

Intergenerational Mobility and Macroeconomic History Dependence*

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Abstract

That historical inequality can affect long run macroeconomic performance has been argued by a large literature using models of indivisibilities in occupational choice, combined with credit constraints. Most of these models are characterized by a continuum of steady states, and absence of mobility in any steady state. We augment such a model with heterogeneity in agents' abilities in order to generate the possibility of occupational mobility in steady state. Traditional results concerning steady state multiplicity and history dependence are then considerably weakened. We provide a class of conditions under which there is a unique steady state with mobility. In general such steady states are (generically) locally unique and finite in number, implying that small temporary shocks to steady states do not have permanent macro effects. A 'little' bit of heterogeneity may also cause competitive equilibrium dynamics to fail to converge.

1 Introduction

The role of history in powerfully shaping the nature of economic development many centuries later has been argued by many recent authors (Acemoglu, Johnson and Robinson (2001); Banerjee and Iyer (2002); Engerman and Sokoloff

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(1997)). These authors describe how historical inequality associated with colonial institutions appear to explain differences in economic backwardness even long after these institutions have disappeared. This raises the question: what prevents such countries from catching up thereafter with more developed countries?

Theoretical explanations of the role of historical inequality in determining long run macroeconomic performance have been based on indivisibilities in occupational choice combined with credit constraints. In the models of Banerjee and Newman (1993), Galor and Zeira (1993) and Ghatak and Jiang (2003), equal and unequal steady states co-exist, with historical distributions determining which steady state the economy converges to. In other literature in this field (e.g., Ray (1990), Ljungqvist (1993), Bandyopadhyay (1993, 1997), Freeman (1996), Mookherjee and Ray (2003)) there is a continuum of steady states, all of which are unequal, and involve zero mobility.¹ The steady states are ordered by per capita income, consumption, and wage inequality; those with higher per capita income also involve lower inequality (and so are representative of more developed countries). Historical inequality determines which steady state the economy converges to. Since there is a continuum of such steady states, small temporary shocks or policies to steady states can have permanent macro effects.

The feature of zero mobility in income or occupations is clearly at odds with reality: even the most unequal societies are typically characterized by *some* mobility. One would expect that it would be relatively straightforward to explain the presence of occupational mobility by enriching these models to allow heterogeneity of agents' characteristics, in the style of Becker and Tomes (1979), Loury (1981) or Maoz and Moav (1999). For instance, if children's learning abilities are subject to randomness, occupational mobility would emerge owing to the tendency for unusually gifted children in poor families to acquire education (and conversely for untalented children in rich families to fail to become educated).

This paper studies the implications of introducing such forms of heterogeneity in order to create a model with positive mobility in steady state. We find that the history dependence property is typically lost. In a large class of cases there is a unique steady state with mobility. In general there cannot be a continuum of mobile steady states: these are (generically) locally unique and finite in number. This implies that previous literature has exaggerated the extent of hysteresis: small temporary shocks to steady states can no longer have permanent macroeconomic effects.

Our model (very similar to Maoz and Moav (1999)) has two occupations, one unskilled (requiring no education) and the other skilled (requiring a certain level

¹The continuum of steady states also appears in Ghatak and Jiang (2003) and Galor and Zeira (1993). A more detailed description of the existing literature is provided in Section 5.

of education to be acquired). Agents are heterogenous with respect to their cost of getting educated; these costs are drawn from an i.i.d. distribution. Parents cannot borrow against their children’s future earnings: this is the key capital market imperfection. In the benchmark case of agent homogeneity, elaborated in Section 2, all agents incur the same (exogenous) cost of education. As in Mookherjee and Ray (2003), both occupations are essential in production, so in the case of homogenous costs there is a continuum of steady states, all of which involve zero occupational mobility and persistent inequality in earnings and utility.²

The intuition for the uniqueness results is somewhat akin to the effects of enriching the occupational space to allow diversity of occupations (i.e., removing the indivisibility in investment options). As shown by Mookherjee and Ray (2003), steady states with occupational diversity are characterized by incentive constraints in the form of equality rather than inequality constraints: agents in any given occupation have to be locally indifferent between their own occupation and neighboring occupations. These equality constraints pin down the steady state uniquely. In this paper we retain occupational indivisibilities and instead introduce heterogeneity in education cost. Steady states with mobility are characterized by interior threshold education costs for unskilled and skilled households (with education acquired if and only if the education cost falls below the corresponding threshold). In particular, agents at the threshold costs must be indifferent between educating and not educating their children. Hence the steady state is characterized by incentive constraints for the threshold type that take the form of equality constraints. This removes the scope for local multiplicity of mobile steady states: small perturbations to the skill ratio cause the steady state conditions to be violated. Under appropriate conditions that rule out multiple reversals of investment preferences of unskilled households as the skill ratio in the economy rises, such local uniqueness also implies global uniqueness.

Section 2 explains the basic assumptions of the model, and the baseline case of homogenous agents. Section 3 introduces heterogeneity, and considers the question of steady state uniqueness first with an example, followed by a sequence of cases of increasing generality. Towards the end of this section we also discuss properties of steady states in terms of the relation between equality and mobility. Section 4 then presents a preliminary discussion of non-steady state dynamics: an example is presented where the competitive equilibrium may fail to converge with the introduction of a ‘small’ amount of heterogeneity.

²The baseline model in this paper differs from Mookherjee and Ray (2003) only with respect to the bequest motive: instead of a dynastic bequest motive it is assumed that parents care only about the incomes earned by their children (apart from their own consumption). This simplifies the model, without altering any of the essential qualitative results.

Section 5 describes how this paper relates to existing literature in some detail. Section 6 concludes with a discussion of future research questions concerning occupational mobility.

2 Basic Model with Agent Homogeneity

Consider a continuum of families indexed by $i \in [0, 1]$. At each date $t = 0, 1, 2, \dots$ family i is represented by an agent who lives as an adult for one period. This agent is also referred to by i . Any generation- t agent i has an occupation $o_t^i \in \{n, s\}$, referring to either *unskilled* or *skilled* labor. The fraction of skilled agents in period t is denoted by λ_t .

The economy produces a single consumption good under perfect competition. Output is given by a production function f which is assumed to be twice continuously differentiable, strictly concave in both types of labor, has constant returns to scale, and satisfies Inada end-point conditions. Thus the marginal products of the unskilled and skilled respectively are given by functions $f^n(\lambda)$ and $f^s(\lambda)$ respectively, where f^n is strictly increasing and f^s is strictly decreasing, and $f^n(0) = 0 = f^s(1)$; $f^n(1) = f^s(0) = \infty$.³

Skilled workers can choose whether to work as skilled or unskilled employees, while the reverse is of course not true. This implies that the skilled wage can never fall below the unskilled wage. Let $\bar{\lambda} \in (0, 1)$ be defined by the property that $f^n(\bar{\lambda}) = f^s(\bar{\lambda})$ ($= \bar{w}$ say). Then letting w^n and w^s denote wages of the unskilled and skilled respectively, it follows that for all $\lambda \geq \bar{\lambda}$, both w^n and w^s equal \bar{w} . Of course, such high skill ratios will not emerge in equilibrium.

Every worker has a zero reservation wage, though in the next section we shall consider the consequences of a positive reservation wage. Then the wages of the skilled and unskilled will simply equal their marginal products at all skill ratios below $\bar{\lambda}$. We therefore obtain $w^n(\lambda)$ increasing from 0 to \bar{w} as λ increases from 0 to $\bar{\lambda}$, and equal to \bar{w} thereafter. Conversely $w^s(\lambda)$ decreases from ∞ to \bar{w} as λ increases from 0 to $\bar{\lambda}$, and is equal to \bar{w} thereafter.

Work in the skilled profession requires an educational investment of $x > 0$ units of the consumption good. These pecuniary costs of education are assumed to be exogenously given and (in this section) identical for all agents in the basic model. Parents have to finance their child's education but cannot borrow against their descendent's income. So education for a generation- t agent i has

³We therefore follow the normal convention that even if there are no skilled workers in the economy the production activities in the economy will not cease. If skilled workers are truly essential the economy may still be able to function if it has the opportunity of importing skilled workers from other countries at a sufficiently large wage. This enables the economy to avoid a degenerate equilibrium where production is never viable owing to the complete absence of skilled workers.

to be paid from its parent's income w_{t-1}^i ($t \geq 1$). There is no way to transfer wealth between generations apart from parents' educational investment.⁴ The investment needed to work in the unskilled profession is zero.

Let I_t^i equal 1 if period- t agent i decides to invest in his child's education and 0 otherwise (corresponding to $o_{t+1}^i = s$ and $o_{t+1}^i = n$, respectively). We assume that parents' bequest motive takes the form of *paternalistic altruism*, where they care about the wealth of their children, apart from their own consumption. This represents a bequest motive less far-sighted and sophisticated than a Barro-Becker dynastic motive (where parents care about the utility of their child, and thus indirectly about the consumption of all their future descendants). But it is more sensitive to the consequences of bequests for the well-being of their children, compared to a 'warm-glow' bequest motive (where they care only about the size of the bequest apart from their own consumption) traditionally assumed in much of the literature (e.g., Banerjee-Newman (1993) or Galor-Zeira (1993)). In dividing income w_t^i between own consumption and the possible investment in education, each agent i therefore maximizes

$$(1) \quad u(\underbrace{w_t^i - xI_t^i}_{\equiv c_t^i}) + v(w_{t+1}^i),$$

where u and v are both strictly increasing, continuously differentiable functions, u is strictly concave, and w_{t+1}^i is determined by I_t^i and equilibrium skill ratio in the economy at $t + 1$.

The concavity of u represents the key property that poorer parents need a higher sacrifice in utility terms to undertake a given educational cost, resulting in a positive correlation between parental income and educational investments. We additionally restrict agent consumption to be non-negative, and assume that the marginal utility $u'(0)$ at zero consumption is infinite. This implies that parents may be too poor to afford education: e.g., if $w_t^i < x$. This creates the possibility of a low level poverty trap, where certain families cannot simultaneously afford to educate their children and meet minimum consumption standards. With heterogenous educational costs (to be introduced in the next Section), however, this possibility is weakened: with a sufficiently gifted child educational costs are low enough to be affordable by poor parents.

Given an arbitrary skill ratio λ_t at any given date t , a competitive equilibrium is defined by a skill ratio λ_{t+1} at the following date, with the property that upon foreseeing λ_{t+1} correctly each agent i selects education for his child to maximize (1), and λ_{t+1} is exactly the resulting fraction of households that educate their children. Proceeding recursively, this determines a competitive

⁴Consequences of allowing supplemental financial bequests are discussed in Mookherjee and Ray (2002).

equilibrium skill ratio λ_T for all successive dates $T = t + 1, t + 2, \dots$. A *steady state* is a sequence of such competitive equilibria with a stationary skill ratio λ .

Note that in order to provide educational incentives the skilled wage must lie above the unskilled wage, so the equilibrium skill ratio at any date must always lie between 0 and $\bar{\lambda}$. In order to allow upward mobility to be feasible, we assume that the educational cost x is less than \bar{w} , so it is feasible for currently unskilled workers to purchase education at some skill ratios.⁵

As explained in detail in Mookherjee and Ray (2003), every steady state in this model must involve zero mobility and persistent inequality of earnings and utility across occupations.⁶ The absence of mobility follows from the fact that skilled families must earn higher incomes, and thus encounter a lower utility sacrifice in educating their children (given the missing capital market). So they have a bigger incentive to educate their children than do unskilled parents. If an unskilled parent decides to educate his child, so must every skilled parent. It is therefore not possible for the child of an unskilled parent to become skilled, while at the same time the child of a skilled parent becomes unskilled.

Hence every steady state must involve all descendants of a skilled (resp. unskilled) parent remaining skilled (resp. unskilled). A necessary and sufficient condition for λ to be a steady state is therefore

$$(2) \quad \underbrace{[u(w^s(\lambda)) - u(w^s(\lambda) - x)]}_{\equiv C^s(\lambda)} \leq \underbrace{[v(w^s(\lambda)) - v(w^n(\lambda))]}_{\equiv B(\lambda)} \leq \underbrace{[u(w^n(\lambda)) - u(w^n(\lambda) - x)]}_{\equiv C^n(\lambda)}.$$

$C^j(\lambda)$ denotes the utility sacrifice or subjective cost of education for parents from occupation j . $B(\lambda)$ is the (common) utility benefit from having a skilled rather than unskilled child. These functions are depicted in Figure 1 (where L_p denotes λ_p , $L(x)$ denotes λ , L^* denotes λ^* , and L_u denotes $\bar{\lambda}$).

The fact that earnings differ across occupations and u is concave imply that in any steady state λ^* at least one of the two inequalities in (2) must be strict. If for instance skilled parents are indifferent between acquiring education and not acquiring it for their children, then unskilled parents must strictly prefer not to acquire education. This implies that a left or right neighborhood of λ^* must consist of steady states, too. For instance, if skilled parents are

⁵In the absence of this assumption, unskilled workers can never afford education, and skilled workers can afford it only if the skill ratio falls below some threshold. Then the set of steady state skill ratios is some interval of the type $(0, \lambda_p]$, where the skilled have sufficient incentive to acquire education for their children. Unskilled families fall into a poverty trap.

⁶The only significant difference from the model in that paper is the nature of the bequest motive. But essentially the same arguments apply. One difference is that with a dynastic bequest motive, the skilled must have higher consumption in any steady state, and this may not be true in the current formulation. However, in the case where the bequest motive takes the form $v = \delta u$ with $\delta < 1$, skilled households also have higher levels of consumption in every steady state.

indifferent, then an entire left neighborhood (where skilled parents are slightly richer and unskilled parents are slightly poorer) will consist of steady states where each type of parent has strict incentives to either acquire or not acquire education. Hence there will always be a continuum of steady states, with interior steady states characterized by incentive constraints that take the form of strict inequalities. This slack in the incentive constraints in turn permits such a large set of steady states to exist.

Note that steady states with higher skill ratios are associated with higher levels of per capita income, utilitarian welfare, and lower (income or utility) inequality. It can be shown that starting from any initial skill ratio, there is a unique dynamic competitive equilibrium which converges to a steady state. If the economy starts at a steady state skill ratio, then it stays there for ever. Hence small perturbations to an initial skill ratio that constitutes an interior steady state, will result in permanent macroeconomic effects. A small one time education subsidy for instance will raise the economy's long term level of human capital and per capita income. The model suggests that historical inequality and lack of human capital may lock countries permanently into low living standards.

Note also that it is possible for the set of steady states to not be connected, i.e., to be the union of disjoint intervals, as in Figure 1. This owes to a non-monotonicity of the educational incentives of unskilled with respect to the skill ratio. As λ increases the benefit from educating children decreases, lowering their educational incentive. At the same time, the unskilled wage increases, making them less poor, and lowering the sacrifice entailed in educating their children. So the cost and benefit functions for the unskilled can intersect more than once. These possible nonmonotonicities will play an important role in the next section once we introduce agent heterogeneity. But for now note that there is also scope for the model to allow multiple intervals of steady states, so allow a cross section of countries to be characterized by multiple clusters of living standards and human capital levels.

In particular, there is always an interval $(0, \lambda_P]$ of steady state skill ratios which has the characteristic of a poverty trap — for sufficiently low skill ratios the wages of the unskilled fall below education costs, so there is no question of them being able to purchase any education. At the same time the skilled are sufficiently rich (that the sacrifice entailed in educating children is small), while the disparity between skilled and unskilled is sufficiently large that the benefit of education is large enough for the skilled to want to educate their children.

At skill ratios higher than λ_P , however, unskilled wages may be high enough for them to want to acquire education. So intermediate skill ratios may not constitute steady states, while even higher skill ratios may — where the benefit of education has declined sufficiently to choke off the demand from the unskilled. There may thus be an interval of high skill ratios that also constitute steady

states, which co-exist with the poverty trap steady state set.

3 Model with Heterogeneous Ability

3.1 Perturbing the Homogeneous Agent Model Slightly: The Genius-Idiot Example

To illustrate the impact of introducing heterogeneous abilities, consider the following variation on the homogeneous agent case, where a small fraction of children are unusual with regard to their learning abilities. A fraction γ of children are *geniuses*, with an educational cost of 0, so it will be optimal for even unskilled parents to educate them at every date. Another fraction ι of children are *idiots*, with an educational cost so large that their parents will never want to try to educate them, no matter how rich they may be. The remaining children are *normal* and have a common (fixed) educational cost of x , as described in the previous section. Let the highest steady state interval in the homogeneous agent economy be denoted by $[\lambda_1, \lambda_2]$. We shall assume that the proportion of geniuses and idiots in the population is small enough in the sense that $\gamma < \lambda_2, \iota < 1 - \lambda_1$.

Consider the case where $\frac{\gamma}{\iota}$ lies in between $\frac{\lambda_1}{1-\lambda_1}$ and $\frac{\lambda_2}{1-\lambda_2}$. In this case define λ^* by the property that $\frac{\lambda^*}{1-\lambda^*} = \frac{\gamma}{\iota}$, which then falls in between the two endpoints λ_1 and λ_2 of the steady state interval. Then for any λ to the left of λ^* , there will be a ‘rightward’ drift in the skill ratio owing to the presence of the unusual children. There will be an upward flow of geniuses from unskilled families into the skilled profession, and a reverse downward flow of idiots from skilled families to the ranks of the unskilled in the next generation. To the left of λ^* the upward flow will dominate the downward one, inducing the skill ratio to move rightwards. So even if there is no occupational mobility among ‘normal’ children (i.e., if λ happens to fall within a steady state interval in the homogeneous agent economy), this rightward drift will tend to move the skill ratio upward in the economy. And if λ happens to fall outside a steady state interval for the homogeneous agent economy (but is less than λ^*), then (as explained in the previous section) even normal children of unskilled families will tend to switch to the skilled profession while normal children of skilled professions will not want to switch. In this case the skill ratio will rise owing to the decisions of both the normal and unusual children.

Conversely, if λ lies above λ^* then the downward movement of the idiot children of the skilled will dominate the upward flow of the talented from unskilled families, to produce a leftward drift in the skill ratio. So no such skill ratio can constitute a steady state. In this economy there is a unique steady state at λ^* . In effect the perturbation created by introduction of a few unusual children has the effect of singling out a unique steady state from the continuum of steady

states in the homogenous agent economy.

3.2 The General Case with Heterogenous Ability

Now consider the general case of heterogeneous learning abilities. These are reflected by *random costs of education* \tilde{x} that are distributed independently and identically across families and generations according to a cumulative distribution function F defined on $[0, \infty)$. We make no restrictions whatsoever on the distribution of these costs. Parents observe the realization x of \tilde{x} for their child before deciding whether to invest in their education. We make the iid assumption for simplicity, in order to generate enough heterogeneity to allow occupational mobility. Allowing persistence of ability across generations would perhaps be more realistic, as argued by Becker and Tomes (1979) or Bowles and Gintis (2001). We suspect that many of our results will extend to the case where ability follows a Markov process within any family (where the correlation of traits across successive generations within any given family is not too large), but this remains to be verified. The effects of alternative forms of heterogeneity (e.g., in traits that directly affect earnings apart from education) also need to be considered in future work.⁷

The range of the education cost distribution is $[\underline{x}, \bar{x}]$, where $\underline{x} > 0$ and $\bar{x} < \infty$. We take \underline{x} and \bar{x} to satisfy (i) $x > \underline{x} \Rightarrow F(x) > 0$ and (ii) $x < \bar{x} \Rightarrow 1 - F(x) > 0$. The case $\underline{x} = 0$ is ruled out to avoid further case distinctions: a mass at $\underline{x} = 0$ would eliminate the poverty trap discussed below. Allowing for atoms in this distribution, we let $F^o(x)$ denote the measure of agents with costs *strictly* smaller than x .

Denote by

$$(3) \quad g^n(\lambda_t, \lambda_{t+1}; x) \equiv \underbrace{[v(w^s(\lambda_{t+1})) - v(w^n(\lambda_{t+1}))]}_{B(\lambda_{t+1})} - \underbrace{[u(w^n(\lambda_t)) - u(w^n(\lambda_t) - x)]}_{\equiv C^n(\lambda_t; x)}$$

the difference between subjective benefits and costs of investing in education for an unskilled parent who has a child with learning cost x in period t . Define $g^s(\lambda_t, \lambda_{t+1}; x)$ analogously.

Next, define the corresponding net benefit functions when the current skill ratio is unchanged from the current generation to the next:

$$h^j(\lambda; x) \equiv g^j(\lambda, \lambda; x) = B(\lambda) - C^j(\lambda; x).$$

For there to exist a steady state with occupational mobility, a necessary condition is the following: that there exists some skill ratio $\lambda \in (0, \bar{\lambda})$ at which some

⁷In the case of logarithmic inequality, the effect of (multiplicative) income risk is equivalent to uncertain learning ability. So in that case our results apply also to the case of idiosyncratic income risk.

unskilled families want to purchase education while at the same time some skilled families do not:⁸

$$(4) \quad h^s(\lambda; \bar{x}) < 0 < h^n(\lambda; \underline{x}).$$

Essentially this requires a sufficient spread between the highest and lowest training costs in the population. So in what follows we shall assume that this condition holds.

The first question pertains to the nature of competitive equilibrium in this economy. Given an arbitrary skill ratio $\lambda_t < \bar{\lambda}$ at date t , we can define a competitive equilibrium as before: a skill ratio at the following date λ_{t+1} which if anticipated by all agents at t would cause exactly that fraction of households in the economy to invest in education at that date. Similarly, a steady state is a dynamic competitive equilibrium sequence of skill ratios which is stationary. Clearly a steady state skill ratio must always lie below $\bar{\lambda}$. It is a *steady state with mobility* if a positive fraction of unskilled families in any generation become skilled at the following generation (with a reverse flow of skilled families that become unskilled).

To study the nature of equilibrium, note that education strategies are defined by appropriate cost thresholds. Both g^n and g^s are continuous, and strictly decreasing in x . Therefore, for any λ_t and λ_{t+1} less than $\bar{\lambda}$, there exist well-defined threshold costs $\hat{x}^n(\lambda_t, \lambda_{t+1})$ and $\hat{x}^s(\lambda_t, \lambda_{t+1}) \in (0, \infty)$ such that

$$(5) \quad g^j(\lambda_t, \lambda_{t+1}; x) \begin{cases} > 0 & \text{for } x < \hat{x}^j(\lambda_t, \lambda_{t+1}), \\ = 0 & \text{for } x = \hat{x}^j(\lambda_t, \lambda_{t+1}), \\ < 0 & \text{for } x > \hat{x}^j(\lambda_t, \lambda_{t+1}). \end{cases}$$

with $j = n, s$.

Given the current skill ratio, λ_t , and expecting λ_{t+1}^e for the next period, a total measure of

$$(6) \quad \sigma(\lambda_t, \lambda_{t+1}^e) \equiv (1 - \lambda_t)F^o(\hat{x}^n(\lambda_t, \lambda_{t+1}^e)) + \lambda_t F^o(\hat{x}^s(\lambda_t, \lambda_{t+1}^e))$$

agents will therefore strictly prefer to invest. A measure of

$$(7) \quad \iota^n(\lambda_t, \lambda_{t+1}^e) \equiv (1 - \lambda_t) [F(\hat{x}^n(\lambda_t, \lambda_{t+1}^e)) - F^o(\hat{x}^n(\lambda_t, \lambda_{t+1}^e))]$$

of unskilled agents are indifferent between investing or not and, similarly, a measure of

$$(8) \quad \iota^s(\lambda_t, \lambda_{t+1}^e) \equiv \lambda_t [F(\hat{x}^s(\lambda_t, \lambda_{t+1}^e)) - F^o(\hat{x}^s(\lambda_t, \lambda_{t+1}^e))]$$

⁸Strictly speaking, this requires the weak inequality version of this condition, but we use the strict version instead to avoid some knife-edge cases that complicate the statement of some results.

of skilled agents are indifferent. Hence λ_{t+1} is a competitive equilibrium skill ratio at the following date if there exist numbers α and β both in $[0, 1]$ such that

$$(9) \quad \lambda_{t+1} = \sigma(\lambda_t, \lambda_{t+1}) + \alpha \iota^n(\lambda_t, \lambda_{t+1}) + \beta \iota^s(\lambda_t, \lambda_{t+1}).$$

As a preliminary to our analysis of steady states, it is useful to note:

LEMMA 1 *The economy has a unique (perfect-foresight) competitive equilibrium $\{\lambda_t\}_{t \geq 0}$ from any initial λ_0 .*

The proof is illustrated in Figure 2 (where L_{t+1} denotes λ_{t+1}). Define the non-empty, compact, and convex-valued correspondence $\phi_{\lambda_t} : [0, 1] \rightrightarrows [0, 1]$ by (10)

$$\phi_{\lambda_t}(\lambda_{t+1}^e) = \{\lambda : \lambda = \sigma(\lambda_t, \lambda_{t+1}^e) + \alpha \iota^n(\lambda_t, \lambda_{t+1}^e) + \beta \iota^s(\lambda_t, \lambda_{t+1}^e) \text{ for } \alpha, \beta \in [0, 1]\}.$$

For given λ_t , ϕ_{λ_t} maps expected skill ratio λ_{t+1}^e to the set of all actual skill ratios λ_{t+1} that could result from its anticipation. The latter need not be a singleton if the skill distribution has atoms.

Since threshold levels $\hat{x}^n(\lambda_t, \lambda_{t+1}^e)$ and $\hat{x}^s(\lambda_t, \lambda_{t+1}^e)$ are continuous, $\sigma(\lambda_t, \lambda_{t+1}^e)$ is continuous in λ_{t+1}^e *except* at points where a positive mass of agents switches from a strict preference for or against investment to indifference. Moreover, *except* at these points, $\phi_{\lambda_t}(\lambda_{t+1}^e) = \{\sigma(\lambda_t, \lambda_{t+1}^e)\}$ and thus ϕ_{λ_t} is continuous, too. If at λ_{t+1}^e a mass of agents becomes indifferent between investing or not, all or none of them may invest. So for arbitrary λ_{t+1}^e and arbitrary sequences $\{\lambda^{e^n}\}_{n \geq 0}$ and $\{\lambda^n\}_{n \geq 0}$ with $\lambda^{e^n} \rightarrow \lambda_{t+1}^e$, $\lambda^n \in \phi_{\lambda_t}(\lambda^{e^n})$, and $\lambda^n \rightarrow \lambda'$ it is true that $\lambda' \in \phi_{\lambda_t}(\lambda_{t+1}^e)$. In other words, ϕ_{λ_t} is upper semi-continuous and, by Kakutani's theorem, must have a fixed point $\lambda_{t+1} \in \phi_{\lambda_t}(\lambda_{t+1})$.

Uniqueness follows from the fact that ϕ_{λ_t} is a decreasing correspondence over the range $[0, \bar{\lambda}]$, i.e., that for all $\lambda \in \phi_{\lambda_t}(\lambda_{t+1}^e)$ and $\lambda' \in \phi_{\lambda_t}(\lambda_{t+1}^e)'$: $\lambda_{t+1}^e \leq \lambda_{t+1}^e' < \bar{\lambda} \iff \lambda \geq \lambda'$. This follows from the fact that a higher expected skill ratio decreases the benefits of child education, while the costs are determined by the currently given skill ratio at t .

3.3 The Case of Monotone Educational Incentives

In this section we analyze the case where educational incentives of both skilled and unskilled households are monotone in the skill ratio (assuming this stays constant from this generation to the next). As discussed in the previous section, the incentives for the skilled are always monotone decreasing in the skill ratio, since the benefit of education decreases, while the parental sacrifice increases. But the incentives of the unskilled may not be monotone in general, as the benefit and sacrifice of education move in opposite directions. Nevertheless

under certain conditions, their incentives will also be monotone decreasing in the skill ratio.

Given that there is a lower bound to education costs \underline{x} which is strictly positive, sufficiently low skill ratios will continue to give rise to a poverty trap for the unskilled. If at the same time there is a finite upper bound \bar{x} to education cost, there will continue to exist an interval $(0, \lambda_P]$ of steady states with no mobility at all, where unskilled families cannot afford education at all, while even the \bar{x} type of child in a skilled family receives education. The existence of such a ‘low level trap’ will give rise to nonmonotone incentives for the unskilled, since condition (4) implies that at a higher skill ratio the unskilled wage will rise sufficiently for some unskilled families to want to educate their children. So to allow monotone incentives we need to rule out the possibility of such poverty traps.

In order to do so, we introduce a positive reservation (or subsistence) wage w smaller than \bar{w} , representing the value of leisure or an alternative occupation such as self-employment to all workers.⁹ This then provides a lower bound to the wages of the unskilled in any competitive equilibrium. The poverty trap ceases to exist if the reservation wage is large enough to make it optimal for some unskilled families to educate their children.

Given that a poverty trap has been ruled out, it is now possible for the educational incentives of unskilled families to be monotone decreasing in the skill ratio over the relevant range $[\underline{\lambda}, \bar{\lambda}]$. In the following subsection we shall provide sufficient conditions on the parameters of the model for this property to hold:

SCP An unskilled family’s net benefit from investment, $h^n(\lambda, x)$, is strictly decreasing on $(\underline{\lambda}, \bar{\lambda})$, for any $x \in [\underline{x}, \bar{x}]$.

SCP (for ‘single crossing property’) formalizes that as λ increases, the discouraging incentive effect of a narrowing wage gap dominates the beneficial wealth effect for the unskilled. This implies that h^n crosses zero *at most* once. In particular, combined with the assumption (4) made above, it follows that the demand for education among the unskilled is positive at the (skill ratio corresponding to the) subsistence wage, and decreasing thereafter as the skill ratio rises. Under these conditions, there turns out to be a unique steady state:

PROPOSITION 1 *Assume (4) and SCP hold. Then there exists a unique steady state. It must involve positive mobility.*

⁹Letting λ denote the skill ratio at which the marginal product of the unskilled equals w , it is easy to verify that there will continue to be a unique competitive equilibrium with positive employment in the economy given any initial skill ratio between $\underline{\lambda}$ and $\bar{\lambda}$, if and only if the fraction of households that want to purchase education at the skill ratio $\underline{\lambda}$ is at least λ .

This result is proven as follows (and illustrated in Figure 3, where u and d denotes upward and downward flows respectively, L_b denotes $\underline{\lambda}$, L^* denotes λ^* , and L_u denotes $\bar{\lambda}$). Denote by

$$\iota^n(\lambda) \equiv (1 - \lambda) [F(\hat{x}^n(\lambda, \lambda)) - F^o(\hat{x}^n(\lambda, \lambda))]$$

the measure of unskilled agents that are indifferent between investing and not investing in a potential steady state λ . Then,

$$(11) \quad \mu^\uparrow(\lambda) \equiv \left\{ \mu: \mu = (1 - \lambda) [F^o(\hat{x}^n(\lambda, \lambda)) + \alpha \iota^n(\lambda, \lambda)] \text{ for } \alpha \in [0, 1] \right\}$$

maps skill ratio λ to all possible measures of upward-moving families, i. e. unskilled agents who invest in the education of their children. Noting that for λ below $\underline{\lambda}$, wages are exactly the same as at $\underline{\lambda}$, the upward flow is defined for all $\lambda \in [0, \bar{\lambda}]$. This *upward flow* comprises all unskilled families with strict preference for investment and any number of families who are indifferent. Since the upward flow is scaled by the fraction of households that are currently skilled (which is strictly decreasing in λ), SCP ensures that the correspondence μ^\uparrow is *strictly* decreasing wherever it is positive.

Analogously, for all $\lambda \in [0, \bar{\lambda}]$ define $\iota^s(\lambda)$ and *downward flow* correspondence $\mu^\downarrow(\lambda)$, the fraction of families that seek to transit from skilled to unskilled status. The downward flow is strictly increasing wherever it is positive (since $C^s(\lambda; x)$ increases in λ for all x and $B(\lambda)$ decreases; moreover, the total measure of skilled agents of any type x in the support of \tilde{x} strictly increases in λ).

A steady state λ^* by definition involves equality of downward and upward flows, i. e., satisfies $\mu^\uparrow(\lambda^*) = \mu^\downarrow(\lambda^*) \equiv m(\lambda^*)$. Suppose that mobility $m(\lambda^*) = 0$. Then, $\mu^\uparrow(\lambda) = 0$ for $\lambda \geq \lambda^*$ and $\mu^\downarrow(\lambda) = 0$ for $\lambda \leq \lambda^*$ by monotonicity of the two flows. This contradicts the assumption (4) that there exists some skill ratio where both flows are positive.

A steady state exists because the upward flow is positive at $\lambda = 0$, while the downward flow is zero, and the converse is true at $\bar{\lambda}$. Uniqueness of the steady state skill ratio also follows from the monotonicity of the two flows, combined with the property that every steady state involves positive flows where the flows are strictly monotone in opposing directions. This completes the proof of Proposition 1.

This proposition establishes that monotonicity of education incentives implies steady state uniqueness, in striking contrast to the case of agent homogeneity. In that context, monotone incentives would give rise to a single interval of steady states. The effect of introducing agent heterogeneity is to shrink this entire interval to a single point. The surprising feature of this result is that it holds almost without any restrictions on the distribution of learning costs, except to require that its range be wide enough so as to render occupational

mobility feasible (i.e., assumption (4)). The necessity to balance upward and downward flows ties down the steady state ratio to a single point: at other skill ratios there tends to be either upward or downward drift in the skill ratio. This was clearly illustrated in the genius-idiot example considered above. More generally, if the learning cost distribution has no atoms, the unique steady state is characterized by the condition

$$(12) \quad \frac{\lambda^*}{1 - \lambda^*} = \frac{F(x^n(\lambda^*, \lambda^*))}{1 - F(x^s(\lambda^*, \lambda^*))}$$

Even if the distribution of education costs has no mass in a neighborhood of the steady state so that the upward and downward flow *rates* are unchanging in λ , the fact that these flow rates have to be weighted by the relative proportion of skilled and unskilled households respectively ties this proportion to the corresponding ratio of the (locally constant) flow rates. Hence the steady state must be locally unique. Global uniqueness then follows from the monotonicity of the flows in opposing directions.

An alternate intuition for the uniqueness result is that it must involve an indifference property for a suitable pair of threshold cost-types of unskilled and skilled families. Hence analogous to the effect of adding a rich space of occupations without any gaps in learning costs as in Mookherjee and Ray (2003), steady state conditions are characterized by incentive constraints that are equality rather than inequality constraints. These equality constraints tie down the steady state skill ratio uniquely, if educational incentives are monotone.

3.4 Double Crossing

We now consider the question of how restrictive the monotonicity constraint on education incentives of the unskilled is, and whether the uniqueness property is robust with respect to this property. The monotonicity property is unlikely to hold in the absence of a subsistence sector that ensures a minimal income sufficient to pay for education at least for the most talented children, whence a poverty trap exists. So let us now confront the consequences of a zero (or sufficiently low) reservation wage.

Let $\lambda(x)$ denote the skill ratio at which the unskilled wage equals x . An unskilled family with education cost x can then afford education only when the skill ratio is at least $\lambda(x)$. Its net benefit $h^n(\lambda; x)$ from investing in education is then defined over the range of (constant) skill ratios above $\lambda(x)$. At a skill ratio exactly equal to $\lambda(x)$, it would not want to invest because that would drive its consumption down to zero. But it is possible that at higher skill ratios it may want to invest, while of course if the skill ratio rises sufficiently (e.g., close to $\bar{\lambda}$) it would not as the benefit from education would tend to vanish. Then the net

benefit from education for the unskilled would cross zero at least twice, causing violation of the SCP property. This suggests that the case considered in the previous subsection would not apply. What can we say now about the set of steady states?

The following assumption (called the ‘double crossing property’) states that the net benefit from investment for the unskilled crosses zero *at most* twice.

DCP For any $x \in [\underline{x}, \bar{x}]$ if there is a skill ratio $\lambda > \lambda(x)$ where an unskilled family with education cost x wants to invest in education, then the range of skill ratios (above $\lambda(x)$) for which it has a positive net benefit from investing is a subinterval $(\lambda_1^n(x), \lambda_2^n(x))$ of $(\lambda(x), \bar{\lambda})$.

It turns out that DCP is satisfied if agents have constant elasticity utility functions with relative risk aversion at least one, and Cobb-Douglas technology:

LEMMA 2 *Let the economy be defined by a Cobb-Douglas production function*

$$f(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$$

for $\alpha \in (0, 1)$, utility function

$$u(w_t^i - xI_t^i) + \delta u(w_{t+1}^i)$$

with $\delta \in (0, 1)$ and

$$u(c) = \ln(c) \quad \text{or} \quad u(c) = \frac{c^{1-\rho}}{1-\rho}$$

with $\rho > 1$. Then DCP is satisfied.

The proof is provided in the Appendix. If the upper bound of education costs is not too high, then DCP continues to imply that there is at most one steady state with mobility:

PROPOSITION 2 *Suppose (4) and DCP hold, and that the upper bound \bar{x} for education cost is not too high in the sense that at some (constant) skill ratio $\hat{\lambda} < \bar{\lambda}$ every unskilled family would want to invest in education. Then:*

- (a) *there is a unique steady state with mobility;*
- (b) *if in addition the subsistence wage \underline{w} is smaller than the lowest education cost \underline{x} , there is also a nonempty interval $[\underline{\lambda}, \lambda_1^n(\underline{x})]$ of steady states without mobility;*
- (c) *there are no other steady states.*

The proof is simple, and is illustrated in Figure 4 (where u and d denotes upward and downward flows respectively, L_b denotes $\underline{\lambda}$, L_1^n denotes $\lambda_1^n(\underline{x})$, L^H denotes $\hat{\lambda}$, L^* , L^{**} denote λ^* , λ^{**} respectively, and L_u denotes $\bar{\lambda}$). At skill ratio $\hat{\lambda}$, every unskilled family wants to invest, and hence so must every skilled family. The downward flow μ^\downarrow is therefore zero at every λ below $\hat{\lambda}$. So there cannot be any steady state with mobility at or below $\hat{\lambda}$.

Next, note that at $\hat{\lambda}$, the upward flow μ^\uparrow is strictly positive. For all higher skill ratios, DCP implies that the upward flow must be strictly decreasing in λ whenever it is positive. This is because at $\hat{\lambda}$ all unskilled families want to invest, so $\hat{\lambda}$ exceeds $\lambda_1^n(\underline{x})$ where the net gain of the highest cost type first turns positive. So further increases in λ above $\hat{\lambda}$ cannot increase the upward flow rate (i.e., the fraction of all unskilled families that want to invest) any further, and must be weakly decreasing thereafter. If the rate is positive then the flow, which equals $1 - \lambda$ times the flow rate, must be strictly decreasing. Eventually as λ keeps increasing, the benefit of education drops low enough that the upward flow must hit zero at some skill ratio less than $\bar{\lambda}$, and stay equal to zero thereafter.

For the same reason, skilled families will also lose their incentive to invest if λ increases sufficiently. So the the downward flow $\mu^\downarrow(\lambda)$ which is an increasing u.s.c. correspondence, must be strictly positive for λ close enough to $\bar{\lambda}$. Since the upward flow is a decreasing u.s.c. correspondence above $\hat{\lambda}$, it follows there must exist at least one steady state above $\hat{\lambda}$.

We claim that there is a unique steady state above $\hat{\lambda}$, and this must involve positive mobility. If every steady state above $\hat{\lambda}$ is associated with zero mobility, it must be the case that the downward flow is still zero when the upward flow hits zero (at L^{**} in Figure 4). Then there does not exist any skill ratio at which both flows are simultaneously positive, contradicting the necessary condition (4) for mobility. Hence there must be a steady state above $\hat{\lambda}$ with positive mobility. At any such steady state both flows are strictly monotone, so there must be a unique steady state above $\hat{\lambda}$. This establishes part (a).

Parts (b) and (c) follow from the fact that the downward flow is zero everywhere below $\hat{\lambda}$, and in this region the upward flow is zero if and only if λ also falls below $\lambda_1^n(\underline{x})$. So the only steady states below $\hat{\lambda}$ must lie in the interval $(\underline{\lambda}, \lambda_1^n(\underline{x}))$, and any λ in that interval constitutes a steady state. This completes the proof of Proposition 2.

It follows that the steady state with mobility is not necessarily the only steady state: if the subsistence wage is low enough that there is no demand for investment by the unskilled at that wage, then there will additionally be a continuum of steady states constituting a poverty trap with no mobility. In this case the economy will either be in this low level trap, or at a higher steady state with positive mobility. The latter will involve higher per capita income, lower inequality, and higher mobility.

On the other hand, if the subsistence wage is high enough so as to generate a positive demand for education from the unskilled at that wage, then there will be no poverty trap. Then the mobile steady state is globally unique. If the subsistence wage is so high that it elicits investment demand from *every* unskilled family, then the DCP property reduces to the SCP property, and we are back in the environment of the previous subsection where educational incentives are monotone.

3.5 The General Case

What can be said for the case where education costs can be high enough that some unskilled families never want to invest? This will then serve to raise the downward flow as well, which can be positive even over the region where the upward flow is increasing in λ . In that case we cannot rule out the possibility of multiple steady states with mobility. In addition if DCP is violated, investment demand from the unskilled may be subject to more than one reversal, as shown in Figure 5, increasing the potential for multiplicity of mobile steady states.¹⁰

What *can* be said, however, is that unlike the case of homogeneous agents, there will typically be a finite number of mobile steady states, all of which will be locally unique. To make this precise, we invoke standard differential arguments, so need to make a number of regularity assumptions ensuring that upward and downward flows are smooth (continuously differentiable) functions. Sufficient conditions ensuring this are the following: the production and utility functions are twice continuously differentiable, and the education cost distribution has a continuous positive density throughout the range $[\underline{x}, \bar{x}]$.

Suppose also that we scale the degree of parental altruism by a parameter $\delta > 0$, so that the function v is replaced by δv . Increasing δ then raises the degree of altruism, and thus the demand for education. Specifically, an increase in δ will lower the cost thresholds for both skilled and unskilled families (given arbitrary λ_t, λ_{t+1} less than $\bar{\lambda}$).

Then at any mobile steady state λ^* , both upward and downward flows are positive, implying that both cost thresholds $x^s = \hat{x}^s(\lambda, \lambda), x^n = \hat{x}^n(\lambda, \lambda)$ lie in the interior of the range of the cost distribution.¹¹ Since the cost distribution has a positive density everywhere on its support, this implies that the upward flow must be strictly increasing in the altruism parameter δ at λ^* , and the downward flow must be strictly decreasing. So the difference between the upward and

¹⁰In Figure 5, u and d denote upward and downward flows, and L_i^n denotes $\lambda_i^n(\underline{x})$.

¹¹Note that x^s must be bigger than x^n . So if $x^n \geq \bar{x}$, then $x^s > \bar{x}$ and the downward flow must be zero. Hence $x^n < \bar{x}$. On the other hand the requirement of a positive upward flow implies that $x^n > \underline{x}$. A similar argument implies that x^s must be interior.

downward flows must be strictly increasing in δ at any mobile steady state λ . Standard transversality arguments then imply:

PROPOSITION 3 *Suppose that marginal productivity and utility functions are continuously differentiable, and the cost distribution has a continuous positive density everywhere on its range. Then except possibly for a zero Lebesgue measure set of altruism parameters δ , every mobile steady state λ is locally unique, and there are at most a finite number of such steady states.*

Finally, in the case of multiple steady states with mobility, it is interesting to note that steady states are ordered with respect to the extent of mobility:

PROPOSITION 4 *Suppose there are two steady states with positive mobility. Then the steady state with higher skill ratio has higher mobility, lower wage inequality, and higher per capita income.*

This follows from the fact that the downward flow correspondence is strictly increasing in λ . Hence comparing across two economies with exactly the same characteristics but operating at two distinct steady states, equality and mobility will be positively related. Richer countries will tend to be more equal and more mobile. However the positive correlation between equality and mobility may not obtain when examining comparative static properties of a given steady state. For instance, if we start at a given steady state and raise the altruism parameter δ , then the upward flow correspondence will rise, and the downward flow will fall, at every λ . The steady state skill ratio will move to the right, but the effect on mobility is ambiguous, depending on which flow moves to a greater extent. If the downward flow falls locally ‘by more’ than the increase in the upward flow, the net effect will be to lower steady state mobility. Intuitively, the greater incentive of the unskilled families to invest in their children’s future is outweighed by the greater reluctance of skilled families to not perpetuate their choice of a skilled occupation. This may be relevant in understanding cross-country differences in mobility. For instance, Checchi, Ichino and Rustichini (1999) find that Italy is characterized by a lower level of mobility than the US, despite a more generous public education program: our model provides a potential explanation of this finding.

4 Non-Steady State Dynamics: The Genius-Idiot Example Again

Agent heterogeneity also complicates the nature of competitive equilibrium dynamics considerably. We noted that in the homogenous agent case, the competitive equilibrium always converged to some steady state, irrespective of initial

conditions. This property does not carry over in general to the case of heterogeneity. In particular even if there is a unique steady state, competitive equilibria can fail to converge in some cases.

To illustrate this, we return to the genius-idiot example, where the nature of the competitive equilibrium dynamic can be explicitly derived. In order to do so, we need the following notation. Let $\lambda^i(\lambda)$ denote the skill ratio at the following date (given a skill ratio λ prevailing at the current date) which would make a family with skill level i today and a ‘normal’ child indifferent between educating and not educating that child. Specifically:

$$B(\lambda^i(\lambda)) = C^n(\lambda; x).$$

Hence if the equilibrium skill ratio at the next generation exceeds (resp. falls below) $\lambda^i(\lambda)$, such a family would not (resp. would) optimally educate its child. Since skilled families are richer than unskilled families (given $\lambda < \bar{\lambda}$), we know that the threshold is higher for skilled families: $\lambda^s(\lambda) > \lambda^n(\lambda)$. Moreover, λ^s is a decreasing function, while λ^n is an increasing function.

Define the drift function $D(\lambda) \equiv \lambda + (1 - \lambda)\gamma - \lambda_1$, the dynamic of the skill ratio driven by the unusual children alone (if all normal children follow their parents’ occupations), i.e., by the upward flow of geniuses from unskilled families, less the downward flow of idiots from skilled families. Then it is easy to verify that the competitive equilibrium dynamic will be as follows. If the current skill ratio is λ , then the ratio λ' at the next generation will be given by $D(\lambda)$ if this lies in between $\lambda^n(\lambda)$ and $\lambda^s(\lambda)$. Otherwise if $D(\lambda)$ is less than $\lambda^n(\lambda)$, then λ' equals $\lambda^n(\lambda)$ — in this case unskilled families with normal children must be indifferent between educating and not educating their children, and a fraction of them will educate their children. On the other hand if $D(\lambda)$ is bigger than $\lambda^s(\lambda)$, then λ' equals $\lambda^s(\lambda)$ — in this case a positive fraction of skilled families with normal children will educate their children, while all unskilled families with normal children will not educate them.

Let λ_1 and λ_2 denote the endpoints of the rightmost steady state interval in the baseline case where there are no geniuses or idiots. Note that at the stationary skill ratio λ_1 (resp. λ_2), unskilled (resp. skilled) parents are indifferent between educating and not educating their children. Hence $\lambda^n(\lambda_1) = \lambda_1$ and $\lambda^s(\lambda_2) = \lambda_2$. See Figures 6 and 7 (where the letter L denotes the symbol λ).

Note that if the dynamic of skill ratios were to follow the λ^n function over the entire range, then the skill ratio would converge to λ_1 . But if instead it were to follow the λ^s function, then it will converge (to λ_2) only if the slope of this function in the neighborhood of λ_2 is less than one in absolute value. If the slope exceeds one then the steady state λ_2 of the λ^s function is locally unstable. Whether the slope of the λ^s function exceeds or falls below one depends on the parameters of the model, e.g., on the strength of the altruism motive. If this

motive is sufficiently weak (i.e., the altruism parameter δ sufficiently small), then the slope will exceed one for a large neighborhood of λ_2 .

However, as we have noted above, the global dynamic will not follow either the λ^n or λ^s function throughout. It will switch between these two functions and the drift function, depending on the relative values of these functions. So in order to characterize the global dynamic, we need to distinguish between different cases, which are represented in Figures 6 and 7 respectively.

In Figure 6 the unique steady state λ^* (defined by the condition $\frac{\lambda^*}{1-\lambda^*} = \frac{\gamma}{1}$) lies in the interior of (λ_1, λ_2) . Here the competitive equilibrium converges to the steady state from arbitrary initial conditions. In the neighborhood of the steady state the drift function lies in between the λ^n and λ^s functions, so represents the local dynamic. The equilibrium sequence converges because the drift function has a positive slope less than one (note that D is linear with a slope of $1 - \gamma - 1$).

In Figure 7 we represent the case where the proportion of geniuses is much larger, and $\frac{\gamma}{1} > \frac{\lambda_2}{1-\lambda_2}$. Now the unique steady state skill ratio is at λ_2 , because the rightward drift is positive even at λ_2 , and a positive fraction of skilled families with normal children must disinvest in order to counterbalance the rightward drift.¹² The competitive equilibrium dynamic in the neighborhood of the steady state must follow the λ^s function (since the drift function lies above it in such a neighborhood). Recall from the above discussion that the λ^s function may well have a slope exceeding one in absolute value, in which case the steady state is unstable. Any slight perturbation of the steady state will lead to a dynamic sequence which will perpetually oscillate around the steady state, and never converge to it.

The failure to converge is reminiscent of failures of various learning algorithms to converge to mixed strategy equilibria. Note that certain modifications of the dynamic would be likely to restore convergence. For instance, it is easy to see that the competitive equilibrium dynamic has the general property that if the current skill ratio is less (resp. greater) than the steady state, then the skill ratio at the next date will be higher (resp. lower) than the one prevailing today. In other words, the skill ratio moves in the direction of the steady state. The failure to converge stems from a tendency to overshoot. Such a tendency would be considerably mitigated in the presence of inertia. For instance, suppose that only a positive fraction of families consider switching occupations in any generation, the size of which is an increasing function of the payoff benefit from the switch (with nobody switching if the current occupation of the parent is optimal for the child). If the fraction of switchers is scaled down enough (for any given payoff benefit) as a result of inertia, then the overshooting can be

¹²There cannot be a steady state to the right of λ_2 because there *all* skilled families with normal children will want to disinvest. Then the skill ratio at the following date will be γ , which is less than λ_2 by assumption.

moderated. With sufficient inertia we conjecture that the equilibrium sequence will be convergent. However a systematic and general analysis of the non-steady state dynamics remains to be carried out.

5 Relation to Existing Literature

Models analyzing steady state mobility include Aghion and Bolton (1997), Banerjee and Newman (1993), Maoz and Moav (1999) and Piketty (1997). Banerjee-Newman consider a model with four occupations and three income classes, with risky income patterns within each occupation. Mobility is induced by sufficient variability in *ex post* incomes, somewhat analogous to the variability in education costs in our model. Positive income shocks allow the poor to escape poverty and switch occupations. These shocks eliminate steady states with zero mobility. Banerjee and Newman find that their model is typically characterized by two steady states with mobility, varying with respect to per capita income, equality and mobility.¹³ This is similar to our finding that there are typically a finite number of isolated steady states with mobility. Steady state multiplicity arises for similar reasons: a less equal distribution prevents the poor from making the investment necessary to switch occupations. However, in our model there is a range of cases where the steady state is unique. And owing to our assumption that each occupation is essential, the steady states in our model necessarily involve inequality, whereas in their model one of the steady states is characterized by income equality. The papers of Aghion and Bolton (1997) and Piketty (1997) share many of the features described above for Banerjee and Newman (1993)¹⁴ and so bear the same relation to this paper.

Maoz and Moav (1999) consider a model very similar to ours, but do not address the question of steady state multiplicity, nor the convergence properties of competitive equilibria. Instead they focus on the qualitative features of a ‘development process’, characterized by the movement from a low non-steady state skill ratio to a steady state skill ratio. They also analyze the effects and design of redistributive taxes along such a development process.

The relation to Ray (1990), Ljungvist (1993), Freeman (1996) and Mookherjee and Ray (2002, 2003) is that steady states in those models are characterized by zero mobility, and a continuum of unequal steady states in the presence of occupational indivisibility. This paper can be interpreted as studying the effect of augmenting such a model with heterogeneity in agents’ education costs, and

¹³Using a continuous time dynamic they avoid the overshooting problems described in the previous section, so competitive equilibria converge in their model.

¹⁴The Aghion-Bolton paper however studies a model with a unique mobile steady state, while Piketty’s model is characterized by multiple mobile steady states.

showing that the effect can be to dramatically reduce the extent of steady state multiplicity.

Galor and Zeira (1993) do not allow any heterogeneity or income risk in their models, and consequently the steady states in their model do not involve any mobility. Their first model is characterized by complete absence of pecuniary externalities, so each family follows an independent dynamic. The income dynamic has two steady states, hence at the macro level there is a continuum of steady states varying with respect to the proportion of families at different steady state income levels. In that context it is evident that adding heterogeneity of income or education costs would typically eliminate this indeterminacy, and give rise to a unique steady state at the macro level. So the conclusions of our paper would (trivially) apply to that context as well. In the subsequent model in their paper which incorporates pecuniary externalities, Galor and Zeira describe two classes of steady states (associated respectively with a developed and underdeveloped economy), but do not address the question of local indeterminacy.

Ghatak and Jiang (2003) consider a simplified version of the Banerjee-Newman model, with three occupations, homogenous agents and absence of income risk. In their model also there is a continuum of steady states, and absence of occupational mobility in steady state. We presume that our results concerning the implications of introducing heterogeneity or income risk on steady state multiplicity will also apply to their model.

Finally, the earlier work of Becker and Tomes (1979) and Loury (1981) was based on a version of the neoclassical growth model where there are no indivisibilities or nonconvexities in the investment technology. Their models incorporate agent heterogeneity in order to explain the persistence of income inequality. They are characterized by a unique steady state with mobility.

6 Conclusion

This paper has argued that when occupational mobility is sought to be explained by agent heterogeneity, the history dependence property of steady states tends to be severely restricted, even in the presence of nonconvexities in occupation choice and presence of borrowing constraints. We provided examples of large classes of economies where mobile steady states are globally unique. More generally, they tend to be locally unique. This is in contrast to the case of homogeneity in agents' tastes and abilities that most of the existing literature has focused upon, where steady states form a continuum. The introduction of heterogeneity and occupational mobility not only suggests that the conclusions concerning persistence of long run inequality between families has been overstated by previous literature, but also inferences concerning history dependence. This suggests that models based on political economy rather than market fail-

ure may be more relevant in understanding the role of history in perpetuating underdevelopment.

Many questions remain to be explored. For one, non-steady state dynamics need to be studied in greater generality. For instance under what general conditions can convergence to steady states be ensured?

Second, there is the question of how robust the results are with respect to the bequest motive. We suspect that they carry over to the context of a dynastic bequest motive, but are yet to complete our analysis of this question. Robustness with respect to state dependence of ability within families or alternative formulations of agent heterogeneity is also an interesting question.

Third, what is the role of interventionist policies in a world with occupational mobility? The results suggest that temporary policies (of education subsidies for instance) are considerably less effective in affecting long term levels of human capital, per capita income, or inequality. But there may still be a role for long term policies. Is it possible that steady states involve too little investment in human capital from an efficiency standpoint? Also as discussed in the text, permanent education subsidies may raise steady state levels of human capital and per capita income while reducing cross-sectional inequality, but their effects on mobility are considerably less clear-cut. Issues concerning effects and optimal design of public policies in contexts involving occupational mobility constitute an important research agenda for the future.

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Appendix

Proof of Lemma 2: If $\rho > 1$,

$$(13) \quad h^n \equiv (1-\sigma)^{-1}[\delta\alpha^{1-\sigma}\eta^{(1-\alpha)(1-\sigma)} - (1+\delta)(1-\alpha)^{1-\sigma}\eta^{-\alpha(1-\sigma)} + \{(1-\alpha)\eta^{-\alpha} - x\}^{1-\sigma}]$$

where $\eta \equiv \frac{1}{\lambda} - 1$. Hence in this case h^n is nondecreasing if and only if

$$(14) \quad m(\eta) \equiv \{(1-\alpha)\eta^{-\alpha} - x\}^{-\rho} - \delta\alpha^{-\rho}\eta^{1-\rho(1-\alpha)} - (1+\delta)(1-\alpha)^{-\rho}\eta^{\alpha\rho} \geq 0.$$

It can easily be verified that the same is true when the utility function is logarithmic (putting $\rho = 1$).

Claim: If $\rho \geq 1$ and $m(\eta) \geq 0$, then $m'(\eta) > 0$.

To prove the claim, note that the condition $m'(\eta) > 0$ is equivalent to

$$(15) \quad \alpha\rho(1-\alpha)\eta^{-1-\alpha}\{(1-\alpha)\eta^{-\alpha} - x\}^{-1-\rho} > \alpha\rho(1+\delta)(1-\alpha)^{-\rho}\eta^{\alpha\rho-1} + \delta(1-\rho+\alpha\rho)\alpha^{-\rho}\eta^{\alpha\rho-\rho}$$

Now $\rho \geq 1$ implies that $\alpha\rho \geq \alpha\rho + 1 - \rho$, so $m(\eta^*) \geq 0$ implies that the R.H.S. of (15) cannot exceed $\alpha\rho\eta^{-1}\{(1-\alpha)\eta^{-\alpha} - x\}^{-\rho}$, which in turn cannot exceed the L.H.S. of (15). This establishes the claim.

The claim implies that if $m(\eta^*) \geq 0$ then $m(\eta) > 0$ for all $\eta > \eta^*$. Using (14) this implies that if $h' \geq 0$ at λ^* , then $h' > 0$ at all $\lambda < \lambda^*$. If $\rho \geq 1$ then we know that h^n is negative for λ close to $\lambda(x)$, and also close to $\bar{\lambda}$. Hence if there exists some $\lambda \in (\lambda(x), \bar{\lambda})$ where $h^n > 0$, then h^n must have a unique maximum λ^* over this range, and will be rising from $\lambda(x)$ to λ^* and decreasing thereafter. Hence h^n is inverse U-shaped: it is positive over a subinterval of $(\lambda(x), \bar{\lambda})$, and negative outside it.

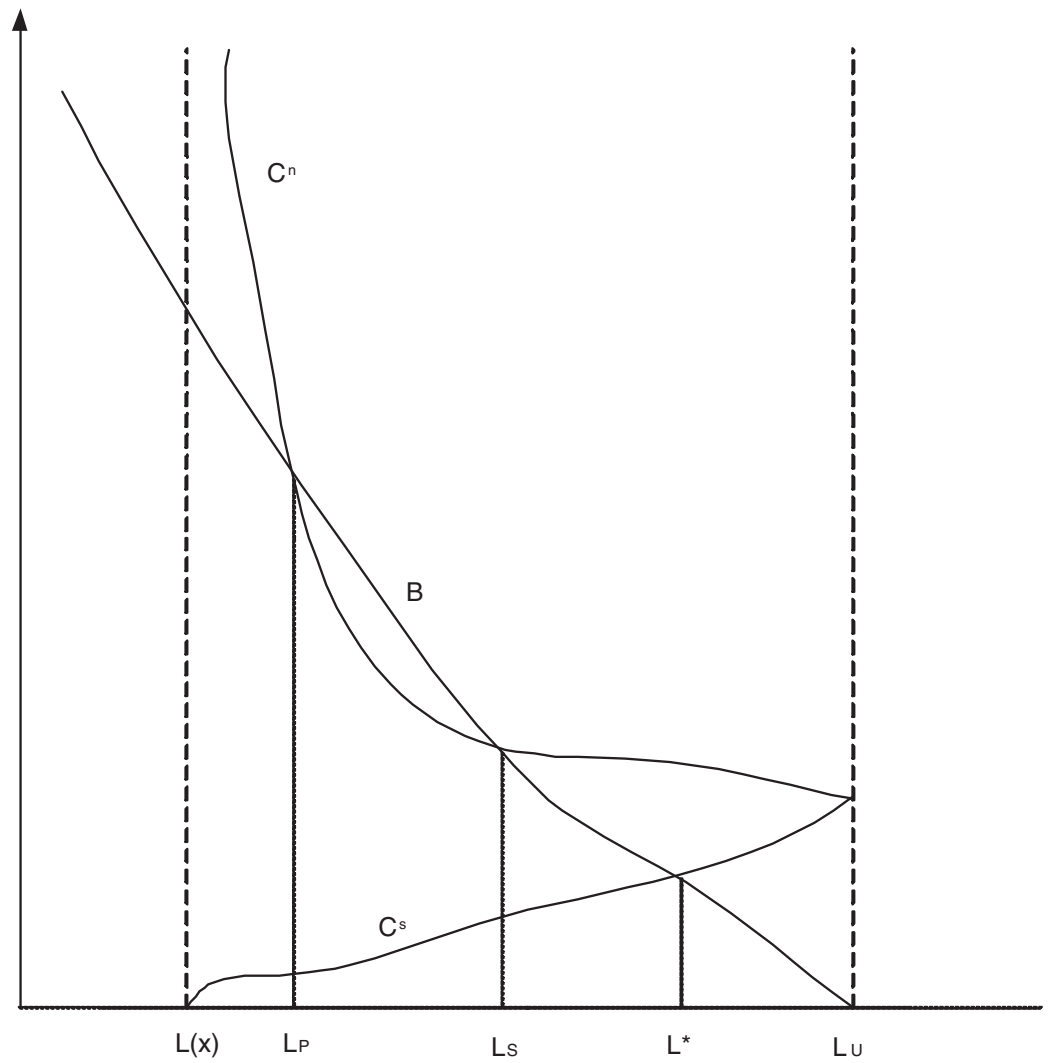


Figure 1. HOMOGENEOUS AGENTS

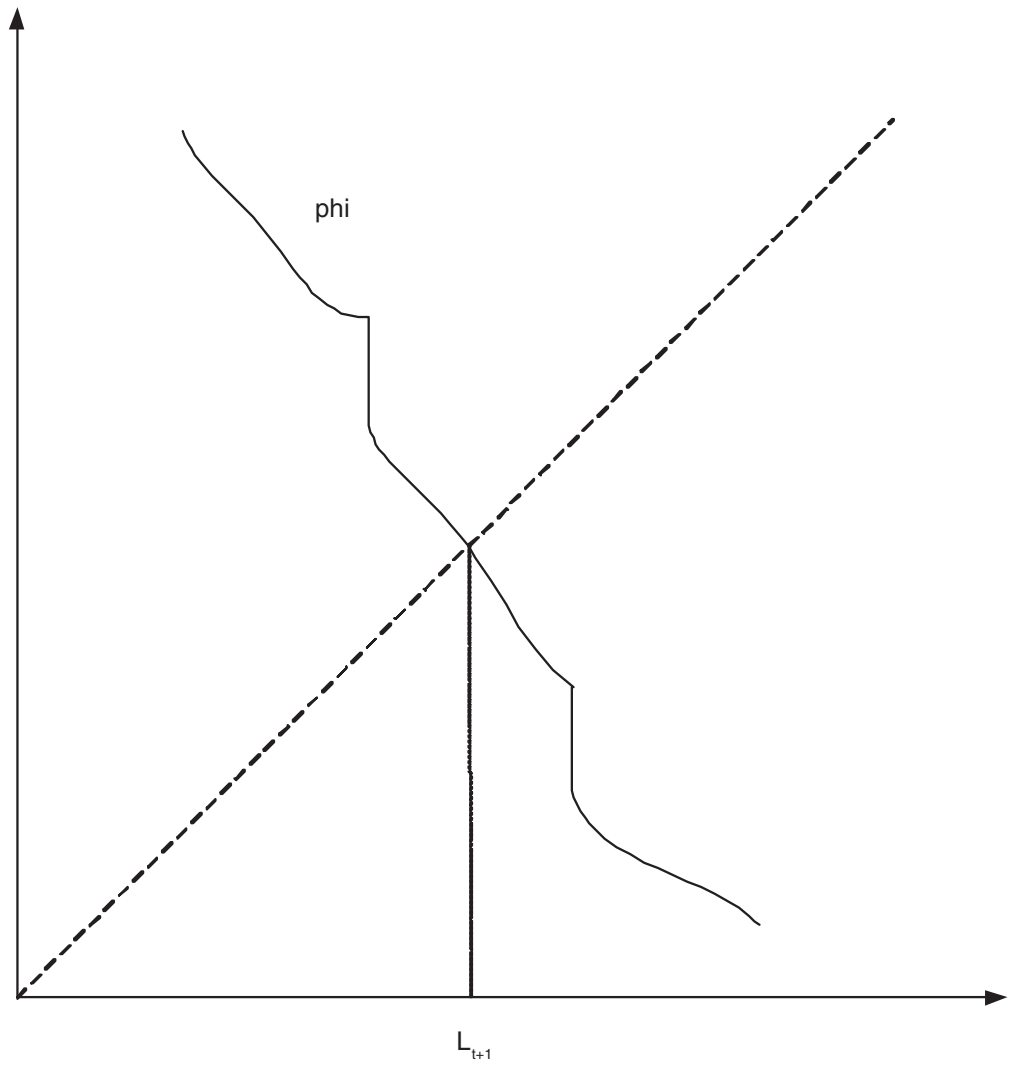


Figure 2. COMPETITIVE EQUILIBRIUM GIVEN ARBITRARY CURRENT SKILL RATIO

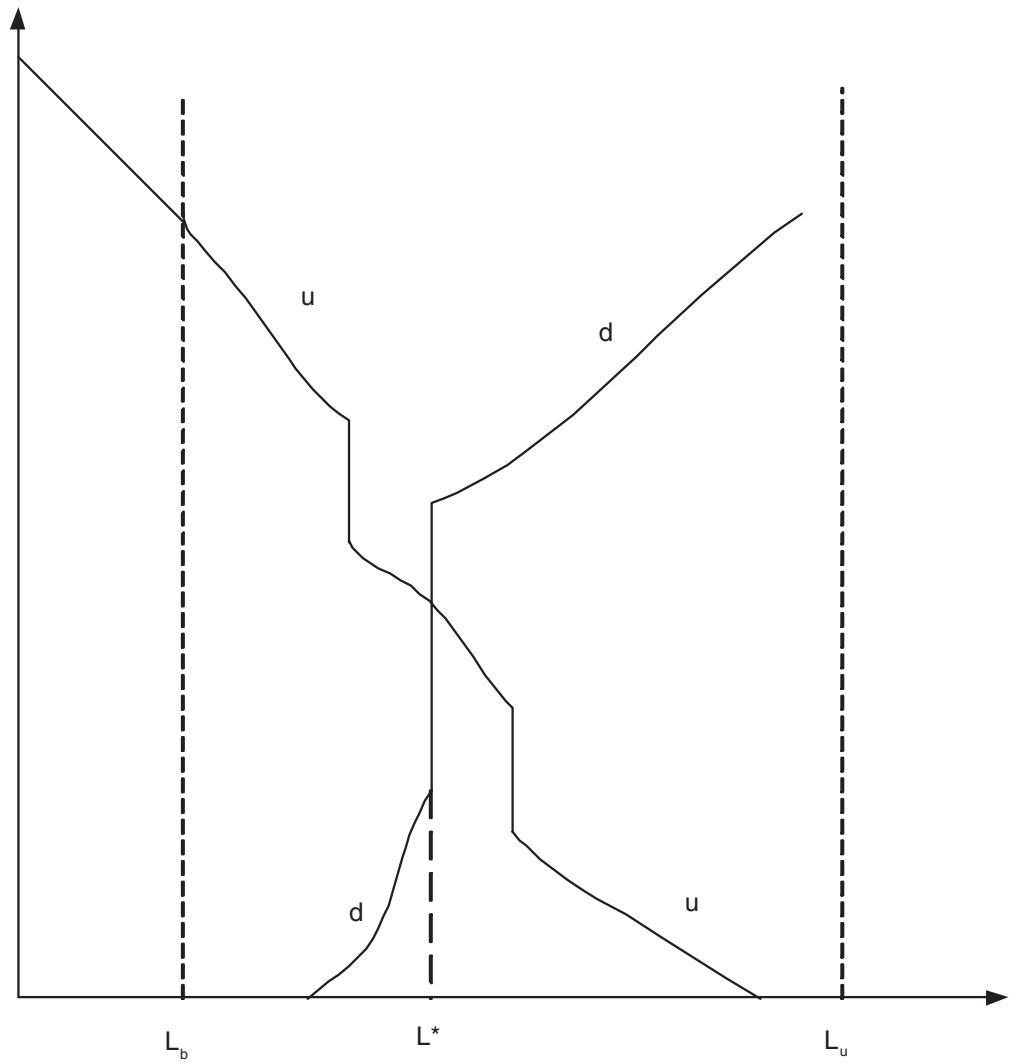


Figure 3. STEADY STATE WITH SINGLE CROSSING PROPERTY

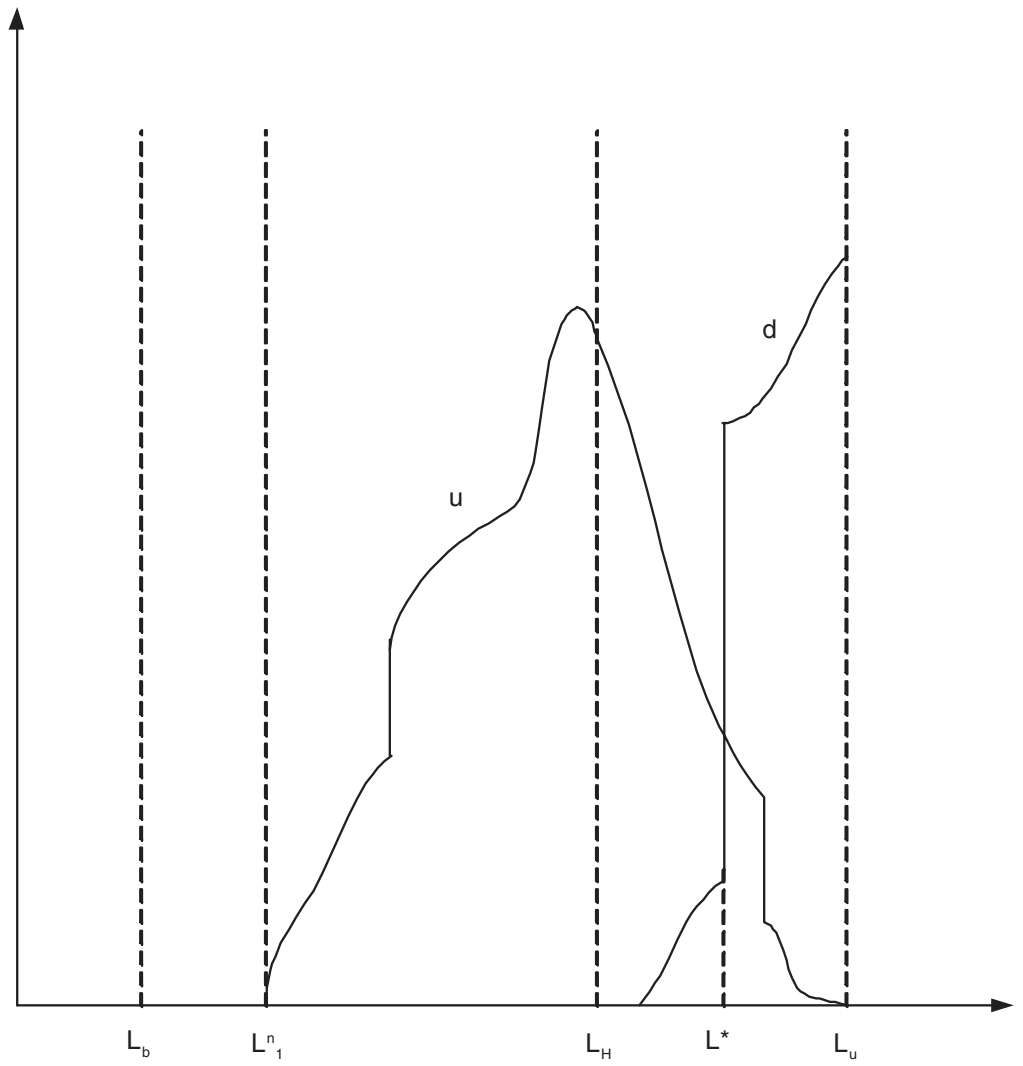


Figure 4. STEADY STATES WITH DOUBLE CROSSING PROPERTY

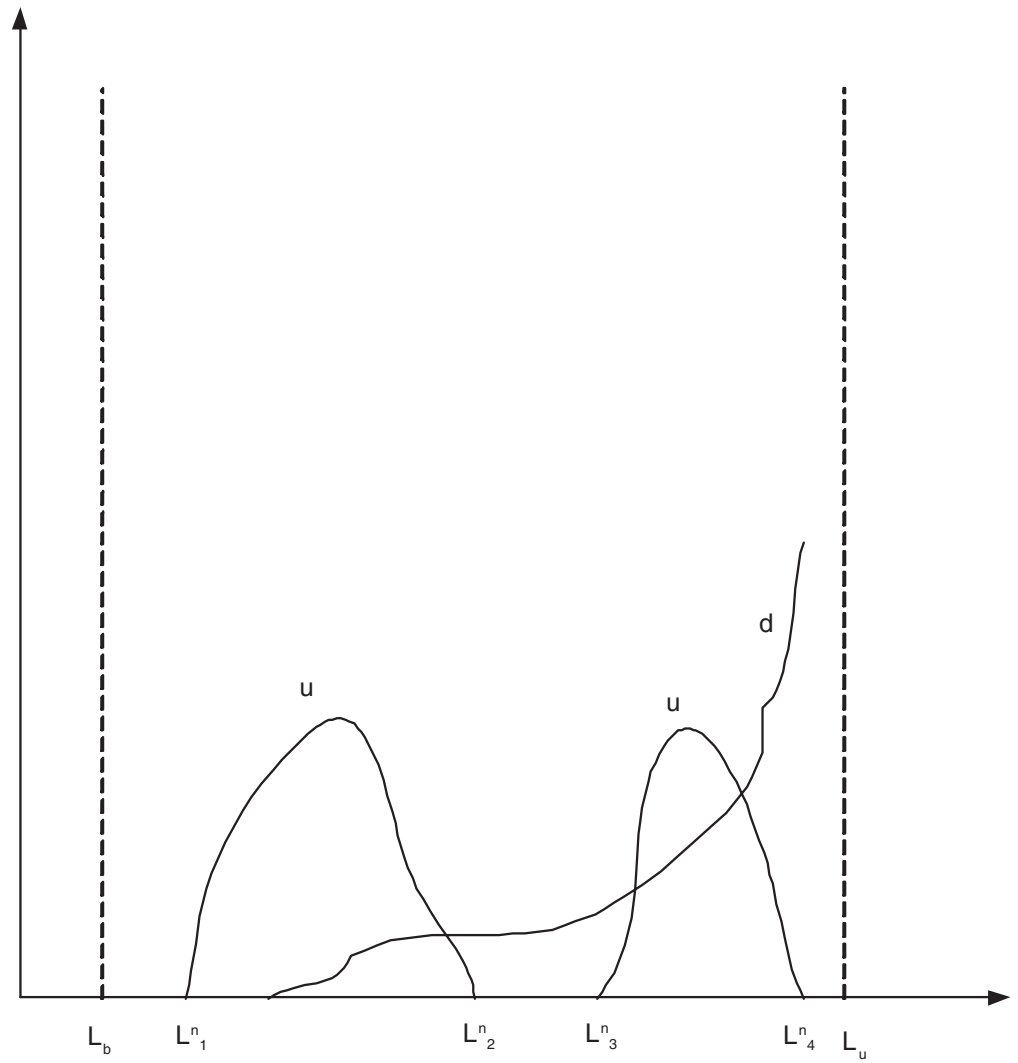


Figure 5. STEADY STATES: GENERAL CASE

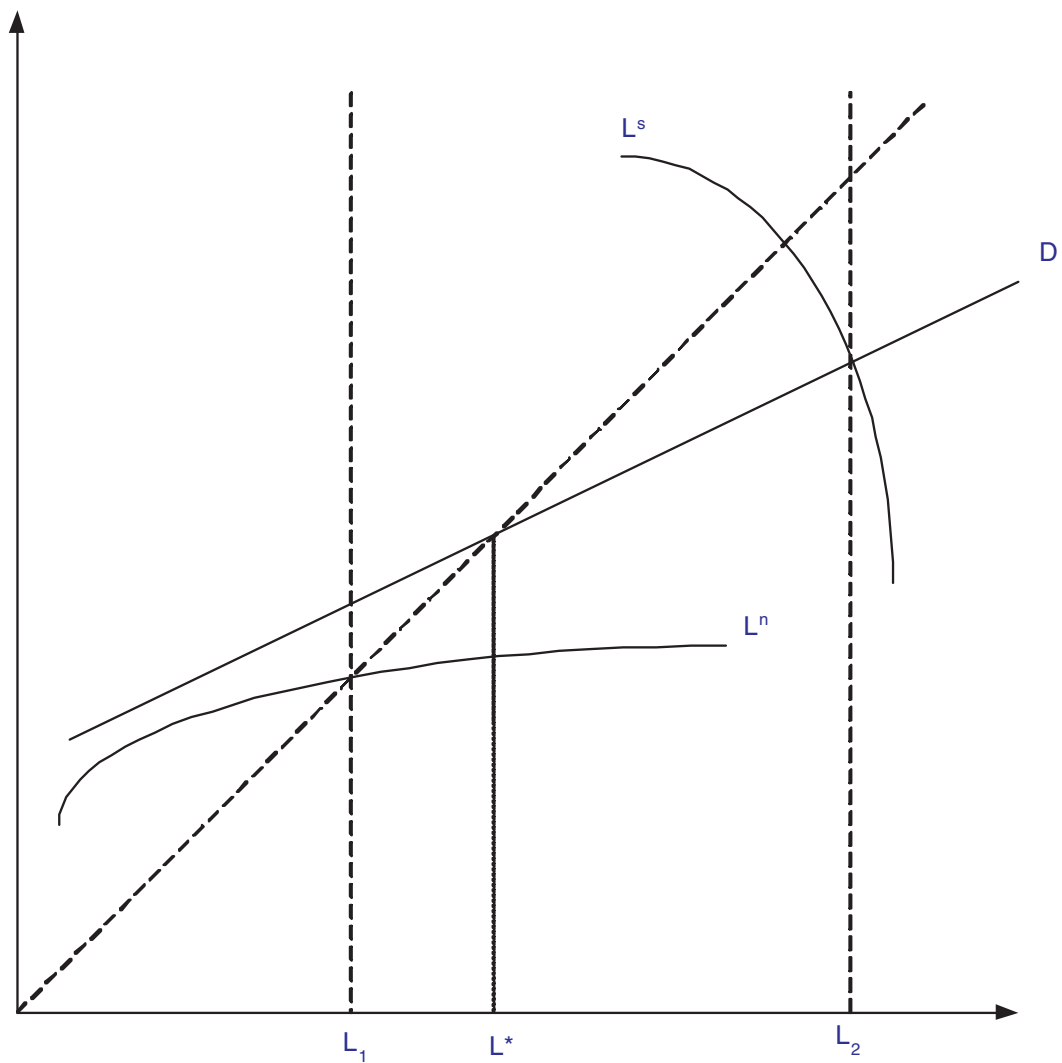


Figure 6. GENIUS-IDIOT EXAMPLE: CONVERGENT DYNAMICS

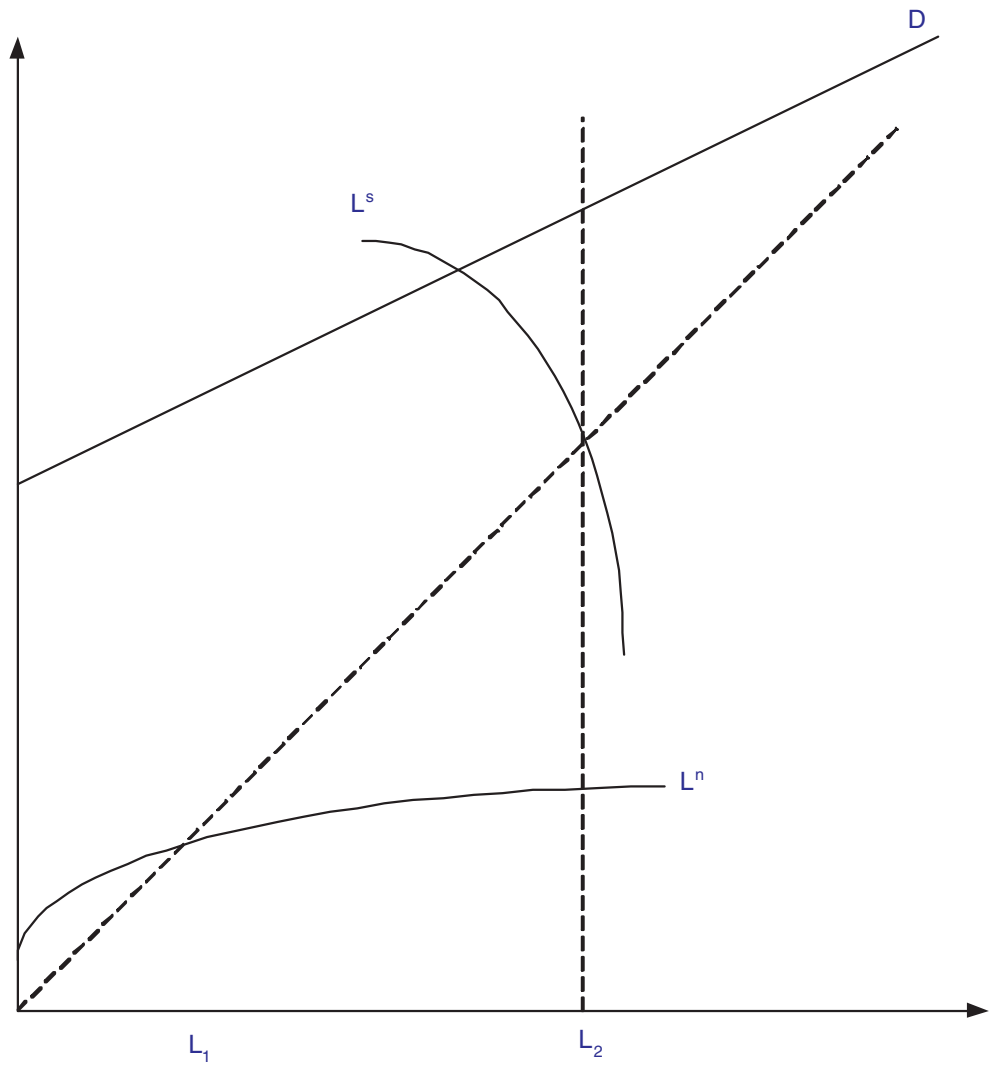


Figure 7. GENIUS-IDIOT EXAMPLE: DIVERGENT DYNAMICS