$\log\left(\frac{C_{t+1}}{C_t}\right) = \alpha + \beta r_{t,t+1}^{\text{real}} + \varepsilon_{t+1.}$$

It follows from result 1 that this regression is misspecified and that the true regression, assuming constant η_{it} and σ_t, is

$$\log\left(\frac{C_{t+1}}{C_t}\right) + \frac{1}{\sigma}\sum_{i=1}^{N}\eta_i s_{it}\left(\widehat{P}_i^{t+1} - \widehat{P}^{t+1}\right) = \alpha + \frac{1}{\sigma}\beta r_{t,t+1}^{\text{real}}$$

or in the end

$$\begin{aligned}\log\left(\frac{C_{t+1}}{C_t}\right) &= \alpha + \frac{1}{\sigma}r_{t,t+1}^{\text{real}} - \frac{1}{\sigma}\sum_{i=1}^{N}\eta_i s_{it}\left(\widehat{P}_i^{t+1} - \widehat{P}^{t+1}\right) \\ &= \alpha + \frac{1}{\sigma}\left[r_{t,t+1}^{\text{real}} - \sum_{i=1}^{N}\eta_i s_{it}\left(\widehat{P}_i^{t+1} - \widehat{P}^{t+1}\right)\right] \end{aligned} \tag{2.4}$$

I thus run this regression. (Note that one can interpret it as saying that the inflation rate must be corrected.)[2]

(Of course, it is also possible to use GMM to estimate the moments corresponding to various returns (also possibily using conditioning information). In this case the covariance of the return with the "correction" term $\zeta_t = \sum_{i=1}^{N}\eta_i s_{it}\left(\widehat{P}_i^{t+1} - \widehat{P}^{t+1}\right)$,would appear in the formulas. I plan to do this in future work.)

Relation with separability [to be improved]

Most asset pricing papers emphasize that the key part is that utility is separable between the two goods considered (e.g., housing and the rest of nondurable consumption and services, or

[2] The intuition for why this formula would work better empirically is that the price of "luxuries" is countercyclical, the right-hand side will be more procyclical. It is known that the price of durables, which are identified as luxuries in my regressions (see below), is countercyclical. [But is it true for other luxuries?]

durables and the rest of consumption): $u_c(c_t, h_t)$ will move more than is explained by changes in c_t if h_t changes over time. These papers do remark that relative prices vary but do not insist on homotheticity.[3] I believe that the key difference is that these papers emphasize the effect of a change in h on the marginal utility of c whereas I look at the marginal utility of total aggregate consumption $c + Rh$. If we assume that people can choose their combination of (c, h) each period, this measure, as shown in the computations above, seems to be the relevant one.

Intertemporal Elasticity of Labor Supply

Note that a similar argument can be made for the intertemporal elasticity of labor supply. Economists routinely run equations such as

$$d\log N_{t+1} = \alpha + \beta d \log \left[W_{t+1}/\overline{P_{t+1}}\right] + \gamma r^{\text{real}}_{t,t+1} + \varepsilon_{t+1},$$

where the wage and interest rate are transformed in real terms using the CPI inflation. Clearly, the same correction will need to be done to modify the CPI inflation to add the correction ζ_t. Estimates will be affected in as much as ζ_t covaries with the changes in hours worked.

3 Empirical Investigation

I start an empirical investigation to examine if the effects of this correction are important.

Data

I use the tables 2.4.4S and 2.4.5S (Supplementary tables of the National Income and Product Accounts from the BEA), which break down consumption into around 300 products. The breakdown is done at three (sometimes four) levels; for instance, durables→motor vehicles→ new autos→foreign autos; or nondurable consumption→food→food purchased for home consumption→cereals. These tables provide annual expenditure and price data. Quarterly and monthly frequencies are also available and will be used in future work, though there may be more measurement essor. Data ranges years from 1959 to 2003.[4] I will consider three levels of breakdown: a first one with 14 goods, a second one with 61 goods, and a third one with 171 goods. (Ideally of course, I would like to group goods by their income elasticity.)

[3] Pako (2004) argues that durables are a luxury, but Yogo (2004) and Piazessi et al. (2004) for instance use CES preferences, which are homothetic.

[4] In previous work, I used the tables 2.3, 2.4 and 2.5 from the NIPA, but they offer less detail than the supplementary tables, which however may be more poorly measured. The sample for this data is somewhat longer (1947 to 2003).

Summary statistics

I provide three decompositions, by differing degrees. Some summary statistics for the first level of disaggregation (the smallest number of goods considered) is given in table 1. The table gives the average expenditure share of each category, and the average and volatility of the growth rate of quantities and of relative prices.[5]

Item	Share	$Ed\log Q$	$\sigma(d\log Q)$	$Ed\log P$	$\sigma(d\log P)$
Motor Vehicles	5.8	5.1	9.1	-1.0	1.8
Furniture	5.0	6.9	4.8	-3.5	2.2
Other Durables	2.2	5.7	4.9	-1.1	1.5
Food	18.8	2.1	1.5	0.2	1.2
Clothing	6.2	4.2	2.6	-2.1	1.9
Gasoline, Fuel	3.8	1.8	3.0	0.9	9.6
Other Non-Durables	7.8	3.6	2.3	0.1	1.4
Housing	14.9	3.4	1.4	0.4	1.6
Electricity and Gas	2.7	3.0	2.4	0.5	3.5
Other Household Op.	3.4	4..0	2.0	-0.3	2.0
Transportation	3.8	3.3	3.4	0.8	1.5
Medical Care	11.3	4.5	2.0	1.9	1.5
Recreation	2.9	5.1	1.8	0.3	1.3
Other Services	11.4	3.7	2.7	0.9	1.1

Table 1: summary statistics for the first level of disaggregation

Estimating income elasticities

Estimating income elasticities is a complex task. I discuss some difficulties in this section. In light of the results below however, the procedure I use does not seem to be terribly important. I take a first-order approximation of a demand curve $Q_{it} = D_i(P_{1t}, ..., P_{Nt}, M_t)$ to obtain the equation

$$d\log Q_{it} = \sum_j \varepsilon_{ij} d\log P_{jt} + \eta_i d\log M_t.$$

I use homogeneity to express the right-hand side in real terms:

$$d\log Q_{it} = \sum_j \varepsilon_{ij} d\log\left(\frac{P_{jt}}{P_t}\right) + \eta_i d\log\left(\frac{M_t}{P_t}\right).$$

[5]Note that these numbers are not per capita, which explains the relatively high growth rates of quantities.

(Note that this demand curve takes into account our separability assumption that prices in other periods do not affect today's demand).

Except for the first disaggregation level, where I have 14 goods and 44 years, it is not possible to run this regression with all the good prices, because of the excess of right-hand side variables. More fundamentally, the standard simultaneity problem needs to be confronted: do the prices shifts originate from supply or demand shocks? We would need an instrument for prices. At first sight, none seems particularly attractive. As a very rough first cut, I suppress all prices from this equation and run simply

$$d\log Q_{i,t} = \alpha_i + \eta_i d\log\left(\frac{M_t}{P_t}\right) + \varepsilon_{i,t}. \tag{3.1}$$

The results from these equations are displayed in table 2 for the first level of disaggregation.

Item	$\widehat{\eta}_i$	$\sigma(\widehat{\eta}_i)$	R^2	DW
Motor Vehicles	4.71	0.44	73.0	2.11
Furniture	1.93	0.33	43.8	0.97
Other Durables	2.15	0.32	51.7	1.75
Food	0.59	0.10	44.3	1.55
Clothing	1.05	0.19	42.3	1.60
Gasoline, Fuel	1.44	0.17	62.4	1.77
Other Non-Durables	0.75	0.18	29.3	1.43
Housing	0.25	0.12	9.0	0.68
Electricity and Gas	0.48	0.21	11.0	1.68
Other Household Op.	0.57	0.16	22.7	1.19
Transportation	1.31	0.24	41.2	1.23
Medical Care	0.45	0.17	13.5	0.95
Recreation	0.10	0.17	0.7	1.40
Other Services	1.04	0.20	39.9	2.10

Table 2: regression results for the first set of goods considered. OLS of quantity growth on real consumption growth, 1959-2003.

With these estimates in hand, we can proceed to compute the "correction" to the marginal rate of substitution:

$$\zeta_t = \sum_{i=1}^{N} s_{it}\widehat{\eta}_i \left(\widehat{P}_{it} - \widehat{P}_t\right),$$

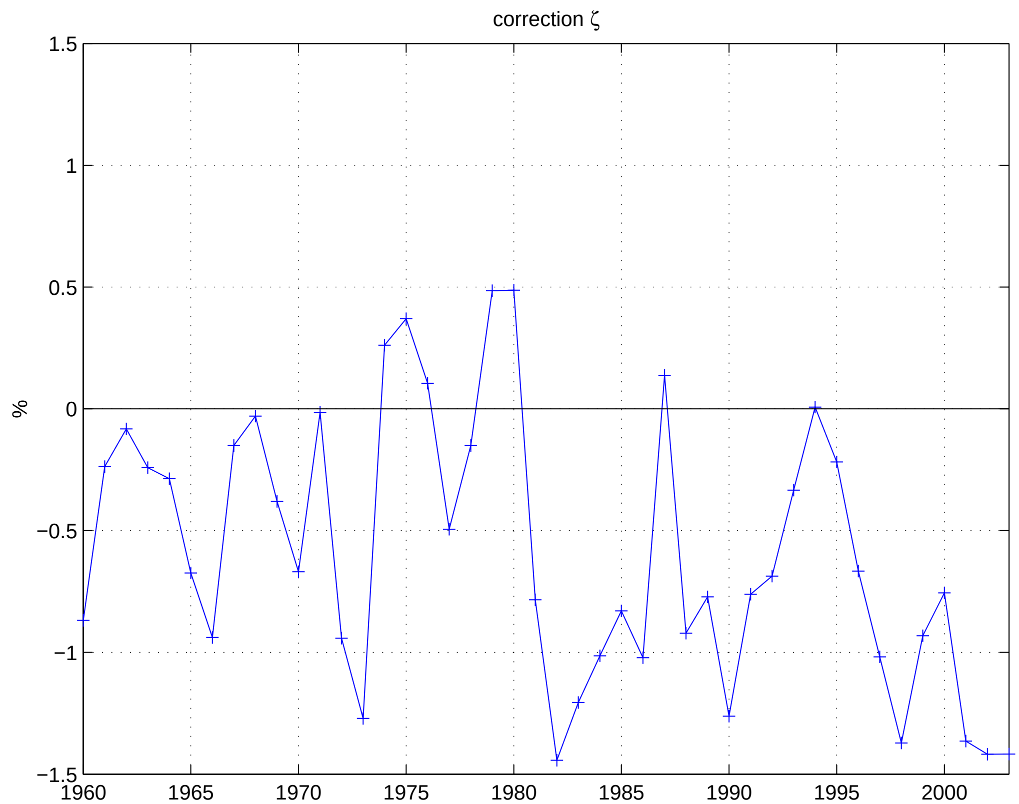

Figure 1: The correction ζ_t. Disaggregation with 14 goods. (in % per year).

which is the share- and elasticities- weighted sum of relative price changes derived in result 1:

$$\log m_{t,t+1} = \log \beta - \sigma \Delta \log c_{t+1} + \zeta_{t+1}.$$

Figure 1 plots this "correction term" ζ_t. Figure 2 plots also the consumption growth rate and the real return on a T-Bill for comparison. Figure 3 plots this correction, computed for the 3 levels of disaggregation.[6]

First, note that the correction term ζ_t is not very volatile, with a standard deviation of around 0.55% annually or so (as compared to 1.6% for consumption growth in this sample) so that by the logic of Hansen and Jagannathan (1991) we know that this the discount factor will not be volatile enough to be consistent with both the risk-free return and the stock return. Second, the correlation of ζ_t with aggregate consumption growth is negative, as in equation (2.4), but is weak (-0.17). Table 3 and 4 give the summary statistics for various level of disaggregation.

[6]Note that I rescale the elasticities by a factor λ_t each period so that the adding-up constraint $\sum s_{it}\eta_{it} = 1$ is preserved, with $\eta_{it} = \eta_i \lambda_t$. Also, I checked that the decomposition $\sum s_{it} P_{it}$ indeed gives the inflation rate, so that the disaggregation is correct. Finally, note that I did not (yet) compute consumption in per capita terms.

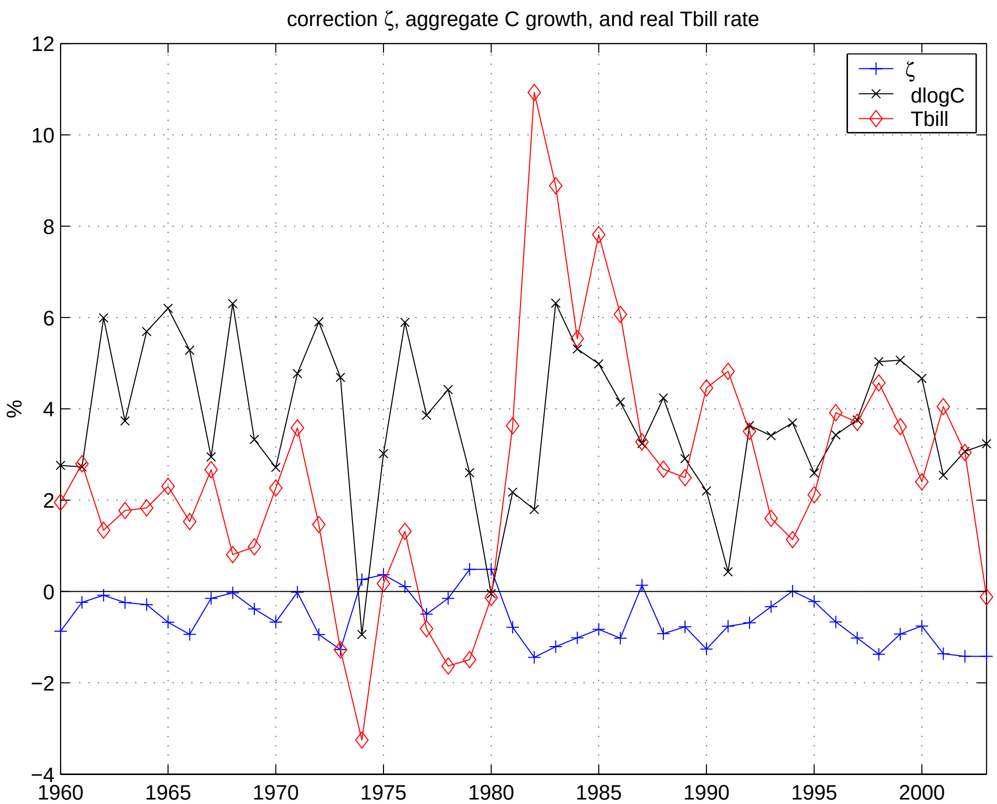

Figure 2: The correction ζ_t, Aggregate real consumption growth, and the real Tbill rate.

% per year	$\mathbb{E}\zeta_t$	$\sigma(\zeta_t)$	$Corr(\zeta_t, d\log C_t)$	$Corr(\zeta_t, r_t^{tbill} - \pi_t)$	$Corr(\zeta_t, r_t^{nyse} - \pi_t)$
14 goods	-0.59	0.55	-0.17	-0.55	-0.20
61 goods	-0.14	0.75	-0.30	-0.27	-0.01
171 goods	-0.02	0.88	-0.46	-0.40	-0.19

Table 3: Statistics for the "correction" term $\zeta_t$for 3 levels of disaggregation

	14 goods	61 goods	171 goods
14 goods	1	–	–
61 goods	0.39	1	–
171 goods	0.57	0.92	1

Table 4: Correlation matrix of the three correction terms

Regression results

I next run regressions as in (2.4); results are given in table 5 below. The first two lines reproduce results of Hall and others: the estimate $\widehat{\beta}$ is, for the TBill rate, insignificantly different

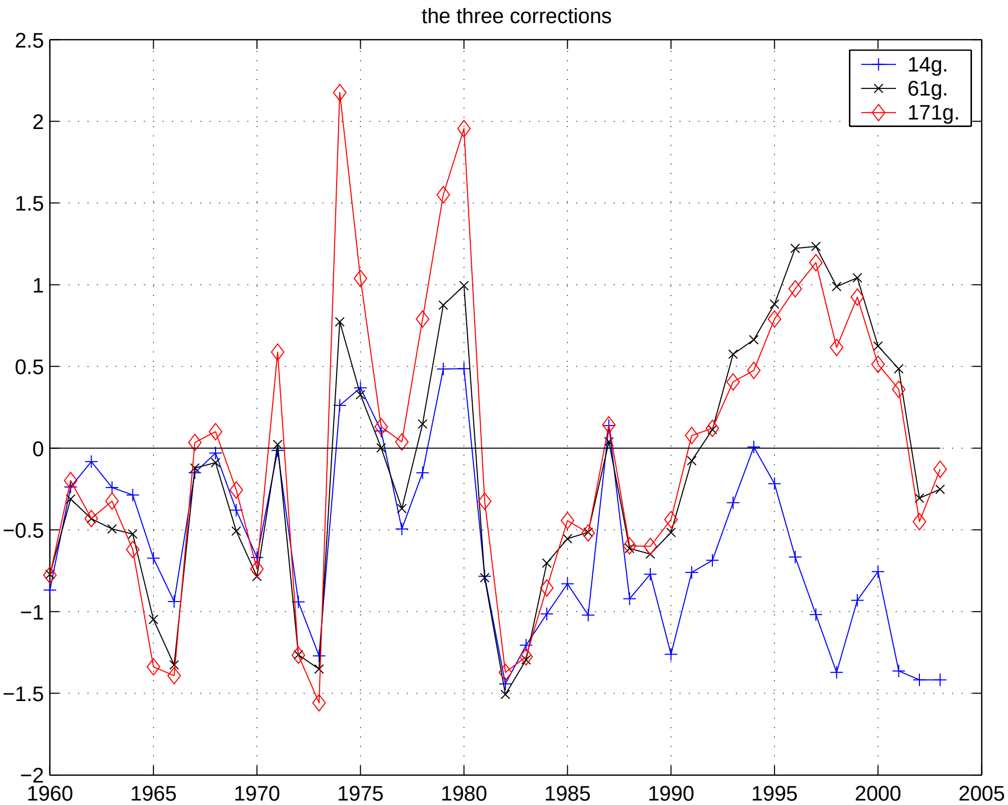

Figure 3: The three corrections, for different levels of disaggregations.

from zero. ($\widehat{\beta}$ estimates $1/\sigma$, the elasticity of substitution; I also give results for the NYSE return, but they are completely insensitive to the inclusion of the correction). The estimates for the IES become more positive (and the standard deviation falls) when one disaggregates at a finer level: $\widehat{\beta}$ is now one and a half standard deviations above zero. Moreover the R^2 increase also a little. However, these effects are minor: the absolute changes in the estimates are small.

$d\log C_t$ on...	$\widehat{\beta}$	$\sigma(\widehat{\beta})$	R^2 (%)	DW
real Tbill	**0.087**	0.092	2.0	1.54
real NYSE	0.048	0.013	22.4	1.27
real Tbill minus correction, disaggregation 1	**0.085**	0.082	2.5	1.54
real NYSE minus correction, disaggregation 1	0.048	0.013	22.7	1.27
real Tbill minus correction, disaggregation 2	**0.111**	0.083	4.0	1.54
real NYSE minus correction, disaggregation 2	0.048	0.013	23.6	1.27
real Tbill minus correction, disaggregation 3	**0.126**	0.078	5.8	1.54
real NYSE minus correction, disaggregation 3	0.049	0.013	24.2	1.27

Table 5: Regression results for the IES. Annual data from NIPA.

Whis is the correction small?

The discussion of part 1 showed that it was important to have goods with income elasticities quite different from one, with large expenditure shares and volatile relative prices. The summary statistics of table 1 show that relative prices are not very volatile. Moreover, few goods have income elasticities very different from 1, as estimated from equation (3.1) and reported in table 2. This seems to explain the results of this first cut.

To be added (GMM estimates of simultaneous system) (Estimates of IES labor supply).(DURABLES to handle better).

4 Concluding remarks & Future work

The current estimates suggest that the correction is modest, for the simple reason that relative prices are relatively smooth, and do not correlate strongly with aggregate consumption growth. There are two simple possible lines of future research.

First, one might be worried that the level of disaggregation does not capture well the distinction between luxuries and necessities. Concretely, we need to have categories such that a consumer will switch from one to the other as he gets wealthier. However, many of our categories are purpose-designed, not quality-designed: for instance, we have one item "books", but no distinction between rare books and paperbacks; we have "foreign cars", but not BMW versus Korean cars. I will investigate whether better data is available for this analysis. (Also, I will use other frequencies as well - monthly or quarterly.)

Second, looking at the subcategories of consumption raises the question, which ones is it possible to increase easily in response to changes in income? For instance, shifting education from a public to a private school is not something that can be easily done at any time (e.g. in the middle of a school year). Similarly, it would seem that many items require adjustment costs, either monetary costs, or through the creation of long-lived "tastes" - I do not enjoy wine now as I am poor, but if I become wealthy I will learn and start spending my money by drinking high-quality wines. If many items are actually predetermined because of either "friction", the remaining items, which are free to adjust (clothing? food?) will have to bear more of the adjustment. This may also be worth exploring. (This it could also help explain some divergences between people's perceived increase in the cost of living, and the BLS computations (see e.g. the Michigan survey?).)

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5 Appendix

To be added: full results for each level of disaggregation.