Occupational Diversity and Endogenous Inequality

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Abstract

This paper constructs a theory of equilibrium investment allocation between human capital and financial assets in the presence of borrowing constraints, and resulting implications for the dynamics of distribution of income and wealth. The theory generalizes most existing models of occupational choice, including Becker-Tomes-Loury models in which markets are inherently equalizing, endogenous inequality models where they are inherently disequalizing, and 'new classical' models in which either can happen depending on historical conditions. Which view turns out to be correct is shown to depend on two attributes of occupational diversity: the range of training cost differentials between least skilled and most skilled occupations, and the richness of occupational structure which affects the divisibility of human capital investment opportunities.

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1 Introduction

A fundamental question in the theory of income distribution is whether free markets have an innate tendency to equalize or disequalize the fortunes of different households or societies. In order to pose this question clearly, one needs a definition of what it means to "equalize" or "disequalize". In our view, the appropriate definition must be set in a context in which the dynamics of income distribution are generated only by the operation of the market, in the (hypothetical) absence of any heterogeneity or stochastic shocks in abilities, tastes or the luck of different participants. This allows a clear conceptual separation between the role of the market *per se* and heterogeneity or random events in generating or propagating inequality.

In the case of inter-household distributions, this permits us to pose the question in the following way: comparing two families with similar abilities and tastes but different wealth in a given generation, does the market by itself (in the absence of any stochastic shocks) induce the wealths of their descendants to draw ever closer together, and their difference to eventually vanish? If so, say that the market is *equalizing*. Conversely, the market is *disequalizing* if it causes the wealths of equal or near-equal households to separate over subsequent generations, and wealth differences between households to persist in the long run (once again, in the absence of any heterogeneity or stochastic shocks).[1]

Existing interhousehold distributional theories provide — explictly or implictly — answers to these questions, and on that basis we can classify such theories into three broad groups. First, there are what one might call the "exogenous inequality" models of intergenerational transmission of earnings and wealth (Becker and Tomes (1979, 1986), Loury (1981)), which predict that markets are equalizing in the sense described above.[2] In this view, long run inequality results from exogenous, ongoing random shocks to abilities or opportunities ("luck", generally speaking), the disequalizing tendencies of which the market subsequently tends to iron out.

At the other extreme are recent models of "endogenous inequality" (Ray (1990), Ljungqvist (1993), Freeman (1996), Bandyopadhyay (1997), Mookherjee and Ray (2003)) which argue that markets are disequalizing: despite the absence of stochastic shocks, *all* steady states must involve interhousehold inequality that persists across generations. Even if all households start equal, their fortunes must subsequently separate in order to induce sorting of households into different occupations. And such differences must persist indefinitely in order to sustain investment incentives, with no movement of families across occupations with disparate earnings. Among other things, these models rely on "symmetry-breaking": the economic need for a diversity of occupations forces individuals in identical or near-identical situations to make distinct choices of profession, with implications for the subsequent emergence of inequality.[3]

In between these two approaches is a third collection of models which may be described as

[1]Such a inquiry has an obvious extension to the macroeconomic level: we may ask whether there is any automatic tendency for economies with similar characteristics (i.e., tastes and technology) but different historical conditions to converge or diverge. However, while we believe that the analysis in this paper is relevant for that inquiry, a closer study must await another paper.

[2]We avoid the term "neoclassical" here, laden as it is with all sorts of preconceptions about what such a methodology entails. Moreover, there are many different versions of the neoclassical model — some in which markets are complete, others where they are incomplete, with differing implications for the dynamics of inequality. We wish to focus on theories with successive generations where the endowments of children are determined by parental investments or bequests, and where parents cannot borrow from their children. We will also be focusing on versions with a determinate theory of distribution, as will be explained further below.

[3]This "broken symmetry" is also studied in a related context by Matsuyama (2002).

theories "neutral towards inequality" (Banerjee and Newman (1993) and Galor and Zeira (1993), among others). These models permit both initial inequalities and initial equalities to persist. Equal and unequal steady states co-exist, and history determines where a society ends up in the long run.[4]

The purpose of this paper is to describe an inclusive framework which embeds all three approaches as special cases, thus allowing us to interpret the key differences in underlying assumptions. Our first objective is to understand what accounts for the two different forces of equalization and disequalization.

All the theories (or versions theoreof) have capital market imperfections, so this is not where the discrepancy resides. We will also see that the indivisibility of occupational choices is not crucial to the matter. We will argue, instead, that the differences arise from assumptions regarding the set of investment opportunities available to market agents, and the manner in which the returns to such investments are determined. Models of exogenous inequality are based on the assumption of some given, convex investment technology. Investment in human capital may occupy center stage, but some aggregation procedure is presumed to exist so that different "amounts" of human capital have the same relative price. In other words, all human capital is — by assumption — commonly expressible in some common efficiency unit.[5] This approach is nicely summarized by Becker and Tomes in their 1986 paper:

> "Although human capital takes many forms, including skills and abilities, personality, appearance, reputation and appropriate credentials, we further *simplify* by assuming that it is homogeneous and the same "stuff" in different families." (Becker-Tomes (1986, p.56), emphasis ours)

As it turns out, this "simplification" turns out to not be so innocuous. The literature on endogenous inequality recognizes that different "units" of human capital often correspond to different occupational choices, and that the relative returns to such professions are fundamentally endogenous. The "divisibility" of human capital investment is not at issue here: the endogeneity of relative returns across occupations is perfectly compatible with an arbitrarily rich range of occupational choices (or a continuous investment space, expressed in monetary terms). It turns out that these two views of investment have markedly different implications.

We illustrate the difference by using an example from Mookherjee and Ray (2003). Suppose there are two occupations, both essential to production: skilled (S) and unskilled (U) where S entails a higher training cost. Suppose, moreover, that labor earnings constitute the only source of income. Now observe that even if all individuals in the economy are ex-ante identical (in terms of wealth), they cannot make the same investment choices. If some occupational slot is

[4]There are also models in which the steady state distribution of wealth is fully indeterminate — see, e.g., Chatterjee (1994). Here the investment frontier that each household faces is linear (rather than strictly convex as in Loury (1981)), so with a dynastic bequest motive each family wants to maintain its wealth indefinitely, and a steady state is compatible with *any* interfamily wealth distribution. If the bequest motive is modified, as in Becker and Tomes (1979), to a paternalistic motive where parents care about the wealth of only their next generation, rather than the infinite succession of subsequent generations, then this arbitrariness disappears; steady state is consistent only with perfect interfamily wealth equality (under the assumptions made by Becker and Tomes on the strength of parental altruism). Alternatively, if investments are subject to diminishing returns at the househol level of each household, then again perfect long run equality is predicted, irrespective of the bequest motive. We focus on versions with a determinate theory of distribution.

[5]So, for instance the investment choice in Loury's model is interpreted as a choice of "how much" education to acquire: there is no formal difference between human and physical capital. The so-called endogenous growth models (see, e.g., Lucas (1988)) continue, by and large, to retain this shorthand.

left unfilled in the aggregate, the rate of return to that occupation will be extremely high. To provide incentives for at least some families to train their children for S, it must be the case that earnings/living standards/utility in occupation S are higher than in U. Hence even if all families start equal in generation 0, in the next generation their earnings must separate. These two outcomes are utility-equivalent for generation 0, but they are not utility-equivalent for generation 1! Furthermore, in succeeding generations wealthier parents will have a greater incentive to train their children for S, so that the "primitive inequality" that sets in at the first generation will be reinforced: children of skilled parents will be more likely to acquire skills themselves. In any steady state, there is persistent inequality.

It is clear from the preceding argument that the divisibility or otherwise of investment opportunities is immaterial: the argument applies more generally irrespective of how many occupations there are. What *is* needed is some minimal degree of occupational diversity (at least two essential occupations must entail distinct training costs). This is precisely the condition that fails in the models of Loury and Becker and Tomes. because different levels of human capital are made of "the same stuff", all occupations are perforce perfect substitutes, and no occupation is essential.[6]

However, the assumption that labor earnings constitute the sole source of income *is* essential to the argument. This means that the only way for parents to transfer wealth to their children is through investment in their education. If financial bequests could supplement educational expenditures, they could be used to offset the inequality induced by differences in educational investments. For instance, if all parents started with equal wealth, they would have similar preferences over their children's future wealth *vis-a-vis* their own consumption. They could simultaneously sort into different occupations while ensuring their children all end up with the same wealth — those selecting less skilled occupations for their children could compensate them via higher financial bequests. The need to ensure occupational diversity would then no longer necessitate inequality.[7]

However, observe that financial bequests must *entirely* redress the inherent inequality in earnings. One of the main purposes of this paper is to precisely study this issue: to what extent can bequests play an offsetting role, and thus enable equalization despite the need to induce occupational sorting incentives?

Our model thus permits physical and human capital to co-exist. "Physical capital" may be viewed as also including the aggregative view of human capital in the exogenous inequality literature. In contrast, what we call "human capital" takes on the form typically assumed in the endogenous inequality literature: a conglomerate of different occupations which supply distinct labor inputs to the production process, inputs that are *not* perfect substitutes for one another. Inter-occupational earning differences are then endogenously determined by relative supplies of people in different occupations, among other things. Such a formulation is supported by considerable empirical evidence.[8]

In all other respects our model is deliberately standard. We study agents embedded in *dynasties*, sequences of individuals connected by intergenerational altruism. We formulate the bequest motive as in Becker and Tomes (1979), mainly to facilitate comparison but also for the sake of

[6]Likewise, in Loury's model, different amounts of capital are to be interpreted as representing distinct occupations, but there will be perfect substitution across those "occupations".

[7]Formally, an "occupation" is now to be interpreted as a pair: the first entry being financial holdings, the second being the occupation in the standard usage of that word. The diversity condition will now have to be imposed on these "occupations", and it may well fail.

[8]See e.g., Katz and Murphy (1992) for responsiveness of US skill premia to relative supply of skilled workers.

simplicity (relative to dynastic specifications). In this formulation parents are concerned about the wealth of their children, apart from their own consumption. This is more sophisticated than the "warm-glow" bequest motive often invoked in the literature, since parents care about the consequences of their bequests for their children, while limiting the range of their concern to consequences only for the following generation. Such altruism manifests itself in two ways: parents can make human capital investments in their progeny and can also pass along financial bequests in addition to such investments. We assume (again in line with the literature) that capital markets are imperfect, so that parents cannot borrow to finance these investments.

It turns out that a simple condition suffices to establish whether the market is equalizing or disequalizing. Roughly speaking, we need to compare the *span* of the occupational structure with the *strength* of the bequest motive. By "span" we refer to the range of training cost differences between the most- and least-skilled occupations. By "strength of the bequest motive" we mean both the desire and the ability to leave large bequests to succeeding generations. The former is, of course, a matter of preferences (such as the degree of altruism). The latter depends, in part, on the productivity of physical capital which via its effect on the equilibrium rate of return determines the efficacy of financial bequests.

Continuing informally for the time being, if span is wide relative to bequest strength, then markets are disequalizing and all steady states must involve persistent inequality: the results of the endogenous inequality models are correct. Put another way, a necessary condition for equalization to be the norm is that occupational investments with endogenous (relative) rates of return be small compared to the more traditional forms of aggregative bequests.

Our condition has some interesting implications. Perhaps dominant among these interpretations is that poverty is correlated with disequalization. First, the strength of the bequest motive will generally be lower in such economies, because the need for current consumption is that much higher. Second, to the extent that poorer econonomies are more unproductive, the rate of return on physical capital is likely lower, once again rendering the condition for endogenous inequality more likely.[9]

A second objective of the paper is to study history-dependence more closely, and in doing so place the "neutral inequality" models in perspective. That multiple equilibria (depending on belief systems) and multiple steady states (depending on historical performance) may explain differences across ex-ante identical societies is a powerful idea in development economics, and can be traced back many decades in academic development thought, to the work of Paul Rosenstein-Rodan and Albert Hirschman and perhaps earlier still. The co-existence of several steady states — some equal and others not — emphasized in the "neutral inequality" models is a particular instance of this. Can history have long run macroeconomic consequences?

It turns out that whether or not there are multiple steady states is related to a different feature of occupational structure: the *richness* of occupational choices. The question is whether the set of human investment options is "diverse" in the following sense: between any pair of occupations differing in required training investment, there always exist occupations with intermediate levels of training costs. If so, the set of occupational choices is said to be perfectly rich; otherwise there are indivisibilities or "gaps" between subsets of investment opportunities. Notice that richness has nothing to do with *holding* multiple occupations: in our model, each person chooses one and only one occupation (though by a judicious definition of "occupation" one could squeeze in

[9]There are other implications as well. For instance, a country that suddenly opens up to a globalized range of products and services will find itself attempting to produce at least some products in that widened range, thereby expanding the list of occupations. If this widens occupational span, inequality is more likely to emerge.

part-time occupations here). The question is whether — in the financial space *outlays* to acquire occupations, her set of choices is fully continuous or not.

We prove that — conditional on the interest rate — richness of occupational structure implies uniqueness of the steady state. That steady state may exhibit persistent inequality (if the span condition described earlier is met), but such inequality is now to be regarded as the *inevitable* outcome for the society. There is no other steady state.[10]

What drives the uniqueness result? Suppose that the occupational structure is perfectly rich, so that there is a continuum of distinct occupations. Now imagine the incentives that will sort our families into these different occupations. For any occupation to be selected by any family, that family must be locally indifferent between the chosen occupation and neighboring ones. This indifference condition ties down the resulting gradient of earnings with respect to training costs, and thence the entire earnings function by the additional requirement of profit maximization in the production sector.

We reiterate that this uniqueness result is orthogonal to matters of equalization. For instance, if occupational span is narrow, then the unique steady state exhibits perefct equality, and the results of the exogenous inequality literature are vindicated. If, on the other hand, the occupational span is wide, then the unique steady state must entail inequality, with a nonconvex investment frontier (and a higher rate of return on human capital over some range than on financial assets). Then the results of the endogenous inequality models apply.

It is also to be emphasized that uniqueness is conditional on the interest rate. It is possible that endogenous variation in the interest rate will create multiple steady states. We have not fully explored this question in the paper.

Next we drop the richness assumption. We show that steady state multiplicity (of the sort favored by the "neutral-to-inequality" models) then appears. Indeed, if occupational span is narrow (so that the endogenous inequality condition above is not met), and if occupational structure is significantly discrete, then the "neutral" theories come fully into their own: equal and unequal steady states co-exist, and the market can be equalizing or disequalizing, depending on initial conditions.

We examine one such special case in more detail. Consider the sparsest possible occupational situation: one skilled and one unskilled occupation. Here equal and unequal steady states can co-exist, provided the occupational span is narrow. In such cases long run outcomes depend on initial conditions: the predictions of the new classical theories apply. For the market to be equalizing, the unskilled must start with at least a certain minimal wage, which requires the economy to start with at least a minimum proportion of skilled families (i.e., absence of extreme inequalities in human capital or wealth). Then unskilled families can progressively catch up in terms of their overall wealth via a sequence of rising financial bequests over successive generations. The rising wealth of the unskilled sets off a process of rising human capital in the economy, and falling inequality, thus reinforcing the process of convergence. In this case the "market equalizing" view prevails.

On the other hand if the economy starts with too few skilled families, this causes the skill premium to be extremely high, trapping the vast majority of unskilled households in low wages,

[10]This observation has very different implications for policy. If there is a unique steady state (especially one involving inequality), then it paints a somewhat more depressing picture of the world: apart from direct and ongoing policies designed to shift the *parameters* of the model, certain inequalities will keep recurring. The view of policy as a shifter of initial conditions, which then drive the economy of its own accord into a salubrious steady state, is then no longer valid — unless occupational span and richness ae themselves products of history.

and preventing them from being able to invest in their children's education. This traps the entire economy in a low-level of human capital and high wage inequality in perpetuity. Unskilled households become progressively poorer owing to shrinking financial bequests, while the opposite is the case for wealthy skilled families. In this case the market disequalizing forces dominate.

The paper is organized as follows. Section 2 lays out the model. Section 3 analyses conditions for existence of an equal steady state. Section 4 addresses uniqueness in the case of perfect divisibiity. Section 5 treats the opposite case of two occupations. Finally, Section 6 concludes.

2 Model

We extend the model of our earlier paper (Mookherjee-Ray (2003)) to incorporate financial bequests. It nests both Becker-Tomes (1979, 1986) and our earlier model: it reduces to Becker-Tomes in the absence of human capital, and to a variant of our earlier model in the absence of financial bequests. The main difference is in the nature of the bequest motive: we assume here — in line with Becker-Tomes — that parents care only about the wealth of their children, apart from their own consumption.

There is a single consumption good, one physical capital good, and a set of occupations $\mathcal{H}$ (unrestricted for now). The technology is described by an aggregate CRS production function $y = f(k, \lambda)$, where k denotes physical capital, and $\lambda \equiv \{\lambda_h\}_{h \in \mathcal{H}}$ is the occupational distribution. All occupations are essential: f satisfies Inada conditions with respect to λ for any given k.[11] Moreover, occupations are imperfect substitutes: f is strictly quasiconcave in λ.

There is an exogenous training cost $x(h)$ for occupation h, denominated in units of the consumption good.[12] Not investing in human capital at all is also an option, so formally there is an occupation with zero training cost. Recall for later use that the two salient attributes of "occupational diversity" will be *span* and *richness*. The former will be given by the range of training costs, and the latter by the variety of different costs in that range.

The variable k may represent physical capital used in production sector of a closed economy, in which case the rate of return r on capital will be endogenous and pinned down by profit maximization. Alternatively, there may be an international capital market (for production) with perfect mobility of capital, in which case the interest rate is fully pinned down. A formally equivalent way of describing production function would then be as $f(k, \lambda) = rk + g(\lambda)$, where g is CRS and strictly quasiconcave, satisfying Inada conditions.

The economy is perfectly competitive. Let $\mathbf{w} \equiv \{w(h)\}$ denote the wage vector, r the interest rate, and $\mathbf{p} \equiv (r, \mathbf{w})$ the factor price vector. Profit maximization in the production sector implies that factor prices $\mathbf{p}$ are determined by marginal productivity. Moreover, CRS implies that the unit cost function $c(\mathbf{p})$ is linearly homogeneous. It is useful to note the following fact: the profit maximization condition $1 = c(\mathbf{p}) \equiv rk + \int_{\mathcal{H}} w(h) d\lambda$ implies that there cannot be two equilibria $\mathbf{p}, \mathbf{p}'$ with $\mathbf{p} \geq \mathbf{p}'$, i.e., given r, two wage functions representing distinct equilibria must cross.

[11] Formally, fix any value of $k > 0$ and any full support $\boldsymbol{\lambda}$. Now consider a positive scalar s multiplying this human input composition. We will assume that output strictly increases with s, with marginal gains going to infinity as s goes to zero and converging to 0 as $s \to \infty$.

[12] Our earlier paper generalized this to allow training costs to depend on the pattern of wages. We suspect the results of this paper will continue to apply when similarly extended, but this remains to be verified.

2.1 Households

There is a continuum of families $i \in [0, 1]$, each represented by single member in each generation $t = 0, 1, 2, \ldots$ In generation t, the representative of family i inherits a financial bequest $b_t(i)$ and occupation $h_t(i)$ from its parent. Aggregating across all families, we obtain the economy-wide factor aggregates: k_t, λ_t, which in turn determine factor prices $\mathbf{p}_t$ according to their respective marginal products. The implied wealth of i at t is $W_t(i) \equiv (1 + r_t)b_t(i) + w_t(h_t(i))$.

Each family then anticipates factor prices $\mathbf{p}_{t+1} \equiv (r_{t+1}, \mathbf{w}_{t+1})$ for the next generation $t + 1$, and selects $b_{t+1}(i), h_{t+1}(i)$ to maximize

$$U(W_t(i) - x(h_{t+1}) - b_{t+1}) + V((1 + r_{t+1})b_{t+1} + w_{t+1}(h_{t+1})) \tag{1}$$

subject to the borrowing constraint $b_{t+1} \geq 0$. We assume that U, V are twice continuously differentiable, strictly concave, strictly increasing functions. The bequest motive is intermediate between 'warm-glow' and 'dynastic', with parents concerned about wealth (rather than consumption or utility) of their children. A special case that we shall often use to illustrate our results is one where U and V both exhibit the same constant elasticity: $U(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma > 0, \neq 1$, and $V \equiv \delta U$, where $\delta \in (0, 1)$ represents the extent of parental altruism.

A *competitive equilibrium* given historical wealth distribution $\mathbf{W}_0 \equiv \{W_0(i)\}_{i \in [0,1]}$ is a sequence of factor prices $(r_t, \mathbf{w}_t), t = 0, 1, 2, \ldots$ and associated $(k_t, \lambda_t), t = 0, 1, 2, \ldots$ such that:

(a) each family i selects bequests and occupations optimally;

(b) these decisions aggregate to k_t, λ_t at each t, and

(c) k_t, λ_t is profit-maximizing at $r_t, \mathbf{w}_t$.

A *steady state* is a stationary competitive equilibrium: $(r_t, \mathbf{w}_t) = (r, \mathbf{w}), (k_t, \lambda_t) = (k, \lambda)$ for all t.

A useful preliminary result is the following.

PROPOSITION 1 *The wealth of (almost) all families is stationary in any given steady state.*

The proof is as follows. Let $W_{t+1} \in F(W_t)$ denote the wealth of a dynasty at $t + 1$, given wealth W_t at t. Suppose first that $W_{t+1} < W_t$. We claim that $W_{t+2} \leq W_{t+1}$. Otherwise $W_{t+2} > W_{t+1}$. Then a parent with wealth W_{t+1} induces his child to have a wealth of W_{t+2}, while a (richer) parent of wealth W_t induces his child to have a lower wealth $W_{t+1} < W_{t+2}$, which contradicts concavity of the utility function.

Hence the wealth of the family is nonincreasing in this case. Since it is bounded below by 0 it must converge.

Next suppose that $W_{t+1} > W_t$. Then reversal of the same logic as above implies that family wealth is nondecreasing across generations. It cannot grow indefinitely for any positive measure of families since average wealth in the population is bounded. So it must converge for all but a measure zero of families.

In what follows, an *equal steady state* will refer to a steady state with a degenerate wealth distribution; all other steady states are called *unequal*.

2.2 Only Financial Bequests: The Becker-Tomes Model

Suppose there are only financial bequests: everyone earns an exogenously fixed wage ω. Also suppose the interest rate r is fixed. In that case a parent with wealth W selects $b \geq 0$ to maximize $U(W - b) + V(\omega + (1 + r)b)$. Let the resulting wealth of the child be denoted $W' \equiv \omega + (1 + r)b$. It is evident that $W'(W; r, \omega)$ is increasing in W. Becker and Tomes (1979) impose the following restriction on bequest behavior, on the basis that this is supported by empirical studies:

Limited Persistence (LP). $\frac{\partial W'}{\partial W} \in (0, 1)$

LP implies that $W'(W; r, \omega)$ is a contraction, with a unique fixed point, denoted $\Omega(\omega; r)$. It leads directly to the conclusion that the market is equalizing.

In their subsequent 1986 paper, Becker and Tomes extend this model to include human capital, "by assuming that it is homogeneous and the same 'stuff' in different families." Different forms of human capital are thus perfect substitutes for one another, implying that relative earnings are exogenously determined by the technology. With appropriate convexity assumptions on the cost of imparting human capital to children by parents, their 1986 model effectively reduces to that of Loury (1981), with a convex investment technology. Hence the market equalization result continues to obtain.[13]

2.3 Human Capital Bequests: The Mookherjee-Ray Model

Now assume that (a) there is no physical capital or financial assets; (b) different occupations produce human capital that are not perfect substitutes for one another, and (c) there is a minimal extent of occupational diversity, in the sense that at least two occupations requiring different training costs are essential in the production process.

Given the absence of financial wealth, zero wealth mobility reduces to zero earnings mobility. Moreover, occupational diversity implies persistent inequality in earnings, and hence in wealth and utility. In this case the market is disequalizing, owing to the need to provide incentives to some parents to select occupations with higher training costs, in the form of additional earnings. Even if parents have equal wealth their children's wealth must diverge, *given the assumption that there is no source of wealth apart from earnings.*

2.4 When Financial Wealth and Human Capital Co-exist

In the general case the Becker-Tomes logic no longer applies, because returns to human capital investment are endogenously determined, so the investment frontier may fail to be convex. Neither does the logic of our earlier paper apply, since it is possible for parents training children for occupations requiring less training to compensate them with higher financial bequests.

To ensure consistency with the Becker-Tomes model, we shall impose a weaker version of limited persistence:

Weak Limited Persistence (WLP). For any r, ω the following is true: if $W'(W; r, \omega) \leq W$ then $W'(\hat{W}; r, \omega) < \hat{W}$ for all $\hat{W} > W$.

[13] This pertains to the bottom end of the distribution where returns to human capital exceed the rate of interest and are strictly diminishing. At higher levels the return on human capital becomes equal to the interest rate and the investment frontier becomes linear, so wealth inequality generated by differing patterns of financial bequests become possible.

Under this assumption, it is still the case that $W'(W;.)$ has a unique, globally stable fixed point, possibly at ∞. The virtue of WLP is that it does not impose an upper bound on the interest rate, unlike LP — so is the appropriate generalization of LP to the case of endogenous interest rates.

To illustrate this, consider the example with isoelastic utility and a constant rate of discount: $U(c) \equiv \frac{c^{1-\sigma}}{1-\sigma}$ for $\sigma > 0, \neq 1$, and $V = \delta U$ with $\delta \in (0,1)$. Let $\rho \equiv [\delta(1+r)]^{\frac{1}{\sigma}}$, which can be taken as a measure of the strength of the bequest motive. Then in the absence of any human capital the Becker-Tomes wealth transmission pattern is as follows:

$$W' = \frac{(1+r)\rho}{1+\rho+r} W + \frac{\rho}{1+\rho+r}\omega \tag{2}$$

if $W \geq \frac{\omega}{\rho}$ and ω otherwise.

If $\rho \leq 1$, then steady state wealth takes the form $\Omega(\omega; r) = \omega$: the bequest motive is not strong enough to sustain any bequests in steady state. If $\rho \in (1, 1+\frac{1}{r})$ then $\Omega(\omega; r) = \frac{\rho}{1-r(\rho-1)}\omega$: in this case there are financial bequests in steady state, and each family attains a finite wealth. Finally if the bequest motive is so strong that $\rho \geq 1 + \frac{1}{r}$ the wealth of each family is unbounded and $\Omega(\omega; r) = \infty$.

In this example, LP imposes an upper bound on the interest rate, while WLP is satisfied at every interest rate, and so represents an assumption on preferences alone.

3 Conditions for Existence of an Equal Steady State

Given interest rate r, say that a wage function $w(h)$ is *r-linear* if $w(h) = \underline{w} + x(h)(1+r)$ for all h, for some $\underline{w}$ — i.e, if the rate of return to human capital investment equals r everywhere. Clearly, given any r, there exists at most one r-linear wage function that is consistent with profit maximization.

Call an interest rate r *allowable* if r and its associated r-linear wage function supports some nonzero production vector as profit-maximizing. In the case where r is fixed by the international capital market, only that value of r is allowable. On the other hand, if f satisfies Inada conditions with respect to physical capital, all interest rates are allowable.

Given an allowable r and its associated r-linear wage function, let $Y(r)$ denote the set of profit-maximizing net outputs, $\bar{w}(r), \underline{w}(r)$ the highest and lowest wage, and $\Omega^*(r) \equiv \Omega(\underline{w}(r), r)$ the steady state wealth in a Becker-Tomes world where only financial bequests are possible, with a constant flow earning of $\underline{w}(r)$.

PROPOSITION **2** *An equal steady state exists if and only if there exists an allowable r such that: (i) $\Omega^*(r) \geq \bar{w}(r)$ and (ii) $\Omega^*(r) \in Y(r)$.*

Condition (i) above is the condition that occupational span is narrow relative to the bequest motive. To interpret it, go back to the iso-elastic example. If $\rho \equiv [\delta(1+r)]^{\frac{1}{\sigma}} \leq 1$ then there are no financial bequests in steady state in the Becker-Tomes world; $\Omega^*(r) = \underline{w}(r) < \bar{w}(r)$, and the narrow span condition cannot be satisfied. With $\rho \in (1, 1+\frac{1}{r})$, the span condition reduces to

$$\sup_h x(h) \leq \frac{\rho - 1}{1 + r(\rho - 1)} \underline{w}(r) \tag{3}$$

i.e., the span of training costs should not be too large, relative to the strength of bequest motive and the interest rate. Essentially, the strength of the bequest motive should be such as to generate a steady state wealth in the Becker-Tomes world at the given interest rate, which exceeds the earning differential between the least skilled and most skilled occupations. That way unskilled agents can be compensated with enough financial bequests to equalize their wealth with the most skilled agents in the economy.

Condition (3) is more likely to hold true in rich economies than poor economies, in the following sense. Suppose economies differ in their level of development solely due to TFP differences, which scale the productivity of all factors uniformly. Then the condition will be satisfied only in countries beyond some threshold TFP level. Intuitively, parents in richer countries will want to invest more in their children, thus making it more likely that the span condition (i) is satisfied.[14] On the other hand, if growth (from neutral technical progress) causes wages to grow at a uniform rate, then the span condition is less likely to hold in fast growing countries, since higher growth in wages across generations will dull the level of desired bequests. To the extent that poorer countries grow faster owing to a 'catch-up' phenomenon in technology, the span condition is less likely to be satisfied in such countries.

The argument for necessity of (i) is straightforward. Suppose that an equal steady state exists with wealth W^* at interest rate r. In that steady state all households will have the same bequest preferences, and yet must sort into different occupations. They must therefore be indifferent between all occupations, and (given the option of selecting an occupation which necessitates no training at all) between investment in human capital and financial assets. Hence the rate of return on every human capital investment must equal r, i.e., the wage function must be r-linear. This means that the investment problem faced by each family in steady state must be the same as in a Becker-Tomes world with no human capital, and constant flow earning of $\underline{w}(r)$. The common wealth W^* of households must then equal $\Omega^*(r)$. The nonnegativity of financial bequests for the most skilled occupation households implies that this wealth cannot fall below their earnings, implying condition (i).

If r is exogenously fixed then it is easy to see why the span condition (i) is sufficient as well: an equal steady state can be constructed as above. Set the wage function to be the r-linear wage function, so that all households face a linear investment frontier with rate of return r. Let all households have the same wealth $\Omega^*(r)$. Then they are all willing to spend a total of $\beta^* \equiv \frac{\Omega^*(r)-\underline{w}(r)}{1+r}$ on their children, and are indifferent between investing in human capital and financial assets. Divide all households between different occupations according to the pattern of demand for workers arising from the production sector. This is feasible because of condition (i), as the highest human capital investment required among all occupations does not exceed β^*, what parents are willing to spend on their children. All labor markets then clear, and since the market for financial assets clears automatically (with the international capital market absorbing all the excess demand at rate r), we have constructed a competitive equilibrium in which the wealth of each family will remain stationary at $\Omega^*(r)$.

In the case of a closed economy, we have to ensure that the capital market clears as well. This is the role of the additional condition (ii) in Proposition 2. Given (ii), we can find a profit-

[14] Of course this relies on the assumption of exogenous training costs. If instead rising wages raise the cost of education proportionately then educational span (the left hand side of (3)) will also rise in step with the level of development, in which case the level of development will have no effect on the validity of (3). But the result is restored if education costs rise less than proportionately than wages, which will be the case if there are possibilities of substitution of teacher inputs by material inputs.

maximizing production vector at interest rate r and the associated r-linear wage function, such that $\Omega^*(r)$ equals the sum of the (undepreciated) capital stock k^* and associated net output of the consumption good y^*. Let the associated wages and occupational distribution be denoted by $w^*(h), \lambda^*(h), h \in \mathcal{H}$. Then a household selecting occupation h seeks to supplement its human capital investment with a financial bequest of $\frac{\Omega^*(r)-w^*(h)}{1+r}$, implying that the supply of financial investment funds from households equals

$$\begin{aligned}\int_{\mathcal{H}} \frac{\Omega^*(r) - w^*(h)}{1+r} d\lambda^*(h) &= \frac{1}{1+r}[\Omega^*(r) - \int_{\mathcal{H}} w^*(h) d\lambda^*(h) \\ &= \frac{1}{1+r}[k^* + y^* - \int_{\mathcal{H}} w^*(h) d\lambda^*(h) \\ &= \frac{1}{1+r}[k^* + rk^*] \\ &= k^*,\end{aligned}$$

the demand for physical capital from the production sector. Hence the capital market also clears, and we have a stationary competitive equilibrium. The necessity of (ii) is clear from the reverse of this argument.

Condition (ii) is somewhat less transparent to interpret than the occupational span condition (i). It is essentially a condition on the nature of the technology, specifically the productivity of physical capital.

4 Uniqueness of Steady State with Perfect Richness

The result of the preceding section showed that wealth inequality is inevitable in steady state if occupational span is large, while an equal steady state does exist when the span is small. In the former case, the results of the endogenous inequality literature apply, rather than the neoclassical or new classical models. To look for domains where the latter two approaches might still continue to be valid, we therefore need to consider economies where the narrow span condition is satisfied. In such contexts an equal steady state exists. But what about unequal steady states: when do they also exist?

The following result answers this question for the case of perfect richness of educational investment opportunities, conditional on a given interest rate. The corresponding steady state wage function turns out to be unique. Hence with a narrow occupational span the neoclassical rather than the new classical theory is correct. The proposition applies more generally to situations where the narrow span condition is not satisfied, and shows that even there a unique steady state wage function exists (corresponding to a given interest rate). Moreover, it provides an explicit characterization of the steady state. Hence if the narrow span condition is violated and investment opportunities are perfectly divisible, the results of the endogenous inequality literature apply, i.e., steady states must involve persistent inequality.

PROPOSITION **3** *Suppose occupational investments are perfectly divisible: X is an interval $[0, \mathbf{x}]$. Then for any given interest rate r, there is at most one steady state.*

In this steady state, the rate of return on human capital equals r for occupations upto training cost $\theta \equiv \frac{\Omega(\underline{w};r)-\underline{w}}{1+r}$; families located in such occupations all have wealth $\Omega(\underline{w};r)$ and leave financial *bequests if* $\Omega(\underline{w};r) > \underline{w}$*. If the narrow occupational span condition fails to hold then there are occupations with training costs above* θ*. Over this latter range families make/receive no financial bequests, receive (unequal) wages satisfying the differential equation*

$$w'(x) = \frac{U'(w(x) - x)}{V'(w(x))}, \tag{4}$$

and the marginal rate of return on human capital exceeds r *(for almost all* x *in this range).*

The result identifies a curious feature of the steady state: financial bequests arise if at all, only at the bottom of the wealth distribution. In that case there is a mass point at the very bottom, at a wealth of $\Omega(\underline{w};r)$. If the narrow span condition applies, this wealth is large enough to encompass the entire earnings distribution, and we obtain the equal steady state. In the other case, the wealth of the poorest families falls below the earnings of high skilled occupations. Rich families (those in occupations with training cost exceeding θ) rely only on human capital investments, as the marginal rate of return on human capital investments beyond θ lies strictly above the interest rate, while below θ they equal the interest rate. Inequality stems from unequal labor earnings, owing to a nonconvexity in human capital investment returns.

The detailed argument for Proposition 3 is provided in the Appendix. Here we provide a broad outline. The first step is to note that every family must have steady state wealth at least $\Omega(\underline{w})$, since all families always have at least the same investment opportunities as in the Becker-Tomes world. The second step is to note that if $\Omega(\underline{w}) > \underline{w}$ then the rate of return on occupations with training cost upto θ must be r. This follows from the first fact: everyone in the population has wealth at least $\Omega(\underline{w})$, so those selecting occupations with training cost less than θ must also be selecting positive bequests. They must then be indifferent between investing in training and in financial assets, so the rate of return on human capital investment upto θ must be r.

The third step is to check that the wage function as constructed in the statement of the Proposition does indeed constitute a steady state. The steady state is constructed on the assumption that those selecting occupations with training cost above θ will select no financial bequests at all, so the wage function must be such as to induce a parent with a given occupation to select the same occupation for its child. The requirement that the chosen occupation be optimal within a local neighborhood of occupations ties down the slope of the wage function as given in (4). Along with the boundary condition provided by the wage corresponding to training cost θ, this differential equation pins down the entire wage function. It can subsequently be checked that the function as constructed provides a marginal rate of return on educational investments strictly above r for (almost) all such occupations. In turn this implies that those families will not want to supplement their educational investments with financial bequests, so the wage function as constructed does constitute a steady state.

It remains to check there cannot be any other steady state with the same interest rate. This is broken down into two steps. First, no other steady state starting with the same wage $\underline{w}$ for the least skilled occupation can arise. By the arguments above, any such steady state must coincide exactly upto θ. Moreover if over some range of occupations above θ there are no financial bequests then the slope of the wage function must satisfy the same differential equation (4) over that range. And over any range where there are financial bequests the marginal rate of return has to equal r. Combining these two facts it is easy to infer that any such wage function — if distinct from the one constructed above — must lie below the latter for some occupations, and coincide for all others. This contradicts the requirement that the wage functions be consistent with profit maximization. Indeed, this argument establishes that all steady states must be of the form described in Proposition 3, with financial bequests upto θ, and none above, and wage functions respectively r-linear upto θ, and governed by the differential equation (4) thereafter.

The final step is to verify that there cannot be any other steady state wage function of this form starting with a different wage for the least skilled occupation. Profit maximization requires these functions must cross, and they cannot cross at any occupation where there are no financial bequests in both steady states (because they must follow the same differential equation (4) at any such occupation). Neither can they cross at an occupation at which the wage functions are both locally r-linear, since they are then locally parallel. So they must cross at an occupation in which the wage function is locally r-linear in one steady state, and not in the other. But this implies that the poorest families in the former steady state are investing more in total than families with the same level of wealth in the other steady state, despite obtaining a lower marginal rate of return.

One important qualification to the statement of uniqueness is worth noting: it is conditional on the interest rate. If the interest rate is exogenously fixed then it does reduce to a statement that the steady state is unique. But if the interest rate is endogenous, there is a possibility of multiple steady state interest rates (as in optimal growth models with nonconcave production functions, or Piketty (1997)). We hope to address this issue in future research.

5 Occupational Sparseness

We turn now to a model in which occupations are extremely sparse: there are just two occupations, one skilled (S) which requires a training cost of x, and the other unskilled (U) which requires no training. Also in this section we shall assume that the interest rate is exogenously fixed at r. Denote the skill ratio by λ, wages in the two occupations by $w_s(\lambda), w_u(\lambda)$ which are respectively decreasing and increasing functions, with $w_s \to \infty, w_u \to 0$ as $\lambda \to 0$.

In this context multiple steady states arise quite naturally. For instance, if x is small enough that the narrow span condition is satisfied, then the equal SS exists (with a skill ratio λ^*, determined by the condition that $w_s(\lambda^*) - w_u(\lambda^*) = (1+r)x$). And unequal steady states also exist, under the mild additional assumption that consumption is constrained to be nonnegative. Define $\tilde{\lambda}$ by the condition that $\Omega(w_u(\tilde{\lambda})) = x$. Then every $\lambda \in (0, \tilde{\lambda})$ constitutes an unequal steady state.[15] Hence a continuum of unequal steady states can co-exist with the equal steady state. Moreover, within the set of unequal steady states, those with a lower skill ratio are associated with lower per capita income and higher inequality. Occupational sparseness can thus precipitate severe history-dependence.

To explore the nature of such history dependence in more detail, we need to describe dynamics out of steady state, so as to follow the consequences of alternative (arbitrary) initial conditions. Particularly interesting is the case where both equal and unequal steady states co-exist. What determines whether the market is equalizing or disequalizing in this case: does the economy converge (if at all) to the equal or an unequal steady state?

So we shall assume for the remainder of this section that the equal steady state exists. We

[15] In any such state, the unskilled wage is low enough that the corresponding Becker-Tomes steady state wealth falls below x, i.e., unskilled families cannot afford education at all. On the other hand, skilled families will want to invest in education: they are willing to do so even at the equal steady state skill ratio, and their incentive is larger when the skill ratio is lower than that.

begin by characterizing unequal steady states. Use $Z(W;\omega)$ to denote the indirect utility of a parent in a Becker-Tomes world with current wealth W and flow earnings ω of the child (at the given interest rate, which we are suppressing in the notation).

PROPOSITION **4** *λ is an unequal steady state skill ratio if and only if*

$$\lambda < \lambda^* \tag{5}$$

and in addition (with $w_u \equiv w_u(\lambda), w_s \equiv w_s(\lambda)$):

$$\max_{b\geq 0}[U(\Omega(w_u) - x - b) + V(w_s + b(1+r))] \leq Z(\Omega(w_u), w_u). \tag{6}$$

The argument is straightforward: if $\lambda = \lambda^*$ then we are effectively in a Becker-Tomes world with a linear investment frontier and a constant rate of return r on all investments, where there cannot be any long run wealth inequality. So $\lambda < \lambda^*$ is necessary for wealth inequality. Unequal steady states must therefore involve a nonconvexity in investment returns: upto x only financial bequests are possible, while at x there is a discontinuous upward jump in investment returns (the size of which depends on the gap between the rate of return on human and financial capital). Since the return on human capital is higher, any parent wishing to invest x or more must first invest in education, and make up the remainder with financial assets. Those investing less than x will only invest in financial assets.

If $\lambda < \lambda^*$ then skilled families have wealth $\Omega(w_s) > \Omega(w_s^*)$. They will want to invest in their children's education because they want to invest at least x if $\lambda = \lambda^*$; now they are even richer and the returns to investing in education are even higher.

Hence one only needs to check the incentives of the unskilled: they must not want to invest x or more. This is the role of condition (6). In steady state unskilled households must have a wealth of $\Omega(w_u)$. The left side is the maximum payoff of a parent with this wealth conditional on investing at least x. The right side is what they attain with financial investments alone. This condition implies that the wealth of the unskilled falls below the skilled wage: $\Omega(w_u) < w_s$.[16] So there must be wealth inequality in these steady states.

Let Γ denote the set of unequal steady state skill ratios. In general Γ consists of a continuum of skill ratios, because it is characterized by a set of inequalities. If λ is an unequal steady state where the constraint (6) holds as a strict inequality, then an open neighborhood of it also satisfies (6) as a strict inequality. On the other hand there is no monotone structure on the set of unequal steady states, because increases in λ lower the cost of investing in education for the unskilled (as they become richer), and also the benefit of education. In general, therefore, Γ is the union of intervals $[\lambda^i, \lambda^{i+1}]$, with $i = 0, 2, 4, ...$ Condition (6) is satisfied as an equality at each λ^i, as a strict inequality in every λ in $(\lambda^i, \lambda^{i+1})$ with i even, and is violated in every λ in $(\lambda^i, \lambda^{i+1})$ with i odd.

Before we proceed to the dynamics, we need the following notation. Let W^i denote $\Omega(w_u(\lambda^i))$, the steady state wealth of the unskilled at the boundary unequal steady state λ^i. Finally, let $\bar{\lambda}$ denote the highest λ in Γ, i.e., is the highest unequal steady state skill ratio. Let $\bar{W} \equiv \Omega(\bar{\lambda})$ denote the corresponding highest unskilled wealth (across all unequal SS's).

[16] Otherwise unskilled parents would be investing at least x in their children, in which case they would be better off educating them rather than provide only financial bequests.

Proposition 5 *Suppose there are two occupations, a fixed interest rate, and both the equal steady state with skill ratio λ^* and a continuum of unequal steady states Γ exist. Then from an arbitrary initial wealth distribution at date 0, the economy converges to a steady state. If the equilibrium skill ratio at date 0 exceeds $\bar{\lambda}$ (the highest skill ratio across all unequal steady states), the economy converges to the equal steady state. Otherwise it converges to an unequal steady state.*[17]

This proposition says that the dynamics depend on the historical wealth distribution, which determines the equilibrium skill ratio at the very beginning. Consider for instance the case where all families start with equal initial wealth.

Corollary. *Suppose all families start with the same wealth W_0. Then there exists a threshold $\bar{W}$ such that if $W_0 \leq \bar{W}$, the economy converges to an unequal steady state, while if $W_0 > \bar{W}$ it converges to the equal steady state.*

Whether an economy that is equal at the outset converges to an equal or unequal steady state depends on how poor it is to start with. If it starts richer than the highest wealth of the unskilled across all unequal steady states, then it will converge to the equal steady state; otherwise to an unequal steady state. Initial and eventual per capita wealth are positively related, and poor countries do not eventually catch up with rich countries. Initial wealth matters because it affects the incentive to invest in human capital, given the presence of borrowing constraints. The capital market imperfection inhibits investment more in poor countries.

In the former case if the economy is rich enough then $\lambda = \lambda^*$ at the very first date itself. There is perfect wealth equality at every date, with financial transfers perfectly offsetting differences in educational investments. If the economy starts rich but not so rich that λ at the first date is less than λ^*, wage inequality falls, while wealth inequality follows an inverted Kuznets-U. Inequality emerges initially, but unskilled wages are high enough that they leave sufficient financial bequests to their children, causing the wealth of the unskilled to grow. In turn this causes the demand for education (and hence λ) to grow, raising unskilled wages even further, and lowering the wage gap between skilled and unskilled. The wealth of the unskilled rise faster than that of the skilled, resulting in convergence. Here financial bequests induce 'trickle down' and the market is equalizing.

In the case of a country that starts poor, the precise dynamics depend on where exactly the economy starts. If initial wealth $W_0 \in [W^i, W^{i+1}]$ with i even, where $W^i \equiv \Omega(w_u(\lambda^i))$, then $\lambda_t = \lambda_1 \in (\lambda^i, \lambda^{i+1})$ for all t. Earnings inequality emerges and remains stationary. In this case the wealth of the unskilled fall, while those of the skilled rise over time, so wealth inequality rises owing to the operation of financial transfers.

Finally, if $W_0 \in (W^i, W^{i+1})$ with i odd, then λ_t rises, and eventually converges to λ^{i+1}. Here earnings inequality emerges initially but falls over time, and the same is true of wealth inequality. Inequality falls but does not vanish ultimately. So we have a Kuznets relationship again. Unlike the case where the economy starts rich enough, however, in the long run the economy remains unequal and poorer on average.

What about the more general case where the initial wealth distribution is non-degenerate? Then initial inequality also matters: even for a country with high initial per capita wealth. If

[17] In the former case the skill proportion rises. In the latter case, the dynamics are as follows. If the date 0 skill ratio is an unequal steady state skill ratio then it converges to that steady state and the equilibrium skill ratio is stationary. If the date 0 skill ratio is not an unequal steady state skill ratio, then it rises over time and converges to the smallest steady state skill ratio lying above it.

this wealth is distributed sufficiently unequally the equilibrium skill ratio at the beginning can fall below $\bar{\lambda}$, causing the economy to converge to an unequal steady state.

In sum, the results that emerge with investment sparseness is that historical conditions matter for the long run, and the market can be equalizing or disequalizing depending on favorable or unfavorable initial conditions, very much along the lines of the new classical models.

6 Conclusion

The aim of this paper was to provide insight into underlying differences between the 'market equalization' view of neoclassical theories of income distribution, the 'market disequalization' of the recent endogenous inequality literature, and the more neutral new classical view that either can happen depending on historical conditions. The results draw attention to an aspect that has received little attention so far in the literature: occupational span. Broadening range of occupations (related to wider range of product varieties resulting from technological change and market openness) and rising training costs relative to other goods and services, may be expected to increase inequality, and make the 'market disequalization' viewpoint more applicable.

Empirically, the equal steady state does not seem particularly relevant: in most economies financial bequests are not that important for most of the population, are positively rather than negatively correlated with earnings, and arise at the top (rather than bottom) end of the wealth distribution. And if financial bequests are relatively unimportant in overall income distribution, the results of our earlier paper on persistent inequality (Mookherjee-Ray (2003)) — which simply assumed that such bequests are unavailable — continue to apply.

On the other hand our theory also runs into the difficulty that it predicts (in the divisible case) that financial bequests arise, if at all, at the bottom end of the distribution. To the extent that bequests are important in modern economies, they arise instead at the very top end.

Two responses may be offered for this anomaly. First, intentional bequests may not be viewed as important at all: e.g., Gokhale *et al* (2001) argue that most financial bequests in the US economy are unintentional, the result of premature death and imperfect annuitization. In the iso-elastic example, this would correspond to the case with ρ falls below unity. Our theory then predicts that there are no intentional bequests anywhere in the wealth distribution, so human capital differences entirely account for all inequality.

The second alternative is to view large financial bequests observed at the top end of the distribution as a form of occupational investment by parents. Presumably the wealthiest sections of the population do not invest much in bank account or Treasury bonds, but instead manage these funds across various high-risk high-return investment opportunities that require large setup investments and access to private information networks. These high end investment activities may themselves be viewed as an occupation, to enter which an agent requires a large upfront investment. Such a model interpreting some high end occupations as financial services or investment activity however needs to be developed and integrated into the 'black-box' model of production used in this paper.

One other issue related to the role of capital in production needs further attention: the possibility that there may be nonunique steady states arising from multiplicity of equilibrium interest rates in economies that are not fully integrated into the international capital market.

Concerning the issue of history dependence at the macro-level, and related issues of convergence or divergence, the results here suggest that such history dependence requires the presence of significant indivisibilities in human capital investment. Causal empiricism suggests that there is little evidence of major gaps in sets of investment opportunities: between most unskilled and skilled occupations one can think of many intermediate occupations. Nevertheless one may argue that there are some key nonconvexities in returns to education that resemble the effect of indivisibilities — e.g. the relative lack of a premium for partial completion of a college degree. In most developed countries with relatively little public subsidization of higher education, the choice of whether or not to go to college or graduate school seems to resemble a significant sparseness. Further though needs to be devoted to the question whether models with indivisibilities are empirically relevant. And if the evidence suggests that they are not, then stories of macroeconomic history dependence will have to be based on political economy channels rather than pure market-based occupational choice mechanisms.

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Appendix: Proofs

Proof of Proposition 3: We divide the proof into a number of steps. Throughout we fix the interest rate at r and suppress it in the notation. Also we shall refer to a BT investment locus as a linear investment frontier with constant rate of return of r, $\underline{w}$ as the wage of the least skilled profession, and $\Omega(\underline{w})$ as the corresponding steady state wealth.

Step 1: given $\underline{w}$ every steady state wage function must be r-linear upto an investment of $\theta \equiv \frac{\Omega(\underline{w})-\underline{w}}{1+r}$.

To prove this note first that every family must have steady state wealth at least $\Omega(\underline{w})$, since they all have access to the linear bequest technology. Hence all those families selecting occupations with training costs below θ must be indifferent between investing in human capital and financial assets. This implies that the rate of return on human capital must be r upto θ.

Step 2: There exists a wage function of the form described in Proposition 3 which constitutes a steady state.

This is proven through a sequence of Lemmas.

Lemma 1. Suppose that the wage function over the range $x \geq \theta$ is described by the differential equation (4), with $w(\theta) = \Omega(\underline{w})$. Then for any $x \in (\theta, \mathbf{x}]$ consider a parent with wage $w(x)$ who is choosing a level of investment $x' \in (\theta, \mathbf{x}]$ for its child, and financial bequests are not allowed. It is optimal for such a parent to select $x' = x$.

Let $M(x, x') \equiv U(w(x) - x') + V(w(x')$ denote the expected utility of such a parent who selects x'. And define $N(x) \equiv M(x, x)$. Then N is differentiable by construction and (4) implies that $N'(x) = U'(w(x) - x)w'(x)$, so

$$N(x) = \int_{\theta}^{x} U'(w(a) - a)w'(a)da. \tag{7}$$

Now take any $x' \in (\theta, x)$: we shall show that $N(x) \equiv M(x, x) \geq M(x, x')$. Then

$$\begin{aligned} M(x,x) &= M(x',x') + \int_{x'}^{x} U'(w(a) - a)w'(a)da \\ &\geq M(x',x') + \int_{x'}^{x} U'(w(a) - x')w'(a)da \\ &= M(x',x') + U(w(x) - x') - U(w(x') - x') \\ &= M(x,x'), \end{aligned}$$

where the step involving the inequality uses the concavity of U and the fact that $x' < x$. A similar argument in the reverse direction shows the same result with respect to any $x' \in (x, \mathbf{x}]$. This proves Lemma 1.

Lemma 2. $w'(x) > 1 + r$ for (almost) all $x \in (\theta, \mathbf{x}]$.

To prove this, note first there cannot be any $\epsilon > 0$ such that $w(x) - w(\theta) \leq (1+r)(x-\theta)$ for all $x \in (\theta, \theta+\epsilon)$. The reason is that $w(\theta) = \Omega(\underline{w})$, the unique steady state wealth on the BT investment frontier corresponding to $\underline{w}$. If a parent with wealth $w(x)$ where $x \in (\theta, \theta+\epsilon)$ were confronted with this investment frontier, the child would attain a wealth strictly less than $w(x)$. Given a rate of return for investments beyond θ which is r or worse, the child would be left with even less, i.e., the parent would select $x' < x$. This contradicts Lemma 1.

Next note that there cannot exist any interval (x_1, x_2) with $x_2 > x_1 > \theta$ such that $w(x)$ is locally r-linear over this interval. Corresponding to the BT investment frontier that coincides with this section of $w(.)$, there can be at most one steady state wealth. But Lemma 1 shows that for every x in this interval, a parent with wealth $w(x)$ leaves his child with the same wealth, and we obtain a contradiction.

Finally we show there cannot be any interval (x_1, x_2) with $x_2 > x_1 > \theta$ such that $w'(x) < 1+r$ for all x in this interval. Suppose otherwise. By the argument two paragraphs above, there must exist points x in (θ, x_1) where $w'(x) > 1+r$. Since $w'(x)$ is continuous by construction, there must exist $x_3 < x_4 < x_5$ such that $w'(x)$ equals $1+r$ at $x = x_4$, exceeds $1+r$ over (x_3, x_4), and is less than $1+r$ over (x_4, x_5). Consider the BT investment frontier passing through $(x_4, w(x_4)) \in \Re^2$. Since a parent with wealth $w(x_4)$ selects $x' = x_4$ in the problem described in Lemma 1, the marginal rate of substitution of that parent equals $1+r$ at a bequest of x_4. Hence the wealth $w(x_4)$ must be the unique steady state wealth associated with this BT investment frontier. Now consider any $x" \in (x_4, x_5)$. Confronted with the BT investment frontier passing through $(x_4, w(x_4))$, such a parent would have invested less than $x"$. With the marginal rate of return for investments beyond x_4 strictly less than $1+r$, such a parent would invest even less. This contradicts the result of Lemma 1.

Hence there cannot exist any non-degenerate interval above θ where $w'(x) \leq 1+r$, which establishes Lemma 2.

Lemma 3. Given the opportunity to supplement educational investments with a financial bequest, no parent with $x > \theta$ would prefer to do so.

This follows from the result of Lemma 2, which implies that the marginal rate of substitution for every parent in occupation $x > \theta$ is at least $1+r$.

Lemma 4. Given the opportunity to invest less than θ, no parent in occupation $x > \theta$ would prefer to do that instead of selecting the same occupation for its child.

This follows from the fact that any such parent is wealthier than those with wealth $w(\theta)$, and the latter prefer θ to any lower investment.

Combining the above set of Lemmas, it follows that the wage function constitutes a steady state, in the sense that it is globally optimal for every parent to choose the same occupation for its child, with only those in occupations below θ investing in financial assets, just enough to leave them with the same level of wealth as themselves. This completes Step 2.

Step 3. There cannot be any other steady state wage function starting from the same $\underline{w}$ *for the least skilled occupation.*

Suppose otherwise. By Step 1 such a wage function must coincide with the constructed steady state upto θ. Next note that over any range of occupations (x_1, x_2) above θ where parents select positive financial bequests, the slope of $w(x)$ must equal $1 + r$. And if these parents select zero financial bequests, the first-order condition for their occupational choice implies $w'(x)$ must satisfy (4) throughout this range. Hence if a steady state involves no financial bequests above θ, it must coincide with the constructed steady state. A distinct steady state wage function must therefore involve positive bequests over some range above θ, in which case the slope of the wage function over that range lies below that of the constructed steady state. That wage function must lie uniformly below the constructed steady state beyond some occupation. For if they ever crossed they must cross at an occupation that leaves no financial bequest, and at such a point the same differential equation (4) applies. We then obtain a contradiction of the property that two distinct wage functions must cross, in order to be consistent with profit maximization.

Step 4. There cannot be any other steady state wage function.

From the previous steps it follows that a steady state wage function must be of the type constructed: r-linear upto $\Omega(\underline{w})$, and following (4) thereafter. So two distinct steady state wage functions must be of this form, with differing wages for the least skilled occupation ($\underline{w}_1, \underline{w}_2$ say, with $\underline{w}_2 > \underline{w}_1$). Let these wage functions be denoted by $w_1(x), w_2(x)$ respectively, and the corresponding investments of the poorest households by θ_1, θ_2, where $\theta_2 > \theta_1$.

These two wage functions must cross. Clearly they cannot cross at an occupation x at or above θ_2, since such an occupation makes no financial bequest in either steady state, so the slope of both wage functions must be the same at that point. They also cannot cross at an occupation $x \leq \theta_1$ since both wage functions have slope $1 + r$ there. So they must cross at an occupation $x \in (\theta_1, \theta_2)$.

We can now find x_1 in a right neighborhood of θ_1 with $w_1'(x_1) > 1 + r$, such that the wealth $W = w_1(x_1)$ is attained in the other steady state with an investment $x_2 \leq \theta_2$. Since the rate of return on investment above θ_1 is higher in the first steady state, the same wealth is attained with a lower investment: $x_1 < x_2$. This is inconsistent with utility maximization: if the household with wealth W is investing optimally in the $w_2(.)$ steady state with a marginal rate of return equal to r, it is underinvesting in the latter given that it enjoys a higher marginal rate of return. This concludes the proof of Step 4 and hence of Proposition 3.

Proof of Proposition 5.

We begin by noting properties of competitive equilibrium at any given date.

LEMMA **1** *Let the wealth distribution at the beginning of any date t be described by the cdf F_t. Then there is a unique competitive equilibrium at date t giving rise to skill ratio at $t+1$: $\lambda_{t+1} \leq \lambda^*$. If $\lambda_{t+1} < \lambda^*$ the equilibrium is characterized by a wealth threshold W_t (satisfying $\lambda_{t+1} = 1 - F_t(W_t)$) such that an unskilled family with wealth at this threshold is indifferent between educating his child and not at date t:*

$$U(W_t - x) + V(w_s(\lambda_{t+1})) = Z(W_t, w_u(\lambda_{t+1})). \tag{8}$$

Moreover, the competitive equilibrium has the following properties:

(a) $\lambda_{t+1} = \lambda^*$ *if and only if* $1 - F_t(W^*) \geq \lambda^*$ *(where* W^* *is defined by* $I(W^*, w_u^*) = x$*, with* $I(W, \underline{w})$ *denoting the optimal bequest of a parent with wealth* W *facing a BT investment frontier corresponding to* $\underline{w}$*).*

(b) If $F_{t+1}(W_t) < F_t(W_t) = 1 - \lambda_{t+1}$*, then* $\lambda_{t+2} > \lambda_{t+1}$*.*

(c) If $F_{t+1}(W_t) \geq F_t(W_t) = 1 - \lambda_{t+1}$*, then* $\lambda_{t+2} = \lambda_{t+1}$*.*

Proof of Lemma 1: Existence and uniqueness of competitive equilibrium skill ratio at any date with a given wealth distribution follows from the fact that we have a continuum economy, and the demand for education (i.e., for investment of at least x) is decreasing in the skill ratio anticipated for the following date. Specifically, if there are two competitive equilibrium skill ratios $\lambda, \lambda' > \lambda$, then there must exist a positive measure of households who invest when $\lambda_{t+1} = \lambda'$ but not when $\lambda_{t+1} = \lambda$. But the incentive to invest must be lower in the former case as the skill premium is lower — if these households weakly prefer to invest in education when $\lambda_{t+1} = \lambda'$, they must strictly prefer to do so when $\lambda_{t+1} = \lambda$.

At this equilibrium, investment incentives are ordered by wealth. Since both occupations are essential, there must be some households investing and others not investing. Since households differ only by wealth, there must exist a wealth threshold where a family is indifferent between investing and not. Such a household must be indifferent between just investing x and investing some amount less than x — otherwise it prefers to invest more than x to investing exactly x, and such a household must strictly prefer to invest at least x to any amount less than x. Hence the wealth threshold is characterized by (8).

Part (a) follows from the following argument. Households with wealth at least W^* are willing to invest at least x when $\lambda = \lambda^*$. If $1 - F_t(W^*) \geq \lambda^*$ then λ^* is a competitive equilibrium skill ratio. For if we set $\lambda_{t+1} = \lambda^*$, the investment frontier coincides with a BT linear investment frontier corresponding to a constant flow earning of w_u^*. Then all those with wealth at least W^* are willing to invest at least x, and are indifferent between education and financial bequests. So we can select λ^* households from this group, and require them to invest in education, while requiring that none of the remaining households in the economy invest in education. Then each household will be choosing optimally and we have a competitive equilibrium. The converse is obvious.

To prove (b), suppose on the contrary $F_{t+1}(W_t) < F_t(W_t)$ and $\lambda_{t+2} \leq \lambda_{t+1}$. Then any household with wealth above W_t strictly prefers to invest in education at date t, and must continue to do so at $t+1$. But there are more households with wealth above W_t at $t+1$, so $\lambda_{t+2} > \lambda_{t+1}$, a contradiction.

Finally, (c) is proven as follows. Suppose $F_{t+1}(W_t) \geq F_t(W_t)$. Then we cannot have $\lambda_{t+2} > \lambda_{t+1}$, as this would imply that a household with wealth W_t would strictly prefer not to invest in education at $t+1$, and there are more households poorer than W_t at $t+1$ than t, which implies that $\lambda_{t+2} \leq \lambda_{t+1}$. On the other hand if $\lambda_{t+2} < \lambda_{t+1}$ then every skilled family will want to invest in education at $t+1$ (since they want to do so even at λ^*), and there are λ_{t+1} skilled families at $t+1$, implying that $\lambda_{t+2} \geq \lambda_{t+1}$, a contradiction. This concludes the proof of Lemma 1.

In the following Lemma, we shall use W^i to denote $\Omega(w_u(\lambda^i))$.

LEMMA **2** *(a)* $W_t = W^i$ *implies* $\lambda_{t+1} = \lambda^i$*.*

(b) $W_t \in (W^i, W^{i+1})$ *with* i *even implies* $\lambda_t \in (\lambda^i, \lambda^{i+1})$ *and* $W_t \geq \Omega(w_u(\lambda_{t+1}))$*.*

(c) $W_t \in (W^i, W^{i+1})$ *with* i *odd implies* $\lambda_t \in (\lambda^i, \lambda^{i+1})$ *and* $W_t < \Omega(w_u(\lambda_{t+1}))$*.*

Proof of Lemma 2. Recall condition (8) relating the threshold wealth W_t with the competitive equilibrium skill ratio λ_{t+1}. Compare this with the relation between steady state wealth W^i and skill ratio at any boundary (unequal) steady state skill ratio λ^i:

$$Z(W^i, w_u(\lambda^i)) = U(W^i - x) + V(w_s(\lambda^i)). \tag{9}$$

Part (a) follows from comparing these two conditions. For part (b), note that $W_t > W^i$ implies that anticipating the skill ratio λ^i at $t+1$, the threshold wealth type W_t would strictly prefer to invest in education, so $\lambda_{t+1} > \lambda^i$. Conversely this type would prefer not to invest in education anticipating a skill ratio of λ^{i+1}, so $\lambda_{t+1} < \lambda^{i+1}$. Hence λ_{t+1} is an unequal steady state ratio, with

$$\begin{aligned} Z(\Omega(w_u(\lambda_{t+1})), w_u(\lambda_{t+1})) &\geq \max_{b \geq 0}[U(\Omega(w_u(\lambda_{t+1})) - x - b) + V(w_s(\lambda_{t+1}) + b(1+r))] \\ &\geq U(\Omega(w_u(\lambda_{t+1})) - x) + V(w_s(\lambda_{t+1}))]. \end{aligned}$$

Now compare with (8) to infer that $W_t \geq \Omega(w_u(\lambda_{t+1}))$.

Next turn to part (c). The same argument as for part (b) shows that $\lambda_{t+1} \in (\lambda^i, \lambda^{i+1})$. Since i is now odd, λ_{t+1} is not a (unequal) steady state skill ratio. Moreover $\lambda_{t+1} < \lambda^{i+1} < \lambda^*$. Proposition 4 now implies

$$Z(\Omega(w_u(\lambda_{t+1})), w_u(\lambda_{t+1})) < \max_{b \geq 0}[U(\Omega(w_u(\lambda_{t+1})) - x - b) + V(w_s(\lambda_{t+1}) + b(1+r))]. \tag{10}$$

We claim that this implies

$$Z(\Omega(w_u(\lambda_{t+1})), w_u(\lambda_{t+1})) < [U(\Omega(w_u(\lambda_{t+1})) - x) + V(w_s(\lambda_{t+1}))]. \tag{11}$$

Otherwise the maximum on the right hand side of (10) is attained at some positive b: a household with wealth $\Omega(w_u(\lambda_{t+1}))$ prefers to supplement educational investment with a positive bequest. The convexity of preferences then implies that a pure educational investment would in turn dominate any financial bequest less than x, contradicting the hypothesis. Finally the result that $W_t < \Omega(w_u(\lambda_{t+1}))$ follows upon comparing (8) with (11). This concludes the proof of Lemma 2.

To complete the proof of Proposition 5, consider first case (b), in which $\lambda_{t+1} \in (\lambda^i, \lambda^{i+1})$. Lemma 2 shows in this case that $W_t \geq \Omega(w_u(\lambda_{t+1}))$. Then all unskilled households at t (i.e., whose parents had wealth below W_t at t) will also have wealth below W_t at $t+1$, The reason is that those with wealth between W_t and $\Omega(w_u(\lambda_{t+1}))$ will leave less to their children. And those at or below $\Omega(w_u(\lambda_{t+1}))$ will leave more than they themselves inherited, yet their children's wealth cannot exceed $\Omega(w_u(\lambda_{t+1}))$. So the mass of the wealth distribution below W_t is not smaller at $t+1$ than at t. Part (c) of Lemma 1 now implies that $\lambda_{t+2} = \lambda_{t+1}$. In turn this implies that $W_{t+2} = W_{t+1}$. So the same story applies at $t+1$ as at t. The equilibrium skill ratio will remain stationary at λ_{t+1} for all $T \geq t+1$. Along this process, the wealth of the unskilled will converge to $\Omega(w_u(\lambda_{t+1}))$, while those of the skilled will converge to $\Omega(w_s(\lambda_{t+1}))$, so the economy converges to the unequal steady state associated with skill ratio λ_{t+1}.

Next consider case (c), in which $\lambda_t \in (\lambda^i, \lambda^{i+1})$ with i odd and $W_t < \Omega(w_u(\lambda_{t+1}))$. Now the wealth of all unskilled households rises towards $\Omega(w_u(\lambda_{t+1}))$. It is still possible that $W_{t+1} = W_t$ and thus $\lambda_{t+1} = \lambda_t$, but if so their wealths will move even closer to $\Omega(w_u(\lambda_{t+1}))$. Eventually at some date $t+T$, we must have $F_{t+T}(W_t) < F_t(W_t)$. Then Lemma 1 implies that $\lambda_{t+T} > \lambda_t$. But comparing with condition (9) applied to $i+1$, it follows that $\lambda_{t+T} < \lambda^{i+1}$. Applying the same argument from $t+T$ onwards, it follows that the equilibrium skill ratio is a nondecreasing sequence bounded above by λ^{i+1}. So it must converge. It can only converge to a steady state skill ratio, which must therefore be λ^{i+1}.

Finally consider the case where $\lambda_{t+1} > \bar{\lambda}$, in which case $W_t > \Omega(w_u(\bar{\lambda}))$. Then also Lemma 2 implies $W_t < \Omega(w_u(\lambda_{t+1}))$. The same logic as in case (c) now implies the equilibrium skill ratio is a monotone sequence, bounded above by the equal steady state ratio λ^*. So it must converge, and to a steady state skill ratio. Since the only steady state skill ratio above λ_{t+1} is λ^*, this is the skill ratio it must converge to. This completes the proof of Proposition 5.