Multi-dimensional screening in a monopolistic insurance market*

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Abstract: In this paper, we consider a population of individuals who differ in two dimensions: their risk type (expected loss) and their risk aversion. We solve for the profit maximizing menu of contracts that a monopolistic insurer wants to put out on the market. One result is that it is in general optimal to separate individuals with different risk aversion, despite the fact that they have identical risk types and therefore equally costly to a risk neutral insurer. Secondly, a sufficiently high risk aversion heterogeneity means that some high risk people (the risk tolerant ones) will get lower coverage than some low risk people (the risk averse ones). Thirdly, we apply the framework to analyze the welfare consequences of allowing the insurer to practise gender discrimination, and show that when the average man and woman differ only in risk aversion, gender discrimination may lead to a Pareto improvement in the insurance market.

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1 Introduction

People that seek insurance differ from each other in many dimensions. For insurance companies, and for the outcome on the insurance market, at least two of these dimensions are of central importance: the distribution of losses that insurance takers face, and the willingness to bear the risk of those losses.[1] Insurance market theory has focussed on the consequences of private information on either of these dimensions, but has rarely studied the case where private information affects both.[2] Moreover, the analysis of a multi-dimensional private information problem in insurance has been restricted to the competitive setting, i.e., one where several insurers compete for clients. In this paper we study the opposite setting by asking how a monopolistic insurance company would design a profit-maximizing contract menu intended to attract agents who not only hold private information on their loss distribution, but also on their risk preference.

Adding risk aversion heterogeneity to the analysis of insurance markets calls for a multi-dimensional adverse selection model. Such analysis is technically not straightforward, because the existence of private information in two or more dimensions breaks the natural ordering of agents according to their willingness to pay for extra coverage. That is, at a contract offering very partial insurance, a very risk averse agent facing a low risk may be more willing to pay for additional coverage than an agent facing a high risk but who is relatively risk tolerant. Moreover, the situation could be the other way round at a contract offering almost full insurance. Technically, their indifference curves cross twice. Absence of single crossing means that the adjacent downward incentive constraints no longer guarantee overall incentive compatibility.[3]

There is a scant literature solving multi-dimensional screening problems, for instance the "user's guide" by Armstrong and Rochet (1999). As it turns

[1] A third dimension, that will not be discussed here, is the moral stance of insurees, determining the amount of false claims which insurers each year have to deal with.

[2] Rothschild and Stiglitz (1976) analyze a perfect competitive insurance market with private information on the distribution of losses. Stiglitz (1977, sections 3 and 4), and Landsberger and Meilijson (1996) look at a monopolist insurer. When private information is with respect to risk attitude, Stiglitz (1977, section 5) and Landsberger and Meilijson (1994) analyze the outcome under monopoly.

[3] Jullien *et al.* (2007) study whether the single crossing property holds in the general monopolistic screening model with moral hazard and where agents differ on their risk preference. On the role of this property in a competitive insurance market with the same informational assumptions and moral hazard, see De Donder and Hindriks (2009).

out, our insurance problem does not lend itself to be solved by the techniques proposed there because it lacks separability and because the dimensions of the contract space falls short of the dimensions of the type space.[4] Moreover, in this market, the insurer does not directly care about the client's risk aversion. Indeed, unlike the size of the average loss, risk aversion does not in itself determine the profitability of a *given* insurance contract. However, risk preferences do determine the rents that an individual derives from insurance, and since part of these rents are captured by the monopolist, risk preferences will determine the profitability of attracting that individual.[5]

In the analysis of the competitive case, the literature has usually made the assumption that firms offer a single contract each. This *de facto* means that the main results are driven by the lack of order between what we refer to as "intermediate types", that is, those whose indifference curves cross twice. This explains why some authors only consider these intermediate types – see, for instance, Wambach (2000). On the other hand, Smart (2000) and Villeneuve (2005) do consider the full set of types: risk averse high risks, risk averse low risks, risk tolerant high risks, and risk tolerant low risks. However, they still maintain the assumption that each company offers a single contract. In contrast, in a monopolistic setting like ours, such a restriction would render the analysis trivial and utterly unrealistic. By assuming that the monopolist offers a menu of contracts, the relative proportion of the non-intermediate types will play a role that is as crucial as the non-single crossing of intermediate types' indifference curves. Hence, the problem of the failure of the single crossing condition–brought about by the intermediate types– is compounded in the monopolistic setting by the necessity of dealing with the non-intermediate types in the design of the optimal menu of contracts.

Our main objective is to characterize the optimal menu of contracts aimed at attracting the full set of types, with emphasis on whether bunching of some types occurs. We then address two issues that have received a lot of attention lately. The first one is methodological. In testing for the presence of

[4]This difficulty is also present in Armstrong (1999) and Dana (1993). These works deal with a planner (the less-informed party) regulating a firm (the more informed party), and this implies that both dimensions of firm heterogeneity become common values, in the sense explained in the next footnote. In our setting, risk aversion is instead a private value.

[5]Chiappori *et al.* (2006), point out that risk attitude constitutes "private-value", so that heterogeneity in this dimension alone would become irrelevant in the context of perfect competition. They also point out that, even in the context of competition, preference heterogeneity does compound with risk heterogeneity into a more complex problem.

adverse selection in insurance markets, the question is whether the absence of significative positive correlation between risk and coverage (or the presence of significative negative correlation) should be taken as indicative of the absence of adverse selection. Chiappori *et al.* (2006) derive the testable prediction that in a *competitive* insurance market with asymmetric information, the observable risk should be positively correlated with coverage. We show under which conditions this result goes through in our monopolistic setting, and when it doesn't. In this sense, our results corroborate the role of the perfect competition assumption for the Chiappori *et al.* (2006) result.[6] Our analysis also adds the combination of market power with preference heterogeneity to the list of possible explanations for the lack of evidence supporting the existence of adverse selection. Other explanations in the (growing) list are (i) endogenous heterogeneity in risks due to moral hazard (see, e.g., Cutler, Finkelstein and McGarry, 2008 for evidence on moral hazard); (ii) endogenous wealth heterogeneity (Netzer and Scheuer 2007); and (iii) assuming that it is the insurer who has privileged information on risks (Villeneuve, 2005)..

The second issue is a recent policy debate on the pros and cons of gender discrimination in insurance which arose after the European Commission came with a proposal to "implement the principle of equal treatment between women and men in the access to and the supply of goods and services" (Commission of the European Communities, 2003). A ban on gender discrimination would surely affect the insurance sector, because of the common practice to differentiate premia (and other components of the insurance contract) according to gender when underwriting life, health and car accident risks. Regarding life insurance, for example, it is argued that if one controls for lifestyle, environmental factors, and social class, "the difference in average life expectancy between men and women lies between zero and two years" and therefore that "the practice of insurers to use sex as a determining factor in the evaluation of risk is based on ease of use rather than real value as a guide to life expectancy." (Commission of the European Communities, 2003: 6) Not surprisingly, European insurer carriers have reacted fiercely to the proposed ban, arguing that removing gender would weaken their ability to assess risk and that gender-neutral calculation would increase the premia for many of their products, especially for women (Financial Times, November 3, 2003, p. 2). We show that even if gender only provides information on an individual's risk aversion (i.e., no information on underlying risk–as claimed

[6] In fact, Chiappori et al. (2006) propose a local argument indicating that negative correlation between risk and coverage could appear in the case of monopoly.

by the European Commission), then allowing the monopolist to condition the terms of the insurance contract on gender may be Pareto improving. We provide sufficient conditions for such an improvement to arise.

From a technical point of view, we have taken a new approach to the analysis of screening insurance takers that simplifies the problem and is at the same time, we believe, quite appealing. Rather than following the standard set up where the individual faces the possibility of a single monetary loss, we assume that the loss is normally distributed and that agents differ in the expected loss, which can be high or low. If the insurance indemnity is linear in the loss, as is the case under a reimbursement insurance scheme with a constant coinsurance rate, final income will be normally distributed as well. Endowing agents with a utility function that displays constant absolute risk aversion (r), which also can be high or low, their preferences over uncertain income prospects can be represented as

$$\mathrm{E}(final\ income) - \frac{r}{2}\mathrm{var}(final\ income).$$

An important consequence of this approach is that preferences over insurance contracts become quasi-linear in the insurance premium and therefore in the information rent. Readers familiar with contract theory will acknowledge the usefulness of linearity in the information rent in writing down the incentive compatibility constraints. An additional advantage of mean-variance preferences is that it allows for an explicit characterization of the optimal menu of contracts.

The limitations of our approach follow immediately from these assumptions. We do not consider insurance contracts with either a deductible or a cap since that would destroy the normality of net income. Secondly, the normality assumption implies a positive likelihood of negative losses, although this problem may be of second order by considering sufficiently high means and/or low variances for the loss. Perhaps the most important objection is that we have no skewness in the loss distribution, and in particular no strictly positive probability mass for a zero loss. Still we feel these are minor blemishes when compared to the considerable advantages the approach offers for characterizing the solution to a two-dimensional screening problem. To economize on space, our characterization is restricted to three benchmark cases with respect to the correlation between risk aversion and expected loss: (a) perfect positive correlation, (b) perfect negative correlation, and (c) zero correlation. It is worth mentioning here that we show that it is possible to obtain a negative correlation between risk and coverage even in the last case.

The remainder of the paper is organized as follows. In section 2, we model the preferences of insurance takers and specify reimbursement contracts. In section 3 we set up the problem faced by a monopolistic insurer. Section 4 characterizes the optimal menu of contracts when the insurees only differ in risk size or risk aversion, as well as the case of perfect positive correlation. In section 5 we discuss six regimes (contract menus) that may be optimal. For each regime, we characterize the optimal set of coinsurance rates. Next, in section 6, we then ask which regime is dominating for which part of the parameter space. In section 7, we interpret the testable prediction of Chiappori et al (2006) in the light of our results. In section 8 we trace out the consequences of allowing the monopolistic insurer to gender discriminate. Section 9 concludes.

2 Insurance takers and reimbursement contracts

Insurance takers

We assume that individuals are endowed with initial wealth e and a negative exponential vNM utility function defined on final wealth y: $u(y) = -\exp(-ry)$, where $r > 0$ is the (constant) degree of absolute risk aversion. Initial wealth is subject to a random loss ℓ that follows a normal distribution with mean μ and variance σ^2.

Agents have access to reimbursement insurance. A typical reimbursement contract pays out a compensation of β per Euro loss, in return for a premium P. *Ex post*, final wealth is then given by

$$y = e - P - (1 - \beta)\ell, \tag{1}$$

which *ex ante* is also normally distributed. For convenience, we denote by c the coinsurance rate, i.e., $c \overset{\text{def}}{=} 1 - \beta$, and express contracts as pairs of coinsurance rate and premium. We denote contracts by $C = (c, P)$.

It is well known that under the assumptions made, the expected utility of the agent is representable by the mean-variance function $U = \mathrm{E}(y) - \frac{r}{2}\mathrm{var}(y)$. By replacing the mean and variance of final wealth, expected utility is given by

$$U = e - P - c\mu - \frac{r}{2}c^2\sigma^2. \tag{2}$$

From now on, we write $\nu \stackrel{\text{def}}{=} r\sigma^2$, and assume that this product can be either high or low, and likewise for the expected loss: $\mu \in \{\mu_L, \mu_H\}$ and $\nu \in \{\nu_L, \nu_H\}$,where $\mu_L < \mu_H$ and $\nu_L < \nu_H$. The model can thus be interpreted in two ways: either people are equally risk averse but their losses have different variances, or the loss variance is identical but people have different degrees of risk aversion. Throughout, we will stick to the second interpretation.

A person with characteristics (μ_i, ν_j) is said to be of type ij. The share of ij people in the population is given by α_{ij} ($i, j = H, L$, $\sum_{i,j}\alpha_{ij} = 1$). We denote by $\alpha_{k\cdot}$ the fraction of people with expected loss μ_k ($\alpha_{k\cdot} = \alpha_{kL} + \alpha_{kH}$); likewise $\alpha_{\cdot k}$ is the fraction of people with perceived variance ν_k ($\alpha_{\cdot k} = \alpha_{Lk} + \alpha_{Hk}$).

Incentive compatible contracts

When a person of type ij ($i, j \in \{H, L\}$) signs the contract $\mathbf{C} = (c, P)$, her expected utility, is

$$U^{ij}(c, P) \stackrel{\text{def}}{=} e - P - c\mu_i - \frac{1}{2}c^2\nu_j. \tag{3}$$

If instead she decides to remain uninsured, her expected utility becomes $e - \mu_i - \frac{1}{2}\nu_j$, which is of course equivalent to accepting the contract $(c, P) = (1, 0)$, where the agent bears the full loss but pays no premium. The utility *rent* that the agent enjoys from contract (c, P) is then

$$R_{ij}(c, P) \stackrel{\text{def}}{=} U^{ij}(c, P) - U^{ij}(1, 0) = -P + (1 - c)\mu_i + \frac{1}{2}[1 - c^2]\nu_j. \tag{4}$$

Hence, the rent decreases with the coinsurance rate both via the expected loss and via the perceived variance (if $c > 0$).

The marginal willingness to pay for a slightly lower coinsurance rate c is thus

$$MWP^{ij}(c) \stackrel{\text{def}}{=} -\frac{\mathrm{d}P}{\mathrm{d}c}|_{\mathrm{d}U^{ij}=0} = \mu_i + c\nu_j, \tag{5}$$

and increases in c: $\frac{\mathrm{d}MWP^{ij}(c)}{\mathrm{d}c} = \nu_j > 0$.

Indifference curves in the contract space (c, P) are thus concave in c, and downward-sloping for non-negative coinsurance rates. Also, individuals with a higher expected loss and/or a higher risk aversion have a higher marginal willingness to pay. The figure below illustrates the indifference curve that passes through the no-insurance point $N = (1, 0)$. Since the slope of the indifference curve when it passes the P-axis is μ, it is easy to decompose the total willingness to pay for full insurance into the expected loss and the risk premium $\nu/2$.

Figure 1 here.

When agent ij signs a contract intended for agent kl, the rent that the former receives is given by

$$R_{ij}(c_{kl}, P_{kl}) = -P_{kl} + (1 - c_{kl})\mu_i + \frac{1}{2}(1 - c_{kl}^2)\nu_j. \tag{6}$$

It is useful to define the following function:

$$\delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l) \overset{\text{def}}{=} (1 - c_{kl})(\mu_i - \mu_k) + \frac{1}{2}(1 - c_{kl}^2)(\nu_j - \nu_l). \tag{7}$$

Suppose now that type kl is truthful and gets rent $R_{kl}(c_{kl}, P_{kl})$. Which rent does ij obtain when choosing the contract for kl? Using (4) and (7), the answer is given by

$$R_{ij}(c_{kl}, P_{kl}) \overset{\text{def}}{=} R_{kl}(c_{kl}, P_{kl}) + \delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l), \tag{8}$$

i.e., δ is the extra rent that ij earns from kl's contract. A marginal increase in the coinsurance rate for kl, $\mathrm{d}c_{kl} > 0$, has then the following effect on the rent for the mimicker ij:

$$\frac{\partial R_{ij}(c_{kl}, P_{kl})}{\partial c_{kl}}|_{\mathrm{d}R_{kl}=0} = \frac{\partial}{\partial c_{kl}}\delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l) = -(\mu_i - \mu_k) - c_{kl}(\nu_j - \nu_l). \tag{9}$$

Thus the rent for ij goes down to the extent that (i) kls premium goes down with less than the increase in the mimicker's copayment, and (ii) the compensation that kl gets (in the form of a lower premium) for an increased exposure to risk is less than the compensation that the mimicker requires. This explains why increasing a coinsurance rate for some type will lower the rents of all those mimicking (and the mimickers of these mimickers) that have a higher risk, and it will increase the rent of all those mimicking (and the mimickers of these mimickers) that have a lower risk aversion.

From now on we simply write R_{ij} for $R_{ij}(c_{ij}, P_{ij})$ $(i, j = L, H)$. Self-selection between contracts (c_{ij}, P_{ij}) and (c_{kl}, P_{kl}) then requires that

$$R_{ij} \geq R_{kl} + \delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l),$$
$$R_{kl} \geq R_{ij} + \delta(c_{ij}, \mu_k - \mu_i, \nu_l - \nu_j),$$

which taken together imply $0 \geq \delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l) + \delta(c_{ij}, \mu_k - \mu_i, \nu_l - \nu_j)$, or, using (7),

$$\int_{c_{kl}}^{c_{ij}} [(\mu_i - \mu_k) + c(\nu_j - \nu_l)]\, dc \leq 0.$$

A necessary condition for incentive compatibility between contracts Hj and Lj $(j = H, L)$ is that

$$\int_{c_{Lj}}^{c_{Hj}} \Delta\mu dc \leq 0 \Longleftrightarrow c_{Hj} \leq c_{Lj}, \tag{10}$$

with $\Delta\mu \stackrel{\text{def}}{=} \mu_H - \mu_L > 0$. Similarly, incentive compatibility between contracts iH and iL $(i = H, L)$ requires that

$$\int_{c_{iL}}^{c_{iH}} c\Delta\nu dc \leq 0 \Longleftrightarrow c_{iH} \leq c_{iL}, \tag{11}$$

with $\Delta\nu \stackrel{\text{def}}{=} \nu_H - \nu_L$ and where it is assumed that $c \geq 0$ (on which more below).

The double dimensionality leads in general to double crossing of the indifference curves of types HL and LH. Solving $MWP^{HL}(c) = MWP^{LH}(c)$ for c yields $c = \frac{\Delta\mu}{\Delta\nu}$. I.e. in the (c, P)-space, the locus of tangency points between HL's and LH's indifference curves is a vertical line at $\frac{\Delta\mu}{\Delta\nu}$. For lower coinsurance rates, HL's indifference curve crosses that of LH downwards from above, while for higher rates, this happens from below. It is useful to note that if the crossing occurs at some point $(c.P)$ to the left of $\frac{\Delta\mu}{\Delta\nu}$, then the second crossing lies at the same distance d to the right of $\frac{\Delta\mu}{\Delta\nu}$–see the figure below.[7]

Figure 2 here.

Finally, we introduce two crucial variables for characterizing the profit maximizing set of contracts:

$$D \stackrel{\text{def}}{=} \frac{\Delta\mu}{\nu_L} \in (0, \infty) \text{ and } x \stackrel{\text{def}}{=} \frac{\nu_L}{\nu_H} \in (0, 1].$$

[7]The first crossing allows us to equate $e - P - c\mu_i - \frac{1}{2}c^2\nu_j$ to a fixed U^{ij}. We do the same for an ji agent. Then we solve for P in both equations. Since P is common, we can write $e - c\mu_L - \frac{1}{2}c^2\nu_H - U^{LH} = e - c\mu_H - \frac{1}{2}c^2\nu_L - U^{HL}$. Solving this last expression for c yields two solutions that can be expressed as $\frac{\Delta\mu}{\Delta\nu}(1 - \varepsilon)$ and $\frac{\Delta\mu}{\Delta\nu}(1 + \varepsilon)$, as claimed, and where $\varepsilon = \sqrt{1 + \frac{2\Delta\nu(U^{HL} - U^{LH})}{\Delta\mu^2}}$.

The ratio D measures in a unit free fashion the difference in risk between the two types.[8] The ratio x measures the degree of similarity along the risk aversion dimension. The locus of tangency points is therefore located at $D\frac{x}{1-x}$ so that for sufficiently small x the tangency of the intermediate types' indifference occurs at a coinsurance rate below 1.

3 The insurance company

We consider a single, risk-neutral insurer with monopoly power on the market for reimbursement contracts. Her expected profits when an agent of type ij has accepted a reimbursement contract (c, P) is given by

$$\pi^{ij}(c, P) = P - \beta\mu_i = P - (1-c)\mu_i. \tag{12}$$

Therefore, the iso-profit associated to type ij has slope $-\mu_i$ in the contract space (c, P).

With *full* information, the monopolist will provide ij with full insurance ($c_{ij} = 0$) at a premium that sets her rent equal to zero. Hence using (4), $P_{ij} = \mu_i + \frac{1}{2}\nu_j$. This yields a per capita payoff equal to $\pi = \frac{1}{2}\nu_j$. The tangency line in Figure 2 thus corresponds to the maximal iso-profit line, and the profit which the insurer makes can be read of from the dashed vertical axis on the right hand side. Under full information, the insurer can extract the entire risk premium $\nu/2$. In the remainder, we will characterize the optimal coinsurance rates and the optimal rents. The corresponding premia can then found with the help of (4).

Using (12), the insurer's total profit is equal to $\sum_{i,j}\alpha_{ij}\Pi^{ij}(c_{ij}, P_{ij})$. Using (4) and (12)–both evaluated at (c_{ij}, P_{ij})–and recalling that we write R_{ij} for $R_{ij}(c_{ij}, P_{ij})$ (i.e., type ij's rent when truthful), we can express the insurer's total profit as

$$\sum_{i,j}\alpha_{ij}\left[\frac{1}{2}[1-c_{ij}^2]\nu_j - R_{ij}\right]. \tag{13}$$

This objective function is to be maximized with respect to (c_{ij}, R_{ij}) ($ij = H, L$), subject to the usual voluntary participation and incentive compatibility constraints.

As in most of the literature (Picard, 2000), to these constraints we add two additional sets of constraints that are needed to avoid false claims. If a

[8] Since ν_L is twice the risk premium (RP_L) an individual is willing to pay for full insurance, $D = \frac{\Delta\mu}{2RP_L}$.

coinsurance rate is negative, the insurer refunds losses for more than 100%, and the insuree will obviously have a strong incentive to overstate the size of the loss. On the other hand, if a coinsurance rate is larger than one, the agent will have to be paid to accept such a contract (that is, a negative premium). Once the agent has accepted the insurance, he would have to pay the insurer on top of bearing the loss once it occurs. It is clear that he would have strong incentives to understate the size of the loss (or even hide the loss altogether). Hence we constrain coinsurance rates to lie in the interval [0, 1].

The monopolist thus solves the following problem:

$$\max_{\{c_{ij},R_{ij}\}} \sum_{i,j} \alpha_{ij} \left[\frac{1}{2}[1 - c_{ij}^2]\nu_j - R_{ij} \right], \text{ s.t.} \tag{14}$$

$$R_{ij} \geq 0 \qquad (i, j, = H, L) \tag{15}$$

$$R_{ij} \geq R_{kl} + \delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l) \quad (i, j, k, l, = H, L) \tag{16}$$

$$0 \leq c_{ij} \leq 1 \qquad (i, j, = H, L) \tag{17}$$

The first set of constraints ensures voluntary participation, while the second ensures that all types self-select. The third set consists of the (reduced form) *ex ante* and *ex post* moral hazard constraints.

The following theorem provides the usual result of no-distortion-at-the-top (full insurance for the HH type) and no-rents-at-the-bottom (the proof is given in Appendix A).

Theorem 1 *At the optimum solution,(i)* $c_{HH} = 0$ *and (ii)* $R_{LL} = 0$.

Before characterising the rest of the solution to the bi-dimensional screening problem, it is useful to first consider the single dimension case.

4 One-dimensional screening

There are three instances where the screening becomes unidimensional. In the first one, all agents have the same risk aversion, i.e., let $\nu_H = \nu_L = \nu$. This is the standard monopoly problem with just two types where insurees either bear a low or a high expected loss. The type distribution can be described by a single parameter $\alpha_{H\cdot}$, the proportion of high risks in the population. We have the following theorem (with proof in Appendix A).

Theorem 2 *When all agents have the same risk aversion, the optimal menu has* $c_{H\cdot} = 0$ *and* $c_{L\cdot} = \min\{\frac{\Delta\mu}{\nu}\frac{\alpha_{H\cdot}}{1-\alpha_{H\cdot}}, 1\}$.

The full insurance contract giving L zero rent will be selected by H as well. At zero coinsurance rate, the slope of Hs indifference curve is steeper than that of L. If the insurer increases $c_{L.}$ above zero, this will create a second order reduction in profit from L, but a first order gain in profit from H because the latter can be charged a strictly higher premium (for full insurance). Hence, it pays off to start distorting Ls contract. The optimal coinsurance rate balances the gain in profit from H $(\alpha_{H.}\Delta\mu)$ with the loss in profits from L $((1-\alpha_{H.})\nu)$. Notice that may pays off to exclude type L whenever $\alpha_{H.} \geq 1/(1+\frac{\Delta\mu}{\nu})$, i.e., whenever the proportion of low loss people is small enough–as expected.

The second instance where the screening problem becomes unidimensional is when individuals differ in risk aversion only. Let $\alpha_{.H}$ instead be the proportion of highly risk averse types, i.e., those with $\nu = \nu_H (> \nu_L)$. We have the following theorem (with proof in Appendix A):

Theorem 3 *When all agents face the same expected loss, the optimal menu has* $c_{.H} = 0$ *and* $c_{.L} = 0$ *if* $x > \alpha_{.H}$ *and* 1 *otherwise.*

This result is less standard. With only differences in risk aversion, the optimal solution is always corner. Either the low type is excluded or he receives full insurance. In other words, for the insurer, distorting the contract intended for the less risk averse type (L) is either uniformly beneficial (if either the proportion of the latter types is small and/or the difference in risk aversion is small) or uniformly harmful.

The reason for this knife edge solution is that, unlike in the different risk scenario, at a zero coinsurance rate both Hs and Ls indifference curve are *tangent* to one another. Hence distorting Ls contract by rasing the coinsurance rate now results in a *second order* gain in profit from H. Hence it is the second order condition that determines whether $c_{.L} = 0$ is a local maximum or minimum.

The final instance is where risk level and risk aversion are perfectly correlated. As it transpires from (5), we have that $MWP^{HH}(c) > MWP^{LL}(c)$ for any c. The two types are therefore once again unambiguously ordered.

Theorem 4 *When the two characteristics are perfectly correlated, the optimal menu has* $c_{HH} = 0$ *and* $c_{LL} = \min\{D\frac{\alpha_{HH}x}{x-\alpha_{HH}}, 1\}$ *if* $x > \alpha_{HH}$ *and* 1 *otherwise.*

We now turn to the two-dimensional screening problem.

5 Two-dimensional screening

From now on, we let people not only differ in their risk level, but also in their risk aversion. The insurance company then faces a bivariate probability distribution of types:

	ν_L	ν_H	
μ_L	α_{LL}	α_{LH}	$\alpha_{L\cdot}$
μ_H	α_{HL}	α_{HH}	$\alpha_{H\cdot}$
	$\alpha_{\cdot L}$	$\alpha_{\cdot H}$	1

The correlation between risk (μ) and risk aversion (ν) plays an important role in the analysis. It is given by

$$\text{corr}(\mu,\nu) = \frac{E(\mu - E\mu)(\nu - E\nu)}{\sigma_\mu \sigma_\nu} = \frac{\alpha_{HH}\alpha_{LL} - \alpha_{LH}\alpha_{HL}}{\sqrt{\alpha_{L\cdot}\alpha_{H\cdot}}\sqrt{\alpha_{\cdot L}\alpha_{\cdot H}}}$$

In the remainder, we let ρ represent the numerator of the correlation expression, viz. $\rho \stackrel{\text{def}}{=} \alpha_{HH}\alpha_{LL} - \alpha_{LH}\alpha_{HL}$.

To parameterise the distribution of types, we will use ρ, $\alpha_{H\cdot}$ and α_{HH} ($\leq \alpha_{H\cdot}$), and have the remaining fractions determined by

$$\alpha_{HL} = \alpha_{H\cdot} - \alpha_{HH} \tag{18}$$

$$\alpha_{LH} = \alpha_{HH}\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} - \frac{\rho}{\alpha_{H\cdot}}, \text{ and} \tag{19}$$

$$\alpha_{LL} = (\alpha_{H\cdot} - \alpha_{HH})\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} + \frac{\rho}{\alpha_{H\cdot}}. \tag{20}$$

Non-negativity of α_{LH} and α_{LL} requires that $-\alpha_{HL}(1-\alpha_{H\cdot}) \leq \rho \leq \alpha_{HH}(1-\alpha_{H\cdot})$.

In our analysis, we will focus on two special cases: *perfect negative correlation* ($\alpha_{HH} = \alpha_{LL} = 0$, and $\rho = -\alpha_{HL}(1-\alpha_{HL})$ so that corr $= -1$) and *independence* ($\rho = 0$). The reason for this restricted focus is mainly to economise on space. In both these two cases, the typology of the equilibrium set of contracts is already intricate and our study of intermediate forms of correlation has revealed that the main qualitative features of the contract menus are preserved.

The monotonicity conditions (10) and (11) imply that there are only two possible orderings of coinsurance rates:

$$\text{Order 1: } 0 = c_{HH} \leq c_{HL} \leq c_{LH} \leq c_{LL} \leq 1, \tag{21}$$

$$\text{Order 2: } 0 = c_{HH} \leq c_{LH} \leq c_{HL} \leq c_{LL} \leq 1. \tag{22}$$

Lemma 1 *If order 1 applies with $c_{HH} < c_{LH}$, it is optimal to pool HL with HH iff $x > \frac{\alpha_{HH}}{\alpha_{H\cdot}}$. If order 2 applies with $c_{HH} < c_{HL}$, the optimal coinsurance rate for LH is given by* $\min\{\frac{\Delta\mu}{\nu_H}\frac{\alpha_{HH}}{\alpha_{LH}}, c_{HL}\}$.

This result is intuitive. With order 1, the only type that may envy the contract for HL is HH. Thus, the choice of c_{HL} is only governed by weighing the profits from these two types. Since they have the same risk size, we may apply Theorem 3 on this sub group. Likewise, if order 2 applies, the choice of c_{LH} should balance the increase in premium income from HH with the loss in profits from LH. Since these types have the same risk aversion, Theorem 2 applies.

In Appendix B, we show that at most five regimes may solve the monopolist's problem. By a regime we mean a menu of contracts satisfying certain pooling or separation properties. The five regimes are:

- $\{HH, HL\}, \{LH, LL\}$: **Regime A**
- $\{HH, HL\}, \{LH\}, \{LL\}$ with $c_{LH} + c_{LL} < 2\frac{\Delta\mu}{\Delta\nu}$ and $c_{LL} = 1$: **Regime M**
- $\{HH, HL\}, \{LH\}, \{LL\}$ with $c_{LH} + c_{LL} = 2\frac{\Delta\mu}{\Delta\nu}$: **Regime B**
- $\{HH, HL, LH\}, \{LL\}$ with $c_{LH} + c_{LL} > 2\frac{\Delta\mu}{\Delta\nu}$: **Regime C**
- $\{HH\}, \{LH\}, \{HL, LL\}$: **Regime E**

Thus **Regime A**, e.g., is where HH and HL are pooled, and so are LH and LL. Note that full separation is *never* optimal (cf Lemma B.14 in Appendix B), and that **Regime E** distinguishes itself from the others in that order 2 applies.

In Appendix C, we derive for each regime the optimal values of the contract terms as a function of the model's parameters, as well as the ensuing maximal profit. Typically, some regimes will take the shape of other regimes once the relative measure for risk aversion, x, falls outside a certain range. In the main text, we will limit our description for each regime to those parameter ranges where a regime may have a chance of dominating any of the other regimes (a full description is given in Appendix C). This is the first step which is done in the remainder of this section. In a second step we will explain when it pays for the insurer to move from one regime to another (Section 6).

- **Regime A**

This regime pools the high risk types at full insurance, and the low risk types at high, but partial, insurance. The figure below illustrates (in this figure and those that follow, solid/dashed indifference curves refer to high/low risk aversion, while bold/thin indifference curves refer to high/low risks).

Figure 3 here

Denoting the coinsurance rate for the low risk types as $c^A_{L\cdot}$, **Regime A** is described by

$$c^A_{L\cdot} = \min\{D\frac{\alpha_{H\cdot}}{1-\alpha_{H\cdot}}, 1\}, \tag{23}$$

$$c^A_{HH} = c^A_{HL} = 0. \tag{24}$$

This policy corresponds to the one when people differ only in the risk dimension (Theorem 1). Below, we will argue that **Regime A** will be optimal for when x is sufficiently large, more specifically when $x \geq 1 - \alpha_{LL}$. From now one, we make

Assumption N $\quad D < \frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}}$.

This assumption rules out exclusion of the low risk types when people are almost equally risk averse. Since $1 - \alpha_{LL} > \alpha_{H\cdot}$, it follows that when **Regime A** applies the pooling of the low risk types happens at a "low" coinsurance rate, viz., $c^A_{L\cdot} < D\frac{x}{1-x} (= \frac{\Delta\mu}{\Delta\nu})$.

- **Regime M**

This regime pools the high risk types at full insurance, insures LH at a small but positive coinsurance rate, and excludes LL. The figure below illustrates.

Figure 4 here

Defining $\overline{x}^M(D) \stackrel{\text{def}}{=} \frac{1-\alpha_{LL}}{\alpha_{H\cdot}(1+D)}$, the optimal values for the coinsurance rates are given by

$$c^M_{LH} = D\frac{\alpha_{H\cdot}.x}{\alpha_{H\cdot}.(1-x)+\alpha_{LH}}, \tag{25}$$

$$c^M_{LL} = 1,\ c^M_{HH} = c^M_{HL} = 0, \tag{26}$$

and the parameters x and D need to satisfy

$$x \leq 1 - \alpha_{LL}, \tag{27}$$

$$x \geq x_{BM}(\alpha_{H\cdot}, \alpha_{LH}, D), ^{9}$$

$$D \geq \underline{D}^M \overset{\text{def}}{=} \frac{(1-\alpha_{H}\cdot)\alpha_L L}{\alpha_L H + (1-\alpha_{H}\cdot)(1-\alpha_L L)} \tag{28}$$

Condition (27) ensures that it is not optimal to pool LL with LH; condition (**??**) ensures that the right hand crossing happens at a coinsurance rate above 1 (formally that $2\frac{\Delta\mu}{\Delta\nu} - c_{LH}^M \geq 1$); while condition (28) guarantees that the previous two conditions are compatible with each other.

Two remarks are in order. First, we can rule out the corner solution $c_{LH}^M = 1$. Indeed, this would require that $x \geq \frac{1-\alpha_{LL}}{\alpha_{H\cdot}(1+D)}$. But by Assumption N, $\alpha_{H\cdot}(1+D) < 1$, making the *rhs* of the previous inequality to exceed 1. Second, with perfect negative correlation $\underline{D}^M = 0$ and condition (28) is automatically fulfilled.

- **Regime B**

Like in **Regimes A** and **M**, the high risk types are pooled at full insurance. But the low risk types are separated on each side of the double crossing of HLs and LHs indifference curve. That is, they satisfy

$$c_{LH} + c_{LL} \equiv 2\frac{\Delta\mu}{\Delta\nu}. \tag{29}$$

We may distinguish between **Regime Bf** and **Regime Bp**, depending on whether LH gets full insurance ($c_{LH} = 0$) or partial insurance ($c_{LH} > 0$), respectively. For the latter regime we can also make a distinction on whether LL people are excluded (**BpX**: $c_{LL} = 1$) or included (**BpI**: $c_{LL} < 1$) from insurance. These are illustrated in the three panels of the figure below.

Figure 5a, 5b, 5c here

Defining $\overline{x}^{Bp}(D) \overset{\text{def}}{=} \frac{1-\alpha_{H\cdot}-D(\alpha_{H\cdot}+\alpha_{LH})}{1-\alpha_{H\cdot}-\alpha_{H\cdot}D}$ and $\overline{D}^{Bp} \overset{\text{def}}{=} \frac{\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$, **Regime B** may be summarised as follows. If $D \leq \overline{D}^{Bp}$

$$c_{LL}^B = \begin{cases} 1 & \text{if } x \geq \overline{x}^{Bp}(D) & \textbf{(BpX)} \\ D\frac{2\alpha_{LH}+\alpha_{H\cdot}(1-x)}{(1-\alpha_{H\cdot})(1-x)} & \text{if } \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} \leq x < \overline{x}^{Bp}(D) & \textbf{(BpI)} \\ 2D\frac{x}{1-x} & x < \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} & \textbf{(Bf)} \end{cases},$$

and if $D > \overline{D}^{Bp}$,

$$c_{LL}^{B} = \begin{cases} 1 & \text{if } x \geq \frac{1}{1+2D} \quad \textbf{(BpX)} \\ 2D\frac{x}{1-x} & \text{if } x < \frac{1}{1+2D} \quad \textbf{(Bf)} \end{cases} .$$

Moreover, $c_{HH}^{B} = c_{HL}^{B} = 0$ and $c_{LH}^{B} = 2D\frac{x}{1-x} - c_{LL}^{B}$.

- **Regime C**

Regime C is one where everybody is given full insurance, except for the LL people who face a very high coinsurance rate (**CI**) or are even excluded (**CX**):

$$c_{LL} \geq 2\frac{\Delta\mu}{\Delta\nu}.$$

This regime is illustrated in the figure below.[10]

Figure 6 here

Regime C thus balances a high premium income from the 'upper' types with the loss in profit from distorting LL's contract. Intuitively, with few LL people around, such distortion is attractive, and with very few of them around, it is even optimal to exclude them all together.

More formally, letting $\overline{D}_C \overset{\text{def}}{=} \frac{\alpha_{LL}}{1-\alpha_{LL}}$, we can summarise **Regime C** as follows. If $D < \overline{D}_C$,

$$\begin{aligned} c_{LL}^{C} &= D\frac{1-\alpha_{LL}}{\alpha_{LL}} \text{ if } x \leq \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \quad \textbf{(CI)} \\ &= 2D\frac{x}{1-x} \text{ if } x > \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \quad \textbf{(CI)} \end{aligned}$$

[10] In the appendix, we define **Regime C** more generally as the solution to

$$\max_{c_I, c_{LL}} \pi_{tot} \text{ s.t.}$$
$$0 \leq c_I \leq c_{LL} \quad (\lambda_1)$$
$$2\frac{\Delta\mu}{\Delta\nu} - c_{LL} \leq c_I \quad (\lambda_2)$$

where c_I is the common coinsurance rate for the intermediate types, LH and HL. Thus, it is not *a priori* imposed that $c_I = 0$. **Regime B** is a special case when constraint (λ_2) is binding. For expository reasons, we discuss here in the main text **Regime C** in the strict sense ($c_I = 0$) in order to distinguish it from **Regime B**.

Thus for $x \geq \frac{1-\alpha_{LL}}{1+\alpha_{LL}}$, it is optimal to set c_{LL} equal to $2\frac{\Delta\mu}{\Delta\nu}$, and we are back in **Regime Bf** (i.e. it is not attractive to distort LL's contract with a coinsurance rate above $2\frac{\Delta\mu}{\Delta\nu}$).

If $D > \overline{D}_C$, and $x \leq \frac{1}{1+2D}$, it becomes optimal to exclude LL:

$$c_{LL}^C = 1. \quad (\mathbf{CX})$$

The other types are fully insured: $c_{HH}^C = c_{HL}^C = c_{LH}^C = 0$. Note that if $D > \overline{D}_C$ and $x > \frac{1}{1+2D}$, **Regime C** is no longer incentive compatible as it will make the LH type to strictly prefer no insurance rather than full insurance. The only way of restoring incentive compatibility is to offer LH partial insurance at $2\frac{\Delta\mu}{\Delta\nu} - 1$. Then we are back in **Regime BpX**.

- **Regime E**

A common feature of all regimes discussed so far is that order 1 applies ($c_{HL} \leq c_{LH}$). In **Regime E**, the opposite is true: HLs contract is now severely distorted in order to make room for increasing the distortion on LH; this in turn allows the insurer to extract more rent from the HH people. The figure below illustrates.

Figure 7 here

Denoting the common coinsurance rate for HL and LL as $c_{\cdot L}$, and defining $\underline{x}_E(D) \overset{\text{def}}{=} \frac{\alpha_{\cdot H}}{1-D\alpha_{HL}}$ the optimal coinsurance rates are

$$c_{\cdot L}^E = \begin{cases} D\frac{x\alpha_{HL}}{x-\alpha_{\cdot H}} & \text{if} \quad x > \underline{x}_E(D) \quad (\mathbf{EI}) \\ 1 & \quad\quad x \leq \underline{x}_E(D) \quad (\mathbf{EX}) \end{cases}, \tag{30}$$

$$c_{LH}^E = D\frac{\alpha_{HH}x}{\alpha_{LH}}, \tag{31}$$

$$c_{HH}^E = 0. \tag{32}$$

Moreover, we need that

$$2D\frac{x}{1-x} - c_{\cdot L}^E \leq c_{LH}^E \leq c_{\cdot L}^E. \tag{33}$$

The left hand inequality ensures that the coinsurance rate for LH does not get too small for HL to be attracted to LHs contract. This constraint can be ignored when characterising the global optimum. The reason is that if the lower inequality constraint is binding, **Regime E** would have LH and HL at the opposite crossings of the double crossing, and profits could obviously be be increased by pooling HL with LH rather than LL. The right hand inequality in (33) ensures monotonicity. In **Regime EX**, Assumption N ensures that this will never be binding.[11] In **Regime EI**, pooling of LH with the low risk averse types becomes optimal if $D\frac{\alpha_{HH}x}{\alpha_{LH}} \geq D\frac{x\alpha_{HL}}{x-\alpha_{.H}}$, which translates as

$$x \geq \alpha_{.H} + \frac{\alpha_{HL}\alpha_{LH}}{\alpha_{HH}}.$$

With perfect negative correlation, this is violated for any x below 1 (since $\alpha_{HH} \to 0$). In appendix C we show that with zero correlation, the range for x where pooling of the three lower types may be more profitable than **Regime C** is a strict subset of $[0, \alpha_{.H} + \frac{\alpha_{HL}\alpha_{LH}}{\alpha_{HH}}]$ and therefore that this pooling solution can never constitute a global optimum. We can therefore safely ignore (33).

This concludes the discussion of the five regimes. For each regime, we have determined its profit maximising shape. In the next section, we will compare the optimal version across regimes.

6 Comparison of regimes

Having established the optimal coinsurance structure for each regime, we can now ask for which (D, x)-combinations each of the regimes becomes optimal. The precise comparisons are relegated tot the appendix. Here, we will limit ourselves mainly to a graphical presentatition by partitioning the (D, x)-space into subsspaces according to which regime secures the monopolist the highest profit.

Inspection of (30) shows that for small x, it is optimal to exclude the two low risk averse types (HL and LL) in **Regime E**. For the other regimes, one of the risk tolerant types (i.e., HL) continues to buy insurance. But if x gets very small, the willingness to pay for insurance by high risk averse types (HH and LH) gets infinitely larger than that by low risk averse types.

[11] A necessary condition for pooling of LH with the low risk averse types to be optimal in **Regime EX**, is that $x \geq \frac{\alpha_{LH}}{D\alpha_{HH}}$. Using (19), the *rhs* can also be written as $\frac{1}{D}\frac{1-\alpha_{H.}}{\alpha_{H.}} - \frac{1}{D}\frac{\rho}{\alpha_{HH}\alpha_{H.}}$. Condition 1 implies that the first term exceeds one. Non-positive correlation thus ensures that the entire expression exceeds one. Hence, there does not exist an $x \in (0, 1]$ for which $x \geq \frac{\alpha_{LH}}{D\alpha_{HH}}$ holds.

Therefore, it cannot be optimal to keep providing the latter with insurance, as this constrains the premia that can be charged to the former types.

Proposition 1 *As $x \to 0$, the optimal contract menu is defined by regime E.*

Before we consider when the other regimes become optimal, we mention here that because regimes A, M, B, and C all share the same order (order 1), the transition from one regime into an adjacent one takes place in a continuous way in the sense that at least one of the coinsurance rates changes continuously. **Regime E**, on the other hand, makes use of order 2. The move from this regime into an adjacent goes together with a discontinuous behaviour in all the coinsurance rates (except c_{HH} which is always equal to zero). Identification of the border line of regime E is then only possible by comparing the maximal profit functions.

6.1 Independence of characteristics

When there is absence of risk aversion heterogeneity, we know from Theorem 2 that **Regime A** is optimal. By a continuity argument, this is also true for small differences between ν_H and ν_L. Low risk types will be partially insured while high risks get full insurance.

Proposition 2 *Suppose that characteristics are independently distributed. As $x \to 1$, the optimal contract menu is defined by regime A.*

In the appendix we have derived the (x, D)-locus that ensures indifference between **Regimes E** and **C** as well as between **Regimes E** and **B**:

$$\pi_{tot}^{E} \gtrless \pi_{tot}^{C} \Longleftrightarrow x \lesseqgtr x_{EC}(\alpha_{H\cdot}, \alpha_{HH}, D)$$
$$\pi_{tot}^{E} \gtrless \pi_{tot}^{Bf} \Longleftrightarrow x \lesseqgtr x_{EBf}(\alpha_{H\cdot}, \alpha_{HH}, D)$$

We can show that there exists a critical value $f(\alpha_{H\cdot})$ such that

$$\alpha_{HH} < (>) f(\alpha_{H\cdot}) \Longrightarrow x_{EC}(\alpha_{H\cdot}, \alpha_{HH}, D) < (>) x_{EBf}(\alpha_{H\cdot}, \alpha_{HH}, D) < (>) \frac{1-\alpha_{LL}}{1+\alpha_{LL}},$$

so that if $\alpha_{HH} < f(\alpha_{H\cdot})$, **Regime EI** takes over from **Regime CI** if $x < x_{EC}(\alpha_{H\cdot}, \alpha_{HH}, D)$ while if $\alpha_{HH} > f(\alpha_{H\cdot})$, **Regime EI** takes over from

Regime Bf if $x < x_{EBf}(\alpha_{H\cdot}, \alpha_{HH}, D)$.[12] The function $f(\alpha_{H\cdot})$ is displayed in the figure below. Thus if there are few high risk people in the population and if most of them have a high risk aversion, **Regime C** will cease to be optimal.

Figure 8 here.

To fix ideas, suppose now that $\alpha_{HH} < f(\alpha_{H\cdot})$. Then the optimal policy can be summarised by the following figure in the (D, x)-space.

Figure 9 here

Proposition 3 *When the characteristics are independently distributed and* $\alpha_{HH} < f(\alpha_{H\cdot})$ *the choice of menu structure is as displayed in figure 9.*

Recall that D measures the incentive for μ_H-people to mimic μ_L-people, normalised by (twice) the risk premium of the latter. A high coinsurance rate discourages the former group of applying for the contracts intended for the latter, and thus allows to charge them more for full insurance.

On the other hand, x measures the similarity in term of risk aversion in terms of the willingness to pay for (full) insurance. A low similarity warrants a contract menu that screens low from high risk averse consumers. The latter group has indeed a much higher willingness to pay for insurance coverage, and the monopolist takes advantage of this. Such screening is absent in **Regime A** and maximal in **Regime EX** where all risk tolerant people are excluded. The result is a market with only highly risk averse customers, with private information on their expected loss. The standard screening problem then applies again. The figure below describes the optimal coinsurance rates as a function of x, (assuming $D < \underline{D}^M$).

Figure 10 here.

6.2 Perfect negative correlation of characteristics

With perfect negative correlation, only intermediate types are around and $\alpha_{LL} = \alpha_{HH} = 0$. The three vertical lines of figure 9 then coincide with the

[12] Since $x_{EBf}(\alpha_{H\cdot}, \alpha_{HH}, D) < \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$, there is no possibility that **Regime EI** dominates **Regime Bf**.

vertical axis, and the horizontal lines at $1-\alpha_{LL}$, $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$, and $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$ coincide with the horizontal line at 1. The result is that **Regime A** completely vanishes, while the other regimes are–not surprisingly–all characterised by exclusion of LL (since there are no LL people to derive profit from). Comparing the maximal profit for **Regime CX** with that of **Regime EX** shows that

$$\pi_{tot}^{EX} \gtrless \pi_{tot}^{CX} \Longleftrightarrow x \lesseqgtr x_{ECX}(\alpha_{HL}, D) \stackrel{\text{def}}{=} \frac{1-\alpha_{HL}}{1+2D(1-\alpha_{HL})}$$

and since $x_{ECX}(\alpha_{HL}, D) < \frac{1}{1+2D}$ there is no doubt that this is the right comparison. The figure below then summarises the insurance company's optimal contract menus. It ignores Assumption N, and includes the boundary $\frac{1}{\alpha_{HL}(1+D)}$ above which it becomes optimal to exclude LH in **Regime M** (a possibility that Assumption N rules out).

Figure 11 here.

Proposition 4 *With perfect negative correlation of hidden characteristics, the choice of menu structure is as displayed in figure 11.*

Loosely speaking, with perfect negative correlation, a sufficiently large heterogeneity in the risk size dimension (high D) will favour full insurance of HL and exclusion of LH, while the opposite is true for a sufficiently large heterogeneity in the risk aversion dimension (small x).

7 The positive correlation test

Chiappori *et al.* (2006) have shown that a common prediction of any model of a competitive insurance market with asymmetric information is that observable risk should be positively correlated with coverage. Conditional on the competition assumption, their result is very general; it holds for any combination of moral hazard and adverse selection in underlying risk.[13] As

[13] Moral hazard in fact reinforces the positive correlation because individuals enjoying more coverage have less of an incentive to take precautionary behavior, which makes them *observationally* more risky. Of course, one of the econometric issues is that, even after obseving some positive correlation it is hard to disentangle the adverse selection and the moral hazard effects.

it turns out, the empirical evidence on such correlation is somewhat weak, and is even reversed in some markets.[14]

As mentioned in the introduction, there have been several theoretical attempts to provide an explanation for this lack of evidence. One is the so called "cherry picking argument" (Chiappori and Salanié, 2000) or "propitious selection" (Hemenway, 1990), which combines adverse selection in risk preference (but not in the underlying risk) with moral hazard. The argument is that if individuals choose precautionary effort after having purchased some coverage level, then more risk averse individuals will both purchase more coverage and exert more effort, everything else equal. This may then result in a negative correlation between observed risk and coverage.[15] We show here that exogenous risk heterogeneity in a monopolistic market can also result in a negative correlation. Chiappori *et al.* (2006) already argued that in a monopoly, the the sign of the correlation may be turned around. And they do this by starting with a model where only preference heterogeneity exists (cf section 4), and the introduce infinitesimal amount of exogenous risk heterogeneity is introduced with a negative correlation between risk and risk aversion (i.e., the more risk-averse agents have a slightly smaller accident probability). What we now will show is that the positive correlation hypothesis will fail to hold, once **Regime E** applies, even if underlying risk and risk preference are completely independently distributed.

Translated to our setting, the Chiappori *et al.* (2006) proposition reads as follows:

Consider two contracts C_a and C_b that are offered on the market. Suppose that (i) C_a gives more coverage then C_b, i.e., $c_a < c_b$, and (ii) the per capita profit generated by contract a does not exceed that of contract b, $\pi(C_a) \leq \pi(C_b)$. Then (iii) the expected loss to those consumers signing up for contract a should exceed the expected loss of those consumers signing up for contract b, i.e., $\mu(C_a) \geq \mu(C_b)$.

It is easy to see that the property (iii) is satisfied in all regimes except for **Regime E**. In that regime the contract for LH has more coverage than that for the low risk averse people (LL and HL). The positive correlation

[14] The later phenomenon is termed "advantageous selection". It has been observed, for instance, in life insurance (Cawley and Philipson, 1999 and McCarthy and Mitchell, 2003) and in long-term care (Finkelstein and McGarry, 2006).

[15] See for instance Jullien et al. (2007), De Donder and Hindricks (2009), Finkelstein and McGarry (2006).

property would then require that

$$\mu(C_{LH}) = \mu_L > \mu(C_{\cdot L}) = \left(\frac{\alpha_{HL}}{\alpha_{\cdot L}}\mu_H + \frac{\alpha_{LL}}{\alpha_{\cdot L}}\mu_L\right),$$

which is obviously violated. The reason that the property does not work is that condition (ii) is not satisfied: C^E_{LH} generates a higher per capita profit than $C^E_{\cdot L}$ does. This can be seen as follows

$$\begin{aligned}
\pi(C^E_{\cdot L}) &= \frac{\alpha_{HL}}{\alpha_{\cdot L}}\left\{\frac{1}{2}[1-(c^E_{\cdot L})^2]\nu_L - (1-c^E_{\cdot L})\Delta\mu\right\} + \frac{\alpha_{LL}}{\alpha_{\cdot L}}\frac{1}{2}[1-(c^E_{\cdot L})^2]\nu_L \\
&= \frac{1}{2}[1-(c^E_{\cdot L})^2]\nu_L - \frac{\alpha_{HL}}{\alpha_{\cdot L}}(1-c^E_{\cdot L})\Delta\mu \\
\pi(C^E_{LH}) &= \frac{1}{2}[1-(c^E_{LH})^2]\nu_H - \frac{1}{2}[1-(c^E_{LH})^2]\Delta\nu \\
&= \frac{1}{2}[1-(c^E_{LH})^2]\nu_L
\end{aligned}$$

Since $c^E_{LH} < c^E_{\cdot L}$, it follows that $\pi(C_{LH}) > \pi(C_{\cdot L})$, irrespective of which optimal values the coinsurance rates take in **Regime E**.

Proposition 5 *For a sufficiently large heterogeneity in the risk aversion dimension, a positive correlation between coverage and risk size no longer holds in a monopolistic insurance market.*

8 Gender discrimination

In this section, we explore the effects of gender discrimination on market efficiency. Let us write $p(\mu,\nu,g)$ as the likelihood function that an arbitrary insuree has expected loss μ, a risk aversion ν and gender $g \in \{m,w\}$.

A monopolist who is allowed to condition on gender will for each of the two genders g design an optimal contract menu based on the risk aversion ratio x and the probability matrix[16]

$$\begin{pmatrix} p(\mu_L,\nu_L|g) & p(\mu_L,\nu_H|g) \\ p(\mu_H,\nu_L|g) & p(\mu_H,\nu_H|g) \end{pmatrix}.$$

[16]Here, we assume that the support of the distribution of types does *not* vary with the signal. Alternatively, the support could be made dependent on the signal. This, in effect, amounts to assuming that the support consists of more than four (μ,ν)-pairs, some of which have zero probability mass, depending on the observation of the signal.

We now make the assumption that risk aversion is a sufficient statistic for gender w.r.t. expected loss:

Assumption S $\quad p(g|\nu, \mu) = p(g|\nu).$

Assumption S means that if expected loss of a person could be observed, this carries no extra information about the gender of that person on top of the observation of the person's risk aversion. In general, sufficiency is not enough to break the link between gender and expected loss. If female drivers are highly risk averse, and if this attitude leads them to careful driving, then there will still be a connection between gender and expected loss. This last connection is broken by the assumption that expected loss is independently distributed of risk aversion–that risk aversion has no impact on driving. The following result then immediately follows:

Lemma 2 *If the likelihood function $p(\cdot)$ satisfies (S) then with zero correlation, expected loss and gender will be independently distributed: $p(\mu|g) = p(\mu)$.*

Thus, under these assumptions one should be led to the same conclusion as the European Commission–that a person's gender is insignificant in explaining her risk type.

Since a gender discriminating firm will use the probability functions $p(\mu, \nu|g)$ $(g = m, w)$, rather than the single function $p(\mu, \nu)$ to design menus, and since profits and consumer rents depend on the coinsurance rate c_{LL}, it is important to trace out changes in $p(\cdot)$ on c_{LL}. Let us assume that $D < \underline{D}_M$ so that we can ignore **Regime M**. From proposition 3, it follows that without discrimination, the upper boundaries of regimes **C**, **Bf**, **BpI** are determined by the parameters α_{LL} and α_{LH}. Fixing x, $\alpha_{L\cdot}$,(and thus $\alpha_{H\cdot} = 1 - \alpha_{L\cdot}$), it is possible to trace out the optimal value of c_{LL} as a function of α_{LL}. Doing so yields the figure below where it is assumed that $\frac{1-x}{1+x} < \frac{1}{2}(1 + \alpha_{L\cdot})(1 - x)$. This means that curve for LL's optimal coinsurance rate is flat for some range of α_{LL} values; it is equivalent to assuming that

$$\frac{1 - \alpha_{L\cdot}}{1 + \alpha_{L\cdot}} < x. \tag{34}$$

Figure 12 here.

Let us define ω_L (ω_H) as the likelihood that an arbitrary person with low (high) risk aversion is a female, i.e. $\omega_L \stackrel{\text{def}}{=} p(w|\nu_L)$ and $\omega_H \stackrel{\text{def}}{=} p(w|\nu_H)$. With half of the population consisting of women, we have that $p(w) = \omega_H \alpha_{\cdot H} + \omega_L(1 - \alpha_{\cdot H}) = \frac{1}{2}$.

There is now ample evidence women are on average more risk averse than men.[17] For our model, this means that $\omega_L < \frac{1}{2} < \omega_H$. When the insurance company is allowed to gender discriminate, it will upon observation of a customer's gender g update the probability α_{LL} in the following way:

$$\alpha_{LL|w} \overset{\text{def}}{=} p(\mu_L, \nu_L|w) = 2\omega_L \alpha_{LL} \ (< \alpha_{LL}), \text{ and}$$
$$\alpha_{LL|m} \overset{\text{def}}{=} p(\mu_L, \nu_L|m) = 2(1 - \omega_L)\alpha_{LL} \ (> \alpha_{LL}).$$

By lemma 2, we also have that $p(\mu_i|g) = p(\mu_i)$ $(i = L, H)$, meaning that $\alpha_{L\cdot}$ and $\alpha_{H\cdot}$ do not change when gender is observed.

Suppose now that (34) holds, that the fractions of LL people for the entire population, the population of men and that of women, are $\alpha_{LL}, \alpha_{LL|m}$, and $\alpha_{LL|w}$, respectively, and that these fractions are as in the figure below.

Figure 13 here.

Then we can conclude that since

$$\frac{1-x}{1+x} < \alpha_{LL|w} < \alpha_{LL} < \frac{1}{2}(1 + \alpha_{L\cdot})(1 - x), \tag{35}$$

the coinsurance rate for LL-women will remain at its no-discrimination value, and the rents of LH-, HL- and HH-women will not change due to discrimination (and LL-women continue to receive zero rent). On the other hand, because

$$\frac{1-x}{1+x} < \alpha_{LL} < \frac{1}{2}(1 + \alpha_{L\cdot})(1 - x) < \alpha_{LL|m}, \tag{36}$$

the optimal coinsurance rate for LL-men will drop below its no-discrimination value, and therefore all men will get more rent when offered the optimal contract menu for men (except LL-men, who continue to receive zero rent). Finally, the insurance company will increase total profit since it found it optimal to choose a new menu for its male clientele–it could have stuck to same menu as in the no-discrimination case. Thus, a (weak) Pareto improvement is possible by allowing for gender discrimination. As can be seen from the figure, conditions (35) and (36) are not only sufficient for a (weak) Pareto improvement, but also necessary. We summarise this result as

[17] See Hartog, Ferrer-i-Carbonell & Jonker (2002), Eckel & Grossman (2003), Kimball, Shapiro and Sahm (2008) and Aarbu and Schroyen (2009).

Proposition 6 *Suppose that (34) holds. For given values of* $x, \alpha_{L\cdot}$, *and* D, *allowing for gender discrimination will lead to a (weak) Pareto improvement in the insurance market if and only if conditions (35) and (36) hold.*

Condition (34) will be satisfied when the fraction of low risk people, $\alpha_{L\cdot}$, is not too small in relation to x.

9 Conclusion

In this paper we have studied the outcome on a monopolistic insurance market when the insurer is only aware of the statistical distribution of the expected loss and the level of risk aversion of its customers. We have formulated a mean-variance model that results in quasi-linear preferences over contracts, identified five contract menus that will occur in equilibrium, and for each menu derived the optimal level of coinsurance rates. Next, we identified for each regime the set of parameter values for which it is optimal. We did this for three scenarios: perfect positive, perfect negative and zero correlation of the two characteristics. In the latter two scenarios, the analysis is intricate because the two-dimensional nature of the private information means that the types lack a natural ordering in terms of their marginal willingness to pay for coverage.

We find that

- it is never optimal to fully separate all the types. In other words, there will always be some pooling of types;
- the larger the heterogeneity in the expected loss, the more it pays to screen the low risk from the high risk types, by imposing a high coinsurance rate on the former;
- the larger the heterogeneity in terms of risk aversion, the more it pays to screen the low risk averse from the high risk averse by imposing a high coinsurance rate on the former;
- positive correlation between coverage and expected loss does no longer provide a test for the hypothesis of adverse selection.

We have also identified an open set for parameter values such that when gender (only) affects people risk aversion, allowing for gender discrimination will result in a weak Pareto improvement in this market. Our analysis thus points out that one should be careful when abolishing gender categorisation: even when gender itself does not affect (in a statistical way) the expected level of losses or claims, it may affect the outcome in an imperfectly competitive insurance market in a way where nobody gains and some participants become worse off.

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Appendix

1. Details of Regime B

For Regime **Bp**, the condition $c_{LH} \geq 0$ is not binding. Defining $\overline{x}^{Bp}(D) \stackrel{\text{def}}{=} \frac{1-\alpha_{H\cdot}-D(\alpha_{H\cdot}+\alpha_{LH})}{1-\alpha_{H\cdot}-\alpha_{H\cdot}D}$, the optimal value for c_{LH}^{Bp} is given by

$$c_{LL}^{Bp} = D\frac{2\alpha_{LH}+\alpha_{H\cdot}(1-x)}{(1-\alpha_{H\cdot})(1-x)} \text{ if } x < \overline{x}^{Bp}(D) \ \ \textbf{(Regime BpI)} \tag{37}$$

$$= 1 \text{ if } x \geq \overline{x}^{Bp}(D). \ \ \textbf{(Regime BpX)} \tag{38}$$

For Regime **Bp** to have a chance of being optimal, we need that $c_{LH} = 2\frac{\Delta\mu}{\Delta\nu} - c_{LL}^{Bp} \geq 0$. For the interior solution (**BpI**), this means that

$$\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} < x,$$

and compatibility with the condition $x < \overline{x}^{Bp}(D)$ requires that D does not get too high:

$$D < \overline{D}^{Bp}(\alpha_{LL}, \alpha_{LH}). \tag{39}$$

where $\overline{D}^{Bp} \stackrel{\text{def}}{=} \frac{\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$. For the exclusion solution (**BpX,** $c_{LH}^{Bp} = 1$), condition $2\frac{\Delta\mu}{\Delta\nu} - c_{LL}^{Bp} \geq 0$ is equivalent with

$$\frac{1}{1+2D} < x. \tag{40}$$

For Regime **Bf**, the condition $c_{LH} \geq 0$ is binding. The optimal value for c_{LL}^{Bf} is then given by

$$c_{LL}^{Bf} = 2D\frac{x}{(1-x)} \text{ if } x < \frac{1}{1+2D} \quad \textbf{(Regime Bf)} \tag{41}$$

If $x > \frac{1}{1+2D}$, the only way **Regime Bf** may be implemented is by having $c_{LL} > 1$, which is ruled out by the moral hazard constraint. The only way of restoring incentive compatibility is to offer LH partial insurance at $2\frac{\Delta\mu}{\Delta\nu} - 1$. Then we are back in **Regime BpX**.

Finally, it can be shown that

$$D > (<)\overline{D}^{Bp} \Longrightarrow \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} > (<)\frac{1}{1+2D} > (<)\overline{x}^{Bp}(D).$$

Regime B can then be summarised as in the text.

2. Comparison of regimes

We here discuss the comparison of the different regimes.

Start from **Regime A**, with $c^A_{L\cdot}$ defined by (23). By Assumption N, this is an interior solution. We now ask when $2\frac{\Delta\mu}{\Delta\nu} - c^A_{L\cdot} \geq 1$. The answer is if

$$x \geq x_{AM}(\alpha_{H\cdot}, D), \tag{42}$$

where $x_{AM}(\alpha_{H\cdot}, D) \stackrel{\text{def}}{=} \frac{1-\alpha_{H\cdot}(1-D)}{D+(1-\alpha_{H\cdot})(1+D)}$. If this is the case, we can move from **Regime A** into **Regime M** by increasing c_{LL} up to 1, and increasing the premia, and therefore the profits, of all the other types–see the figure below.

Figure 8 here

This increase in profits will outweigh the fall in profits from type LL (which fall down to zero since she is excluded) if there are sufficiently few LL people; formally if $1 - \alpha_{LL} > x$. Thus, the move from **A** to **M** is feasible and desirable if and only if

$$x_{AM}(\alpha_{H\cdot}, D) \leq x \leq 1 - a_{LL}.$$

For this interval to be non-empty, it is required that $D > \underline{D}^M$.

Lemma 3 ***Regime A*** *outperforms* ***Regime M*** *if and only if* $D > \underline{D}^M$ *and* $x \leq 1 - a_{LL}$.

Suppose now that the inequality (42) is *not* satisfied. Then it is feasible to move from **Regime A** into **Regime BpI** by increasing c_{LL} up to $2\frac{\Delta\mu}{\Delta\nu} - c^A_{L\cdot}$. The ensuing loss in profit from LL will outweigh the increase in profits from the other types, if $x > 1 - \alpha_{LL}$. Thus if

$$1 - \alpha_{LL} < x < x_{MA}(\alpha_{H\cdot}, D),$$

moving from **Regime A** to **Regime BpI** is feasible but not profitable. For this interval to be non-empty, the condition is that $D < \underline{D}^M$. On the other hand, if $x < 1 - \alpha_{LL}$, this move also becomes profitable. Thus **Regime BpI** takes over from **Regime A**. See the figure below.

Figure 9 here.

Lemma 4 *Suppose that* $D < \underline{D}^M$. ***Regime B*** *outperforms* ***Regime A*** *if and only if* $x \leq 1 - a_{LL}$.

Next, consider **Regime B** with $c_{LL}^B = 1$ (also called **BpX**). Recall from the discussion of this regime that we need $x \geq \max\{\frac{1}{1+2D}, \overline{x}^{Bp}(D)\}$. Let us now ask what happens to total profit if c_{LH} is unilaterally lowered below $2\frac{\Delta\mu}{\Delta\nu} - 1$ and the premium charged to LH is raised to keep this type on the same indifference curve, as shown in the figure below.

Figure 10 here.

By this reform we end up in **Regime M**. Premia on (and therefore profits from) HH and HL will be lowered, but profits from LH are increased and the total effect on profits is

$$-\frac{\partial \pi_{tot}}{\partial c_{LH}}|_{c_{LH}^B} = \alpha_{LH} c_{LH}^B \nu_H - \alpha_{H\cdot}(\Delta\mu - c_{LH}^B \Delta\nu) \geq 0$$

$$\Updownarrow$$

$$D\frac{\alpha_{H\cdot} x}{\alpha_{H\cdot}(1-x) + \alpha_{LH}} > c_{LH}^B.$$

Since the *lhs* is c_{LH}^M and the *rhs* equals $2\frac{\Delta\mu}{\Delta\nu} - 1$, this condition amounts to (**??**). For this lowering of c_{LH} to be feasible, we should have that $c_{LH}^B > 0$, or $x > \frac{1}{1+2D}$. Since $\frac{1}{1+2D} < x_{BM}(\alpha_{H\cdot}, \alpha_{LH}, D)$ for all parameters and since $\overline{x}^{Bp}(D) < x_{BM}(\alpha_{H\cdot}, \alpha_{LH}, D)$ iff $D > \underline{D}^M$, moving from **BX** into **M** is optimal if and only if

$$x_{BM}(\alpha_{H\cdot}, \alpha_{LH}, D) < x \text{ and } D > \underline{D}^M.$$

Lemma 5 ***Regime BpX*** *outperforms* ***Regime M*** *if and only if* $D > \underline{D}^M$ *and* $x < x_{BM}(\alpha_{H\cdot}, \alpha_{LH}, D)$.

In the discussion of **Regime C** we already pointed out that this regime coincides with **Regime B** if $x \geq \frac{1-\alpha_{LL}}{1+\alpha_{LL}}$ when $D < \overline{D}_C$ and if $x \geq \frac{1}{1+2D}$ when $D > \overline{D}_C$. For values of x below these upper limits, **Regime C** performs better than **Regime B** since it exploits the fact that c_{LL} is not constraint at $2\frac{\Delta\mu}{\Delta\nu}$.

Lemma 6 ***Regime C*** *outperforms* ***Regime B*** *if and only if* $x \leq \min\{\frac{1-\alpha_{LL}}{1+\alpha_{LL}}, \frac{1}{1+2D}\}$.

Finally, we should compare the profits generated by **Regime E** with those of the other regimes. Unlike the previous comparisons, there is no continuity between this regime and the others in the sense that none of the coinsurance rates move continuously when switching into or out of **Regime**

E. This is due to a non-convexity of the feasible set. Identifying when **Regime E** starts to be optimal thus requires a comparison of the maximal profit functions.

The maximal profit functions for **Regimes C** and **E** are as follows

$$\pi_{tot}^{CI} = \pi_{tot}^{C}|_{(c_I=0,c_{LL}=\frac{\Delta\mu}{\nu_L}\frac{1-\alpha_{LL}}{\alpha_{LL}})} = \nu_L\left[\frac{1}{2} - \alpha_{H\cdot}D + \frac{1}{2}D^2\frac{(1-\alpha_{LL})^2}{a_{LL}}\right]$$

$$\pi_{tot}^{CX} = \pi_{tot(c_I=0,c_{LL}=1)}^{C} = \nu_L\left[\frac{1}{2} + \alpha_{LH}D - \frac{1}{2}\alpha_{LL}\right]$$

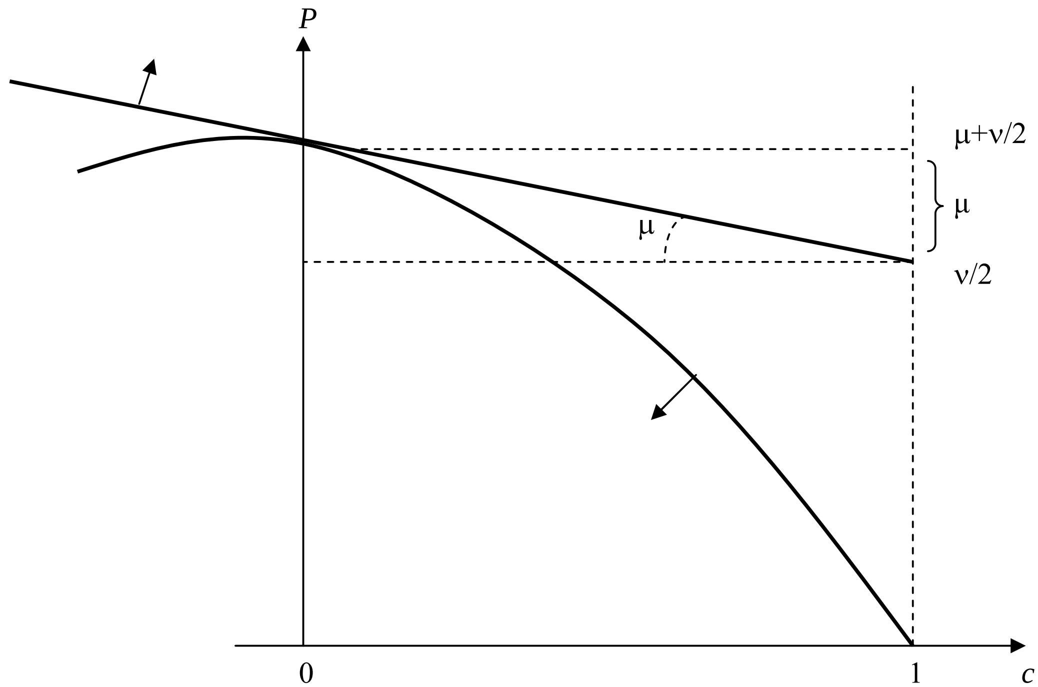
$$\pi_{tot}^{EI} = \pi_{tot}^{E}|_{(c_{LH}=D\frac{\alpha_{HH}}{\alpha_{LH}}x,c_{\cdot L}=D\frac{x\alpha_{HL}}{x-\alpha_{\cdot H}})} = \nu_L\left[\frac{1}{2} - \alpha_{H\cdot}D + \frac{1}{2}D^2\left(\frac{x\alpha_{HL}^2}{x-\alpha_{\cdot H}} + \frac{x\alpha_{HH}^2}{\alpha_{LH}}\right)\right]$$

$$\pi_{tot}^{EX} = \pi_{tot}^{E}|_{(c_{LH}=D\frac{\alpha_{HH}}{\alpha_{LH}}x,c_{\cdot L}=1)} = \upsilon_L\left[\frac{1}{2} - \alpha_{HH}D - \frac{1}{2}D^2\frac{x\alpha_{HH}^2}{\alpha_{LH}} + \frac{1}{2}\frac{\alpha_{HH}+\alpha_{LH}-x}{x}\right]$$

In the figure below, the borderline between **CI** and **EI** is found by solving $\pi_{tot}^{CI} = \pi_{tot}^{EI}$ for x. This gives a value that is independent on D (the horizontal piece of line that holds for low D values). It can be shown that this critical x-value is smaller than $\frac{1-\alpha_{LL}}{1+\alpha_{LL}}$ iff $(\alpha_{H\cdot}, \alpha_{HH})$ lies outside the shaded area of figure 8.

For larger D values, the relevant comparison is between π_{tot}^{CI} and π_{tot}^{EX}, which gives again a quadratic equation in x. For D-values above $\frac{\alpha_{LL}}{1-\alpha_{LL}}$, we need to solve $\pi_{tot}^{CX} = \pi_{tot}^{EX}$. This is still another quadratic equation in x. The roots for all three quadratic equations are computed for the case of independence, i.e., $\alpha_{LH} = \alpha_{HH}\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}}$, and $\alpha_{LL} = (\alpha_{H\cdot} - \alpha_{HH})\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}}$. Since the expressions are very long, we do not report them here.

Figure 17 here.



Figur 1. An indifference curve and an iso-profit line in the (c,P)-space.

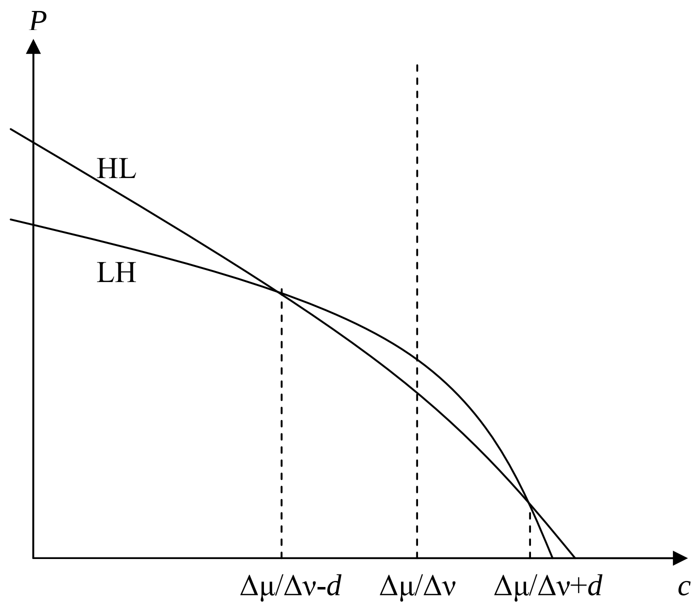


Figure 2. The indifference curves of HL and LH cross twice.

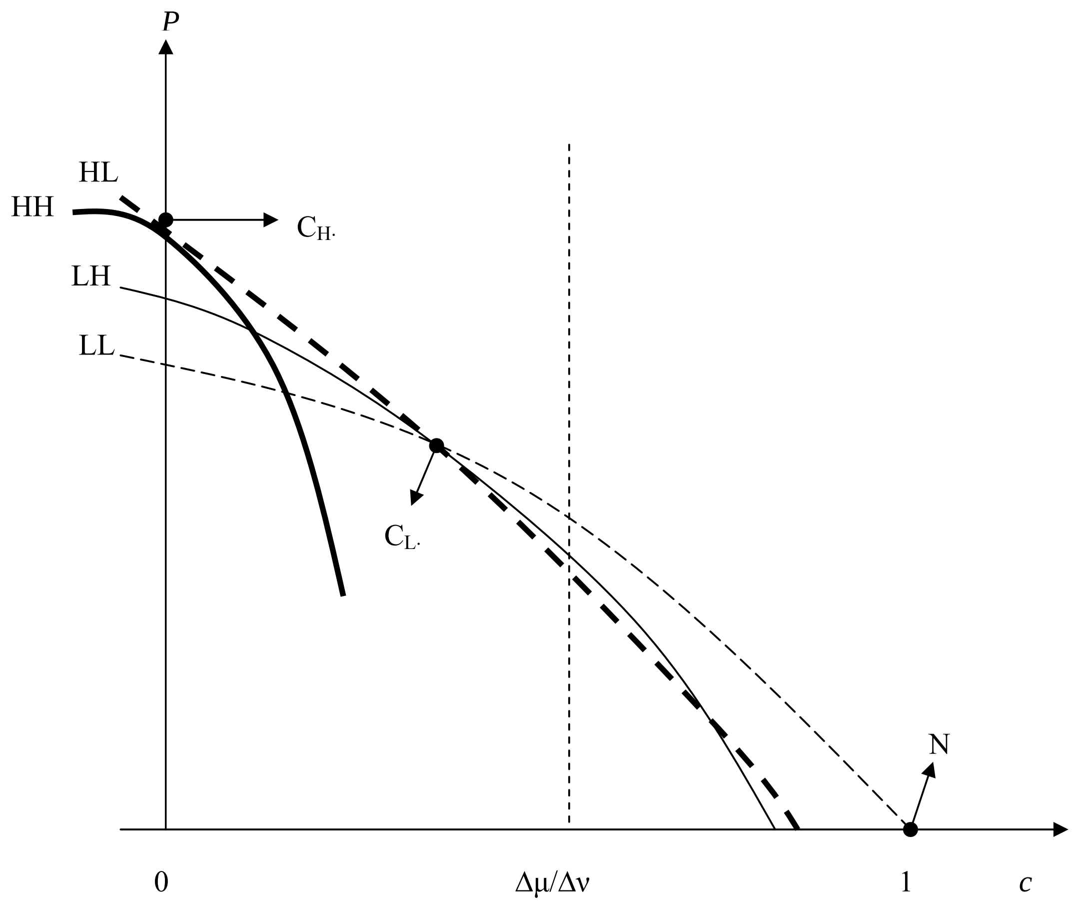
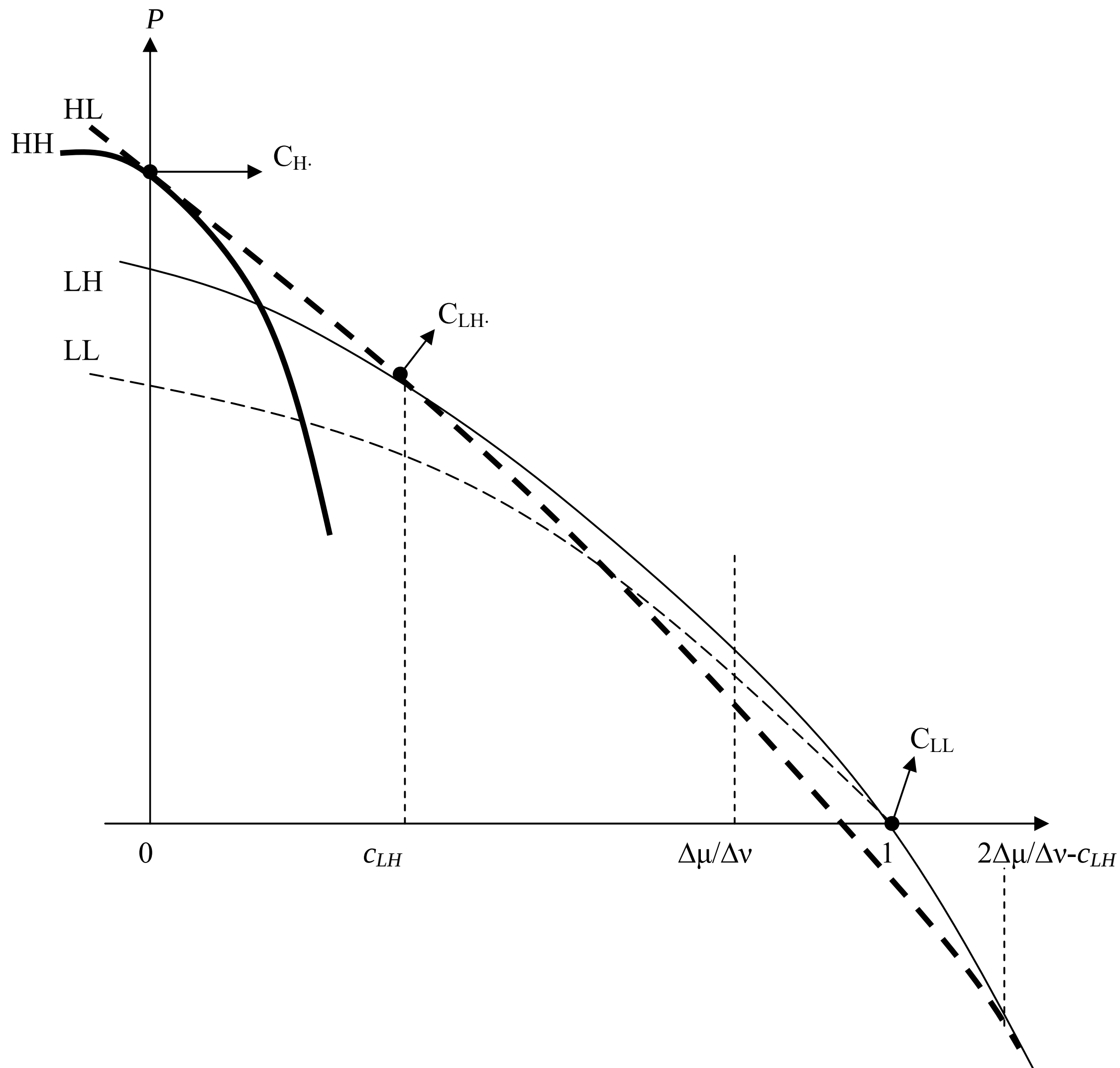


Figure 3. Regime A.



Figur 4. Regime M.

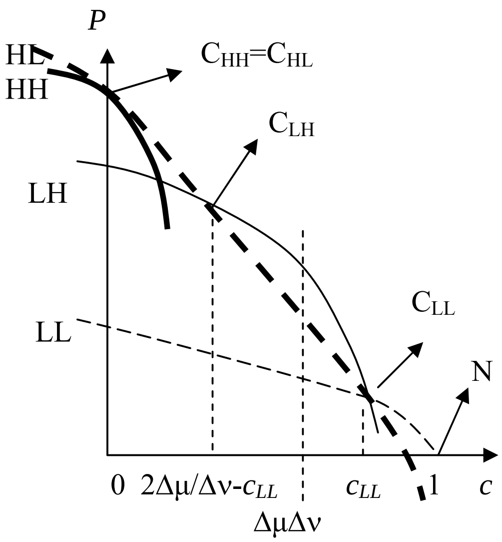


Figure 5a. Regime BpI

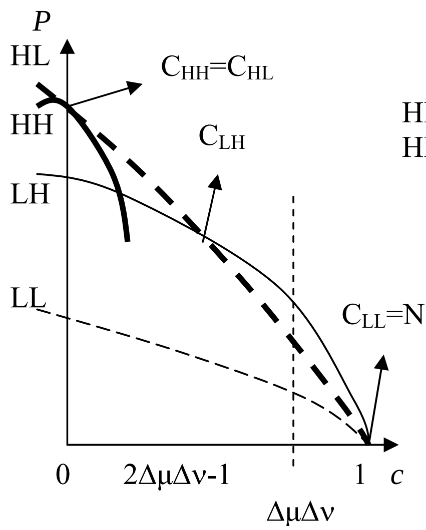


Figure 5b. RegimeBpX

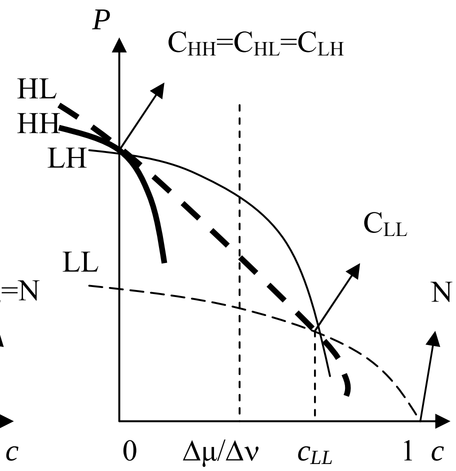


Figure 5c. Regime Bf

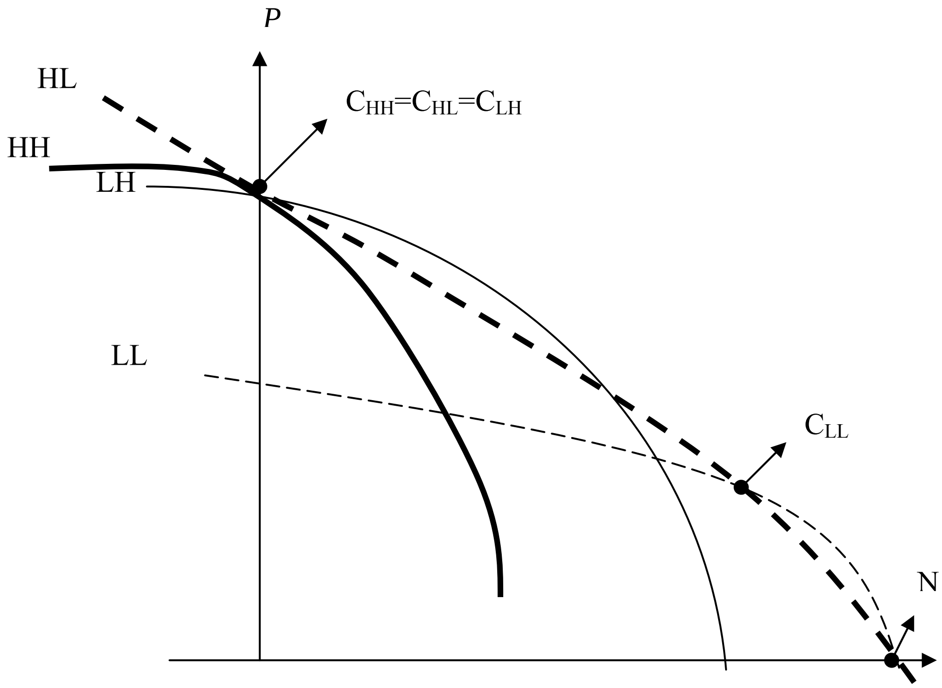


Figure 6. Regime C.

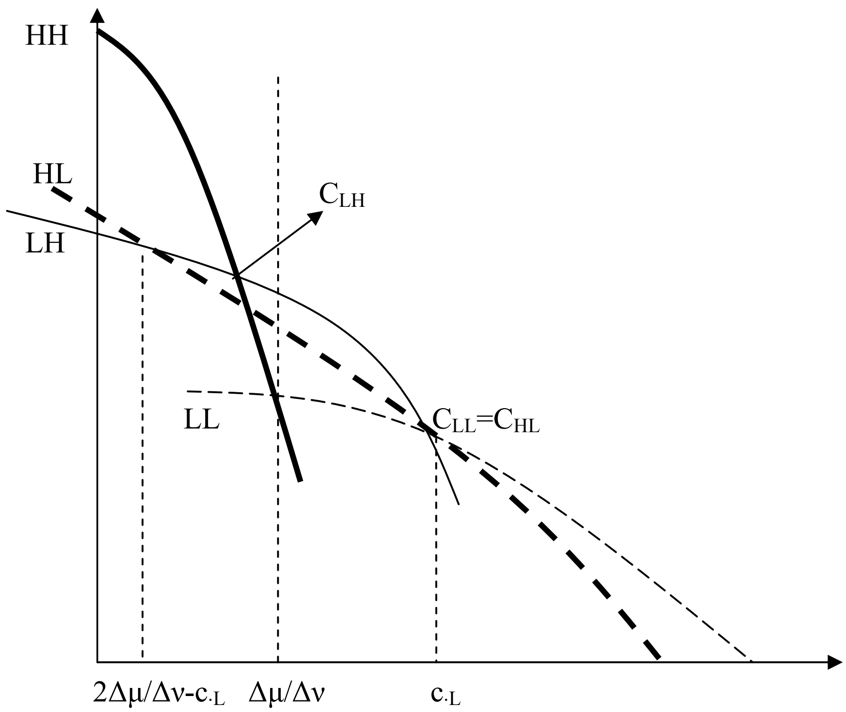


Figure 7. Regime E: {HH},{LH},{HL,LL}.

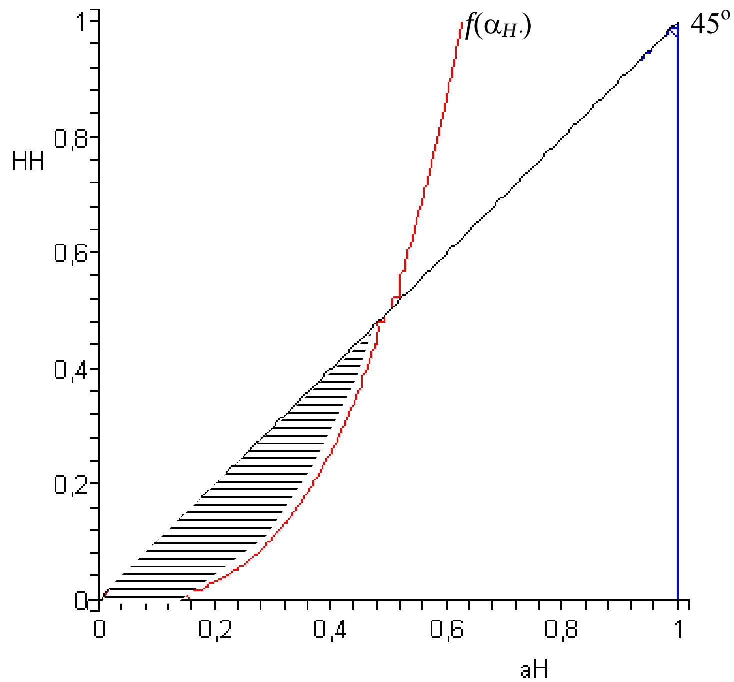


Figure 8. For $(\alpha_{H\cdot},\alpha_{HH})$-values in the shaded region, Regime C ceases to occur.

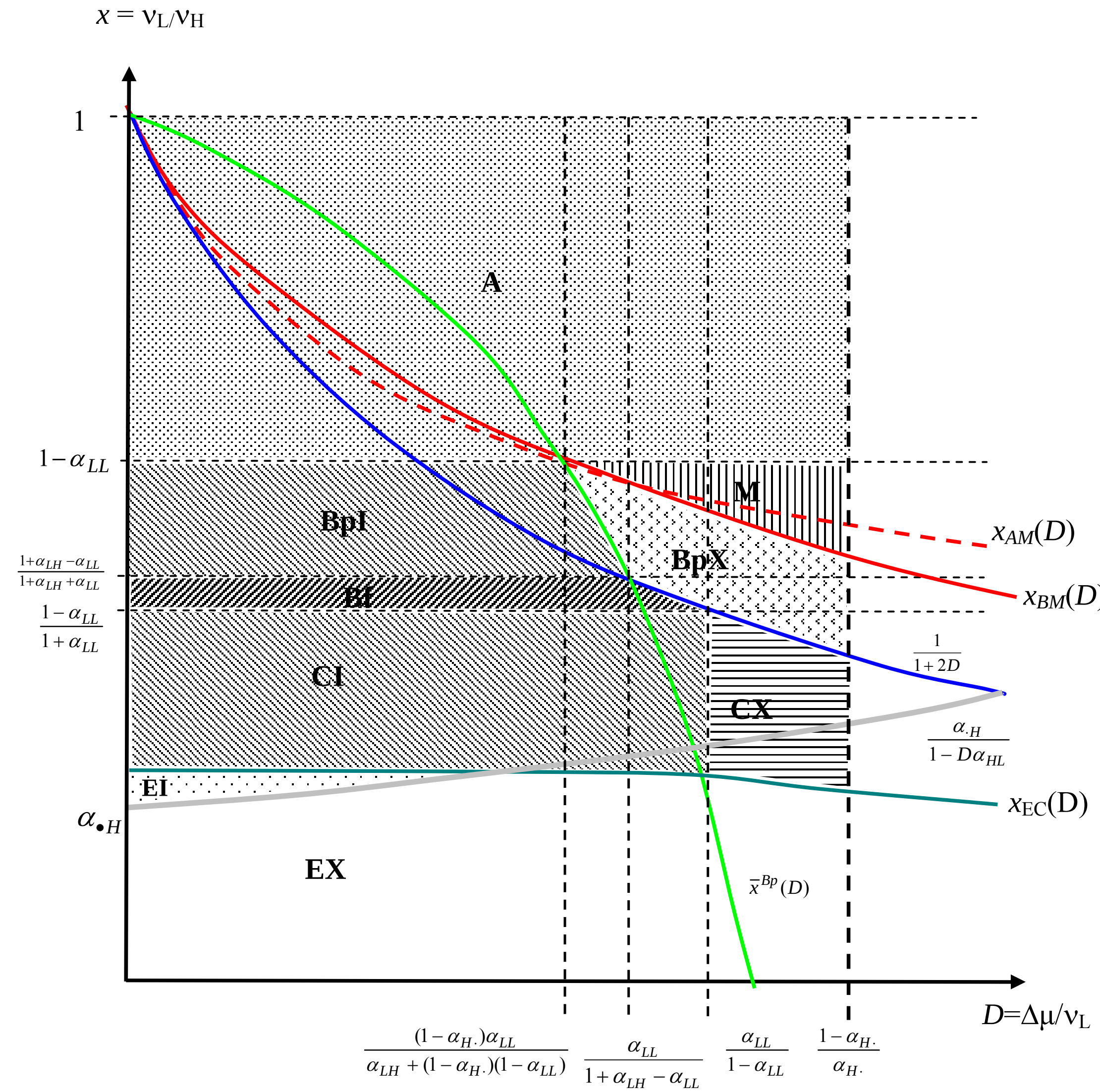


Figure 9. Optimal regimes in the (D,x)-space. The case of independence.

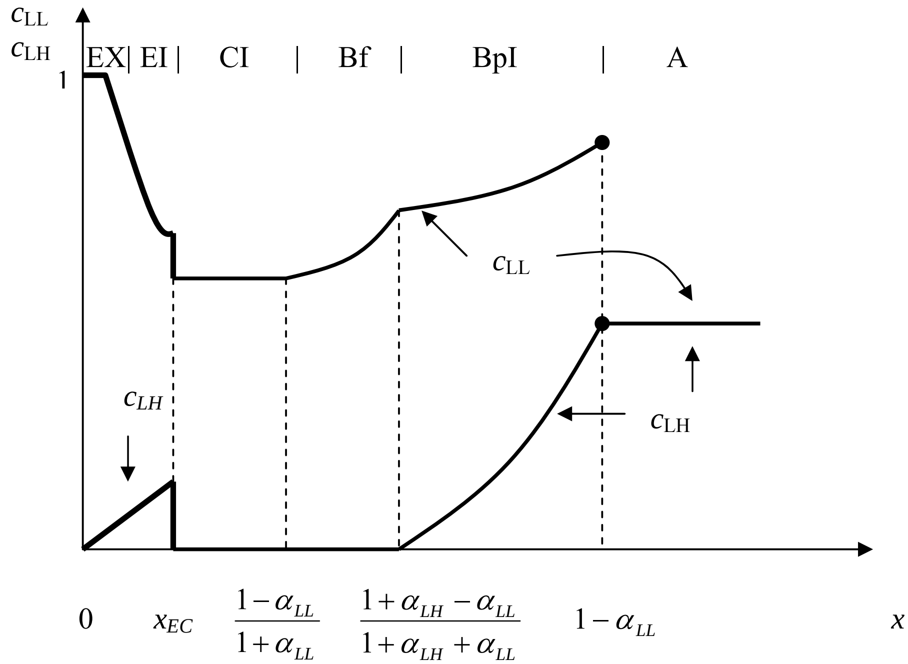


Figure 10. Optimal coinsurance rates as a function of *x*.

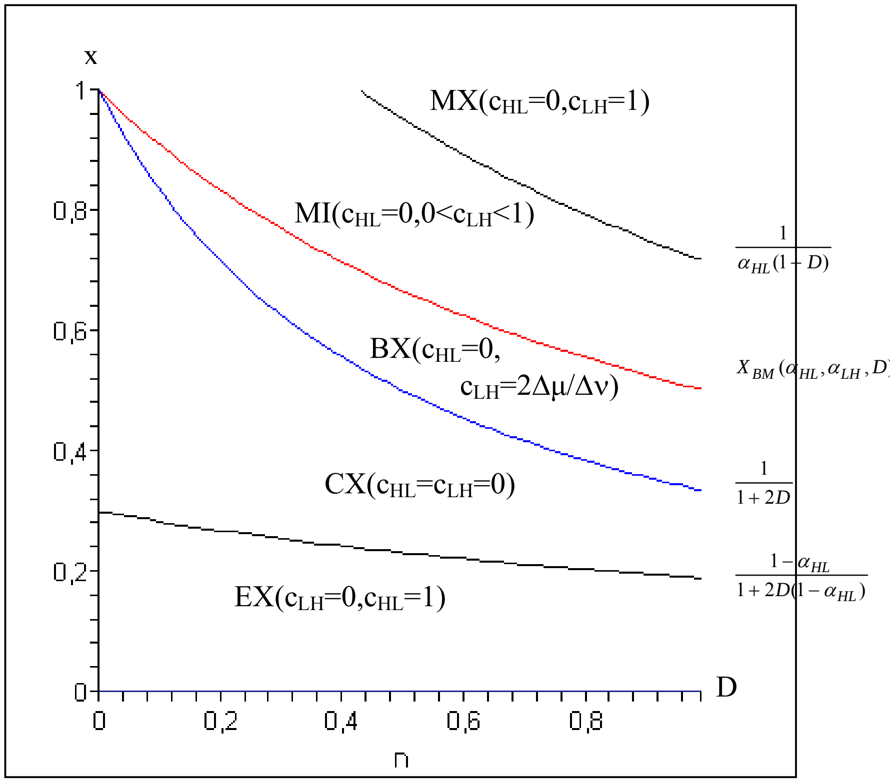


Figure 11. Optimal regimes in the (*D*,*x*)-space. The case of perfect negative correlation.

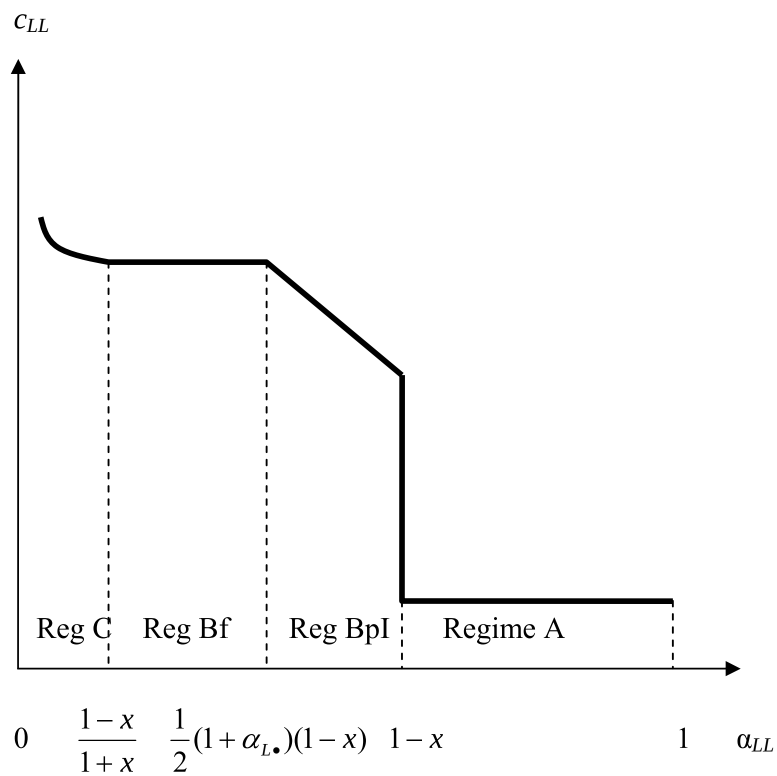


Figure 12. The optimal coinsurance rate on LL as a function of α_{LL}.

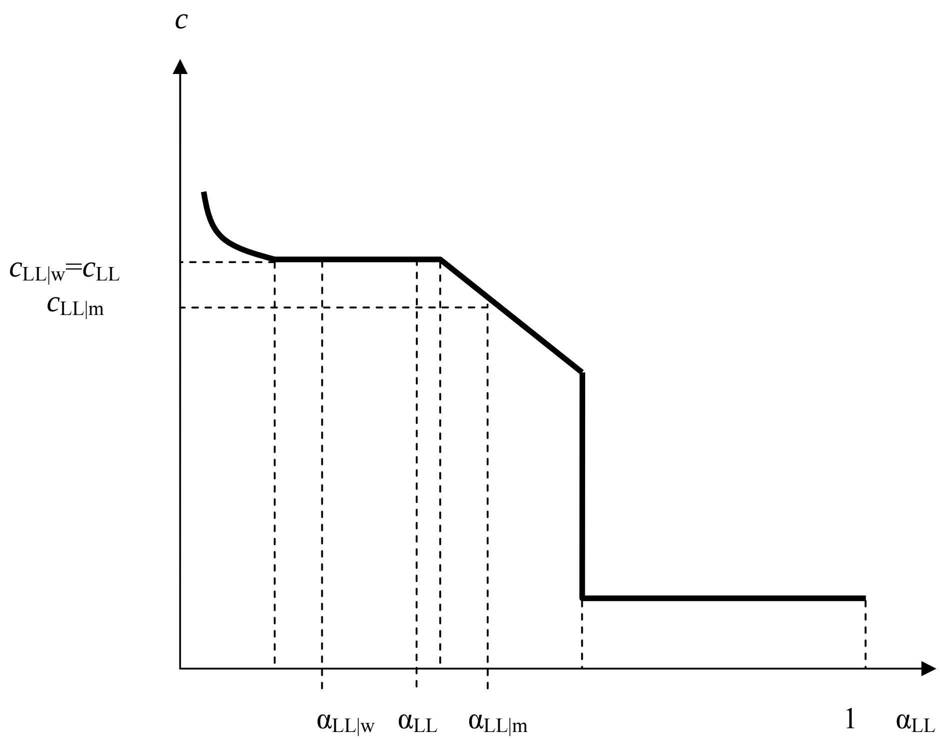


Figure 13. *A priori* and updated probability of type *LL* and corresponding coinsurance rates.

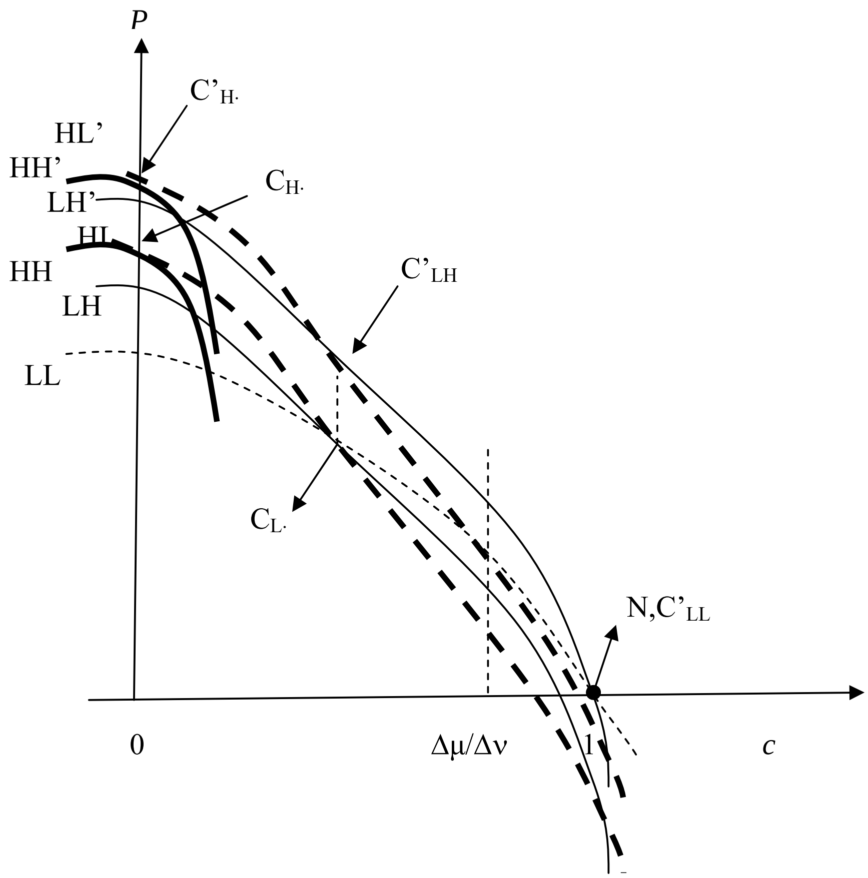


Figure 14. Moving from Regime A ({HH,HL},{LH,LL}) to Regime M ({HH',HL'},{LH'},{LL=N}).

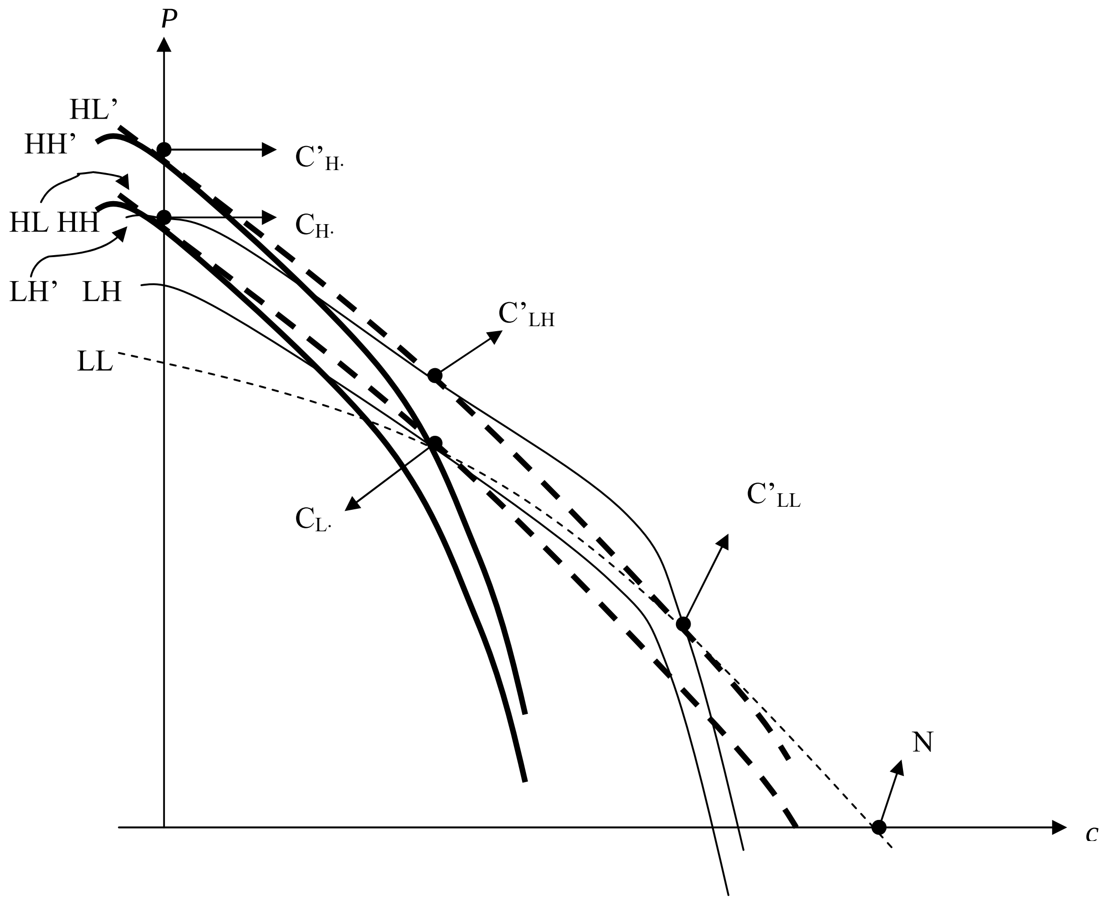


Figure 15. Moving from Regime A ({HH,HL},{LH,LL}) to Regime BpI ({HH',HL'},{LH'},{LL'}).

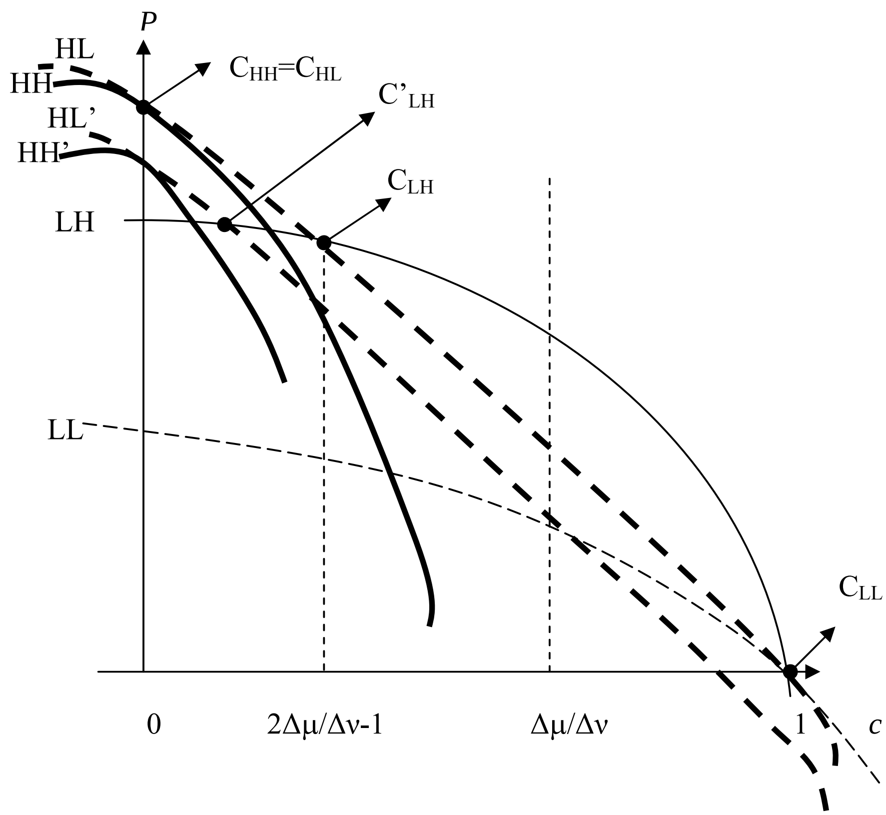


Figure 16. Moving from Regime BpX ({HH,HL},{LH},{LL=N}) to Regime M ({HH',HL'},{LH'},{LL=N}) .

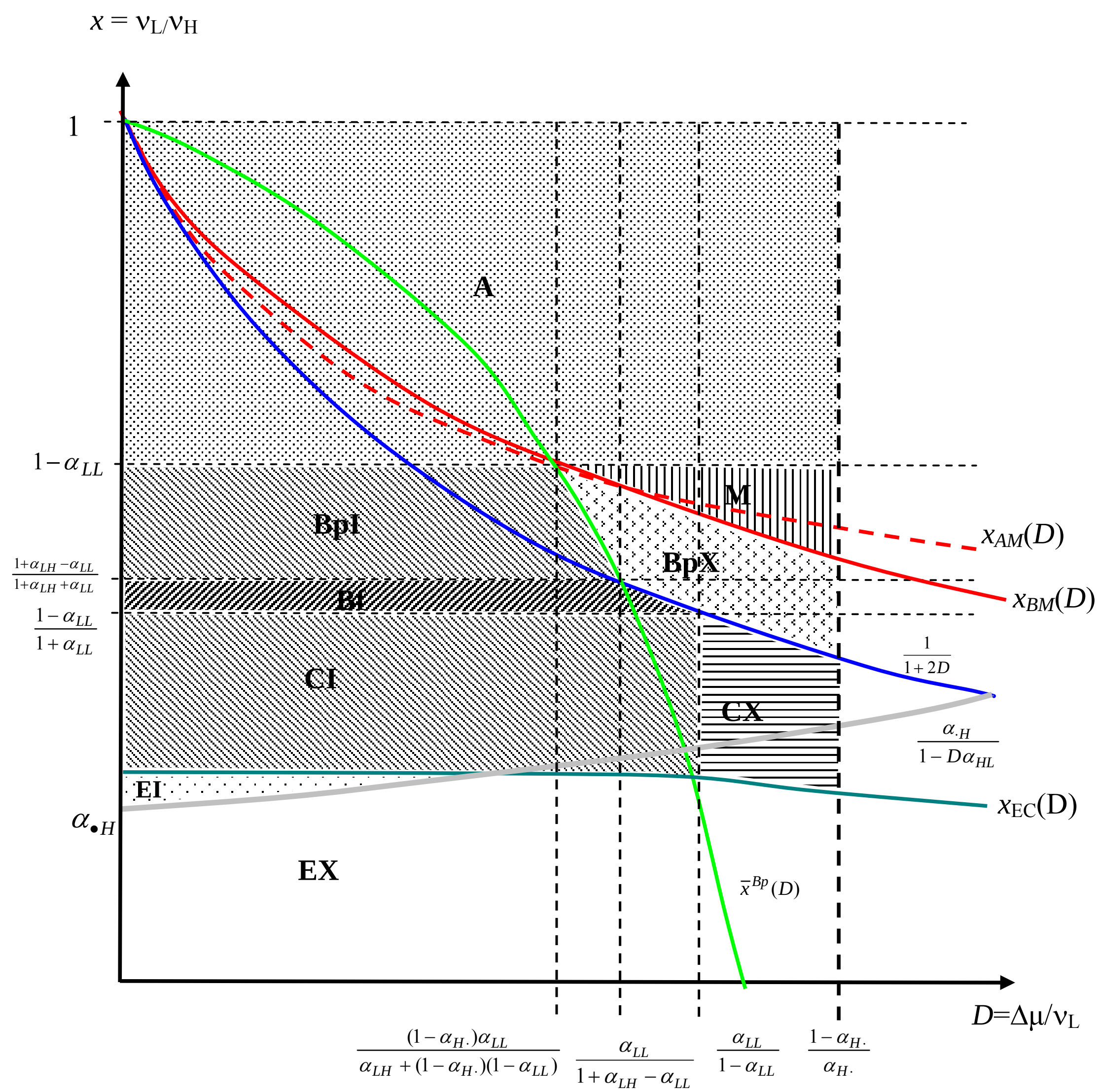


Figure 17. Partitioning of the parameter space (D,x) into different regimes.