BANKRUPTCY LAW, BONDED LABOR AND INEQUALITY

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Abstract

Should the law restrict liability of defaulting borrowers? We abstract from possible benefits arising from limited rationality or risk-aversion of borrowers, contractual incompleteness, or lender moral hazard. We focus instead on general equilibrium implications of liability rules with moral hazard among borrowers with varying wealth. If lenders are on the short side of the market, weakening liability rules lower lender profits, may cause additional exclusion among the poor, but generate additional rents for wealthier borrowers. For certain changes in liability rules (such as a ban on bonded labor, or weakening bankruptcy rules below a wealth threshold) they also raise productivity among borrowers of intermediate wealth. Hence they can be interpreted as a form of efficiency-enhancing redistribution from lenders and poor borrowers to middle class borrowers. Our model provides a possible rationale for why weaker liability rules are observed in wealthier countries.

1 Introduction

Legal limits on the liability of borrowers vary extensively across countries, and are the subject of extensive policy debates. Some poor countries such as Nepal exhibit and have until recently legally permitted contracts with bonded labor provisions, which are banned in most countries. Nevertheless many poor countries frequently exhibit occurrence of bonded labor, especially with respect to children.[3] Most countries operate under some form of bankruptcy law that limit borrower liability. The extent of bankruptcy protection varies considerably across countries: e.g.,

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[3]For a description of bonded labor contracts in different developing countries see U.S. Department of Labor (2005). The Nepalese Kamaiya system is a well-known example (see also Joshi (2003) and Edmonds and Sharma (2005)). These contracts were declared illegal by the Nepal government in 2000. As the US Department of Labor report states: "Loans are a central feature for maintaining the Kamaiya system. Since Kamaiyas are generally not

traditional Chapter 7 bankruptcy rules in the US limit liability much more than in Germany. Defaulting borrowers with wealth below a limit can essentially walk away from their debts under Chapter 7, while in German law a substantial fraction of their future earnings are required to be transferred to their creditors.[4]

From the perspective of contract theory, it is not evident that there should be any legally imposed limits on borrower liability at all. Borrowers and lenders could select liability provisions themselves and write them directly into credit contracts. Legal restrictions on these liabilities effectively disallow certain contracts with stricter liability provisions. If agents are rational and can foresee the future consequences of contractual provisions, it is not clear why the law needs to restrict their freedom to choose strict liability provisions.

Indeed, it is frequently argued that these liability restrictions are unfair, inefficient and significantly impair the functioning of credit markets. The inability of a given borrower-lender pair to choose contracts with strict liability provisions restricts the set of feasible contracts, resulting in a Pareto-inferior outcome. Weak liability rules limit the capacity of borrowers to precommit to repaying their loans, causing many to be excluded from the credit market. Others who do manage to secure credit are provided lower incentives to prevent default, raising default risk anticipated by lenders and the interest rates they are charged. These effects are more severe, the poorer the borrowers are. Empirical evidence supporting these claims have been made both in cross-country as well as cross-state analysis of correlations between lender rights and access of borrowers to finance (La Porta *et al* (1997, 1998), Gropp, Scholz and White (1997), Berkowitz and White (2000)). From this perspective, the best legal regimes are those with the strongest

paid enough to meet their basic needs, many have no choice but to take loans from their master. Many also carry inherited debts, sometimes going back for three or four generations, in addition to their own. A Kamaiya burdened by debt must continue to work for the same landlord until the debt has been repaid. The Kamaiya remains bound to the landlord unless, at markets held each winter, the Kamaiya finds a new master to pay off his debt or the original master sells off the Kaimaya and his family to a new master".

[4]Recent reforms in personal bankruptcy law in the US have, however, imposed exemption limits for Chapter 7 filings above which debtors can file for bankruptcy only under Chapter 13 which requires a repayment plan. In German law, every bankrupt debtor has to hand over a well defined fraction of his income for the next six years. For example, a single person is allowed to keep all of his income up to a limit of 930 Euros. 70% of the income between 930 and 3020 Euros, and 100% income above 3020 Euros, has to be handed over to debtors for the next four years, with 10–15% lower rates for the fifth and sixth years. In the seventh year, the bankrupt person is debt free and can undertake a "fresh start".

possible protection of lender rights.

Carrying this logic to an extreme, one is also led to question the existence of restrictions on bonded labor. From a consequentialist perspective, it makes little sense to not allow rational borrowers to commit themselves to providing indentured services to lenders should they lack the resources to repay their loans. The normative argument for banning bonded labor would have to be based either on lack of rationality or foresight of borrowers, or on deontological freedom-based principles. From a positive standpoint, the widespread prevalence of weak liability rules could be explained by the strong tension between their *ex post* costs and *ex ante* benefits. Once borrowers are in a state of distress, courts, politicians, the media and public opinion tend to be reluctant to allow well-to-do lenders and banks from implementing draconian measures to recover their debts. Yet economists argue that such practices are short-sighted and mistaken, weakening *ex ante* incentives that end up hurting borrowers in general.

Some arguments have been advanced in the literature for limiting lender rights. Manove *et al* (2001) argue that lenders may also be subject to incentive problems with respect to adequate screening and monitoring activities, which necessitate exposing them to some costs when the projects of their borrowers fail. Bolton and Rosenthal (2002) make a case for weak lender protection when credit contracts are incomplete and borrowers are risk-averse. Limiting borrower liability may then provide them useful insurance. Additional arguments for borrower protection may be advanced if borrowers do not rationally evaluate future consequences of strict liability rules.

In this paper we argue that even with complete contracts, rational risk-neutral agents, and absence of any incentive problems for lenders, there can be a coherent rationale for limiting borrower liability under specific circumstances (relating to the distribution of wealth and concentration in the credit market). The rationale arises from general equilibrium effects of liability restrictions in the presence of borrower moral hazard, which cause redistribution of rents from lenders to borrowers. Weaker liability rules limit the ability of lenders to extract rents from borrowers, causing equilibrium profit rates earned by lenders to decline. In a moral hazard environment, this (indirect) general equilibrium redistribution effect tends to increase borrower *ex ante* utility and effort levels, in contrast to the (direct) adverse effect of weaker precommitment ability of borrowers when liability rules are weakened.

We construct a model where profit rates are determined by stable matching allocations of borrowers and lenders, similar to two-sided matching market models of Gale and Shapley (1962) and Roth (2002). With heterogeneity of wealth among borrowers, the equilibrium profit is equal to that in the monopolistically optimal contract for lenders with respect to the poorest borrower that receives a credit contract. Owing to competition among lenders, wealthier borrowers capture all the rents from their superior ability to post collateral. Restricting liability of borrowers then reduces the ability of lenders to extract rents from poorer agents, thus reducing the equilibrium profit rate, which benefits wealthier borrowers and encourages them to select higher effort (owing to lower 'debt overhang'). At the same time the lower ability to precommit to repayment adversely affects their utility and effort. The latter effect is more severe, the poorer the borrower. The poorest borrowers may end up getting excluded altogether from the credit market. But the direct effect of reduced commitment ability is weaker for wealthier borrowers, for whom the favorable indirect general equilibrium effect is just as strong. So wealthier borrowers may benefit from weaker liability rules. In general, weaker liability rules result in a redistribution from lenders (and possibly poor borrowers) to 'middle-class' borrowers. When general equilibrium effects are incorporated, they do not result in a Pareto-inferior outcome.

An important question then concerns the efficiency effects of this form of redistribution. We consider two different contexts. In the first, borrowers are protected by bankruptcy law and bonded labor is banned. We consider the effects of weakening provisions of bankruptcy law, e.g., for the fraction of subsequent earnings of defaulting borrowers required to be transferred to their debtors. This generally hurts poor borrowers and benefits wealthier borrowers in terms of their *ex ante* utilities. The corresponding effort effects depend on the precise manner in which the law is weakened. If a constant fraction has to be transferred, lowering the fraction causes effort of all borrowers to decline. If on the other hand, it is lowered only for poor borrowers, then effort of wealthier borrowers is raised — since their repayment commitment is unaffected, so only the favorable general equilibrium effect operates. Somewhat paradoxically, weakening the law selectively for the poor ends up benefiting the non-poor more.

In the second setting, we assume that a strong bankruptcy law is in place, and evaluate the effect of bonded labor. When permitted, bonded labor provisions arise only for agents with wealth below a threshold. For wealthier borrowers a ban on bonded labor has no direct adverse impact, since they do not employ any. If marginal borrowers are poorer than the threshold, and the lenders are on the short side of the market, lender profits decline owing to the restriction on their ability to extract rents from the poorest borrowers. For borrowers with wealth above the threshold, then, only the general equilibrium effect operates, and both effort and utilities rise. Poor borrowers on the other hand may suffer as they may lose access to credit, and effort declines owing to the primacy of the direct effect of weaker precommitment ability. If the wealth distribution of borrowers is more concentrated among 'middle class' rather than the poor, a ban on bonded labor (similar to weakening bankruptcy law only for the poor) can then be a form of redistribution which also raises efficiency.

In either setting, the presence of sufficient 'middle class' borrowers can provide the basis for weakening liability rules. If on the other hand poor borrowers predominate, concern for their welfare can motivate strong liability rules. In poor countries, thus, bonded labor may be justified as a means of widening credit access. More developed countries with a large middle class may instead seek to ban bonded labor, and use bankruptcy law to limit borrower liability. Within such countries, the argument for or against weakening bankruptcy law depends on the distribution of wealth, on how concentrated credit markets are, and societal trade-offs between efficiency and redistribution.

The paper is organized as follows. Section 2 discusses relation to existing literature. Section 3 introduces the model. Section 4 considers the case of bankruptcy law, when bonded labor is banned. Section 5 studies the effect of banning bonded labor. Section 6 discusses some extensions and applications.

2 Related Literature

Several studies examine cross-sectional variation across different countries or states states with differing personal bankruptcy laws. La Porta *et al* (1997) use empirical measures of protection of lender rights to explain cross-country variation in availability of external financing to the private sector. They find that countries with less extensive lender rights (such as those with legal origins in French civil law) have narrower debt markets. Gropp, Scholz and White (1997) use 1983 *Survey of Consumer Finances* data to examine effects of differing bankruptcy law in different US states on debt and interest rates. Their main findings are that poor agents are adversely affected by a weak bankruptcy law. Interest rates are higher (only) for poorer agents if bankruptcy law is weak. Moreover, poor agents are more likely to be denied credit if bankruptcy law is weak. At the same time, the propensity of richer agents to receive credit is higher under a weaker bankruptcy law. These findings are consistent with the predictions of our model.

Berkowitz and White (2004) also use the variation in exemption levels but focus on the availability of credit to small enterprises. Since small firms are often not incorporated, their owner is personally liable for all financial obligations of the firm. Since bankruptcy law defines the extent to which the owner can be held liable, bankruptcy law can be expected to affect credit access. They find that small firms are more likely to be denied credit in firms with high exemption levels, i.e., subject to a weaker bankruptcy law. Interestingly, the probability of credit denial is increasing in the exemption level but only for low wealth levels. For firms with high net assets, they find the overall effect is close to zero.

Among theoretical arguments for a weak bankruptcy law owing to incomplete contracts, Bolton and Rosenthal (2002) consider an agricultural economy subject to macroeconomic shocks. While farmers (debtors) and lenders can use debt contracts to enable production and give incentives to work hard, they write insurance contracts that are not contingent on macroeconomic shocks. As a result, *ex post* intervention of the government has a potentially beneficial aspect because it helps provide insurance to farmers. This is true even if the intervention is anticipated. Similarly, insurance advantages of a weak bankruptcy law are stressed by Gropp, Scholz and White (1997) and Fan and White (2003).

Manove, Padilla and Pagano (2001) provide an alternative argument for weak bankruptcy law, in terms of the need to provide banks with incentives to screen investment projects. In their model, only lenders have the expertise to ascertain project quality by engaging in a costly screening process. They show that in a competitive credit market, equilibrium loan contracts will be designed with excessively high collateral requirements that leave lenders with insufficient incentives to screen projects. Legal restrictions on collateral mitigate this inefficiency. If the credit market were more monopolistic, this inefficiency also tends to be mitigated as lenders internalize the effects of superior project quality.

Concerning bonded labor, Braverman and Stiglitz (1982) discuss their role in motivating effort incentives and risk-taking, but do not provide a welfare analysis. Srinivasan (1980, 1989) examines determinants of worker preferences for bonded labor clauses *vis-a-vis* credit contracts where default is followed by denial of credit in the future, and their effects on technological innovation. However, he does not provide a welfare analysis of bonded labor laws. Genicot (2002) provides an argument for banning bonded labor used by a monopolistic landlord to preempt competition from a moneylender who cannot employ bonded labor. In her model, banning bonded labor generates greater competition between the landlord and the moneylender, and thus may increase welfare for poor farmers.

The contrast between partial equilibrium and general equilibrium effects of imposing legal restrictions on contracts is similar to arguments presented by Bardhan (1989), Basu (1989), Basu and Van (1998) and Kanbur (2001). Basu (1989) investigates the welfare implications of banning sexual harrasment on the workplace. In Basu and Van (1998) a ban of child labor is analyzed. While banning child labor renders every family (slightly) worse off in partial equilibrium, these effects might be overturned by the general equilibrium impact on the wage rate. Braverman and Stiglitz (1982) also discuss cases of interlinked labor-credit or tenancy-credit contracts which may be welfare reducing for either landlord or workers when general equilibrium effects are taken into account.

The role of eviction threats as supplementary incentive devices is discussed in Banerjee, Gertler and Ghatak (2002) and Banerjee and Ghatak (2004). These incentive effects are similar to bonded labor but involve no deadweight losses with full commitment on the principal's part. Only with a hold-up problem (limited commitment ability of the principal) do they give rise to *ex ante* inefficiencies (though no *ex post* inefficiencies). A ban on eviction threats (only) in that case can be welfare improving.

3 The model

The economy has a given population of m principals and n agents. A principal and agent pair, once matched, can jointly invest in an upfront investment I to start a project. Each agent can work on at most one project. It is simplest to consider the case where a similar restriction is imposed on principals as well (motivated by the limited time available to the principal to screen loan applicants and monitor progress of the project). However, the results are unaffected as long as there are given capacity limits in terms of the number of projects each principal can fund, and there are at least two distinct principals.[5]

Each agent has a given level of wealth w to start with; without loss of generality this is less than I, the upfront investment needed for the project (since others will self-finance). Principals have enough wealth to pay the upfront investment. In case an agent or principal does not participate in the project they each earn zero from it. Agents are distinguished from one another only by their wealth, while all principals are identical.

Once a principal and agent are matched and participate in the project, they enter into a contract which describes how financing and returns from the project are shared among them. The returns to the project are stochastic. With probability e the project is a success; the state of the world will be called *good*, and denoted by s. In that case the project return is y_s. With probability $1-e$ the project fails, the state of the world is *bad*, denoted by f, and the corresponding return is $y_f < y_s$.

The agent's effort determines the probability of a good state. Investing effort is costly and causes effort costs $D(e)$ which are strictly convex, thrice differentiable, strictly increasing, with $D'''(e) \geq 0$.[6]

We shall additionally assume that (i) $y_s > I > y_f$, so that the project is never going to be viable if the agent selects zero effort, and (ii) there exists a level of effort e at which the project is viable, i.e., $e(y_s - y_f) + y_f > I + D(e)$.

After the project has been completed, the agent can work l hours on a spot labor market and earn an income of $R \cdot l$. This opportunity to work on the spot market arises irrespective of whether the agent participated in the project. Working on the spot labor market causes an additional effort cost $g(l)$ to the agent. In the event that the agent defaulted on the financial contract, the spot market provides an opportunity to supplement earnings that can be used to

[5] Corresponding results for the case of a single monopolist principal can be derived from analysis of the P-optimal contracts. In this case the general equilibrium effects will not appear, and the comparative statics with respect to liability rules will be quite different.

[6] The assumption on convexity of marginal disutility of effort simplifies the analysis considerably, as it implies that the cost of providing the agent with appropriate effort incentives (i.e., inclusive of the incentive rents) is convex.

pay back debt to the principal. Alternatively if bonded labor is permitted, the *ex ante* contract with the principal can specify an extent of labor that the agent is required to provide on behalf of the principal.

3.1 Contracts, Legal Regimes and Sequence of Moves

Once a principal and agent are matched, their interaction is structured as follows.

In Stage 1, the principal and the agent write a contract which specifies their respective contributions to the upfront investment I, financial transfers and (if bonded labor is permitted) spot market labor services to be provided by the agent to the principal *ex post*, contingent on the realized state of the world. Therefore, a contract will be $\{I_A, \tilde{t}_i, \tilde{l}_i\}_{i=s,f}$, where I_A denotes the contribution of the agent to the upfront investment, $\tilde{t}_i$ is the financial transfer from the agent to the principal in state i, and $\tilde{l}_i$ the labor services provided (as a bonded laborer) for the principal. If the law forbids bonded labor, then $\tilde{l}_i$ is constrained to equal zero. Moreover, transfers must be financially feasible for the agent to pay out of the project returns alone, so the contract does not have to condition on future labor market earnings of the agent. Hence $I_A \leq w, \tilde{t}_i \leq w - I_A + y_i$. Note that if the legal regime permits the contract to mandate a payment exceeding $w - I_A + y_i$, this is tantamount to allowing bonded labor. The key assumption is that transfers financed out of project earnings or agent wealth (prior to working on the spot market) entail no deadweight costs, in contrast to any supplementary transfer financed by subsequent earnings of defaulting borrowers.

At Stage 2, the agent selects effort e.

At Stage 3, the state of the world i is realized. The agent and the principal can then renegotiate the financial contract, from $\tilde{t}_i, \tilde{l}_i$ to t_i, l_i, if and only if both agree to the replacement. The only constraint that the new contract must satisfy is that $t_i \leq w - I_A + y_i$.

After this, the subsequent actions and consequences depend on the legal framework within which the principal and agent function. To start with, consider the following two polar legal settings.

In the first, bonded labor is banned and contract liability is enforced by a bankruptcy law, which gives the agent freedom to default on the contract, and decide how much labor to provide subsequently on the spot market. The law mandates some punitive transfers P that the agent must pay to the principal, a function of the ex post wealth $w - I_A$, project returns y_i and spot labor market earnings $R.l_i$ of the agent. There are some deadweight losses associated with the enforcement of these punitive transfers, given by a function $Q(P)$ which is strictly increasing, convex and twice differentiable, with $Q(0) = 0$. The principal thus receives $P - Q(P)$. These include the legal and other costs incurred by the principal in recovering payments from defaulters.

In such a setting, at Stage 4 of the contract the agent decides whether or not to default on the obligation to pay t_i to the principal. In either case, the agent subsequently decides how much labor to provide on the spot market. If he does not default, none of these spot market earnings are owed to the principal: t_i is the actual *ex post* transfer. If the agent defaults, the agent is compelled to pay $P(l_i, w - I_A, y_i)$ to the principal. The punitive transfer must satisfy the constraint that such a payment is *ex post* feasible for the agent, hence $P(l_i, w - I_A, y_i) \leq R.l_i + w - I_A + y_i$. The bankruptcy law specifies the exact formula by which P is determined. In some countries (such as Chapter 7 provisions in the US) it allows transfer of wealth but not of subsequent labor earnings, while in others (such as Germany) it allows for transfer of part of subsequent labor earnings as well. This affects the extent of *ex post* inefficiency associated with default — besides the enforcement deadweight costs — the implicit taxation of subsequent labor market earnings results in undersupply of spot market labor. To illustrate the implications of different bankruptcy laws, we shall consider for most part a linear form for punitive transfers, which consist of the sum of a fixed fraction $\alpha(\leq 1)$ of *ex post* earnings $R.l_i$ and a fraction $\beta(\leq 1)$ of residual wealth $w - I_A + y_i$, so $P(l_i, w - I_A, y_i) = \alpha[R.l_i] + \beta[w - I_A + y_i]$. The parameters α and β then represent the strength of the bankruptcy law.

The other polar setting we shall consider involves strict enforcement of contracts and bonded labor is legally allowed. In these cases, the agent is not free to choose his *ex post* labor supply, and is bound by the contractual terms (for both financial transfers and bonded labor). The law does not stipulate any limits to the agent's liability, apart from the restriction that financial transfers be feasible for the agent, already built into the terms of the contract. Any additional transfers to the principal can be financed by the agent working on the spot market, as stipulated by the bonded labor clauses in the contract. We assume that the enforcement of bonded labor provisions of the contract entails some deadweight costs for the principal. So if the agent provides l_i services to the principal as per the contract, this will be subject to a deadweight loss $q(l_i)$ incurred by the principal, where $q(0) = 0$, q is a strictly increasing, convex and twice differentiable function. The net return to the principal in state i will then be $t_i + R.l_i - q(l_i)$. Further assumptions on these deadweight costs will be presented later, which essentially restrict the curvature of the q function.

In effect, both bonded labor and bankruptcy entail *ex post* deadweight losses, consisting of possible *ex post* oversupply of labor, and enforcement costs. Accordingly contracts will be structured *ex ante* to minimize their use. The interpretation of bonded labor is that it represents a means used by the principal to relax *ex post* limited liability constraints for poor agents, arising out of the need to ensure the agent has nonnegative consumption. Financial transfers will be used to the maximal extent permissible by the agent's wealth and project returns. If these prove insufficient to generate a required rate of return for the principal, supplementary bonded labor obligations are permitted. They will be used, at an *ex post* cost, in order to ensure an adequate *ex ante* return to the principal. When bonded labor is permitted, the contract stipulates these, reducing scope for *ex post* discretion for the agent with respect to subsequent spot labor market supply. When bonded labor is illegal and there is bankruptcy law instead, the agent has greater *ex post* discretion — whether to pay t_i, or seek refuge in bankruptcy which may reduce the transfers that the agent has to eventually pay. The agent retains control over its subsequent spot market labor supply — though these are subject to taxation — and therefore the magnitude of the eventual transfers.

Finally, in order to illustrate the effects of a ban on bonded labor, we shall consider a third, intermediate regime, where the law continues to enforce the financial transfers stipulated by the contract. This corresponds to a world without bonded labor but a strong bankruptcy law which allows no freedom to agents to default on the financial terms of their contract.[7]

We thus consider three different legal settings in all. The first is one where bonded labor is banned, and bankruptcy law may limit *ex post* financial transfers. In this setting the game continues beyond Stage 3 as follows. At Stage 4, the agent decides whether or not to default on the contract. If he defaults by not providing paying t_i to the principal, the law will enforce the

[7] Implicitly this must be backed up by sanctions for default that are sufficiently credible and punitive that default does not arise on the equilibrium path.

mandatory financial transfer rule P. Finally, at stage 5 the agent selects spot market labor supply l_i. If he has defaulted he pays P to the principal (who receives $P - Q(P)$), otherwise he pays only t_i (who receives t_i). The agent ends up with a net payoff of $w - I_A + y_i - P(l_i, w - I_A, y_i) + R.l_i - g(l_i) - D(e)$ in the former case, and $w - I_A + y_i - t_i + R.l_i - g(l_i) - D(e)$ in the latter.

In the intermediate setting with a strong bankruptcy law but with a ban on bonded labor, the agent is compelled to pay t_i as per the terms of the contract at Stage 4. Then at Stage 5, he selects his *ex post* labor supply $\hat{l}_i$, to end up with a net payoff of $w - I_A + y_i - t_i + R.\hat{l}_i - g(\hat{l}_i) - D(e)$.

In the last setting there is a strong bankruptcy law, and bonded labor is allowed without restriction. At Stage 4 the agent is compelled to pay t_i to the principal, as well as provide bonded labor services of l_i. Finally at Stage 5, he decides whether to provide any supplementary labor supply $\hat{l}_i$. The agent then ends up with an *ex post* payoff of $w - I_A + y_i - t_i + R.\hat{l}_i - g(l_i + \hat{l}_i) - D(e)$.

4 Bankruptcy without Bonded Labor

In this section we consider the first legal setting described above, and illustrate the implications of varying provisions of bankruptcy law (e.g., represented by the parameter α).

Consider the decision of the agent at Stage 4 whether to declare bankruptcy. If the agent decides to default, he has to make a payment of $\beta \cdot [w - I_A + y] + \alpha \cdot (R \cdot l)$ to the principal. This gives him $(1 - \beta) \cdot [w - I_A + y] + (1 - \alpha) \cdot (R \cdot l) - g(l) - D(e)$ as utility. His consumption will be $(1 - \beta) \cdot [w - I_A + y] + (1 - \alpha) \cdot (R \cdot l)$.

The utility of the agent depends on the amount of labor he is willing to bring forward in period $t = 4$. After declaring bankruptcy, the agent will choose $l \geq 0$ to maximize $(1 - \beta) \cdot [w - I_A + y] + (1 - \alpha) \cdot (R \cdot l) - g(l)$. The solution is denoted by $l^*(\alpha, R)$ and implicitly given by $(1 - \alpha) \cdot R = g'(l^*(\alpha, R))$. The eventual profit of the principal in case of bankruptcy will be $\beta \cdot [w - I_A + y] + \alpha \cdot R \cdot l^*(\alpha, R) - (I - I_A)$.

If on the other hand the agent does not declare bankruptcy, he will pay t_i to the principal. This allows him to consume $w - I_A - t_i + R \cdot l$. The utility will be $w - I_A - t_i + R \cdot l - g(l) - D(e)$. The optimal level of labor is derived as in the case of bankruptcy and equals $l^*(0, R)$.

The agent will choose not to declare bankruptcy if the no-default condition

$$\begin{aligned}(1-\beta)\cdot(w-I_A)+(1-\alpha)\cdot R\cdot l^*(\alpha,R)-g(l^*(\alpha,R)) \\ \leq w-I_A-t_i+R\cdot l^*(0,R)-g(l^*(0,R))\end{aligned} \quad (1)$$

holds. It turns out that every contract that fulfills the no default constraint will be renegotiation proof, and attention can be restricted to such contracts.

Proposition 1 *(i) On the equilibrium path an agent will not declare bankruptcy.*

(ii) Attention can be restricted to renegotiation proof contracts.

(iii) Contracts are renegotiation proof if and only if they fulfill the no-default condition (1).

Proof. (i) Suppose the agent declares bankruptcy under a given contract. Then the principal will receive a transfer which is at most $\beta\cdot(w-I_A)+\alpha\cdot R\cdot l^*(\alpha,R)$, i.e., the punitive transfers mandated by the law, less the legal cost to the principal of enforcing these transfers. Moreover, owing to $\alpha<1$ the agent will undersupply labor subsequently. Then there exists a Pareto-dominating contract which avoids these distortions, in which the agent is required to make the alternative transfer $\hat{t}_i=\beta\cdot(w-I_A)+\alpha\cdot R\cdot l^*(\alpha,R)+\gamma$, where γ is a small positive number, chosen so that $0<\gamma<[R\cdot l^*(0,R)-g(l^*(0,R)]-[R\cdot l^*(\alpha,R)-g(l^*(\alpha,R)]$. This implies that the new transfer $\hat{t}_i$ is smaller than previously mandated $\tilde{t}_i$, so continues to be a feasible transfer (i.e, $\hat{t}_i<\tilde{t}_i\leq w-I_A+y_i$). Moreover, by construction the new contract satisfies the no-default condition strictly, so the agent will not default on the contract. Then both the agent and the principal are better off.

(ii) Take a contract C that is renegotiated and let the final contract be C'. The contract C' must be renegotiation proof. The initial contract C can now be replaced by the new contract C'.

(iii) If a contract fulfills (1), it is optimal for the agent to not declare bankruptcy, whence no *ex post* inefficiencies occur and there is no scope for renegotiation.

To see that every renegotiation proof contract fulfills (1), suppose to the contrary that this were not true. Then there exists a contract which is renegotiation proof, yet violates (1). The argument in (i) can be used to show that the original contract can be Pareto-dominated by an alternative contract, contradicting the premise that the contract is renegotiation-proof. ■

Every legal contract leads to a utility level the agent can achieve as well as an effort level the agent will choose. Therefore, it is convenient to describe contracts in economic terms: specifying state-contingent indirect utilities for the agent, along with the induced effort $\{v_i, e\}_{i=s,f}$. Indirect utility (net of effort disutility) in the good state of the world is defined as $v_i \equiv y_i + w - I_A - t_i + R \cdot l^*(0, R) - g(l^*(0, R))$. Moreover, the *ex post* surplus is $S_i \equiv y_i + S(R)$, where $S(R) \equiv R \cdot l^*(0, R) - g(l^*(0, R))$. The expected profit of the principal is then $\pi = e \cdot [w - I_A - v_s + S_s] + (1 - e) \cdot [w - I_A - v_f + S_f] - (I - I_A) = w - I + e \cdot (S_s - v_s) + (1 - e) \cdot (S_f - v_f)$. The expected utility for the agent is $V = e \cdot v_s + (1 - e) \cdot v_f - D(e)$.

Using standard arguments, attention can be restricted to contracts characterized by the following constraints.

$$\begin{aligned} \text{IC} &: \quad v_s - v_f = D'(e) \\ \text{LL} &: \quad v_f \geq (1 - \beta) \cdot (w - I_A) + (1 - \alpha) \cdot R \cdot l^*(\alpha, R) - g(l^*(\alpha, R)) \\ \text{PC}_A &: \quad e \cdot v_s + (1 - e) \cdot v_f - D(e) \geq w + S(R) \\ \text{PC}_P &: \quad e \cdot (S_s - v_s) + (1 - e) \cdot (S_f - v_f) - (I - w) \geq 0 \end{aligned}$$

The incentive compatibility constraint IC characterizes the effort chosen by the agent. The constraint LL is the no-default constraint in the bad state, which also implies the validity of the no-default condition in the good state (since w.l.o.g. $v_s > v_f$; otherwise $e = 0$ owing to IC and then the good state never arises). It can be viewed as a limit on the *ex post* liability of the agent. By the previous proposition, it ensures that the contract is also renegotiation proof. The participation constraints ensure that both the agent and the principal receive at least as much as in autarky.

Note that w.l.o.g. the contract will require the agent invest his entire *ex ante* wealth upfront $I_A = w$, with the principal funding the rest. This leaves *ex ante* payoffs of both unaffected, as well as all constraints except LL, which is weakened. From now onwards we shall restrict attention to such contracts. The value of β is then irrelevant; the strength of bankruptcy law is represented solely by the parameter α.

A weaker bankruptcy law (e.g., a lower α) increases the *ex post* utility of the agent associated with the default option, limiting credible promises by the agent to repay the principal, thus ultimately restricting the set of *ex ante* feasible contracts. It appears to be in the interest of neither principal nor the agent, if we abstract from general equilibrium considerations. Note that if the contract could stipulate punitive payments in the event of default, subject to an upper bound set by the law, it would be (privately) optimal for every contracting pair of principal and agent to set default payments at this upper bound.

4.1 Matching

We now analyze the general equilibrium implications of bankruptcy law. Our economy is characterized by a set $A = \{a\}$ of agents with differing wealth w_a and a set $P = \{j\}$ of principals. In what follows we describe the case where each principal can fund at most one project, though the results extend straightforwardly when this is relaxed and there are at least two competing principals. Each matched pair of principal and agent with wealth w can select a contract $\{v_i, e\}_{i=s,f}$ from the set $\Gamma(w)$ of contracts satisfying constraints IC, LL, PC_A and PC_P.

An **allocation** is a matching of principals and agents (with all unmatched principals/agents remaining inactive), and a contract $C \in \Gamma(w_a)$ for every matched (a, j) pair. The resulting payoff for the principal j is $\pi = e \cdot [S_s - v_s] + (1 - e) \cdot [S_f - v_f] - (I - w)$ and for the agent a is $e \cdot v_s + (1 - e) \cdot v_f - D(e)$.

Definition: An allocation is stable if there does not exist any pair (a,j) that could select a deviating contract $\tilde{C} \in \Gamma(w_a)$ that would make both better off.

An agent a is said to be *viable* if there exists a feasible contract for that agent, i.e., $\Gamma(w_a)$ is nonempty. Note that bankruptcy law (represented by α) affects the set of feasible contracts for any given agent, so can alter the set of viable agents. A higher value of α weakens the LL constraint, thus permitting the set of viable agents to expand. For any given α, the set of viable agents is simply the set of agents with wealth above a critical threshold. Let n denote the number of viable agents. Order viable agents in order of their wealth: $w_1 \geq w_2 \geq ...$ Let m denote the number of principals (more generally, the maximum number of projects that can be funded, aggregating across all the principals).

A simple characterization of stable allocations can now be provided. For this we need the following definitions.

P-optimal contract A P-optimal contract for a viable agent with wealth w is the contract that maximizes the payoff of the principal $\pi = e \cdot [S_s - v_s] + (1-e) \cdot [S_f - v_f] - (I - w)$ over the set $\Gamma(w)$ of feasible contracts. The resulting profit will be denoted $\pi^P(w)$.

A-optimal Contract An A-optimal contract corresponding to profit target π is one which maximizes the agent's expected payoff over the set of feasible contracts $\Gamma(w)$ satisfying the constraint that the principal attains an expected payoff of at least π, provided at least one such contract exists. The optimized utility is denoted $U(\pi)$.

Proposition 2 *Stable allocations exist. In every stable allocation*

(A) the number of agents that will be matched is at most $Q = \min\{n, m\}$ and equal to Q for generic distributions of wealth. Moreover, the Q-richest agents are matched with a principal.

(B) All principals obtain a common profit

$$\pi \begin{cases} \in [\max\{\pi^P(w_{m+1}), 0\}, \pi^P(w_m)], & \text{if } m \leq n; \\ = 0, & \text{if } m > n. \end{cases}, \tag{2}$$

(C) Every matched agent a gets an A-optimal contract subject to the constraint that the profit of the principal must be at least π.

The idea is simple — the market consists of a population of n viable agents with heterogeneous wealth, and m identical principals. Competition among the latter implies that all principals must earn the same profit, defined by the marginal viable agent, with all 'rents' for intramarginal agents accruing entirely to those agents. The only source of indeterminacy arises when there is a wealth gap between the last matched agent, and the next wealthiest one — with a large number of agents this indeterminacy shrinks. In what follows we shall resolve this indeterminacy (somewhat arbitrarily) by setting $\pi = \pi^P(w_m)$, i.e., awarding all the surplus between the last matched agent and the principal to the latter. The structure of contracts and payoffs resulting in stable allocations can then be understood as an interaction between those of P-optimal and A-optimal contracts. The contract received by the marginal matched agent is the P-optimal contract for that agent. The corresponding profit $\pi^P(w_m)$ is the profit earned by all principals. All other wealthier agents receive the A-optimal contract corresponding to the minimum profit target $\pi^P(w_m)$. This characterization will be used to derive the implications of changing bankruptcy law (parameter α).

Note that the notion of a stable allocation is related to the core, but not identical to it. It corresponds to the core when only coalitions of a single principal and agent are allowed to form. Coalitions of agents are not allowed to form: conceivably in this setting with wealth constraints there is scope for coalitions of agents to engage in ROSCA-like wealth lotteries. This requires the use of randomized contracts, which remain beyond the purview of this paper.

Before proceeding to examine the effect of varying the strength of bankruptcy law, it is useful to note key qualitative features of P-optimal and A-optimal contracts.

Lemma 3 *(i) In a P-optimal contract, there exist thresholds $w_1, w_2 (> w_1)$ such that (a) for a viable agent with wealth w below w_1 the optimal effort is e^* which maximizes $e(S_s - S_f) - eD'(e)$, and $v_f = L_\alpha$, so LL binds but APC does not; (b) for wealth between w_1 and w_2 the optimal effort is $e_\alpha(w) \in (e^*, e^{FB})$ which solves $w + S(R) = L_\alpha + eD'(e) - D(e)$, and $v_f = L_\alpha$, so both LL and APC bind; (c) for $w \geq w_2$ the optimal effort is e^{FB} and $v_f = w + S(R) - e^{FB}D'(e^{FB}) + D(e^{FB})$, so LL does not bind while APC does.*

(ii) In an A-optimal contract relative to Π, the optimal effort $\hat{e}(\Pi, L_\alpha, w)$ is $\min\{\tilde{e}(\Pi, L_\alpha, w), e^{FB}\}$, where $\tilde{e}(\Pi, L_\alpha, w)$ is the largest e solving

$$S(e, w) \equiv e(S_s - S_f) + S_f - e'D'(e) + w - I = L_\alpha + \Pi, \tag{3}$$

and $v_f = w - I + S(\hat{e}(\Pi, L_\alpha, w), w)$. The agent attains an expected utility of $F(\hat{e}(\Pi, L_\alpha, w), w) - \Pi$, where $F(e, w) \equiv e(S_s - S_f) + S_f + w - I - D(e)$ denotes the first-best surplus.

The nature of optimal efforts is shown in Figure 1. Only agents with wealth above $\underline{w}$ are viable. The P-optimal contract provides rents to poor agents with wealth between $\underline{w}$ and w_1, and implements the same effort e^* for any wealth in this region. In the unsuccessful state the agent gets a utility equal to the liability limit L_α, and an additional incentive rent $D'(e^*)$ in the successful state. This generates expected utility $L_\alpha + e^*D'(e^*) - D(e^*)$ which is higher than the agent's outside option. An agent with wealth w_1 is exactly indifferent between this 'efficiency wage' contract and his outside option. For agents between w_1 and w_2 the principal has to offer a better contract

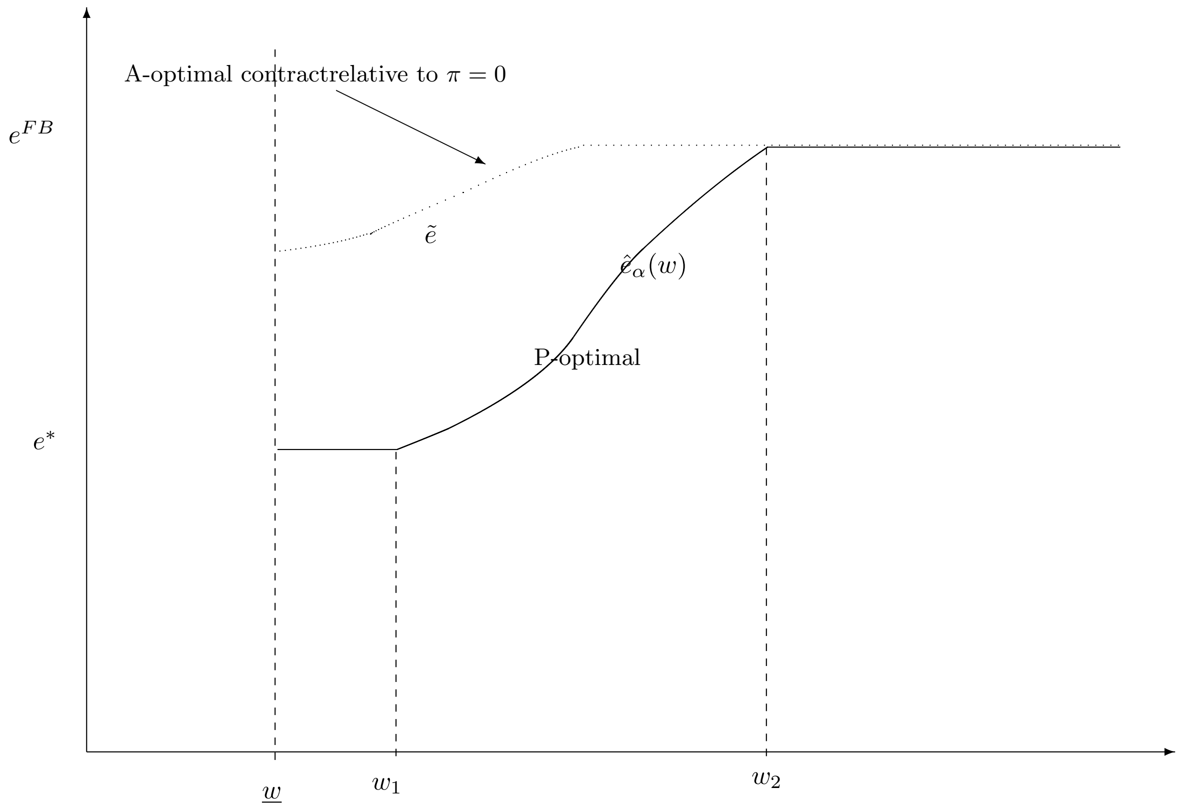


Figure 1: Effort in P-Optimal and A-Optimal contracts

to induce them to participate: accordingly these agents are provided a higher incentive rent in the successful state, motivating them to choose higher effort, while payment in the unsuccessful state is the same as in the efficiency wage contract, so the LL constraint continues to bind. Effort rises with wealth, until wealth w_2 where it equals the first-best effort. Thereafter the LL constraint ceases to bind. Agents with wealth above w_1 do not earn any rents, while those between $\underline{w}$ and w_1 do.

4.2 Effects of Changing Bankruptcy Law

The P-optimal contract is of independent interest insofar as it will be the outcome of a market monopolized by a single principal. Conditional on continuing to receive a contract, agents with wealth below w_1 will be induced to select the same effort e^* (and incentive rent) as before, while their payment in the unsuccessful state will rise, if bankruptcy law is weakened.[8] Contracting with such agents will be less profitable for lenders; the weaker law may cause the poorest agents to lose viability and get excluded from the credit market altogether. For those that still have access, those below w_1 will benefit, while those above w_1 will be unaffected. The converse is true of effort: it remains the same below w_1 and above w_2, but falls for those with intermediate wealth. The weaker law will thus allow some redistribution of rents from the monopoly lender to agents with wealth below w_1. But it comes at the expense of a utility loss for the poorest agents that lose access, and lower productivity amongst agents of intermediate wealth.

When principals compete, however, Proposition 2 shows that the properties of the P-optimal contract are relevant only for the poorest borrower with access to credit. For all others, the A-optimal contract is relevant. Holding the profit rate fixed at a pre-specified level, the properties of the A-optimal contract indicate the nature of credit contracts received by intramarginal borrowers. In an A-optimal contract all incentive rents remain with the agent, while the participation constraint of the principal binds. The agent becomes the residual claimant, so maximizes first-best surplus, subject to a financial viability constraint of leaving lenders with a prespecified profit target. For wealthy borrowers, the first-best effort is financially viable, and is chosen by them. For poorer borrowers, the first-best effort is not viable. In that case they select the highest effort which is viable, given by the function $\tilde{e}$. Figure 2 shows this. The financial viability constraint is represented by the requirement that the inverse-U shaped curve representing the function $S(e, w)$ (which is maximized at e^*) lies above $L_\alpha + \pi_{alpha}$. This reduces to selecting efforts in an interval the highest value of which is given by $\tilde{e}(w)$ for an agent with wealth w. The financial constraint is less binding for wealthier agents since they can post more collateral. Hence wealthier agents choose higher effort.

[8] In this paragraph we omit the qualifications resulting from the fact that the thresholds w_1 and w_2 themselves depend on α. Accordingly, the statements apply to borrowers whose wealth is not close enough to these thresholds that they switch their position relative to the threshold.

In the case of an A-optimal contract relative to a given profit target, a weaker bankruptcy law causes L_α to rise, making the financial viability constraint more stringent. This causes a loss in their expected utility and effort levels (unless they are wealthy enough to not be constrained in the first place). The partial equilibrium effect of a weaker bankruptcy law (i.e., when lender profits are held fixed) is then a Pareto-inferior outcome: the only effect is a weaker precommitment ability of the agents which ends up hurting themselves and lowering their productivity.

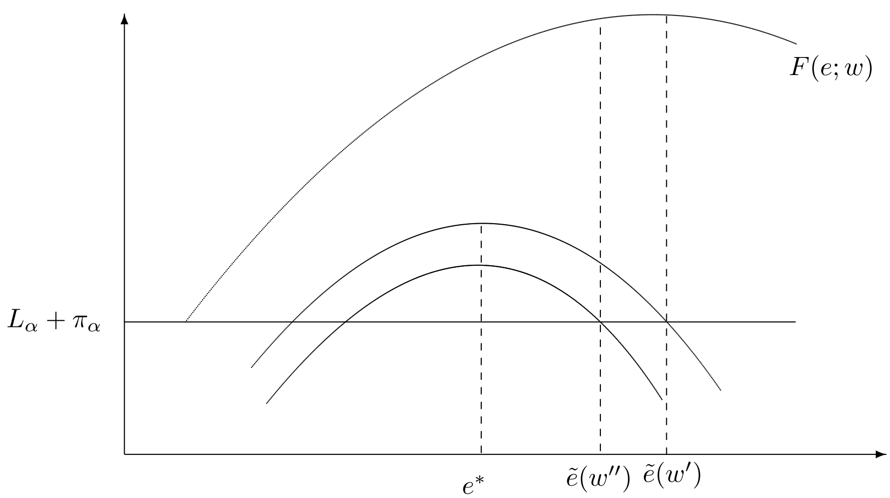


Figure 2: Effort in A-Optimal Contract For Agents with Varying Wealth

Now turn to the general equilibrium implications of varying α in the setting where principals compete. We use Proposition 2 to derive the change in the equilibrium profit rate. In what follows, we shall make explicit the dependence of the (chosen) stable allocation on α. Specifically, there are $n(\alpha)$ viable agents, where an increase in α means bankruptcy law is strengthened, enlarging the set of viable agents, so $n(\alpha)$ is a nondecreasing function. The set of active agents will be $\min\{m, n(\alpha)\}$, also nondecreasing in α. We focus on stable allocations in which the entire surplus with the marginal matched agent goes to the principal, i.e.,

$$\pi(\alpha) = \begin{cases} \pi^P(w_m;\alpha), & \text{if } m \leq n(\alpha); \\ 0, & \text{if } m > n(\alpha). \end{cases} \tag{4}$$

Proposition 4 *Suppose that bankruptcy law becomes weaker, i.e., α falls from $\bar{\alpha}$ to $\underline{\alpha}$.*

(A) The set of active agents either decreases or remains the same.

(B) Principals earn the same or lower profit.

(C) If principals earn zero profit with the stronger bankruptcy law, then each agent is either made worse off or is left unaffected.

(D) For any agent active before and after the change, effort decreases or remains the same.

(E) Suppose $m \leq n(\underline{\alpha})$, i.e. the same number of agents are active both at $\bar{\alpha}$ and $\underline{\alpha}$. Then the marginal agent (with w_m) is either better off or unaffected.

(F) The utility change of all agents with higher wealth than w_m is greater than the utility change of the marginal agent with w_m.

Proof. (A) This is true due to the construction of $n(\alpha)$.

(B) Take any $\bar{\alpha} > \underline{\alpha}$. It has to be shown that $\pi(\underline{\alpha}) \leq \pi(\bar{\alpha})$. If $\pi(\underline{\alpha}) = 0$ this is obvious. If $\pi(\underline{\alpha}) > 0$ then it must be the case that $\pi(\underline{\alpha}) = \pi^P(w_m;\underline{\alpha}) > 0$. Then, $\pi(\bar{\alpha}) = \pi^P(w_m;\bar{\alpha}) \geq \pi^P(w_m;\underline{\alpha}) = \pi(\underline{\alpha})$, because $n(\bar{\alpha}) \geq n(\underline{\alpha}) \geq m$.

(C) From $\pi(\bar{\alpha}) = 0$ it follows that $\pi(\underline{\alpha}) = 0$. As α goes from $\bar{\alpha}$ to $\underline{\alpha}$, some agents active at $\bar{\alpha}$ may become inactive at $\underline{\alpha}$ and these agents cannot benefit.

Next, consider agents who are active both at $\bar{\alpha}$ and $\underline{\alpha}$. They get an A-optimal payoff relative to $\pi = 0$ in both cases, and a lower α cannot make this any higher.

(D) First note that for any w, $L_\alpha + \pi^P(w;\alpha)$ cannot decrease as α goes from $\bar{\alpha}$ to $\underline{\alpha}$. If L_α increases by ϵ the principal has the option of raising v_s and v_f equally by ϵ while leaving effort unchanged, so in the P-optimal problem the fall in the profit $\pi^P(w;\alpha)$ is bounded above by ϵ.

Second, recall from lemma 3 that the A-optimal contract for any agent involves setting $e^A_\alpha = \min\{e^{FB}; \tilde{e}(\pi(\alpha); L_\alpha; w)\}$. Also recall from Proposition 2 and assumption (4) that $\pi(\alpha)$ equals the P-optimal profit with the poorest active agent. Hence if $m \leq n(\underline{\alpha})$, then $\pi(\alpha) = \pi^P(w_m; \alpha)$ and $L_\alpha + \pi(\alpha)$ cannot decrease as α falls. Therefore, $\tilde{e}(\pi(\alpha); L_\alpha; w)$ cannot increase for any active agent (since at the largest solution to (3) $e \cdot (S_s - S_f) - e \cdot D'(e)$ is decreasing in e).

If $m > n(\bar{\alpha})$ then $\pi(\bar{\alpha}) = \pi(\underline{\alpha}) = 0$, $L_\alpha + \pi(\alpha)$ is higher at $\underline{\alpha}$ and the same argument as above applies.

If $n(\bar{\alpha}) > m > n(\underline{\alpha})$ then $\pi_{\bar{\alpha}} \leq L_{\underline{\alpha}} - L_{\bar{\alpha}}$ because the agent who was viable at $\bar{\alpha}$ is no longer viable at $\underline{\alpha}$. This implies that $\pi_{\bar{\alpha}} + L_{\bar{\alpha}} \leq L_{\underline{\alpha}} + \pi_{\underline{\alpha}} = L_{\underline{\alpha}}$. Otherwise, if $\pi_{\bar{\alpha}} > L_{\underline{\alpha}} - L_{\bar{\alpha}}$ then the principal could pay $L_{\underline{\alpha}} - L_{\bar{\alpha}}$ more in both states of the world to the agent and still earn a profit, so the agent would be viable at $\underline{\alpha}$.

(E) An agent with w_m gets the P-optimal contract corresponding to his wealth w_m and α (which is the A-optimal contract corresponding to $\pi^P(w_m)$). We know from above that this agent's effort is the same or lower.

Let $e^* = \text{argmax } e \cdot (s - f) - e \cdot D'(e)$. If $e \geq e^*$ both before and after the change, the agent's utility will be $w + S(R)$, and thus remain unaffected. If $e_{\bar{\alpha}} \geq e^* = e_{\underline{\alpha}}$, the participation constraint is non-binding and the agent earns positive rents at $\underline{\alpha}$, while the participation constraint is binding at $\bar{\alpha}$, then the agent is strictly better off in the former situation.

(F) Take any two wealth levels w' and w'' with $w' > w''$. The agent's benefit from the change to the weak bankruptcy law is non-decreasing in wealth:

$$U(\underline{\alpha}, w') - U(\bar{\alpha}, w') \geq U(\bar{\alpha}, w'') - U(\underline{\alpha}, w'') \tag{5}$$

$$\Leftrightarrow (e^A_{\bar{\alpha}}(w') - e^A_{\underline{\alpha}}(w'))(S_s - S_f) - (D(e^A_{\bar{\alpha}}(w')) - D(e^A_{\underline{\alpha}}(w'))) \leq \tag{6}$$

$$(e^A_{\bar{\alpha}}(w'') - e^A_{\underline{\alpha}}(w''))(S_s - S_f) - (D(e^A_{\bar{\alpha}}(w'')) - D(e^A_{\underline{\alpha}}(w'')))$$

where $e^A_\alpha(w)$ denotes the effort in the A-optimal contract for agent with wealth w with bankruptcy law α.

Since effort is nondecreasing in wealth, $e^A_{\underline{\alpha}}(w') \geq e^A_{\underline{\alpha}}(w'')$ with the strong inequality as long as $e^A_{\underline{\alpha}}(w'') \neq e^{FB}$. Using this information and the fact that $e \cdot (S_s - S_f) - D(e)$ is concave, it follows that $U(\underline{\alpha}, w') - U(\bar{\alpha}, w') \geq U(\underline{\alpha}, w'') - U(\bar{\alpha}, w'')$ if the change in effort is greater for the poorer agent: $(e^A_{\bar{\alpha}}(w') - e^A_{\underline{\alpha}}(w')) \leq (e^A_{\bar{\alpha}}(w'') - e^A_{\underline{\alpha}}(w''))$.

We claim this is indeed the case. Recall that $e^A_\alpha(w) = \min\{\tilde{e}(\pi(\alpha); L_\alpha; w); e^{FB}\}$, where $\tilde{e}(\cdot)$ is the largest solution for e in equation 3. Note that the left-hand-side of this equation is concave in e; so it has two solutions, and is downward sloping at the larger solution, with a slope that is steeper the larger $\tilde{e}$ is (refer to Figure 2). The wealthier agent selects the higher effort, so the slope is steeper there. Since the increase in the right hand side of the equation, $L_\alpha + \pi(\alpha)$, is common to all agents, the required cutback in effort is greater for the poorer agent. This is evident if $e^A_{\bar{\alpha}}(w'') < e^{FB}$. It is also true if $e^A_{\bar{\alpha}}(w'') = e^{FB}$, for then $e^A_{\bar{\alpha}}(w') = e^{FB}$ also, and we get

$$(e^A_{\bar{\alpha}}(w') - e^A_{\underline{\alpha}}(w')) \leq (e^A_{\bar{\alpha}}(w'') - e^A_{\underline{\alpha}}(w'')) \quad (7)$$

$$\Leftrightarrow e^A_{\underline{\alpha}}(w'')) \leq e^A_{\underline{\alpha}}(w')) \quad (8)$$

which is true because $w' > w''$. ■

Corollary 5 *Suppose profits are positive both before and after the weakening of bankruptcy law. Then all agents are weakly better off, while principals are worse off.*

Weakening the bankruptcy law may lead to the exclusion of some active agents, as those agents may now cease to be viable. If the principals are on the short side of the market, so profits are positive to start with, the profit of the principals will decrease (as long as the poorest borrower is credit-constrained in the sense that the effort is less than first-best).

If profits decline, there is a positive general equilibrium effect on the welfares of intramarginal active agents. This runs against the direct negative effect of weaker commitment ability of these agents. The net effect can go either way.

This is illustrated by considering two polar cases. The first is where the principals are on the long side of the market, so profits are zero both before and after the change in the bankruptcy law. Then only the direct effect operates, and no agent can benefit. Some agents will cease to be active, as they are no longer viable. They are rendered worse off if they were previously obtaining positive rents. Other intramarginal agents who continue to be active are also left worse off as long as their wealth is low enough that the limited liability constraint is binding. Their weaker ability to commit limits their access to credit.

The other polar case is where the principals are on the short side of the market and profits are positive both before and after the change. In this case, the welfare impacts on the agents are dramatically different: no active agent loses market access, and all active agents benefit from

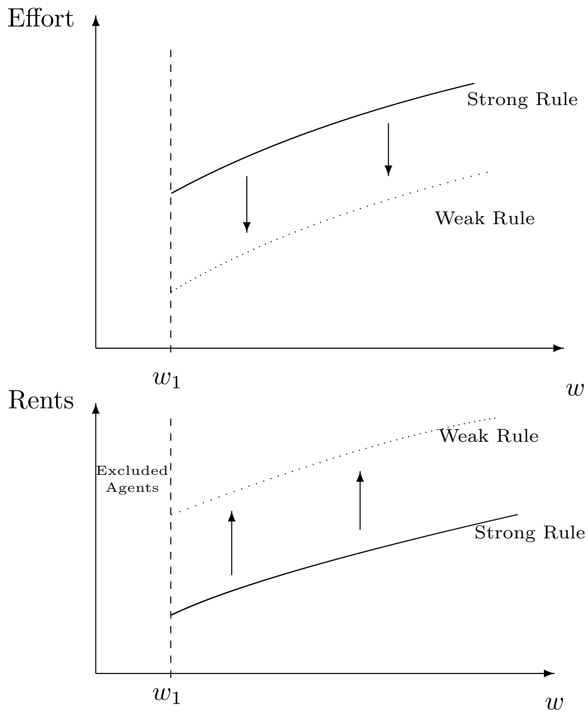


Figure 3: Effects Of Weakening Bankruptcy Rule When Set of Active Agents is Unaffected

the weakening of bankruptcy law. The general equilibrium effect of lowered profit outweighs the direct negative effect of weaker commitment ability of agents. Figure 3 depicts this situation.

In intermediate cases, depicted in Figure 4, the profit rate is positive with the strong bankruptcy law, but drops to zero with the weaker law. In this case some active agents are excluded from the market: these agents may be worse off. Wealthier intra-marginal agents may be better off, owing to the general equilibrium effect. In general, as part (F) of the result shows, the wealthier agents benefit more from the weakening of bankruptcy law, essentially because the negative direct effect resulting from the aggravation of the limited liability constraint is less acute for them. Hence a weaker bankruptcy law results in a redistribution from poorer agents and principals to agents of intermediate wealth (who are not wealthy enough to self-finance, yet are wealthy enough to not be affected substantially by the weaker precommitment ability). Weaker bankruptcy law can be understood as a redistribution in favor of middle class borrowers against the poor as well as lenders.

The contrast between the direct and indirect effects suggests that the effect on productivity

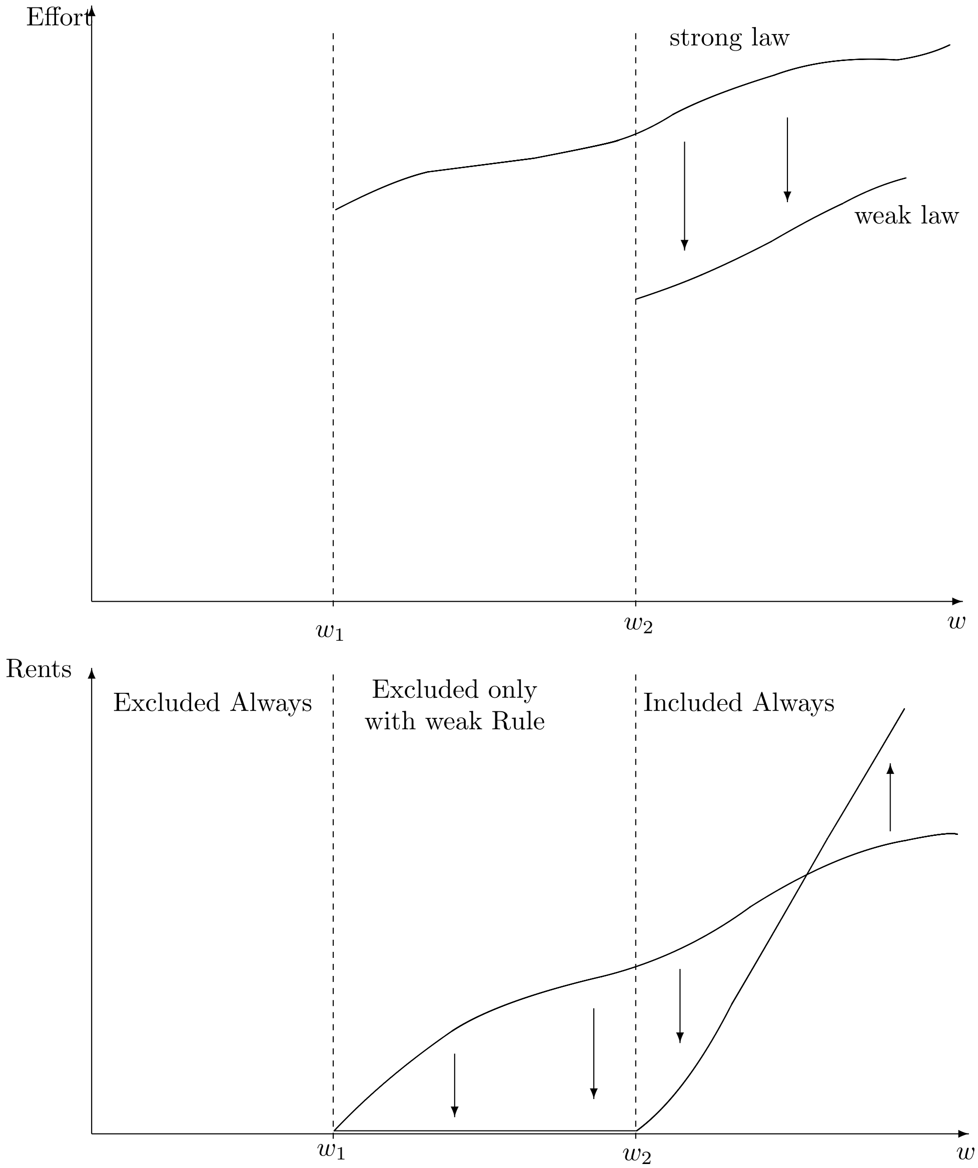


Figure 4: Effects Of Weakening Bankruptcy Rule When Set Of Active Agents Shrinks

could also go either way. The direct effect would tend to reduce effort, while the indirect effect which transfers rents from principals to agents would tend to raise effort (as it is akin to an increase in the agent's wealth). Yet, it turns out that the net effect on effort can never be positive. In contrast to the impact on agent welfare levels, the direct negative effect always outweighs the indirect positive effect on effort of each active agent.

The preceding statements pertain, however, to the situation where there is an across-the-board reduction in α, applicable to all borrowers. Consider instead a selective weakening of the law, applicable only to those with wealth below some threshold.

Proposition 6 *Suppose to start with there is a constant α that applies to all wealth levels, followed by a selective lowering of α for all borrowers below some threshold w^* at which borrowers initially had access to credit and for whom the LL constraint was binding. If the equilibrium profit rate was initially positive, borrowers wealthier than w^* become better off, their effort cannot decrease, and increases strictly for all those with effort below e^{FB} to start with.*

The effects of the relaxation of the liability rule selectively for poor borrowers are somewhat paradoxical: such a seemingly pro-poor reform may end up hurting the poor (by causing many of them to be excluded from the market), but they do benefit the wealthier borrowers not covered by the reform. For the latter are exempted from the harmful partial equilibrium effect of lower commitment, and can only benefit from the general equilibrium effect. The latter will arise as long as the law is weakened for borrowers poor enough that the liability limits are binding, and equilibrium profits were positive to start with. A key difference from the case of an across-the-board reduction in α is that the productivity effects are now positive for wealthy borrowers. This result provides a rationale for selective access of poor borrowers to Chapter 7 rules in the US: they can be viewed as a form of redistribution from lenders (and maybe poor borrowers) to middle-class borrowers. If the wealth distribution is concentrated on the middle, such forms of redistribution can promote efficiency besides being politically popular.

However, the reform that occurred in March 2005 in the US limited access to Chapter 7 filings for borrowers with wealth above a threshold, so was the exact opposite of the reform described in Proposition 6. In the case of a selective strengthening of bankruptcy law above a threshold, there will be no general equilibrium effects (assuming those below the threshold continue to retain access to credit). Only the partial equilibrium effects will appear, and according to our model their effect is described by an increase in α on A-optimal contracts with a fixed profit rate. This results in an *ex ante* Pareto improvement with gains accruing (only) to wealthier borrowers owing to their improved commitment ability.

5 Bonded labor

In this section we consider the effects of bonded labor. As explained previously, we assume that there is strong enforcement of contracts — borrowers are not given any leeway to deviate from terms of the original contract, either with respect to financial transfers or bonded labor. Financial transfers are assumed to be costlessly enforced, while bonded labor services l entail some deadweight enforcement cost $q(l)$, besides possible *ex post* inefficiency if labor market returns are low compared with the agent's effort disutility. Consequently efficient contracts will use bonded labor only after exhausting the scope for financial transfers, i.e., $l_i > 0$ only if $t_i = y_i + w - I_A$. Given this property, there is no scope for a contract with bonded labor to be renegotiated. Lowering the extent of bonded labor to be provided by the latter cannot be accompanied by a supplementary financial transfer to the lender, as the borrower has no funds left. Hence bonded labor may arise in equilibrium despite the attendant deadweight costs. This is in contrast with bankruptcy, which did not actually occur in equilibrium.

We now introduce the notation used to analyse bonded labor contracts. Legally, a contract $\{t_i, l_i\}_{i=s,f}$ specifies transfer payments t_i and an amount of labor l_i the agent has to work for the principal in each state i. Contracts are restricted to satisfy the constraints $t_i \leq y_i + w - I_A$, with equality holding if $l_i > 0$. Given any such contract, an agent will offer supplementary labor $\hat{l}_i$ on the spot market, which solves the problem of maximizing $Rl - g(l_i + l)$ with respect to choice of $l \geq 0$. Let the maximized payoff attained $R\hat{l}_i - g(l_i + \hat{l}_i)$ be denoted by $-G(l_i)$. G can then be interpreted as a net cost to the borrower associated with bonded labor obligation l_i, after incorporating supplemental earnings from the spot market. If $M(R) \geq 0$ denotes the maximum value of $R.l - g(l)$ with respect to $l \geq 0$, it is clear that $-G(0) \equiv M(R)$ and $G(l_i)$ is an increasing function of l_i. In particular it increases at a constant rate of R upto $l_i = \hat{l}(0, R)$ where the marginal disutility $g'(l_i)$ equals R. Over this range the agent supplies an *ex post* efficient level of labor services, with an increase in bonded labor 'crowding out' supplementary spot market

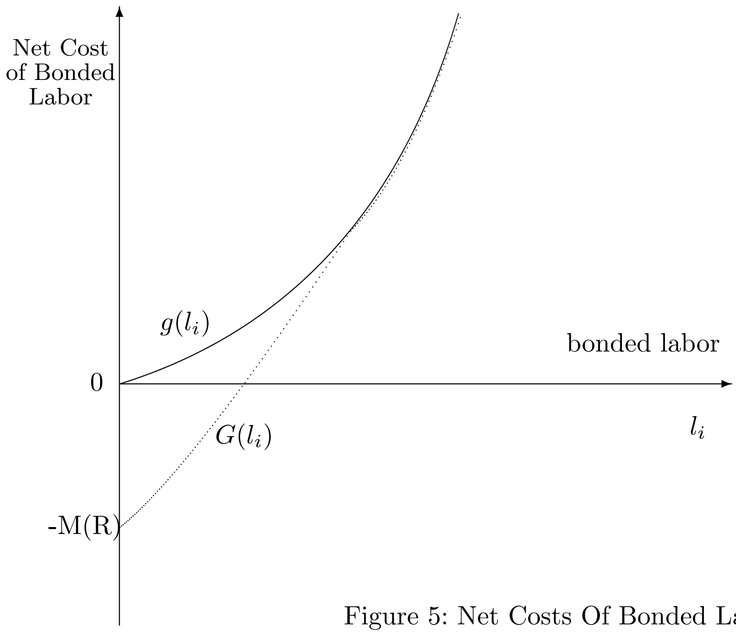


Figure 5: Net Costs Of Bonded Labor

labor one-for-one. For bonded labor obligation exceeding the *ex post* efficient level, the agent will supply no additional labor to the spot market; G will coincide with the labor supply disutility function g from that point onwards. See Figure 5.

The payoff of the borrower in state i (net of *ex ante* effort cost) will then be $v_i \equiv y_i + w - I_A - t_i - G(l_i)$. Using state contingent payoffs to denote the contract, the financial transfer t_i equals $y_i + w - I_A - v_i - G(l_i)$. In addition to this, the lender receives from the bonded labor services worth $x(l_i) \equiv R.l_i - q(l_i)$, owing to need to expend $q(l_i)$ resources to enforce the bonded labor obligation. We assume that $q'(l_i) \in (0, R)$ for all l_i, implying that x is an increasing function, i.e., that the marginal enforcement costs q' are always positive but lie below the return to labor supply R. So increasing bonded labor in any state raises the returns to the principal.

It is also convenient to replace the bonded labor requirement l_i by its net cost to the borrower $G(l_i)$. Since $G(.)$ is strictly increasing, it is an invertible function, and we can express the net return to the lender as a function of the state-contingent cost G_i of bonded labor to the borrower: $r(G_i) \equiv x(G^{-1}(G_i)) \equiv RG^{-1}(G_i) - q(G^{-1}(G_i)$. Since G is also convex, r is a strictly increasing, concave function, with a slope everywhere strictly less than R, owing to the existence of positive marginal costs of enforcing bonded labor obligations.

The net deadweight cost of a bonded labor obligation that costs G_i to the borrower is then given by the function $Q(G_i) \equiv G_i - r(G_i)$ defined over $G_i \geq -M(R)$. $Q(.)$ is convex owing to the concavity of $r(.)$. It is also easily checked that this is a strictly increasing function, with a slope between 0 and 1.[9] In what follows, we shall impose an upper bound on the convexity of this deadweight loss function:

$$\frac{Q''(G)}{Q'(G)} < \frac{Q'(G)\cdot(1-Q'(G))}{D''(1)+D'(1)} \text{ for all } G \geq -M(R) \tag{9}$$

In other words, the marginal deadweight costs of bonded labor should not be increasing at too fast a rate. In particular, it requires that the marginal disutility of effort D' and its rate of increase D'' be bounded. The role of this assumption will be explained in due course.

Using $(\{v_i, G_i\}_{i=s,f}, e)$ to denote a bonded labor contract, we can express the net payoff to the lender in state i as $t_i + r(G_i) - (I - I_A) \equiv y_i + (w - I_A) - v_i - G_i + r(G_i) - (I - I_A) \equiv w - I + y_i - v_i - Q(G_i)$. So the expected profit of the lender is $\pi = w - I + e\cdot(y_s - y_f) + y_f - e(v_s + Q(G_s)) - (1-e)\cdot(v_f + Q(G_f))$, while expected utility of the borrower is $V = e\cdot v_s + (1-e)\cdot v_f - D(e)$. Moreover, the liquidity constraint on financial transfers $t_i \leq y_i + w - I_A$ is equivalent to the constraint that $v_i \equiv y_i + w - I_A - t_i - G_i \geq -G(l_i)$. Therefore a feasible contract is described by the following constraints:[10]

$$\text{LL}: \quad v_s \geq -G_s,\ v_f \geq -G_f, \text{ with } G_i \geq -M(R), i = s, f \tag{10}$$

$$\text{IC}: \quad v_s - v_f = D'(e) \tag{11}$$

$$\text{PPC}: \quad \pi \equiv w - I + e\cdot(y_s - y_f) + y_f - e(v_s + Q(G_s)) - (1-e)\cdot(v_f + Q(G_f)) \geq 0 \tag{12}$$

$$\text{APC}: \quad V \equiv ev_s + (1-e)v_f - D(e) \geq w + M(R) \tag{13}$$

Intuitively, the potential value of bonded labor is that it helps relax the liability limit of the agent, and permits the lender to extract more resources than would be permitted by the agent's project returns and net personal wealth. However, this is subject to a deadweight enforcement cost Q. The optimal design of a bonded labor contract will have to trade off the benefit of relaxing liability limits on the agent with their corresponding deadweight losses.

In the intermediate regime where bonded labor is banned, the additional restriction that $G_f = G_s = -M(R)$ applies. Here the flexibility to adjust the liability limit is no longer available.

[9] Since q is strictly increasing, $x(l_i)$ has a slope less than R. Moreover, G' is never less than R, so the slope of G^{-1} is bounded above by $\frac{1}{R}$. Therefore $r(G_i)$ which is a composition of these two increasing functions, is rising at a rate less than 1. Hence $Q(G) \equiv G - r(G)$ is a strictly increasing function, with a slope less than 1.

[10] The agent's participation constraint reflects the net payoff in the event of not participating in a project, the sum of *ex ante* wealth w and subsequent labor market opportunities $M(R)$.

In what follows we shall assume that the *ex ante* wealth w of the agent is nonnegative. This seems fairly innocuous, though relaxing it may allow additional flexibility to agents in a dynamic setting whereby they may be allowed to carry their debts forward into the future.

The constraints above characterize the set of feasible bonded labor contracts. We can define a contract to be P-optimal if it maximizes π over the feasible set, and a contract to be A-optimal relative to profit target π^* if it maximizes the agent's expected utility over the subset of feasible contracts that generate expected profit of at least π^*. Finally a contract is said to be Pareto optimal if there does not exist another which makes either the principal or agent better off, without making the other worse off.

We start by noting some important qualitative features of optimal contracts.

Lemma 7 *(i) In any Pareto optimal contract, bonded labor does not arise in state s, and the LL constraint binds if at all in state f.*

(ii) APC binds in a P-optimal contract.

(iii) In an A-optimal contract relative to π^*, bonded labor is used in state f for an agent with wealth w if and only if the associated effort e satisfies

$$S(e; w) \equiv w - I + e(y_s - y_f) + y_f - eD'(e) < \pi^* + M(R). \tag{14}$$

If (14) holds, $G_f > -M(R)$ is the unique solution for G in the equation

$$G - (1 - e)Q(G) = \pi^* - S(e; w). \tag{15}$$

Otherwise $G_f = G_s = -M(R)$ and bonded labor is not required in either state.

Bonded labor is used, if at all, only when the project fails. If they are also being used in the successful state, it implies that maximal use of financial transfers are not being made in that state. To provide effort incentives the agent must be better off in the successful state, which requires the agent be left with some financial surplus in that state. A Pareto improving change in the contract is then possible, by lowering bonded labor and raising financial transfers. Property (ii) says that the ability to impose bonded labor without any restriction allows a monopolist principal to fully expropriate the agent. It allows relaxation of the liability restriction on the agent in the unsuccessful state that would arise in the absence of bonded labor. Though this restriction comes at the expense of some deadweight enforcement costs, it is nonetheless profitable for the principal to enforce them. Property (iii) describes when bonded labor would be used in an A-optimal contract in order to implement a given effort level: this happens if the surplus left available after paying the agent incentive rents necessary to implement the given effort level is insufficient to cover the profit target of the principal (besides the spot labor market rents the agent would earn in the event of non-participation).

The set of effort levels that would necessitate the use of bonded labor in an A-optimal contract can be seen in Figure 2. For a given wealth level, the surplus function $S(e, w)$ is concave in effort, with an interior maximum at e^*. The set of effort levels that can be implemented without bonded labor thus constitutes an interval, with a maximum at $\tilde{e}(w; \pi^*)$. Implementation of effort levels higher than this would necessitate bonded labor. A higher level of wealth w raises the surplus function. This suggests that bonded labor will tend to be used for poorer agents, controlling for the effort level. However, the effort level is endogenously determined, an issue we turn to next.

5.1 Choice of Effort in A-Optimal Contracts

Let $K(e; \pi^*; w)$ denote the optimal choice of G_f to implement effort e for an agent of wealth w in an A-optimal contract relative to profit target π^*, as described in Lemma 7. The corresponding choice of v_f is then $v_f = S(e, w) - (1 - e) \cdot Q(K(e; \pi^*; w)) - \pi^*$, yielding the following expression for the expected utility of the agent as a function of the effort level alone:

$$V \equiv v_f + \alpha(e) = S(e, w) - (1 - e) \cdot Q(K(e; \pi^*; w)) - \pi^* + \alpha(e) \quad (16)$$

$$= F(e, w) - (1 - e) \cdot Q(K(e, \pi^*, w)) - \pi^* \quad (17)$$

where $F(e; w) = w - I + e \cdot (y_s - y_f) + y_f - D(e)$ is the first-best surplus function. Expression (17) says that the second-best surplus accruing to the agent is simply the first-best surplus, less the amount π^* that has to be transferred to the principal, and the expected deadweight loss $(1 - e) \cdot Q(K(e, \pi^*, w))$ of bonded labor.

(17) implies that for agents wealthy enough that the first-best effort e^{FB} is implementable without bonded labor, the optimal effort is indeed the first-best level. For all others, implementation of the first-best effort necessitates bonded labor. For such agents the marginal deadweight

loss of bonded labor needs to be incorporated in deriving the second-best solution. Note that the expected deadweight loss $(1-e)Q(K(e,w,\pi^*))$ is nonconvex in e. In particular it is possible that the marginal deadweight loss with respect to higher effort is negative at sufficiently high effort levels, while it is positive at lower levels, since

$$C_e = -Q(K(e,\pi^*,w)) + (1-e)\cdot Q'(K(e,\pi^*,w))\cdot K_e. \tag{18}$$

Since bonded labor is effected only in the unsuccessful state, increasing the effort implies a lower chance of having to be in such a state in the first place. Counterbalancing this is the higher level of bonded labor required, in the event that the project is unsuccessful. If the unsuccessful state is sufficiently unlikely in the first place, the first effect could dominate the second. It is difficult to rule out the possibility that increasing the agent's effort beyond some point actually reduces the expected cost of bonded labor. Indeed, for this reason we cannot guarantee that the second-best effort lies below the first-best level.

Nevertheless, what we need for our purposes is a comparative static property of how second-best effort varies with agent wealth, and this can indeed be shown as a result of the assumption (9) bounding the curvature of deadweight loss function. We show that the bonded labor cost satisfies a single-crossing property with respect to wealth: marginal deadweight costs of raising effort are higher for wealthier agents:

Lemma 8 *Let* $C \equiv (1-e)Q(K(e,\pi^*,w)))$ *denote the expected cost of bonded labor. Then*

$$\frac{\partial}{\partial w}\left(\frac{\partial C}{\partial e}\right) > 0 \tag{19}$$

whenever $C > 0$.

The role of assumption (9) can be explained as follows. Given any desired effort level, the extent of bonded labor required to implement it is higher, the poorer the agent. Increasing the effort level reduces the probability of the unsuccessful state, i.e., the likelihood of having to implement bonded labor. This is a benefit of raising the effort level, which is greater for poorer agents since they involve more bonded labor.

Countering this effect is the fact that higher effort necessitates more bonded labor. The convexity of the deadweight loss function implies this effect is also stronger for poorer agents, since they have a higher level of bonded labor to start with. Assumption (9) restricts the strength of the second effect, allowing the previous effect to dominate.

We are now in a position to prove that poorer agents select higher effort and more bonded labor. Moreover poorer agents derive a greater benefit from the use of bonded labor.

Proposition 9 *Consider any A-optimal contract relative to a given profit target π^*.*

(i) There exists a $\bar{w}$ s.t. all agents with $w < \bar{w}$ select bonded labor and all $w > \bar{w}$ do not select bonded labor.

(ii) Effort is decreasing in wealth among agents with $w < \bar{w}$, nondecreasing among richer agents, and strictly increasing whenever their effort is less than the first-best level.

(iii) Poorer agents derive greater ex ante benefit from the use of bonded labor than wealthier agents.

Proof. We start by proving (iii), amongst the set of agents that select positive bonded labor. Pick an interval in which all agents select bonded labor. Lemma 8 implies that poorer agents within this interval will select a higher effort. Consider wealths w_6, w_7 with $w_7 < w_6$ in this interval. Then

$$e(w_7, \pi^*) > e(w_6, \pi^*) > \tilde{e}(w_6, \pi^*) > \tilde{e}(w_7, \pi^*), \tag{20}$$

where $e(w, \pi^*)$ is the effort chosen by an agent with wealth w if bonded labor is allowed. See Figure 6. If bonded labor is not allowed, then it is easily checked that the optimal effort will be $\tilde{e}(w, \pi^*)$, since this is the largest effort below the first-best effort that is implementable without bonded labor.

We claim that the increased utility achievable from use of bonded labor is greater for the poorer agent w_7. For this agent, bonded labor permits effort to increase from $\tilde{e}(w_7, \pi^*)$ to $e(w_7, \pi^*)$. Using inequality (20) it follows that we can break up this change into the sum of three changes: (a) an increase from $\tilde{e}(w_7, \pi^*)$ to $\tilde{e}(w_6, \pi^*)$, (b) increase from $\tilde{e}(w_6, \pi^*)$ to $e(w_6, \pi^*)$, and (c) increase from $e(w_6, \pi^*)$ to $e(w_7, \pi^*)$. For the richer agent with wealth w_6, (b) represents the only change. Over the change in effort represented in (b), the poorer agent attains a higher utility increase, owing to Lemma 8. Adding to this the effect of changes (a) and (c) which arise only for the agent with wealth w_7, it follows that this agent benefits more from use of bonded labor.

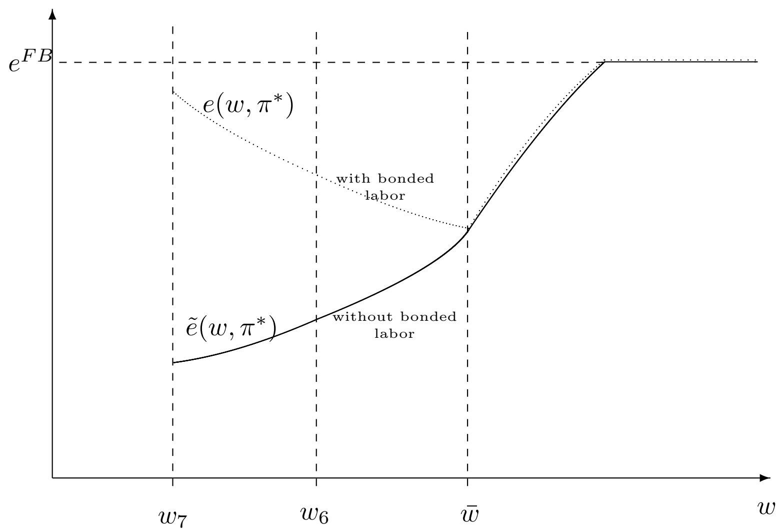
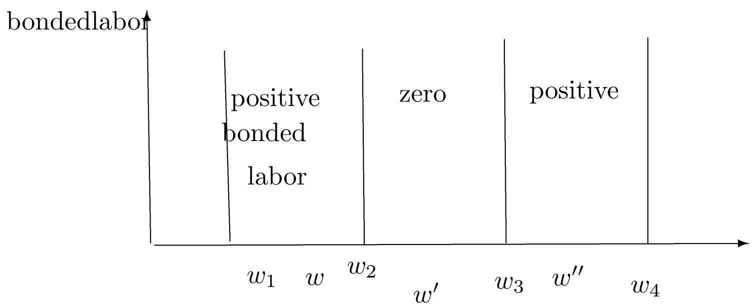


Figure 6: Effort With And Without Bonded Labor For Given Profit Rate

Now turn to (i). If this is false, there exist wealth levels w, w' and w'' with $w'' > w' > w$ where agents with wealth w and w'' select positive bonded labor and w' does not. So we can find wealth levels w_1, w_2, w_3, w_4 with $w_1 < w < w_2 < w' < w_3 < w'' < w_4$ such that agents with wealth in (w_1, w_2) select bonded labor, agents with wealth in (w_2, w_3) do not, and agents with wealth in the interval (w_3, w_4) choose bonded labor (as shown in the following figure):



Since the envelope theorem applies, the indirect utility of agents is continuous in w, with and without the use of bonded labor. Hence the utility gain from use of bonded labor must also vary continuously with w. Remember that all $w' \in (w_2, w_3)$ do not select bonded labor. Taking limits as $w' \to w_3-$ it follows that w_3 derives no benefit from bonded labor. On the other hand, all $w \in (w_3, w_4)$ derive a positive benefit from bonded labor. Moreover, we have shown above that the poorer among them obtain a higher benefit. So then taking limits as w' approaches w_3 from the right, the benefit from bonded labor must be strictly positive. This contradicts the continuity of the benefits in w at w_3.

(iii) now follows for the set of all agents, since those above $\bar{w}$ do not derive any benefit from bonded labor, while the argument provided above applies to all agents below $\bar{w}$. Finally, (ii) follows from combining (iii) and (i). ■

Consider now the effect of a ban on bonded labor. We represent the equilibrium resulting within any legal regime by the stable allocation in the game where principals and agents are matched. The characterization of stable allocations is exactly analogous to that provided for the bankruptcy law regimes. As before, we assume that all the surplus with the marginal agent accrues to the corresponding principal.

Proposition 10 *If bonded labor were to be banned, the effects on the equilibrium in a stable* *allocation would be the following.*

(i) The number of active agents will either decrease or remain unchanged.

(ii) The equilibrium profit π earned by the principals will either decrease or remain unchanged.

(iii) If the number of active agents is unchanged, the utility of the marginal included agent will either increase or remain unchanged.

(iv) If fewer agents are active, the marginal excluded agent is left the same, while the expected utility of intramarginal excluded agents either decreases or is unchanged.

(v) Suppose an agent who remains active both with and without a ban on bonded labor, and is wealthy enough that he would not use bonded labor even when it is allowed. This agent cannot be rendered worse off with the ban, and benefits from it if and only if equilibrium profit π decreases. In the latter case, the agent's effort increases as a result of the ban (if it is less than first-best to start with), and is otherwise unaffected.

(vi) Consider poorer agents who are active both with and without a ban, who use bonded labor when allowed. If the profit rate falls, effort increases for the richest of these agents and is reduced for the poorest of these agents.

Proof. (i) follows from the fact that the set of viable agents shrinks or is unaffected by a ban, and (ii) from the fact that the equilibrium profit (if positive) is determined by the P-optimal contract with the marginal agent. A ban on bonded labor must cause this profit to decline if bonded labor is used with the marginal agent when allowed, and otherwise remains unaffected.

For (iii), if the number of active agents is unaffected then the marginal agent cannot be worse off with a ban, because with bonded labor part (ii) of Lemma 7 implies that the agent earns no rents, i.e., obtains his outside option exactly. So this agent cannot be worse off with a ban. Part (iv) follows from the fact that exclusion results in agents earning their outside options, whereas inclusion implies they attain at least their outside option, so they cannot be rendered better off by being excluded.

For any agent who does not use bonded labor when allowed, the only effect of the ban operates via the change in the profit. There is a change in utility and effort only if the profit declines, in which case both effects are positive (unless the agent is already at the first-bet effort, in which case only utility will rise). This establishes (v).

Finally for (vi), consider first the richest class of agents with wealth in the interval $(\bar{w} - \epsilon, \bar{w})$ for $\epsilon > 0$ sufficiently small, that use bonded labor when allowed. For these agents, utility and effort effects are close to those for an agent with wealth $\bar{w}$ who does not use bonded labor when allowed. From (v) it follows that the effect on these agents' utility and effort must be positive (assuming less than first-best effort). Consider alternately the poorest included agent both with and without the ban, who uses bonded labor when allowed. This agent must be the marginal agent when the ban is in effect, and thus obtains a P-optimal contract in this context. We claim that the corresponding effort e must lie below his effort e' when bonded labor is allowed. Otherwise, suppose $e \geq e'$. Note that this agent receives an A-optimal contract when bonded labor is allowed, hence e' is at least as large as e'', the effort in the P-optimal contract for this agent when bonded labor is allowed. It would then follow that $e \geq e''$, i.e., the effort in the P-optimal contract for this agent is not lowered when bonded labor is banned.

We now show this cannot happen in a P-optimal contract which uses bonded labor. When bonded labor is allowed, a P-optimal contract for an agent with wealth w maximizes $S(e,w) - v_f - (1-e)Q(G_f)$ subject to $v_f \geq -G_f, G_f \geq -M(R), v_f + \alpha(e) \geq w + M(R)$. By Lemma 7 the APC must bind, so $v_f = w + M(R) - \alpha(e)$. The LL constraint then reduces to $w + M(R) - \alpha(e) \geq -G_f$. So G_f will be set equal to $\alpha(e) - w - M(R)$, and the P-optimal problem reduces to maximization of $F(e,0) - (1-e)Q(\alpha(e) - w - M(R))$ with respect to e alone. Note that bonded labor is used if and only if $\alpha(e) > w$, and the expected cost of bonded labor $Z(e;w) \equiv (1-e)Q(\alpha(e) - w - M(R))$ satisfies $\frac{\partial^2 Z}{\partial e \partial w} = Q'(\alpha(e) - w - M(R)) - (1-e)Q''(\alpha(e) - w - M(R))\alpha'(e) > 0$ by virtue of assumption (9). So effort in the P-optimal contract must be decreasing in w over the range where bonded labor is used. On the other hand, when bonded labor is banned, effort in the P-optimal contract is nondecreasing in w. Combining these two facts, it follows that the P-optimal effort must be higher for an agent using bonded labor, compared with P-optimal effort for this agent when bonded labor cannot be used. ■

The welfare effects of the ban are shown in Figures 7 and 8, for the two cases respectively where there is and is not a reduction in the number of active agents (and the equilibrium profit falls as a result of the ban). In Figure 7, agents poorer than w_1 are always excluded. Those

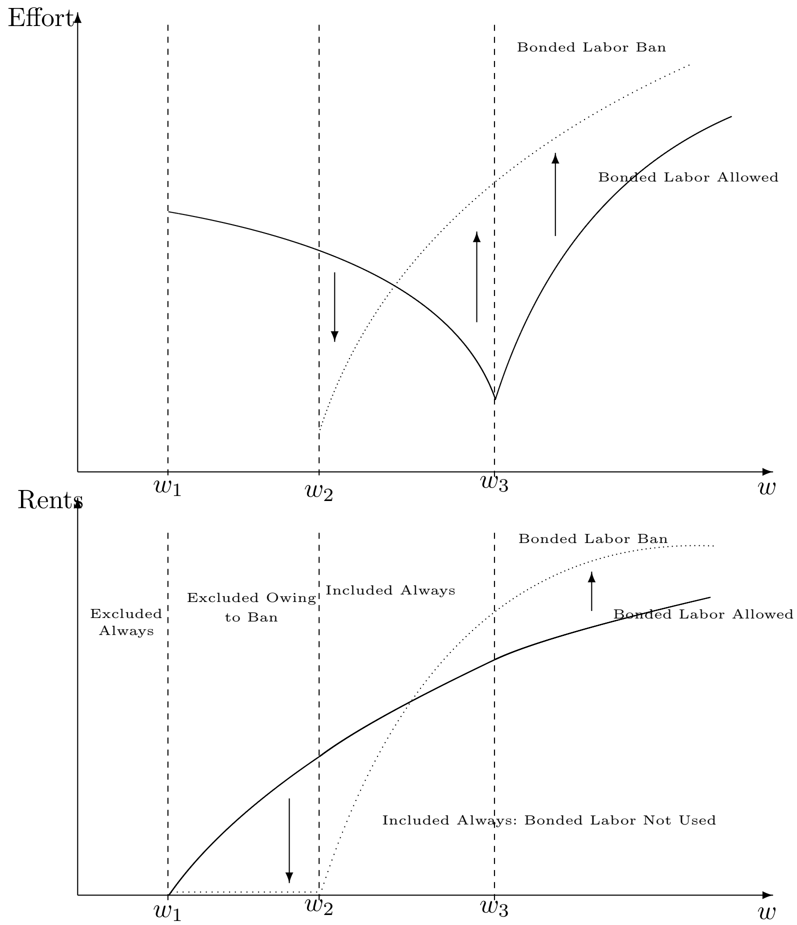


Figure 7: Effects of Banning Bonded Labor When Set of Active Agents Shrinks

between w_1 and w_2 are included when bonded labor is allowed but excluded when it is banned. The marginal agent w_1 when bonded labor is permitted is equally well off, but intramarginal agents between w_1 and w_2 earn some rents when bonded labor is used, so they are worse off being excluded as a result of the ban. Agents between w_2 and w_3 are included in both situations, and use bonded labor when permitted. The poorest among them must be worse off, while the richest among them must be better off. Those richer than w_3 do not use bonded labor when it is permitted, so are not directly affected by the ban. Hence they must benefit from the lower profit rate.

The upper panel of Figure 7 shows the corresponding change in effort levels for the set of included agents. Those above w_3 must choose a higher effort following the ban, as long as it was less than first-best to start with. Since they do not use bonded labor when it is allowed, the ban does not affect them directly (i.e., except through a change in the equilibrium profit rate). The reduced profit rate redistributes rents to these agents, resulting in higher effort. This is an important contrast with the effects of a weaker (linear) bankruptcy rule, where wealthier agents may be rendered better off owing to the general equilibrium effect, but their effort decreases. In that context the bankruptcy constraint is binding for any agent with less than first-best effort. Hence the the weakening of the bankruptcy rule has a direct adverse productivity effect, that eventually outweighs the benefit of a lower profit rate. In the bonded labor context, wealthy borrowers do not themselves use any bonded labor, so there is no direct partial equilibrium effect adversely impacting on their efforts and payoffs. It is similar to a selective weakening of bankruptcy for borrowers below some threshold, as described in Proposition 6.

Amongst agents between w_2 and w_3, effort rises for those near w_3, and falls for those near w_2. These agents use bonded labor when permitted, so there is a direct adverse effect of disallowing bonded labor which offsets the benefit from a lower profit rate. The richer amongst these agents obtain a smaller benefit from bonded labor so for them the profit effect dominates, while the opposite is true for the poorer ones.

Figure 8 shows the productivity and welfare impacts when the ban leaves the set of active agents unaffected. The marginal agent with wealth w_1 may benefit from the ban, as he receives a P-optimal contract in both situations. The ban limits the principal's capacity to extract incentive rents from this agent: if w_1 is smaller than $\alpha^{-1}(e^*)$ (where it may be recalled e^* maximizes $S(e; 0)$) then the marginal agent will earn positive rents in the absence of bonded labor, and thus rendered better off with a ban. Also, agents with wealth above w_4 who do not use bonded labor even when it is permitted, will benefit from the ban. For those in between, i.e., intramarginal agents who do use bonded labor when permitted, may or may not benefit from the ban.[11] Effort rises above w_4, falls at w_1 and may rise or fall in-between. The main difference from the case where the set

[11] If the profit rate were to be unchanged then these intramarginal agents must also be better off. This is because of the preceding result that poorer agents are more adversely affected by a ban of bonded labor, for a given profit rate. A lower profit rate, however, also benefits poorer agents more, by an argument analogous to that used in Proposition 4. Combining the two effects, we cannot infer anything about the relative welfare impact for agents of varying wealth.

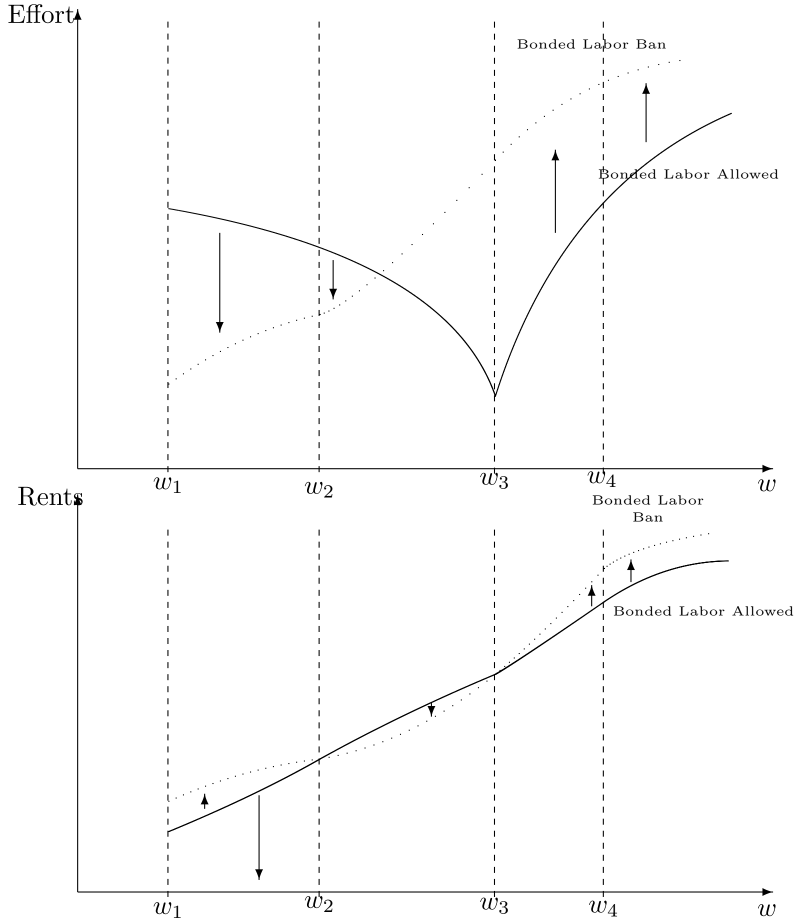


Figure 8: Effects Of Banning Bonded Labor When Set of Active Agents is Unaffected

of active agents shrinks, is that poor agents (at or near w_1) benefit from the ban as long as they manage to continue being active. In the other case they were excluded and became worse off.

6 Concluding Comments

Our results provide some rationale for a greater willingness of poorer countries to tolerate bonded labor. If the wealth distribution is concentrated among poor borrowers who stand to lose credit access if bonded labor were to be banned, and there are relatively few borrowers wealthy enough to benefit from a lower rate of profit while not using bonded labor themselves, banning bonded labor will hurt most borrowers as well as lenders, and is likely to reduce average productivity in the economy.

When the economy has a larger concentration of middle class borrowers who would not use bonded labor themselves, the benefits of banning bonded labor grow. For more developed countries, thus, bonded labor may come to be banned, and replaced by bankruptcy law that provides borrowers *ex post* freedom to choose how much labor services to provide after the project has been terminated. In such countries, selective weakening of bankruptcy provisions for poor borrowers may be both popular among the middle class and raise productivity, but will be resisted by lenders. The relative political strength of the two groups may well determine the nature of the resulting legal rules adopted.

Our model has deliberately abstracted from a number of important considerations which bear on the design of debtor liability rules. These include possible incompleteness of contracts, limited rationality among borrowers, or incentive problems among lenders.

The analysis may have interesting applications to other markets as well, such as rentals for land, housing or productive assets. In the context of labor markets, analogous issues arise in laws concerning indentured labor. Immigrant workers often enter into indentured agreements with agents to cover the costs of moving to a new country. Allowing indentured labor tends to increase immigrant inflows which lower wage rates, which hurt native workers while benefiting employers. A similar general equilibrium effect explains why native workers are often opposed to such indentured contracts among immigrants, while employers favor their use.

Extensions of our model in a number of directions would be interesting. These include the case where borrowers are risk-averse, or where principals do not get all the surplus in their contract with the marginal agent (instead this is replaced by a Nash bargaining solution). With concave utility the most profitable borrowers are not always the wealthiest, owing to income effects on the demand for leisure. Instead agents with intermediate wealths are the most profitable and productive (Mookherjee (1997)). Besides, the analysis of optimal contracts will have to additionally address insurance effects. Whether or how our results get modified is not obvious, and needs further research. The case where the surplus is divided in a different way will not affect results if the number of borrowers is large enough, so the wealth gap between the poorest active borrower and the next poorer agent (who is excluded) is small. Even with a bigger gap, we suspect our results will be qualitatively unaltered if we were to use a Nash bargaining solution.

Further extensions to the context of randomized contracts and joint liability loans would also be of interest; these will involve a more significant extension of the basic model. Another extension might be to a context where different principals vary in their fixed costs, whence changes in the law will affect the extent of concentration on the lender side.

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Appendix: Proofs

PROOF OF PROPOSITION 2 We begin with the following properties of A-optimal contracts.

Lemma 11 *An agent's utility $U(\pi)$ in an A-optimal contract is strictly decreasing in π.*

Proof. Suppose this were not true and $U(\pi) = U(\pi')$ with $\pi > \pi'$. Then, the participation constraint for the principal is not binding at π', which is not possible in an A-optimal contract because v_s and v_f could be increased uniformly without disturbing e or any of the constraints. ∎

Lemma 12 *Fix any $\pi \leq \pi^P(w)$. If there exists a feasible contract where the principal gets π and the agent gets $U_A < U(\pi)$ then there exists a feasible Pareto improving contract.*

Proof. Define payoffs of the agent with wealth w and payoff in the P-optimal contract for this agent as follows: $U^* = e^* \cdot D'(e^*) - D(e^*)$ and $\pi^* = e^* \cdot (S_s - S_f) + S_f - L_\alpha - e^* \cdot D'(e^*) + w - I$, where L_α denotes $(1-\alpha) \cdot R \cdot l^*(\alpha, R) - g(l^*(\alpha, R))$.

Now consider the problem of maximizing the principal's payoff over the set of feasible contracts for agent with wealth w, subject to the additional constraint that the agent attain at least U_A.

$$\pi(U_A) \equiv \max_{C \in \Gamma} \pi \text{ s.t.: } e \cdot v_s + (1-e) \cdot v_f - D(e) \geq U_A.$$

We first argue that the hypothesis that there exists a feasible contract where the principal gets π and the agent less than $U(\pi)$ implies that π must be strictly less than π^*. At the P-optimal contract, standard arguments imply that the agents PC is not binding, and the LL constraint must be binding, with $v_f = L_\alpha$. There cannot exist any contract satisfying the IC and LL constraints where the agent gets less than U^*, while the principal gets π^*. This can be seen from the fact that with $v_s - v_f = D'(e^*)$ and $v_f = L_\alpha$, a decrease in the agent's utility can only be achieved by reducing v_s which also reduces the principal's profits. Therefore, there does not exist a feasible contract which gives the principal π^* and the agent less than $U^* \equiv U(\pi^*)$.

Hence it must be the case that the agent's PC is binding, and $\pi < \pi^*$. The former fact implies that over this range, $\pi(U_A)$ is strictly decreasing locally. Hence $U_A < U(\pi)$ implies $\pi(U_A) > \pi$. Starting with the contract giving the principal $\pi(U_A)$, we can raise v_s and v_f uniformly by a small amount which leaves effort unchanged, which gives the agent a utility slightly above U_A and the principal a profit slightly below $\pi(U_A)$. Then the agent gets more than U_A and the principal also more than π, so this contract Pareto-dominates the original one. ■

This property guarantees that agents who do not receive the A-optimal contract are able to transfer utility to the principal and therefore achieve a Pareto improvement by moving to an A-optimal contract.

Now return to the proof of the Proposition. We show first that stable allocations always exist, by explicitly constructing one. Take any π satisfying (2). Select the Q-richest agents, match them each with a principal, and select an A-optimal contract subject to the constraint that the principal get at least π. Since each selected agent i is at least as rich as the Q-th agent, we have $\pi \leq \pi^P(w_i)$; hence such a contract exists for each selected agent.

We claim that this allocation is stable. Note first that no pair already matched can find a profitable deviation, since this must entail a different contract, and the A-optimal contract is already in force.

Currently matched agents cannot match with a different principal, because such a principal would have to be given a profit $\tilde{\pi} \geq \pi$. If $\tilde{\pi} > \pi$ then the agent will do worse, given Lemma 11. If $\tilde{\pi} = \pi$ the agent cannot do better, given the definition of the A-optimal contract.

Unmatched viable agents exist if $n > m$ and they have wealth $w \leq w_{m+1}$. They cannot match with any principal because in this case every principal is already getting $\pi \geq \pi^P(w_m)$. Hence the constructed allocation is stable.

Next, we argue that every stable allocation must be of this form. Consider first property (A). Suppose there is a stable allocation which violates (A). Then there exists an inactive viable agent and an inactive principal who can be matched. To show that the richest agents must be matched suppose to the contrary that this were not true. Then an inactive wealthier agent can match with a principal currently matching with a poorer agent, and provide the principal with a higher profit, while generating a higher utility than autarky for itself.

To show that B must be satisfied by every stable allocation, we claim first that all principals must obtain the same profit. If this were not true, the principal with the lowest profit could match with an agent who matches with a principal who obtains a higher profit, and the agent could be made better off by increasing v_s and v_f slightly relative to the existing contract.

Second, we claim that $\pi \geq \max\{\pi^P(w_{m+1}), 0\}$ if $m < n$, because every principal has the option of matching with the wealthiest agent $m+1$ who is not matched, or not to contract at all.

Third, we claim $\pi \leq \pi^P(w_m)$. If not, $\pi > \pi^P(w_m)$. However, the principal who is matched with the agent m cannot reach this profit level by contracting with agent m, by the definition of $\pi^P(w_m)$.

Fourth, $\pi = 0$ if $m > n$. This follows from the fact that all principals must obtain the same profit, and some principals must remain inactive and make zero profits ($\pi = 0$) because $m > n$.

Finally, suppose property (C) is violated by a stable allocation. If a matched agent a gets more than the utility from the A-optimal contract relative to π, this violates the definition of the A-optimal contract. If an agent a has lower utility than he can obtain in the A-optimal contract, then Lemma (12) ensures the existence of a Pareto improving contract for the matched principal agent pair. This completes the proof of Proposition 2.

PROOF OF LEMMA 3. (i) The problem of selecting the P-optimal contract can be stated as choosing v_f, e to maximize $S(e,w) - v_f$ subject to $v_f \geq \max\{L_\alpha, w + S(R) - \alpha(e)\}$. Clearly at the optimum $v_f = \max\{L_\alpha, w + S(R) - \alpha(e)\}$, and the problem can be reduced to choosing e alone to maximize $S(e,w) - \max\{L_\alpha, w + S(R) - \alpha(e)\}$, i.e., maximize $e(S_s - S_f) - C_P(e,w)$ where $C_P(e,w) \equiv \max\{L_\alpha + e'D'(e), w + S(R) + D(e)\}$ denotes the expected cost to the principal of implementing e with an agent of wealth w. Hence for $e \leq e_\alpha(w)$ the relevant cost function is $w + S(R) + D(e)$ and for $e > e_\alpha(w)$ it is $L_\alpha + e'D'(e)$. Define w_1, w_2 by the property that $e_\alpha(w_1) = e^*, e_\alpha(w_2) = e^{FB}$. Clearly, the marginal cost equals the derivative of the first-best cost $D(e)$ upto $e_\alpha(w)$, and the derivative of $e'D'(e)$ above $e_\alpha(w)$. Note that the $S(e,w)$ function is maximized at e^* while $F(e,w)$ is maximized at e^{FB}. It follows that for $w < w_1$ the optimal effort is e^* and only LL binds; for $w > w_2$ the optimal effort is e^{FB} and only APC binds, while for intermediate wealths the optimal effort is $e_\alpha(w)$ and both constraints bind.

(ii) The A-optimal contract can be reduced to selection of e, v_f to maximize $v_f + \alpha(e)$ subject to $v_f \geq L_\alpha$ and $S(e,w) - v_f \geq \Pi$. Fix any e: it can be implemented only if $S(e,w) \geq \Pi + L_\alpha$, in which case it is optimal to set $v_f = S(e,w) + \Pi$. Hence the problem reduces to choosing e alone to maximize $S(e,w) + \alpha(e) - \Pi \equiv F(e,w) - \Pi$, subject to $S(e,w) \geq L_\alpha + \Pi$. Hence if e^{FB} is implementable this is the optimal effort, otherwise it is the highest e such that $S(e,w) = L_\alpha + \Pi$. This completes the proof of Lemma 3.

PROOF OF LEMMA 7 (i) Note first that both LL constraints cannot bind simultaneously, as this would cause the contract to violate APC: with $v_i = -G_i \leq M(R)$ the expected utility of the agent is bounded above by $M(R)$. The need to provide the agent with positive effort incentives further implies that $v_s > v_f$, so v_f must be strictly smaller than $M(R)$, and therefore the expected utility of the agent would also be strictly smaller than $M(R)$, compared to the agent's outside option of $w + M(R)$ from not participating. It also follows that the LL constraint binds, if at all, in state f. Otherwise $v_s = -G_s \leq M(R)$ and $v_f < v_s$ and again APC must be violated. So in any feasible contract, $v_s > M(R)$, and LL cannot bind in state s. If there is bonded labor in state s, eliminating it (i.e., setting $G_s = -M(R)$, and adjusting financial transfers so as to leave v_s unaffected) would raise the principal's expected profit without affecting the agent's utility or violating any of the constraints.

(ii) If the APC does not bind in any contract, it is possible for v_s and v_f to be lowered by an equal amount $\epsilon > 0$, leaving e unchanged. Since LL does not bind in state s, the reduction in v_s is feasible without any need to adjust G_s, and the principal's payoff in state s is higher. In state f, if the LL constraint was binding, the reduction in v_f necessitates a concomitant increase in G_f by ϵ. Since $r(G_f)$ is an increasing function, the higher enforcement cost does not outweigh the direct effect of higher bonded labor, and the principal benefits in state f. Therefore the principal's expected profit must rise.

(iii) In order to derive the A-optimal contract, we proceed similar to the analysis of optimal contracting in a standard moral hazard problem. At the first step, we fix an effort level to be implemented and derive the maximum achievable utility to the agent corresponding to this effort. Then at the second step, we select the optimal effort. The property in (iii) alludes to the solution to the first stage problem, where the effort level is fixed.

From the IC constraint it follows that $V \equiv v_f + \alpha(e)$ and $\pi = w - I + e(y_s - y_f) + y_f - e \cdot D'(e) - v_f - (1-e) \cdot Q(G_f)$. Let $S(e; w) \equiv w - I + e \cdot (y_s - y_f) + y_f - e \cdot D'(e)$. The A-optimal contract solves

$$\max_{v_f, G_f, e} v_f + \alpha(e) \tag{21}$$

$$\text{s.t. (PPC'): } S(e, w) - v_f - (1-e)Q(G_f) \geq \pi^*$$

$$\text{(LL') } v_f \geq -G_f.$$

Rewrite PPC' to $S(e,w) - (1-e) \cdot Q(G_f) \geq v_f + \pi^* \geq -G_f + \pi^*$ where the last inequality follows from the (LL') constraint.

This implies that an effort e is implementable by v_f and G_f if

$$S(e,w) - (1-e) \cdot Q(G_f) \geq -G_f + \pi^*$$
$$\Leftrightarrow G_f - (1-e) \cdot Q(G_f) \geq \pi^* - S(e,w). \tag{22}$$

Note that $Q'(\cdot) \leq 1$ and the LHS of (22) is accordingly increasing in G_f and goes to ∞ as G_f goes to ∞. Therefore, any effort e is implementable with sufficiently large G_f. Since the agent's objective function is increasing in v_f he will choose the highest v_f for any given pair e, G_f. The highest achievable v_f satisfies $v_f = S(e,w) - (1-e) \cdot Q(G_f) - \pi^*$. Inserting this into the objective function reveals that the agent tries to pick the smallest G_f for any given e. The agent will choose the smallest implementable G_f (for a given effort e and $v_f = S(e,w) - (1-e) \cdot Q(G_f) - \pi^*$) which is the smallest solution to (22). Note that $G_f \geq -M(R)$, and if no bonded labor is used then $G_f = -M(R)$, whence $Q(-M(R)) = 0$. If at $G_f = -M(R)$, condition (22) is satisfied then no bonded labor will be used. This is equivalent to the condition that $S(e,w) \geq \pi^* + M(R)$. If this condition is not satisfied then bonded labor will be used, and G_f will be set at the unique solution to (15). This completes the proof of Lemma 7.

PROOF OF LEMMA 8. First note that the first term on the RHS of (18) is increasing in wealth because $Q(\cdot)$ is increasing and K is decreasing in wealth:

$$K_w = \frac{-S_w(e,w)}{1-(1-e) \cdot Q'(G)} \tag{23}$$
$$= \frac{-1}{1-(1-e) \cdot Q'(G)} < 0.$$

Remember that K is the solution to G (where $G > -M(R)$) in $G - (1-e) \cdot Q(G) = \pi^* - S(e,w)$ and hence $K - (1-e) \cdot Q(K) - \pi^* + S(e,w) = 0$. Let $P(e) \equiv e \cdot (y_s - y_f) - e \cdot D'(e)$ and we have

$$K_e = \frac{-P'(e) - Q(g)}{1-(1-e) \cdot Q'(g)} \tag{24}$$

Therefore, C_e is increasing in wealth if $(1-e) \cdot Q'(K(e, \pi^*, w)) \cdot K_e$ is also increasing in wealth. This holds if

$$(1-e) \cdot Q'(K(e,\pi^*,w)) \cdot K_e = \frac{-P'(e) - Q(G)}{1-(1-e) \cdot Q'(G)} \cdot Q'(G) \tag{25}$$

is decreasing in G because $K_w < 0$. The derivative of $\frac{-P'(e)-Q(G)}{1-(1-e)\cdot Q'(G)}$ with respect to G is

$$\frac{-Q'(G)(1-(1-e)\cdot Q'(e)) + (P'(e)+Q(G))\cdot(-(1-e)\cdot Q''(G))}{(1-(1-e)\cdot Q'(G))^2}.$$

Differentiating $\frac{-P'(e)-Q(G)}{1-(1-e)\cdot Q'(g)} \cdot Q'(G)$ therefore yields

$$\begin{aligned}
&\frac{-Q'(G)(1-(1-e)\cdot Q'(G)) + (P'(e)+Q(G))\cdot(-(1-e)\cdot Q''(G))}{(1-(1-e)\cdot Q'(G))^2}\cdot Q'(G) \\
&\qquad + \frac{-P'(e)-Q(G)}{1-(1-e)\cdot Q'(G)}\cdot Q''(G) \\
&= (-Q'(G)\cdot Q'(G)(1-(1-e)\cdot Q'(G)) + Q'(G)\cdot(P'(e)+Q(G))\cdot(-(1-e)\cdot Q''(G)) \\
&\qquad +(-P'(e)-Q(G))\cdot Q''(G)(1-(1-e)\cdot Q'(G)))\cdot 1/((1-(1-e)\cdot Q'(G))^2) \\
&= (-Q'(G)\cdot Q'(G)(1-(1-e)\cdot Q'(G)) \\
&\qquad +(-P'(e)-Q(G))\cdot Q''(G))\cdot 1/((1-(1-e)\cdot Q'(G))^2)
\end{aligned} \tag{26}$$

which is nonpositive if

$$\begin{aligned}
&-Q'(G)\cdot Q'(G)(1-(1-e)\cdot Q'(G)) \\
&\quad +(-P'(e)-Q(G))\cdot Q''(G) < 0.
\end{aligned} \tag{27}$$

This inequality holds if

$$\begin{aligned}
&-Q'(G)\cdot Q'(G)(1-Q'(G)) - P'(e)\cdot Q''(G) \le 0 \\
&\Leftrightarrow -P'(e)\cdot Q''(G) \le Q'(G)\cdot Q'(G)(1-Q'(G)) \\
&\Leftrightarrow \frac{Q''(G)}{Q'(G)} \le \frac{Q'(G)(1-Q'(G))}{-P'(e)} \\
&= \frac{Q'(G)(1-Q'(G))}{D'(e)+e\cdot D''(e)-(y_s-y_f)}.
\end{aligned} \tag{28}$$

For any effort $e > e^*$, this is guaranteed by assumption (9). On the other hand, it is easily seen that no $e \le e^*$ will ever be chosen because any such effort is dominated by e^* where $S(e,w)$ is maximized: the first-best surplus is higher at e^* than at any lower effort, and it necessitates less bonded labor as well as a smaller probability of an unsuccessful state. Hence we can restrict attention to effort levels above e^*. This completes the proof of Lemma 8.