

# Wait and See\*

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## Abstract

We study a dynamic cheap talk model with multiple senders where the receiver can choose when to make her decision and communication can take place over time. No player has the ability to commit to any action in the future, in particular, the receiver cannot commit to delay the decision. In contrast to the results in static versions of the model, we show that when the senders have common knowledge about the state of the world, there exists an equilibrium with instantaneous, full revelation irrespective of the size and direction of the senders' biases. We show that the equilibrium is robust to various perturbations including the introduction of noise in the senders' signals about the state.

**Keywords:** multi-sender cheap talk, full revelation, noisy signals, delay

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# 1 Introduction

Consulting with experts before making a decision takes time, and happens over time. Our goal in the present paper is to point out some fundamental consequences of considering information transmission between multiple experts and a decision maker in a *dynamic setting*. We investigate how the threat of costly delay, to which no one can commit, can induce communication between parties with dissimilar interests.

In our model a decision maker (she) can consult multiple experts who have private information about the state of nature. There is a single decision to be made, and neither the payoff-relevant state nor the experts' private signals and biases change over time.<sup>1</sup> At every point in time, the experts can send private or public messages; subsequently (but at the same physical time) the receiver either picks a game-ending action, or chooses momentary inaction. When the game ends, the players receive payoffs representing single-peaked preferences over the action space. Conflict arises because the senders' ideal points differ from the receiver's conditional on the state of nature. All players have strict time preferences, that is, they prefer a given policy to be carried out earlier. We focus on perfect Bayesian equilibria requiring all actions to be rational at every point in time given the history of the play. In other words, no player has commitment power. In particular, the decision maker cannot commit to wait, hence delays on the equilibrium path have to be rationalized.

This model is a dynamic extension of a multi-sender cheap talk game that has been studied extensively in the literature—particularly its special case, where the senders perfectly observe the state.<sup>2</sup> Our approach to incorporating dynamics differs from that of multi-round or long cheap talk models<sup>3</sup> in that in our setup the receiver continuously faces the tradeoff between more communication and earlier decision, and that the time spent on communication (or waiting)

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<sup>1</sup>The state, the senders' information and the preferences do not evolve over time so that we can focus on the *dynamics of communication* in this model.

<sup>2</sup>The static version with perfect observation is studied by Gilligan and Krehbiel (1989), Krishna and Morgan (2001a,b), Battaglini (2002), Ambrus and Takahashi (2007), among others. Battaglini (2004) allows a special type of noisy signal structure in the same setup.

<sup>3</sup>See Aumann and Hart (2003), Krishna and Morgan (2004) and Section 5 of Ambrus and Takahashi (2007), among others.

affects all players' payoffs. However, it will also become clear that “time” in our model is not simply a new dimension in which the receiver can threaten the senders with punishment (albeit without being able to commit); the fact that beliefs can evolve over time in our dynamic game is crucial to the analysis.

For a concrete situation that we intend to model, think of immigration reform in the United States. The decision maker is the median voter in Congress; the senders are various committees, representatives and lobbyists who have relevant private information.<sup>4</sup> The honest opinions of these experts may differ due to their information, but based on what they know it would be possible to determine Congress' ideal policy choice. However, the senders are also *biased* from the median voter's perspective due to their ethical considerations, business interests, etc. Therefore, a disagreement among them may also reflect their desire to shift Congress' decision in their favor. The question is how much information can be transmitted if the legislative process takes place over time—hearings followed by proposals, debates and votes—without the median voter being able to explicitly commit to future actions. The social, political and economic problems stemming from illegal immigration do not improve with time, hence delay is wasteful.<sup>5</sup> Other legislative issues in the U.S. that share the same characteristics include entitlement reform and health care.

We first study the case where the two experts have *common knowledge* about the realization of the state of nature, while the decision maker only knows the state's prior distribution. We prove that, no matter what the state space or the senders' biases or any players' time preferences are, there exists a perfect Bayesian equilibrium where the state of nature is truthfully and immediately reported by both experts, and the decision maker carries out her ideal policy without delay. The senders' equilibrium strategy is to report the state at time 0, and in case of a mismatch, repeat their reports frequently, at specific points in time. The receiver takes an action matching the senders' report if they agree at time 0; in case of a disagreement she waits until they are expected to tell the truth again. Her off-equilibrium inaction is rationalized by the expectation

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<sup>4</sup>Legislative procedures are modeled as cheap talk games in Gilligan and Krehbiel (1989).

<sup>5</sup>Some experts may prefer delay, especially in other applications. We assume the decision maker and at least two well-informed experts have a preference for early action.

(shared among all parties) that both senders will report the true state frequently and almost immediately following a disagreement. Note that the receiver does not have the power to commit to wait; she can choose inaction only if she expects to learn more by waiting. Off the equilibrium path, she believes that learning the state is imminent, hence her momentary inaction is rational.

The result that the receiver can *immediately* extract *all* information from biased experts is remarkable because in the corresponding static multi-sender cheap-talk model the result only obtains under certain conditions. As Battaglini (2002) and Ambrus and Takahashi (2007) have shown, the necessary and sufficient condition for the existence of a fully-revealing equilibrium is that the state space be “relatively large” compared to the senders’ biases.<sup>6</sup> In contrast, our result holds *without any restriction* on the shape or size of the state space, nor does it depend on the direction or size of the senders’ biases. The fully-revealing equilibrium in our model involves no delay, therefore an outside observer would not even realize that the underlying situation is dynamic.

Note that the decision maker can fully extract the senders’ private information even though she is unable to commit to delay her decision. She can achieve the first best by rationally maintaining a positive attitude and believing that the senders will eventually agree. Such beliefs are plausible in the model where the senders have bilateral common knowledge of the state of nature—they *must* be able to agree, and the receiver knows that.<sup>7</sup>

Experts do not always have common knowledge about the state of nature. If so, then “cross-checking” their reports and only acting when they fully agree may not be a sensible course of action for the decision maker. This criticism applies not only to our model with perfect observation, but also to a large chunk of the existing literature on multi-sender cheap talk. In those models, too, the senders observe the state, and the construction of a fully-revealing equilibrium

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<sup>6</sup>The condition identified in the cited papers is discussed in Section 3.1.

<sup>7</sup>Similar equilibrium constructions for dynamic models have been used outside the cheap talk literature. In the durable-good oligopoly model of Ausubel and Deneckere (1987) and Gul (1987) the buyers’ off-equilibrium beliefs discipline the sellers’ dynamic pricing behavior. Marx and Matthews (2000) and Lockwood and Thomas (2002) use a related construction to overcome the free-riding problem in public good provision when contributions are made over time. The investment hold-up problem is resolved by Gul (2001) using repeated contract offers and by Che and Sákovics (2004) using dynamic investment.

hinges on this fact.<sup>8</sup> Another drawback of models with perfect observation is that they are not particularly insightful for analyzing situations with more than two senders. In that case, even without dynamics, a fully-revealing equilibrium trivially exists: If more than two senders are expected to report the state, then a single deviator cannot prevent the receiver from learning the truth.

We extend our model to allow for noisy signals on the senders' part. The main difficulty in constructing informative equilibria in this case is that disagreement between the senders can happen on the equilibrium path, and so it is difficult to detect untruthful reporting. If the receiver threatens with a costly delay as a function of the extent of the senders' disagreement and this threat induces truthful reports, then she has no incentive to carry out the punishment once she receives the senders' messages.

Despite these difficulties we prove that in our dynamic multi-sender cheap-talk model, under certain conditions, there exists an equilibrium with *imperfect revelation* and *positive expected delay*. This equilibrium is sustained by the senders not revealing everything they know all at once, thereby motivating the receiver to wait along the equilibrium path. The delay anticipated by the senders depends on the degree of their disagreement, which in turn provides the necessary incentives for them not to misrepresent their signals. As the senders' signals become arbitrarily precise the equilibrium outcome converges to the one found under common knowledge about the state, that is, complete disclosure and no delay.<sup>9</sup> We discuss whether and how the receiver's eagerness to learn the state of nature in comparison to the senders' biases and impatience matters for the existence of such equilibria. Our result shows that the no-delay, fully-revealing equilibrium under perfect observation is not an aberration as it may be the limit of sensible equilibria as the noise in the senders' signals vanishes.

These results shed light on whether and how delay—to which the decision maker cannot commit—can induce the experts to reveal more information. The insights are applicable in our first motivating example, legislation in Congress:

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<sup>8</sup>Models where the experts do not have common knowledge about the state of nature include Austen-Smith (1993), Wolinsky (2002), Battaglini (2004).

<sup>9</sup>The construction involves either private communication or the use of a public randomization device by the experts.

Repeated hearings and procedural delays in the absence of agreement may enhance communication and hence the quality of the median voter's decision.

Another similar, practical situation where our model applies is one in which a court needs to decide in a dispute between parties with opposing interests. The parties have information about the state, which the court wishes to match with its decision. The court can repeatedly ask the parties to provide (soft) information, but delays hurt the litigants as well as the court. The court cannot commit to costly delay; that can only be the result of the court's equilibrium beliefs that prolonged questioning will lead to information revelation and a socially beneficial outcome. Indeed it will, as our results show.<sup>10</sup>

A similar situation is that of a police interrogator questioning multiple suspects of a crime. The suspects have correlated information about the facts of the case, and the interrogator can ask them repeatedly if their stories differ. Repeated questioning decreases the suspects' utilities as they have to forego other activities while in police custody. However, most interrogators cannot explicitly commit to delay. We show that repeated questioning is indeed a sensible strategy for the interrogator.

In some of the applications it may be reasonable to assume that even in the absence of transfers, the decision maker could impose different waiting costs on the experts. For example, the interrogator could make one suspect's wait a lot less comfortable than another's—her tools may include anything from harassment to outright torture. While we do not explicitly allow this in our base model, the availability of such instruments would only strengthen our results.

As we already pointed out, our paper's main contribution is to the literature on multi-sender cheap talk games. This literature is partly motivated by applications in political theory, and focuses on equilibria and institutions that facilitate information transmission among experts and a decision maker.<sup>11</sup> In

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<sup>10</sup>A real-world example is the recent patent dispute between Research in Motion (RIM, maker of Blackberry phones) and NTP (a patent holding company). NTP sued RIM for patent infringement in January 2000. After a series of claims, counterclaims, court decisions and reversals, the parties settled in March 2006. Delay costs affected both parties (RIM risked losing business, NTP risked losing its patents to invalidation). Commentators found the settlement appropriate (see [http://money.cnn.com/2006/03/03/technology/rimm\\_ntp](http://money.cnn.com/2006/03/03/technology/rimm_ntp)).

<sup>11</sup>See also Austen-Smith (1990, 1993), Wolinsky (2002) and their references.

the static version of the model we study, in the special case of perfect observation, Battaglini (2002) and Ambrus and Takahashi (2007) derive necessary-sufficient conditions for the existence of fully-revealing equilibria. Aumann and Hart (2003) and Krishna and Morgan (2004) argue that multiple rounds of communication can in general expand the set of equilibrium outcomes. However, in the multi-sender cheap talk model that we study, under perfect observation, Ambrus and Takahashi (2007) show that large biases still make it impossible to sustain a fully-revealing equilibrium even with long cheap talk. In contrast, we obtain an *unconditional possibility* result in our baseline dynamic model, in the comparable case of perfect observation.

The explicit modeling of time and the possibility of costly delay in our model is reminiscent from dynamic bargaining models (see Serrano (2007) for an overview). However, our underlying game—multi-sender cheap talk—is rather different. Rubinstein and Wolinsky (1992) add a time dimension to a bilateral trading model and study renegotiation-proof contracts; Artemov (2006) considers Nash implementation with costly delay as a punishment device. Our problem is fundamentally different from these because the mechanism designer can commit to delay and other distortions, while the receiver in our cheap talk game cannot. A contemporaneous paper by Damiano, Li and Suen (2007) studies how the use of commitment to costly delay helps players improve efficiency in a dynamic voting game. Compared to these papers our main contribution is to show that the receiver can credibly carry out the delay even without commitment and that costly delay can be avoided in equilibrium as the senders’ signals become more precise.

Our dynamic model is marginally related to cheap talk models with money-burning (see Austen-Smith and Banks (2000), Kartik (2007)). Delay “burns” the payoffs of all parties, not just the senders’. More importantly, in our setup it is the receiver who can decide on delay, not the senders, and the receiver cannot commit to any period of delay. In contrast, in Austen-Smith and Banks (2000) it is the informed party (the sender) who can *signal* his type by committing to verifiably reduce his payoff. In that model, if the sender’s budget is unlimited, there exists an equilibrium where the sender signals a higher state of nature by burning more money, and all types separate. Our results are different, too:

In our setup there is no private or social loss on the equilibrium path when the senders have common knowledge about the state.

The paper is structured as follows. We set up the model in Section 2. The case where the experts commonly know the state is analyzed in Section 3. We discuss noisy signals in Section 4. Section 5 concludes, and an Appendix contains omitted proofs.

## 2 The model

**Information structure.** The state of nature,  $\omega$ , is a random draw from a closed set  $S \subseteq \mathbb{R}^n$ . There are two experts (senders) and one decision maker (receiver). The senders have private information about  $\omega$ . Denote the random variable that sender  $i$  observes by  $X_i$  and its realization by  $x_i$  for  $i = 1, 2$ ; for simplicity let the signals also belong to the state space,  $x_i \in S$ . Assume that the joint distribution of  $(\omega, X_1, X_2)$  is commonly known.

**Timing and feasible actions.** Time runs continuously from 0 to infinity,  $t \in [0, \infty)$ .<sup>12</sup> At every point in time, each sender (he) can send messages, after which the receiver (she) has the opportunity to take an action. Formally, at every physical point in time, there may be  $R + 1$  rounds of play with  $R \geq 1$ . Time can be thought of as a lexicographically ordered pair  $(t, r) \in \mathcal{T} = [0, \infty) \times \{0, \dots, R\}$ . At  $(t, r) \in \mathcal{T}$  with  $r < R$ , each sender  $i$  simultaneously sends a message  $m_i \in M_i$  either privately (to a designated player) or publicly. The message sets contain (at least) all elements of  $S$  and a ‘null’ message representing silence,  $M_i \supseteq S \cup \{\emptyset\}$ . At  $(t, R) \in \mathcal{T}$  the receiver moves by choosing  $y \in Y$ . The receiver’s action set includes all points in the convex hull of  $S$  and the ‘null’ action (doing nothing at that instant):  $Y \supseteq \text{co}(S) \cup \{\emptyset\}$ . Our assumptions on the receiver’s payoffs (see below) ensure that for any beliefs about the state, the receiver’s ideal point falls in the convex hull of  $S$ . As soon as the receiver picks an action other than inaction the game ends.

In what follows, for simplicity we set  $R = 1$ , i.e., assume a single round of communication (no long cheap talk) at each  $t$ . This does not affect our results.

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<sup>12</sup>We consider a discrete-time variant in Section 3; the results are essentially the same.

**Histories and strategies.** It is a known issue that in continuous-time games unrestricted strategies may not determine a unique outcome (see Bergin and Macleod (1993)).<sup>13</sup> The source of the problem is that in continuous time there is no “first moment” after any given point in time. The solution is to require that strategies exhibit “inertia”: A player must keep playing an action for a short time before switching.<sup>14</sup>

In light of this, we now define histories and strategies (which map histories to actions) so that within any finite interval of time, each player can change his action on only countably many occasions. Let  $h_i : [0, t) \rightarrow M_i$  for  $i = 1, 2$  the *public history* of sender  $i$ 's play up to time  $t > 0$ .<sup>15</sup> We call  $h_i$  admissible if  $h_i^{-1}(M_i)$  is a countable collection of intervals of the form  $[\tau, \tau') \subseteq [0, t)$ . Denote the set of all finite, admissible public histories by  $H$  and the restriction of  $h = (h_1, h_2) \in H$  to  $[0, t)$  by  $h|t$ . A (pure) *strategy* for sender  $i$  maps the realization of his signal and the public history to a message,  $f_i : S \times H \rightarrow M_i$ . We assume that  $f_i$  has *inertia*: For all  $x_i \in S$ ,  $t \geq 0$  and  $h \in H$ , if  $f_i(x_i, h|t) = m_i$ , then there exists  $\delta > 0$  such that  $f_i(x_i, h|\tau) = m_i$  for all  $\tau \in [t, t + \delta)$ . The receiver's strategy is  $f_0 : M_1 \times M_2 \times H \rightarrow Y$ . Recall that the game ends at the earliest time  $t$  such that  $y_0(m_1, m_2, h|t) \in Y \setminus \{\emptyset\}$ . Therefore, the inertia condition for the receiver is that for all  $t \geq 0$ ,  $h \in H$  and  $(m_1, m_2) \in M_1 \times M_2$ , if  $f_0(m_1, m_2, h|t) = \emptyset$  then there exists  $\delta > 0$  such that  $f_0(m'_1, m'_2, h|\tau) = \emptyset$  for all  $\tau \in [t, t + \delta)$  and  $(m'_1, m'_2) \in M_1 \times M_2$ . In words, the receiver cannot terminate the game at the “first moment” after a certain time. Denote  $F_i$  the set of inertia strategies for player  $i$  ( $i = 0$  refers to the receiver,  $i = 1, 2$  to the senders), and let  $F = \prod_{i \in \{0, 1, 2\}} F_i$ .

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<sup>13</sup>To see this, suppose that each sender observes the state and uses the following strategy : “Play  $m_i = \emptyset$  at  $t = 0$  and as long as both senders were silent for all  $t' < t$ ; send  $m_i = \omega$  if either sender has sent  $\omega$  at any  $t' < t$ .” Assume the receiver's strategy is to choose inaction at  $(t, 1)$  if the senders were silent at  $(t, 0)$ , and implement  $y = m_1$  if they agreed on a  $\omega \in S$  at  $(t, 0)$ . Then, for any  $t^* > 0$ , both senders playing  $m_i = \emptyset$  for all  $t \in [0, t^*]$  and reporting  $\omega$  for all  $t > t^*$  is consistent with their strategies making the outcome indeterminate.

<sup>14</sup>The length of the “short period” can depend on the public history, the player's signal and action, as in Bergin and Macleod (1993). Somewhat stronger restrictions are imposed in the continuous-time bargaining games of Perry and Reny (1993, 1994).

<sup>15</sup>It is unnecessary to introduce notation for the history of the receiver's play because the game continues only if she plays  $y = \emptyset$  and it stops as soon as she picks any  $y \neq \emptyset$ .

The assumption that the strategies have inertia does not mean the players can commit to future actions (e.g., maintaining their current actions for a given period of time). This is so because  $F_i$  contains *every* strategy with arbitrarily small inertia ( $\delta > 0$ ). Whether or not commitment is feasible is determined by the equilibrium concept, which is discussed below. The inertia assumptions are made purely for technical reasons, in order to guarantee that any combination of strategies  $f = (f_0, f_1, f_2) \in F$  determines a unique outcome.<sup>16</sup> We denote the play induced by the strategy-tuple  $f$  by  $\bar{h}(f)$ , the physical time when it terminates by  $\bar{t}(f) \in [0, \infty]$ , and the receiver's action at  $\bar{t}(f) < \infty$  by  $\bar{y}(f)$ .

**Payoffs.** The players' payoffs (received when the game ends) depend on the state, the receiver's game-ending action and the physical time of termination:  $V_i(\omega, f) = U_i(\omega, \bar{y}(f), \bar{t}(f))$  for  $i = 0, 1, 2$ . The payoffs are normalized to  $-\infty$  for  $\bar{t}(f) = \infty$ . Otherwise  $U_i$  is bounded, concave and single-peaked in  $y$ ; moreover, preferences over  $y$  only depend on  $\omega$ : for all  $\omega \in S$ ,  $y, y' \in Y$  and  $t, t' \geq 0$ ,  $U_i(\omega, y, t) - U_i(\omega, y', t) = U_i(\omega, y, t') - U_i(\omega, y', t')$ . For notational simplicity we identify the receiver's ideal point with the state,  $y_0(\omega) = \omega$ , and denote sender  $i$ 's ideal point by  $y_i(\omega)$  for  $i = 1, 2$ . Note that no matter what the receiver believes about the state, if she decides to end the game, she picks some element of the convex hull of  $S$ .

All parties have strict time preferences:  $\partial U_i(\omega, y, t)/\partial t < 0$ , that is, players prefer a given action to be carried out earlier. The time preference can be the result of exponential or hyperbolic discounting of a positive utility function, for example  $U_i = u_i(\omega, y)e^{-\rho t}$  where  $u_i > 0$  is single-peaked at  $y = y_i(\omega)$ , or equivalently, due to a cost of waiting, e.g.,  $U_i = -(y - y_i(\omega))^2 - \rho t$ ,  $\rho > 0$ .

We assume that for any two actions  $y$  and  $y'$  such that player  $i$  prefers  $y$  over  $y'$  with no delay,  $U_i(\omega, y, t) > U_i(\omega, y', t)$ , there exists a delay such that  $y$  with delay is equivalent to  $y'$  without delay:  $U_i(\omega, y, t + \Delta) = U_i(\omega, y', t)$  for some  $\Delta > 0$ . That is, a sufficiently long delay can compensate for any utility difference over immediate actions.

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<sup>16</sup>At time 0, the play is uniquely determined by inertia for some period  $[0, \delta)$ . If  $t^*$  is the supremum of  $t$  such that play is uniquely determined on  $[0, t)$ , then by inertia there is a unique continuation until  $t^* + \delta' > t^*$ , which implies  $t^* = \infty$ . For more details see Bergin and Macleod (1993) and Perry and Reny (1993).

We define a couple of quantities related to the players' preferences (or trade-offs) over outcomes and delay.  $\Delta_0$  is the maximum delay that the receiver is willing to bear in order to learn the senders' signals and carry out her ideal point contingent on the state:

$$E[U_0(\omega, \omega, \Delta_0)] = \max_{y \in Y} E[U_0(\omega, y, 0)]. \quad (1)$$

$\Delta_0$  is increasing in the receiver's patience and the variance of the prior distribution of  $\omega$ . For example, if the receiver's utility is represented by a quadratic loss function and a linear cost of delay, then  $\Delta_0$  is proportional to the variance of the prior on  $\omega$ . Let  $\Delta_i(\omega)$  denote the (state-contingent) willingness to wait of sender  $i$  in order to obtain his ideal point instead of the receiver's:

$$U_i(\omega, y_i(\omega), \Delta_i(\omega)) = U_i(\omega, \omega, 0). \quad (2)$$

For example, if the sender's ideal point differs from the one-dimensional state of nature by a constant  $b_i$ , has a quadratic loss function and a linear waiting cost, then  $\Delta_i(\omega)$  is simply proportional to  $b_i^2$ .

**Equilibrium concept.** In the game defined above we study perfect Bayesian equilibria (PBE). Formally,  $f^* \in F$  is a PBE if for all  $t \in [0, \infty)$ ,  $i \in \{0, 1, 2\}$  and  $f_i \in F_i$ ,  $E[V_i(\omega, f^*)|X_i, t] \geq E[V_i(\omega, (f_i, f_{-i}^*))|X_i, t]$ , with the convention  $X_0 = \emptyset$ . More practically, perfection requires that at any point in time, given the player's private information and the history of the play, no player has an incentive to deviate from his or her prescribed strategy. No player can commit to any action in the future; in particular, *the receiver cannot commit to delay*.

In the next section we first consider a model in which each sender observes the state of nature ( $X_i \equiv \omega$ ) and the message space is rich enough for the transmission of the realization of  $\omega$ . We show the existence of an equilibrium in which the state of nature is immediately revealed by the senders and so the receiver carries out  $y = \omega$  without delay. Subsequently, we investigate the robustness of this fully-revealing equilibrium to perturbations of the model. In particular, we investigate the case where the senders' private information stems from noisy signals about the state of the world.

### 3 The perfect observation case

In this section we assume that each sender observes the state of nature. We look for the most informative PBE; in particular, we determine whether or not there exists a *fully-revealing equilibrium*, where the senders report  $\omega$  and the receiver carries out  $y = \omega$ . For comparison, we first review the benchmark results in the *static* version of the game. There, Battaglini (2002) and Ambrus and Takahashi (2007) identified a necessary-sufficient condition for the existence of a fully-revealing equilibrium. After discussing this result we turn to our unconditional possibility result in the dynamic version of the same game. We also discuss the continuity of off-equilibrium beliefs and the issue of discrete time.

#### 3.1 The static benchmark

Assume, in Section 3.1 only, that the entire game is played at a single point in time, that is,  $t \equiv 0$ , and for now assume there is a single round of communication. All other aspects of the game are the same as described in Section 2. The senders observe  $\omega$ , then simultaneously send messages  $m_i \in M_i \supseteq S$ , finally the receiver picks an action  $y \in co(S)$ . The question posed in the literature is whether there exists a perfect Bayesian equilibrium in which, by comparing the senders' messages, the receiver can infer the true state.

Denote the set of receiver-actions that sender  $i$  weakly prefers to  $y = \omega$  in state  $\omega$  by  $B_i(\omega)$ . With the notation introduced in Section 2,  $B_i(\omega) = \{y \in co(S) : U_i(\omega, y, 0) \geq U_i(\omega, \omega, 0)\}$ ,  $i = 1, 2$ . The exact condition for the existence of a fully-revealing perfect Bayesian equilibrium is the following.

**Proposition 1 (Battaglini (2002) and Ambrus and Takahashi (2007))**  
*A fully-revealing perfect Bayesian equilibrium exists in the static game if, and only if, for all  $\omega, \omega' \in S$ ,  $co(S) \not\subseteq B_1(\omega') \cup B_2(\omega)$ .*

This result is due to Battaglini (2002) for one-dimensional state spaces; Ambrus and Takahashi (2007) observed that it is true irrespective of the dimensionality of the state space. The intuition is simple. Consider a fully-revealing equilibrium and suppose that sender 1 sends a message consistent with the state

being  $\omega$  while sender 2 reports as if the true state were  $\omega'$ . The receiver's response to such inconsistent reports, say  $y(\omega, \omega')$ , must lie in  $co(S)$ . However, if  $B_1(\omega') \cup B_2(\omega)$  contains  $co(S)$  then no matter what  $y(\omega, \omega')$  is, at least one of the senders has an incentive to deviate: If  $y(\omega, \omega') \in B_1(\omega')$  then sender 1 prefers to pretend it is state  $\omega$  in state  $\omega'$ , while if  $y(\omega, \omega') \in B_2(\omega)$  then sender 2 prefers to report  $\omega'$  in state  $\omega$ . Therefore there is no fully-revealing equilibrium. In contrast, if the condition in Proposition 1 holds, then it is easy to construct an fully-revealing equilibrium: The senders announce the state and, if they agree, the receiver carries out  $y = \omega$ ; if they disagree (sender 1 reports  $\omega$  while sender 2 reports  $\omega'$ ) then the receiver picks any  $y \in co(S) \setminus (B_1(\omega') \cup B_2(\omega))$ .

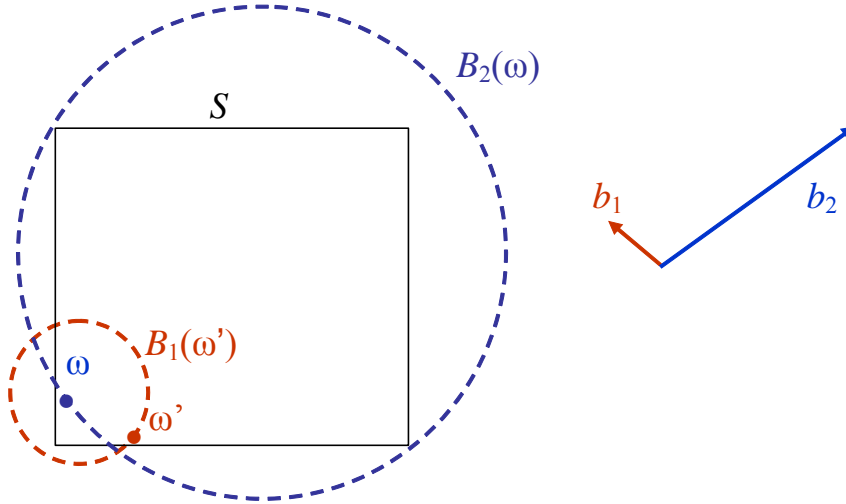


Figure 1: Non-existence of fully-revealing equilibrium in the static game

While the receiver can use the fact that the senders must be able to agree on  $\omega$ , there is a limitation on her ability to cross-check their reports in the static game. If the senders' biases are sufficiently large relative to the state space (like in Figure 1), then it is impossible for the receiver to punish them both upon a disagreement, and so there does not exist a fully-revealing equilibrium. It is important to note that the dimensionality of the state space makes no difference to the analysis.

Multiple rounds of communication (at a single physical point in time, i.e., without costly delay) can expand the set of equilibria, as pointed out by Aumann and Hart (2003), Krishna and Morgan (2004), and, for the multi-sender cheap talk model that we study here, Ambrus and Takahashi (2007). However, Ambrus and Takahashi (2007) also show that in bounded state spaces, for biases that are sufficiently large, fully-revealing equilibria do not exist even if long cheap talk is allowed.

Recall that our setup differs from existing models of cheap talk with one or many rounds of communication in that (i) the receiver can strategically time her decision and allow more or less time for communication, and (ii) time delays are costly. The timing of the receiver’s action could be thought of as just another dimension of her decision problem. However, it is commonly known that the receiver prefers any given action to be carried out earlier, no matter what the other dimensions of the state are—in other words, the “time-coordinate” of the state of nature is commonly known to be zero, even though positive delays belong to the feasible set of actions. Therefore, in the static game with an additional dimension corresponding to “time”, the receiver cannot rationally choose (on or off the equilibrium path) a positive delay. This means that introducing a new dimension to the *static* problem with the properties of “time”—a costly dimension along which the state and all players’ ideal points are commonly known to be zero—would not help the existence of a fully-revealing equilibrium.

Next, we show that in a truly *dynamic* version of the game, the possibility of costly delay (which no one can commit to) does make a difference.

### 3.2 Unconditional possibility in the dynamic game

The main result of this section is that in our dynamic multi-sender cheap talk game there exists an equilibrium where all information commonly known by the senders is immediately revealed. The equilibrium that we construct has other attractive features as well, for example, the receiver’s beliefs are continuous in the senders’ reports at time 0. We discuss these features after the statement and proof of the formal result.

**Proposition 2** *Assume that the senders perfectly observe the state:  $X_1 = X_2 = \omega$ . Then there exists a perfect Bayesian equilibrium where the state of the world is immediately, publicly reported by both senders, and the receiver carries out  $y = \omega$  without delay.*

**Proof.** We construct the equilibrium as follows. For  $i = 1, 2$ , sender  $i$  publicly reports  $m_i = \omega$  in round 0 of time 0, and continues to do the same for all  $t > 0$  provided that both senders reported  $\omega$  at time 0. If the senders send the same message  $m^* \in S$  at time 0 (i.e., do not deviate at time 0), then the receiver carries out  $y = m^*$  in round 1 of all  $t \geq 0$ .

Now we describe the players' off-equilibrium actions and supporting beliefs in case the senders report messages  $\hat{m}_1 \neq \hat{m}_2$  at  $t = 0$ . Denote  $\delta(\hat{m}_1, \hat{m}_2)$  the receiver's willingness to wait in order to find out the exact value of  $\omega$  given that she believes the state is either  $\hat{m}_1$  or  $\hat{m}_2$  with equal probabilities:

$$U_0(\omega, \omega, \delta(\hat{m}_1, \hat{m}_2)) = \max_{y \in Y} E[U_0(\omega, y, 0) \mid \Pr(\omega = \hat{m}_i) = 1/2, i = 1, 2].$$

Define  $T^* \equiv \{\tau \mid \tau = \lambda\delta(\hat{m}_1, \hat{m}_2)/2, \lambda \in \mathbb{Z}^+\}$ . If the senders report messages  $\hat{m}_1 \neq \hat{m}_2$  at time 0, then for any history  $h \in H$  and  $t \in T^*$  they both report  $\omega$  at  $t$ , while for all  $h \in H$  and  $t \notin T^*$  they both "babble" at  $t$  (send random, feasible messages subject to the inertia-constraints). For any history such that the senders report messages  $\hat{m}_1 \neq \hat{m}_2$  at time 0, if  $t \in T^*$  and the senders report the same  $m^* \in S$ , then the receiver believes that  $\omega = m^*$ ; otherwise she believes  $\omega = \hat{m}_1$  or  $\omega = \hat{m}_2$  with equal probabilities. If the senders' messages disagree at time 0, then the receiver chooses momentary inaction at all  $t \geq 0$ , unless  $t \in T^*$  and both senders report  $m^* \in S$  at  $t$ , in which case she picks  $y = m^*$ .

If all play the proposed strategies, then play ends at time 0 and player  $i$  gets a payoff of  $U_i(\omega, \omega, 0)$ ,  $i = 0, 1, 2$ . If the senders report  $\hat{m}_1 \neq \hat{m}_2$  at  $t = 0$ , then the receiver believes  $\omega \in \{\hat{m}_1, \hat{m}_2\}$  with equal probabilities, and that she will learn the truth at exactly  $t = \delta(\hat{m}_1, \hat{m}_2)/2 < \delta(\hat{m}_1, \hat{m}_2)$ , hence it is optimal for her to choose inaction for all  $t' \in [0, \delta(\hat{m}_1, \hat{m}_2)/2)$ . If the senders agree on some  $m^* \in S$  at  $t = \delta(\hat{m}_1, \hat{m}_2)/2$  or any other  $t \in T^*$ , then, according to the receiver's beliefs, it is optimal for her to carry out  $y = m^*$ . If the senders disagree at  $t = \delta(\hat{m}_1, \hat{m}_2)/2$  or any other  $t \in T^*$ , then the receiver continues to

believe that  $\omega \in \{\hat{m}_1, \hat{m}_2\}$  with equal probabilities and that she will learn the truth  $\delta(\hat{m}_1, \hat{m}_2)/2$  time later. Hence it is optimal for her to wait for the next time when she expects the senders to agree.

If the senders disagree at time 0, then in the continuation they expect each other to report  $\omega$  at all  $t \in T^*$  and the receiver to ignore all messages sent at all  $t \notin T^*$  as babble. Therefore neither sender can induce any outcome other than  $y = \omega$  at any  $t \in T^*$ , and their best response is to report  $\omega$  at all  $t \in T^*$ . ■

Since the equilibrium in Proposition 2 exhibits no delay on the equilibrium path, the outcome of our dynamic game is *observationally equivalent* to a fully-revealing equilibrium in a corresponding static environment. As we discussed it in Section 3.1, a fully-revealing equilibrium only exists under certain conditions in the static environment, even with multiple rounds of communication. In contrast, our result regarding immediate full disclosure holds *without any restriction* on the shape or size of the state space, nor does it depend on the direction or size of the senders' biases.

The idea behind the immediate, fully-revealing perfect-Bayesian equilibrium in our dynamic game is the following. In equilibrium, the senders report the state of nature and the receiver picks an action equal to the senders' report right away. In case of disagreement at time 0, the receiver believes the senders will agree "in a moment", which induces her to choose momentary inaction. Knowing that the receiver is always waiting for an imminent agreement, and that the other sender keeps reporting the state, each sender's best response is to tell the truth, which avoids costly delays.

The construction in the proof of Proposition 2 is more involved because of the complications in modeling continuous-time dynamic games. Unrestricted strategies do not always determine a unique play in such games (in particular in our game, as we pointed out in Section 2), therefore we had to restrict the players' strategies in a way that they exhibit inertia.<sup>17</sup> In particular, this restriction implies that if the senders disagree at  $t = 0$ , they cannot suddenly agree for all  $t > 0$ , therefore the receiver cannot rationally believe that she

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<sup>17</sup>It is important to stress that the inertia-assumption is purely technical, to make our game well-defined. The construction of a fully-revealing equilibrium does not exploit this assumption, in fact, inertia only makes the arguments more difficult.

will learn the state *immediately* following an initial disagreement. This issue is resolved by letting the receiver believe that she will learn the true state at a particular point in time after the disagreement, which she is willing to wait for given her beliefs about the state. The senders know that the receiver only pays attention to them at that particular point in time, and only believes them if they agree. Since they expect each other to report truthfully at that time (and babble otherwise), the best they can do once they are off the equilibrium path is to report truthfully when the receiver expects them to do so.

An additional attractive feature of our construction is that in case the senders messages differ only slightly at time 0, then the receiver believes that the true state is also in the neighborhood of the reports. In particular, she believes that if the senders disagree at time 0, then *exactly one of them is truthful*. This implies that the receiver's beliefs, and consequently the overall outcome of the game, do not change discontinuously as a result of a small deviation by the senders from the equilibrium path at time 0, provided that all players follow their equilibrium strategies from then on. This notion of continuity is different from some notions used in the static literature, e.g., Ambrus and Takahashi's (2007) diagonal continuity.<sup>18</sup> However, the most interesting robustness test is not this, but whether fully-revealing equilibria can be approximated even if the senders' signals are noisy, which we investigate in Section 4.

It may be obvious to point out that our unconditional possibility result relies on the unboundedness of the time horizon. If there exists a last point in time by which the receiver has to make a decision, then the sender(s) may be better off by silently waiting until then rather than divulging any information about the state. Of course, our results remain valid if either the stopping time is random (and the probability of continuation is sufficiently high), or if the senders grow infinitely impatient as the end of time approaches.

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<sup>18</sup>Diagonal continuity of an equilibrium requires that the equilibrium outcome be continuous in the senders' observations of the state as they approach perfect observation. Intuitively, this means that the players' strategies remain equilibrium even if the senders' signals are a bit noisy, but all players (incorrectly) commonly believe that the senders perfectly observe the state. Ambrus and Takahashi (2007) prove that in the static version of the game (with additional restrictions), diagonally-continuous fully-revealing equilibria generically do not exist.

Proposition 2 shows the existence of a fully-revealing equilibrium, but there are other informative equilibria of the game as well. For example, any informative equilibrium of the corresponding static game can be sustained in our dynamic setup. The set of equilibria in the static model are not characterized in the literature (the existing papers also focus on fully-revealing equilibria), but it is easy to see that any equilibrium of a corresponding single-sender static cheap-talk game would work with two senders as well because both senders observe the same state of nature. The set of equilibria in the single-sender static game has been characterized for one-dimensional state spaces (Crawford and Sobel (1982)), but it remains an open research topic for multi-dimensional state spaces (Chakraborty and Harbaugh (2007) is the most recent and significant contribution). It is clear that more research needs to be done regarding the set of all equilibria in single- or multi-sender cheap talk games, especially for multi-dimensional state spaces.

The equilibrium constructed in Proposition 2 is striking because it works under very general conditions, without restrictions on the state space, the senders' biases, or anyone's time preferences. In the equilibrium construction we only used public messages. It is also easy to see that the same equilibrium works even if the receiver is uninformed about the senders' biases. In the rest of the section (in fact, the rest of the paper) we discuss whether the outcome, instantaneous full revelation, is robust to certain other perturbations of the model.

### **3.3 Discrete time**

We continue to assume that both senders observe the state, but let time be discrete. Discrete time makes it simpler to construct a fully-revealing equilibrium, but it does not allow the receiver to believe off the equilibrium path that the senders will agree “almost immediately” (in a fraction of a period) in case their reports are only slightly different at time 0. In continuous time this feature ensures that the receiver's beliefs about the state are continuous in the senders' reports at time 0. We show how to modify the equilibrium construction in order to restore the continuity of the receiver's beliefs at time 0 in discrete time.

For the remainder of the section, we assume that there are infinitely many

discrete time periods indexed by  $t = 0, 1, \dots$ . In each period, first the senders simultaneously send messages  $m_i \in M_i$  for  $i = 1, 2$ , then the receiver makes a decision,  $y \in Y$ , where  $y = \emptyset$  corresponds to inaction in the current period. If the receiver picks an action other than  $\emptyset$  at time  $t$  then the game ends and the players receive payoffs  $U_i(\omega, y, t)$ ,  $i = 0, 1, 2$ , with the properties discussed in Section 2. Histories and strategies are defined in the standard way; we do not need any restrictions (e.g., inertia) in discrete time. We only consider perfect Bayesian equilibria; that is, assume that no player can commit to future actions, in particular, the receiver cannot commit to delay.

The following result is the counterpart of Proposition 2 in discrete time.

**Proposition 3** *Assume that time is discrete,  $t \in \mathbb{N}$ , and that the senders observe the state,  $X_i \equiv \omega$  for  $i = 1, 2$ . If  $\Delta_0 \geq 1$  then there exists a perfect Bayesian equilibrium where the state of the world is immediately, publicly reported by both senders, and the receiver carries out  $y = \omega$  without delay.*

**Proof.** The equilibrium is constructed as follows. Sender  $i$  reports  $m_i = \omega$  in all periods, irrespective of the history. At any time such that the senders' reports about the state agree, the receiver picks the action corresponding to their matching reports. At any time such that the senders reports differ, the receiver chooses inaction. The outcome of these strategies is immediate, truthful revelation and  $y = \omega$  at  $t = 0$ .

Neither sender can do better than to report  $\omega$  after any history because the other sender is truthful and the receiver waits until their reports agree. In case of a deviation, assuming that the receiver does not update the prior about the state, the receiver's best response is to wait one period. This is so because she expects to learn the state in the next period and  $E[U_0(\omega, \omega, 1)] > \max_{y \in Y} E[U_0(\omega, y, 0)]$  by  $\Delta_0 \geq 1$  and equation (1). ■

Proposition 3 establishes that when the receiver is willing to wait one “tick of the clock” in order to make a fully-informed decision, there is a fully-revealing equilibrium. The condition  $\Delta_0 \geq 1$  is satisfied if the discrete-time period lengths are sufficiently short (the discount rate is sufficiently high), or if the prior distribution of  $\omega$  is “sufficiently diffuse” (the prior has a sufficiently large variance).

The main difference between the equilibrium construction here and in the proof of Proposition 2 is that in the discrete-time game, the receiver does not update her beliefs about the state following mismatching reports at time 0, while in the continuous-time game, she believes exactly one of the senders reported the true state. (We could not accommodate such an updating rule in discrete time because the delay cannot be less than one period, while  $\delta(\hat{m}_1, \hat{m}_2)$  may be less than one.) This implies that the receiver's beliefs about the state at time 0 are not continuous near equilibrium in the senders' reports.

The problem can be overcome by modifying the equilibrium construction. For simplicity, assume that in each period there are two rounds of communication and that  $\omega$  is uniform on  $[0, 1]$ . Decompose  $\omega$  into  $(\omega_A, \omega_B) \in \{0, 1\} \times [0, 0.5]$  such that  $\omega \equiv 0.5\omega_A + \omega_B$ . Consider the following strategies: In the first round of talk of period 0, and in the first round of talk of each period until their reports match, each sender reports  $\omega_B \in [0, 0.5]$ . In the second round of communication of the period when their reports first match, and from then onwards, the senders report  $\omega_A \in \{0, 1\}$ . The receiver chooses inaction until the senders report the same number in  $[0, 0.5]$  and subsequently the same number in  $\{0, 1\}$ ; when the senders agree on both, she infers  $\omega = \omega_A + \omega_B$  and carries out  $y = \omega$ .

If the receiver is sufficiently patient—more precisely, if she is willing to wait one period to find out whether  $\omega = \omega_B$  or  $0.5 + \omega_B$  conditional on knowing  $\omega_B$ —then these strategies form a fully-revealing equilibrium with no delay such that the receiver's beliefs are continuous in the senders' reports at time 0. To see this, suppose that in period 0 the senders' reports are almost (but not exactly) equal, and assume that the receiver believes one of the senders is truthful (ensuring the continuity of beliefs). Although the receiver can essentially infer  $\omega_B$ , she still does not know whether  $\omega = \omega_B$  or  $0.5 + \omega_B$ . She anticipates that in the following period the senders will both report  $\omega_B$ , and then immediately  $\omega_A$ , so she will learn  $\omega$  by the end of that period. Therefore, if the receiver is willing to wait one period in order to learn  $\omega$  given that she knows  $\omega_B$ , then it is rational for her to choose inaction as long as the senders disagree about  $\omega_B$ .

Perhaps the most interesting issue is to what extent the full-revelation result is robust to the introduction of noise in the senders' signals. We study this in depth in the next section.

## 4 Delay and disclosure with imperfect signals

In this section we investigate the robustness of the possibility result obtained in the perfect-observation model by relaxing the assumption that the senders have common knowledge about the state of nature. In the environment that we shall study, the probability that the senders' signals exactly match (conditional on the realization of the state) is *zero*, and at the beginning of the game it is not commonly known by the senders what the true state is. As a result, equilibrium constructions that rely on the receiver knowing that the senders must be able to agree break down. However, we construct an equilibrium where the senders reveal their signals imperfectly, and there is a positive expected delay along the equilibrium path.<sup>19</sup> We show that as the senders' signals become arbitrarily precise, the outcome of this equilibrium converges to full revelation and no delay—the outcome of the equilibrium found under perfect observation.

The difficulty of constructing this type of equilibrium is the following. In the model with noisy signals the senders' signal reports disagree even if they are truthful. The receiver could threaten the senders with delay as a function of the extent of their disagreement, which may provide the right incentives for them to report their information honestly. However, once the threat of delay induces the senders to be truthful, the receiver has no incentive to carry out any promised delay, and so this type of construction seems unworkable.

We overcome these problems using two “tricks”. First, at time 0, the senders do not disclose their signals publicly, only whether or not they are sufficiently close to each other. They can do so either by “encoding” their signals, or by exchanging private messages before the game starts. Therefore, if they report that their signals are substantially different, the receiver still has an incentive to wait in order to find out exactly how they differ. Second, the delay triggered by substantial disagreement is enforced by the senders themselves: They are both expected to babble during the delay phase, making a profitable individual deviation impossible.

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<sup>19</sup>We show that such equilibrium exists under certain conditions on the biases and patience of the senders. Our equilibrium construction uses either private communication or a certain type of randomization on the part of the senders.

## 4.1 A model with noisy signals

We define a family of information structures where the senders observe correlated private signals about the state of nature. The state of nature is never commonly known by the senders; their signals coincide with probability zero (even conditional on the state). The information structures in this family are naturally (partially) ordered according to the precision of the senders' signals.

Assume that the state space is a sufficiently fine, finite subset of the unit interval. In particular, for a fixed  $N$ , let

$$S = \{\omega_k = (2k - 1)/2^{N+1} \mid k = 1, \dots, 2^N\}.$$

Assume there is a common prior over the possible states in  $S$ . Denote  $a_0 = 0$ ,  $a_k = (\omega_k + \omega_{k+1})/2$  for  $k = 1, \dots, 2^N - 1$ ,  $a_{2^N} = 1$ . Each possible state  $\omega_k$  is the midpoint of the interval  $(a_{k-1}, a_k)$ ,  $k = 1, \dots, 2^N$ . While we assume that the state space is discrete, this is not a great restriction (yet technically convenient, as we shall see below) because the state distribution can approximate any distribution on  $[0, 1]$  arbitrarily closely when  $N$  is sufficiently large.

We assume that the senders' signals,  $(X_1, X_2)$ , are random variables with support  $[0, 1]^2$  and an atomless joint distribution. This guarantees that truthful reports of the signals match with zero probability, and, most importantly, that no realization of  $X_i$  rules out any realization of  $X_j$ . Our analysis remains unchanged (the arguments only become simpler) if the joint distribution of  $(X_1, X_2)$  has an atom or does not have full support.

Let  $\varepsilon$  be the lowest positive number such that the probability of the state and the other sender's signal belongs to  $(a_{k-1}, a_k)$ , conditional on one sender's signal also belonging to  $(a_{k-1}, a_k)$ , is at least  $1 - \varepsilon$ :

$$\varepsilon = \inf \left\{ \varepsilon' : \Pr \left( (\omega, X_j) \in (a_{k-1}, a_k)^2 \mid X_i \in (a_{k-1}, a_k) \right) \geq 1 - \varepsilon', \forall i \neq j, k \right\}. \quad (3)$$

(We omit the natural range of indices such as  $i, j \in \{1, 2\}$ ,  $k \in \{1, \dots, 2^N\}$  for the sake of brevity.) Note that (3) is not an assumption, just a *definition* of  $\varepsilon$ . All joint distributions of  $(X_1, X_2)$  conditional on  $\omega$  are partially ordered by the index  $\varepsilon$ . The smaller the index  $\varepsilon$ , the more precise the senders' signals are.

We make two assumptions on the information structure. First, we assume that nothing can be inferred about the state of nature from the fact that the senders' signals fall in different intervals:

$$\forall q, \Pr(\omega = \omega_q \mid X_1 \in (a_{k-1}, a_k), X_2 \in (a_{\ell-1}, a_\ell), \ell \neq k) = \Pr(\omega = \omega_q). \quad (4)$$

Second, we assume that *if* the senders' signals fall in different intervals, then it is most likely that they fall in adjacent ones:

$$\Pr(|\ell - k| = 1 \mid X_1 \in (a_{k-1}, a_k), X_2 \in (a_{\ell-1}, a_\ell), \ell \neq k) > 1 - \varepsilon. \quad (5)$$

Beyond making these two, rather mild assumptions we do not specify the joint distribution of  $(\omega, X_1, X_2)$  in more detail because that is not necessary for obtaining our results.

Our two assumptions have the following useful implications. By equation (4), if *all* that the receiver learns from the senders' messages is that their signals fall in different intervals, then her beliefs about the state of nature *remain unchanged* (she does not update her prior beliefs). Furthermore, equation (5) implies that the receiver knows, in case there is a disagreement between the senders, it is likely to be small—the signals are most likely to fall in adjacent intervals. If the signal structure is sufficiently precise ( $\varepsilon$  is small), then the true state is likely to be in one of those intervals, too. Therefore, if the state space is also sufficiently dense ( $N$  is large), then learning exactly which (adjacent) intervals the senders' signals fall into is almost equivalent to learning the state of nature. The receiver's expected loss from not knowing which of the two intervals the state falls into becomes negligible for sufficiently small  $\varepsilon$  and large  $N$ . As a result of these two conditions, if the state space is sufficiently dense and the signals are sufficiently precise, then the receiver's willingness to wait to find out which intervals the senders' signals fall into, conditional on knowing that they *do* fall in different intervals, is approximately  $\Delta_0$ .

The sole purpose of assuming (4) and (5) is to ensure that the receiver is willing to incur a delay of approximately  $\Delta_0$  in order to find out how the senders' signals differ (which intervals they fall into) conditional on already knowing that they do.

Information and signal structures that belong to a family described above can be indexed by  $N$  and  $\varepsilon$ , the fineness of the state space and the precision of the senders' signals. When we say that the signal structure converges to complete information regarding  $\omega$  between the senders, we mean that  $\varepsilon$  goes to zero for a given  $N$ .

The signal structure proposed here exhibits the characteristics mentioned at the beginning of the section: There is a positive probability that the senders' signals differ by any amount even conditional on the realization of the state. It is true that as  $\varepsilon$  approaches 0 the senders are almost sure that they observed signal realizations in the same interval  $[a_{k-1}, a_k)$ , but it never becomes commonly known between the two senders that their signals belong to the same partition element. The particular assumptions we made regarding the signal structure (the senders observe a noisy signal of a discrete  $\omega$ ) help in that they provide a natural partition with respect to which the senders' signals can be reported.

## 4.2 Results in the model with noisy signals

Recall from Section 2 that  $\Delta_0$  denotes the amount of delay that finding out the state of nature is worth for the receiver, and that  $\Delta_i(\omega)$  is sender  $i$ 's willingness to wait in order to induce his ideal point instead of the state of nature,  $\omega$ . Define the scalar  $\Delta_i$  as the sender's maximum possible willingness to wait:

$$\Delta_i = \max_{\omega \in S} \{ \Delta_i(\omega) \mid U_i(\omega, y_i(\omega), \Delta_i(\omega)) = U_i(\omega, \omega, 0) \}. \quad (6)$$

Our first result is that if the receiver's maximum willingness to wait exceeds those of the senders and the signal structure is sufficiently precise, then there exists an equilibrium where the senders truthfully report the intervals  $[a_{k-1}, a_k)$  that their signal realizations fall into. This equilibrium involves a positive expected delay. However, for  $N$  sufficiently large, as the signal structure converges complete information regarding  $\omega$  between the senders, the information obtained by the receiver becomes arbitrarily precise regarding the state, and the expected delay tends to zero.

In order to prove this result we assume that the senders can exchange private messages at the interim stage, when they already know the realizations of their

signals but “time” has not started yet.<sup>20</sup> In a subsequent proposition, we shall relax this assumption and show that the same outcome can be implemented even if only one sender has the opportunity to send a private message to the other at the interim stage, or if the senders have access to a certain type of randomization device.

**Proposition 4** *Assume that the senders can simultaneously exchange a private message after they have observed their signals, but before time starts. If  $N$  is sufficiently large and  $\Delta_0 > \max\{\Delta_1, \Delta_2\}$ , then for  $\varepsilon$  sufficiently close to zero, there exists an equilibrium where the receiver learns which interval of the form  $[a_{k-1}, a_k)$ ,  $k = 1, \dots, 2^N$ , each sender’s signal realization falls into and chooses  $y$  optimally subject to this information. The equilibrium expected delay is positive and tends to zero as  $\varepsilon \rightarrow 0$ .*

**Proof.** See the Appendix. ■

The equilibrium construction in the proof of Proposition 4 consists of three steps. First, at the stage when the senders know their signals but time has not started, the senders simultaneously report to each other which possible realization of  $\omega$  their signal falls closest to. These announcements are truthful on the equilibrium path. Second, if the senders’ private reports do not match, then they both babble until a certain point in time,  $t = d$ , which is greater than  $\max\{\Delta_1, \Delta_2\}$  but less than  $\Delta_0$ . During this period of time the receiver infers that the senders’ signals fall closest to different possible realizations of  $\omega$ , but she is willing to wait until  $d$  in order to find out exactly which are those values. Third, if and when the delay is over, the senders simultaneously report the receiver’s ideal point conditional on their (imprecisely-shared) information. If their reports agree, the receiver carries out their recommendation. Otherwise she waits a short amount of time, expecting that the senders will report the “truth” periodically and babble in between. At this point the senders have mutual knowledge about their initial private reports, hence they make the required disclosure immediately and truthfully just like they do in the perfect-observation case (Proposition 2).

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<sup>20</sup>Wolinsky (2002), in a very different model, argues that allowing private communication among the senders can help the decision maker in eliciting information from them.

The significance of Proposition 4 is that it demonstrates the robustness of the full revelation, no-delay equilibrium outcome to a small noise in the senders' signals. Notice that there is *no common knowledge* about the state of the world between the two senders, no matter how small the noise in their signals is. In other words, the receiver is unable to detect lying (deviation from the equilibrium path) with probability 1. Nevertheless, the equilibrium outcome with noisy signals converges to the perfect-observation limit as the noise vanishes.

The construction works because (by assumption) the receiver's willingness to wait to find out the senders' signals is greater than the delay that the senders are willing to suffer in order to implement their own ideal points instead of the receiver's. If this were not the case then a sender could find it profitable to deviate even if that results in a delay equal to the receiver's willingness to wait. If learning the senders' signals does not improve by much the receiver's decision,<sup>21</sup> while at least one of the senders is patient and strongly dislikes the receiver's ideal point, then the equilibrium with almost-immediate, almost-full disclosure is not "credible" in the sense that the patient sender could simply "outwait" the receiver. This argument does not apply in the perfect-observation model—there, immediate truthful reporting *is* and remains a perfect Bayesian equilibrium. However, in a reasonable perturbation of the model like the one of this section—with imperfect signals and delay on the equilibrium path—this type of consideration becomes relevant.

The equilibrium construction provided in the proof of Proposition 4 relies on simultaneous, two-way private communication between the senders. In what follows, we show that this requirement can be relaxed considerably. We show that the result of Proposition 4 can be obtained using either one-way private communication at the interim stage (a single, private message from one sender to the other), or a randomization device that is observed by the two senders but not the receiver. This randomization device could be thought of as an aspect (dimension) of the state about which the senders do have common knowledge—even though they only observe other aspects of the state with some noise.

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<sup>21</sup>For example, because her prior is already precise, and/or the senders' signals are not, and/or the receiver is impatient and/or does not really care about getting her ideal point.

The idea of the equilibrium construction with either one-way private communication or the sender-observed randomization device instead of simultaneous, two-way private communication is the following. First, the senders generate a random key that they commonly know but the receiver does not. This is done either by one sender who privately transmits the key to the other, or by the sender-observed randomization device. The senders encode their signals with the random key when they report their signals. As a result of the encoding, the receiver only learns from the reports whether the signals fall in the same interval. If they do not, then a discrete delay takes place; the threat of delay makes the (encoded) reports truthful. When the delay is over (if there is any) the senders disclose the key and the receiver can carry out her ideal action given the senders' truthful reports.

**Proposition 5** *Assume that the senders can jointly observe the realization of a random variable ( $2N$  i.i.d. uniform random binary variables). If  $N$  is sufficiently large and  $\Delta_0 > \max\{\Delta_1, \Delta_2\}$ , then for  $\varepsilon$  sufficiently close to zero, there exists an equilibrium where the receiver learns which interval of the form  $[a_{k-1}, a_k)$ ,  $k = 1, \dots, 2^N$ , each sender's signal realization falls into and chooses  $y$  optimally subject to this information. The equilibrium expected delay is positive and tends to zero as  $\varepsilon \rightarrow 0$ .*

**Proof.** See the Appendix. ■

In the proof of Proposition 5 we precisely describe a process by which the senders can let the receiver learn whether or not they agree which interval  $[a_{k-1}, a_k)$  their signals belong to without actually reporting  $k$  or the extent of their disagreement. We call this procedure (or reporting protocol) the “ $N$ -question reporting protocol” and assume that it can take place instantaneously at time zero.

This analysis clearly demonstrates that the lack of common knowledge about the state is not an insurmountable impediment to eliciting timely and truthful reports from the experts. The equilibrium constructions that we used relied on either private communication between the senders or the existence of a jointly-observed randomization device.

Our model with noisy signals features a state distribution with finite support. This is a useful technical assumption for two reasons. First, if the signals are sufficiently precise then a sender’s signal “swamps” the prior in a finite-states model.<sup>22</sup> Second, there exists a simple punishment function (fixed delay following any disagreement regarding which realization of  $\omega$  is the most likely) that elicits truthful signal announcements.

Results analogous to those of Propositions 4-5 can be proved with continuous state distributions if the same properties are satisfied. We give one example in the next subsection. This example involves normally-distributed signals with an improper common prior on the state. The information structure is therefore identical to that of Battaglini (2004), which is one of the only models in the comparable static literature that assumes noisy observation of the state.<sup>23</sup> The improper prior assumption implies that the senders’ signals swamp the prior in the sense explained in footnote 22. In addition, we assume that the senders’ biases are equally large (in absolute value), which enables us to find a simple delay function inducing truthful reports.

### 4.3 An example with a continuum of states

Suppose that the state of nature is drawn from a uniform distribution on  $\mathbb{R}$ , that is, the common prior is *diffuse*. Sender  $i$ ’s signal,  $X_i$ , is normal with mean  $\omega$  and variance  $\sigma^2$  for  $i = 1, 2$ . The receiver’s ideal point is  $\omega$ , sender 1’s is  $\omega + b$ , while sender 2’s is  $\omega - b$ , where  $b > 0$ . All players have the same quadratic loss function. Time preference arises as a result of a unit cost of waiting per unit time:

$$U_i(\omega, y, \delta) = -(y - y_i(\omega))^2 - \delta \text{ for } i = 0, 1, 2. \quad (7)$$

Before describing the equilibrium we make some observations. By our assumptions and the rules of updating normal variables, the receiver’s ideal point

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<sup>22</sup>If the state space is finite and the senders’ signals are unbiased and sufficiently precise, then each sender’s “best guess” for the realization of the state is the one nearest to his observed signal value. With a continuum of states (and a proper prior distribution on them), the sender would always “shade” his (honest) best guess towards the prior mean.

<sup>23</sup>For simplicity and without altering our results, we assume that the state space is one-dimensional, while Battaglini’s (2004) is multi-dimensional.

as a function of the realization of the senders' signals is  $E[\omega|X_1 = x_1, X_2 = x_2] = (x_1 + x_2)/2$ . Each sender believes that the state of nature is distributed normally around the realization of his own signal with variance  $\sigma^2$ ; they believe the other sender's signal is distributed around the state of nature with an additional normal noise that has variance  $\sigma^2$ .

Purely as a thought-experiment, consider a *mechanism* where each sender reports the realization of his signal,  $\hat{x}_1$  and  $\hat{x}_2$  respectively, the receiver waits for a period of time  $d(\hat{x}_1 - \hat{x}_2)$ , and then carries out  $y = (\hat{x}_1 + \hat{x}_2)/2$ . The following lemma states that there exists a wait-function  $d$  such that this mechanism is incentive compatible for the senders.

**Lemma 1** *Assume that the senders observe conditionally independent normal signals about  $\omega$  whose prior distribution is diffuse on  $\mathbb{R}$ , and that their biases are of equal size and opposite sign. Consider a relaxed problem where the receiver can commit to a delay  $d$  as a function of the difference of the senders' signal reports before carrying out her ideal action conditional on their reports. Truthful signal reports are elicited by setting  $d(\hat{x}_1 - \hat{x}_2) = 2b \max\{\hat{x}_1 - \hat{x}_2, 0\}$ , where  $\hat{x}_i$  is sender  $i$ 's report for  $i = 1, 2$ .*

**Proof.** See the Appendix. ■

The significance of this mechanism is that its outcome can be replicated in an *equilibrium* with no commitment to delay on the receiver's part. Formally, we prove the following result.

**Proposition 6** *Assume that the senders observe conditionally independent normal signals about  $\omega$  whose prior distribution is diffuse on  $\mathbb{R}$ , and that their biases are of equal size and opposite sign. Then, there exists an equilibrium where the receiver learns the realizations of the senders' signals with a positive expected delay and carries out her full-information ideal action. As the noise in the senders' signals diminishes the expected delay tends to zero, and the equilibrium outcome converges to immediate full disclosure and  $y = \omega$ .*

**Proof.** See the Appendix. ■

The equilibrium is constructed as follows. On the equilibrium path, at time  $t = 0$ , the senders simultaneously and privately report to each other their signals; denote the report of sender  $i$  to the other sender by  $\hat{x}_i$ , for  $i = 1, 2$ . Then, at every point in time (including  $t = 0$ ) both senders send public messages. They independently babble for all  $t \in [0, d(\hat{x}_1 - \hat{x}_2))$  and announce  $m^* = (\hat{x}_1 + \hat{x}_2)/2$  from  $t = d(\hat{x}_1 - \hat{x}_2)$  on. The receiver takes the “null” action as long as the senders’ reports disagree and carries out their report as soon as they agree. Clearly, if both senders report to each other truthfully at time zero and follow their equilibrium strategy thereon then the receiver will learn and implement  $E[\omega|X_1 = x_1, X_2 = x_2]$  with delay  $d(x_1, x_2)$  as claimed. It is also clear that the expected value of  $d(x_1 - x_2)$  goes to zero as the senders’ signals become arbitrarily precise. That is, the equilibrium outcome converges to *full revelation* and *no delay*.

## 5 Conclusion

We have shown that there exists a perfect Bayesian equilibrium with instantaneous, complete revelation in a multi-sender dynamic cheap talk game where the state of nature is commonly known between the senders. In the static version of the model such equilibria only exist under certain conditions on the state space and/or the senders’ biases. The “perfectness” of the equilibrium implies that the construction does not rely on exogenous commitment to delay on the receiver’s part. Instead, the equilibrium is sustained by the receiver’s beliefs that even if the senders have disagreed in the past they will agree soon enough in the future—which they must be able to do as they have common knowledge about the state.

We studied the robustness of this result to certain perturbations of the model. In particular, we have shown that in certain environments where the senders have noisy signals about the state, a sequence of perfect Bayesian equilibria (with endogenous delay on the equilibrium path) converges to instantaneous full revelation as the noise in the senders’ signals vanishes. The key idea there is that even if the senders do not have common knowledge about the state it is possible for them to create mutual knowledge of their signals (at the ex-

pense of some delay), and then all their information can be truthfully reported to the receiver in a perfect equilibrium.

Our result that adding dynamics gives rise to fully-revealing equilibria when they do not exist in static models could be usefully applied in other communication games as well. Consider, for example, communication between a single sender and a receiver, where the sender can send *hard information*.<sup>24</sup> Even though hard signals are available, a fully-revealing equilibrium does not exist in the static game if the sender’s preferences are not monotonic in the receiver’s action or, more generally, if there does not exist a “worst type” for the sender.<sup>25</sup> This is the case, for instance, if the sender’s type (the state of nature) is distributed on the circumference of a circle, and the receiver wants to match her action to the true state that is diametrically opposite to the sender’s ideal point. Both parties’ utilities are positive and decreasing in the distance of the action from their respective ideal points. In this setup, there does not exist a fully-revealing equilibrium because by truthfully revealing the state the sender causes the receiver to implement the worst possible outcome for him. However, there exists a fully-revealing PBE in the dynamic game where the receiver’s belief is that the sender will send a state-revealing hard signal right away as long as he has not done so in the past. This indeed induces immediate full revelation on the sender’s part.

Communication models where the senders observe noisy signals of the state—like the ones studied in Section 4—are realistic and offer a multitude of future research questions. For example, it would be interesting to study realistic dynamic discovery methods under imperfect signals that can involve delay even as the noise becomes arbitrarily small.

## 6 Appendix

**Proof of Proposition 4.** The equilibrium strategies are as follows. Before time starts, but after the senders observe their signals, each sender simulta-

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<sup>24</sup>A hard signal, as opposed to a soft one, can only be sent by the sender of a particular type. See Milgrom (1981).

<sup>25</sup>See Milgrom (1981), Seidmann and Winter (1997).

neously and privately reports to the other sender an integer  $k \in \{1, \dots, 2^N\}$  corresponding to which interval  $[a_{k-1}, a_k)$  his signal realization belongs to. If their reports disagree then the senders delay play by a fixed amount of time  $d$ , to be characterized below, by publicly babbling for all  $t < t^*$ . Either at  $t = 0$  (if the private reports agreed) or at  $t = d$  (if they did not), both senders simultaneously and publicly announce  $\bar{y}_0$ , the expected value of the receiver's ideal point conditional on the signal values privately exchanged at time 0. If the senders' private reports agreed but their public reports at  $t = 0$  do not, then the senders babble until  $t = d$ , and then report  $\bar{y}_0$ . If the game continues beyond  $t = d$  then the senders repeatedly announce  $\bar{y}_0$  at time-intervals  $\delta(\hat{m}_1, \hat{m}_2)/2$ , where  $\hat{m}_i$  is sender  $i$ 's report regarding  $\bar{y}_0$  at  $t^*$ . If the senders' messages agree at time 0 then the receiver carries it out. Otherwise she chooses inaction for all  $t \in [0, d)$ . At  $t = d$ , if the senders' messages agree, then she carries out their recommendation. If they do not (the reports are  $\hat{m}_1 \neq \hat{m}_2$ ) then she repeatedly waits  $\delta(\hat{m}_1, \hat{m}_2)/2$  intervals until the senders reports match at time  $d + \lambda\delta(\hat{m}_1, \hat{m}_2)/2$ , for  $\lambda \in \mathbb{Z}^+$ .

If play reaches time  $t = d$  and the senders' reports disagree about  $\bar{y}_0$ , then the receiver believes that the senders will reveal  $\bar{y}_0$  after a delay of  $\delta(\hat{m}_1, \hat{m}_2)/2$ , where  $\hat{m}_1 \neq \hat{m}_2$  are the senders' reports about  $\bar{y}_0$  at  $t = d$ . Believing that her ideal point is either  $\hat{m}_1$  or  $\hat{m}_2$  with 50-50% chance, the receiver is willing to wait a period of  $\delta(\hat{m}_1, \hat{m}_2)/2$ . Indeed, it follows from the proof of Proposition 2 that the proposed strategies form a PBE from  $t = d$  on. Hence all players expect that  $\bar{y}_0$  will be revealed truthfully by the senders at time  $d$ , in case their private reports disagreed before public communication started.

If the senders' private reports disagree before  $t = 0$  then, for all  $t \in [0, d)$ , neither sender has a profitable deviation because all players believe that both senders babble for all  $t \in [0, d)$ . The receiver is better off choosing inaction during this period rather than any action  $y \in co(S)$  as long as  $d$  is smaller than the delay that makes her indifferent between taking an action immediately, only knowing that the senders' signals fall in different  $[a_{k-1}, a_k)$  intervals, and taking an action with delay  $d$ , but knowing exactly which intervals the senders signals fall into. In the former case, the receiver's best action is approximately  $\max_{y \in co(S)} E[U_0(\omega, y, 0)]$ , because the senders' disagreement does not provide information about the state and  $N$  is large. In the latter case, her best action

is arbitrarily close to  $\omega$ , provided that the signals sufficiently precise and  $N$  is large. Therefore, for  $N$  large and  $\varepsilon$  sufficiently close to zero,  $d$  can be chosen to be arbitrarily close to  $\Delta_0$ .

The senders have no incentive to deviate at time  $t = 0$  given their initial private reports to each other. If their private reports disagreed then we already established that neither sender has a profitable deviation for any  $t \in [0, d]$ . If their private reports agreed then they anticipate that in the continuation the receiver will carry out the state of nature (at least approximately, for  $N$  large and  $\varepsilon$  small). If sender  $i$  behaves as if they disagreed then he induces a delay of length at least  $d$ . The best outcome that this sender can hope for after delay  $d$  is his ideal point,  $y_i(\omega)$ . However, by  $\Delta_0 > \Delta_i$ , we have  $E[U_i(\omega, y_i(\omega), \Delta_0)] < E[U_i(\omega, \omega, 0)]$ , hence a delay approximating  $\Delta_0$  makes such a deviation unprofitable for sender  $i$ . The same argument establishes that neither sender can be better off by lying to the other sender when they send each other private messages at time 0: For  $\varepsilon$  close to 0, each sender anticipates that a truthful private report very likely induces  $y = \omega$  with no delay in the continuation, while a deviation very likely induces delay, which cannot benefit him even if the receiver's eventual action is  $y_i(\omega)$ .

Finally, the expected delay tends to zero as the signal structure converges to the senders having complete information about the state of nature. This is so because the probability that the senders disagree on which realization of  $\omega$  their signals fall closest to goes to zero as  $\varepsilon$  goes to 0. ■

**Proof of Proposition 5.** We only describe the players' moves on the equilibrium path. The proof of Proposition 4 establishes that these actions can be supported in equilibrium.

In the description of the protocol it is convenient to use an alternative notation for the intervals  $[a_{k-1}, a_k)$  for  $k = 0, \dots, 2^N$ . Start with  $S = [0, 1]$ , and define the rest recursively as follows. Suppose that for all  $(j - 1)$ -element sequences of 0 and 1, that is for all  $A \in \{0, 1\}^{j-1}$ , we have already constructed the intervals  $S_A$ , where  $j \geq 1$  is an integer. Then, define  $S_{A0}$  and  $S_{A1}$  as the lower and upper halves of  $S_A$ , the midpoint going to the upper half. For example,  $S_0 = [0, 1/2)$ ,  $S_1 = [1/2, 1]$ ;  $S_{00} = [0, 1/4)$ ,  $S_{01} = [1/4, 1/2)$ , and so on. The intervals  $S_A$  for  $A \in \{0, 1\}^N$  coincide with  $[a_{k-1}, a_k)$  for  $k = 1, \dots, 2^N$ .

The  $N$ -question reporting protocol consists of the following four steps.

Step 1. At time zero  $2N$  independent binary random numbers are generated such that each realization (0 or 1) has a 50% chance; the realizations become known by the senders while the receiver observes nothing. Denote the first  $N$  bits by  $\theta_1^1, \dots, \theta_N^1$  while the last  $N$  bits by  $\theta_1^2, \dots, \theta_N^2$ .

Step 2. At time 0, the senders send  $N$  rounds of binary reports; their reports are simultaneous in every round and the reports of previous rounds are publicly observed. In the first round, sender  $i$  reports  $m_1^i = I((x_i \geq 1/2) \oplus (\theta_1^i = 1))$ , where  $I(\cdot)$  is the indicator function for logical statements and  $\oplus$  is the exclusive disjunction (xor) operator. That is, sender  $i$ 's message contains a coded report whether or not his signal belongs to  $S_0$  or  $S_1$  using  $\theta_1^i$  as the secret key.

Since sender  $i' \neq i$  knows  $\theta_1^i$ , he can infer whether or not  $x_i \geq 1/2$  from  $m_1^i$ . On the other hand, the Receiver learns nothing about whether or not  $x_i \geq 1/2$  by observing  $m_1^i$  but without knowing the realization of  $\theta_1^i$ . This is so because by Bayes' rule,

$$\begin{aligned} \Pr(x_i \geq 1/2 | m_1^i = 0) &= \frac{(1/2) \Pr(x_i \geq 1/2)}{(1/2) \Pr(x_i \geq 1/2) + (1/2) \Pr(x_i < 1/2)} \\ &= \Pr(x_i \geq 1/2), \end{aligned}$$

and similarly,

$$\Pr(x_i \geq 1/2 | m_1^i = 1) = \Pr(x_i \geq 1/2).$$

In round  $j > 1$ , if the senders' past announcements indicate that both  $x_1$  and  $x_2$  belong to the same interval  $S_A$  where  $A \in \{0, 1\}^{j-1}$ , then sender  $i$  reports  $m_j^i = I((x_i \in S_{A1}) \oplus (\theta_j^i = 1))$ . If the senders' past reports indicate that  $x_1$  and  $x_2$  do not belong to the same  $S_A$  then both senders submit the random bits  $\theta_j^1$  and  $\theta_j^2$  from round  $j$  on.

Step 3. Still at time 0, the senders simultaneously publicly announce one more bit:  $m_{N+1}^i = 1$  if and only if their reports in the previous  $N$  rounds indicate that  $x_1$  and  $x_2$  belong to the same interval  $S_A$  for some  $A \in \{0, 1\}^N$ . If they report 0, a delay of length  $d$  is implemented: both senders babble until time  $d$ , and the receiver remains idle during this period.

Step 4: Either at time  $t = d$  (if delay took place) or  $t = 0$  (in case of

no delay) both senders simultaneously announce  $\theta_1^1, \dots, \theta_N^1, \theta_1^2, \dots, \theta_N^2$ . The receiver “decodes” their coded binary messages, and carries out her ideal point given the information. ■

**Proof of Lemma 1.** Suppose that sender 2 reports truthfully,  $\hat{x}_2 = x_2$ , and compute sender 1’s expected utility from reporting  $\hat{x}_1$  when his true signal is  $x_1$ :

$$V_1(x_1, \hat{x}_1) = \int \int \left[ - \left( \omega + b - \frac{\hat{x}_1 + \omega + z}{2} \right)^2 - 2b(\hat{x}_1 - \omega - z)^+ \right] dF(\omega|x_1)dF(z|0),$$

where  $F(\cdot|\mu)$  is the cdf of a normal random variable with mean  $\mu$  and variance  $\sigma^2$ , and the notational convention  $a^+ = a\mathbf{1}_{\{a \geq 0\}}$  is used. Maximizing this in  $\hat{x}_1$  yields the first-order condition

$$\int \int \left( b + \frac{\omega - \hat{x}_1 - z}{2} - 2b\mathbf{1}_{\{\omega \leq \hat{x}_1 - z\}} \right) dF(\omega|x_1)dF(z|0) = 0,$$

which needs to hold at  $\hat{x}_1 = x_1$  for incentive compatibility. (It is easy to check that the second-order condition holds.) Using the facts that  $\int \omega dF(\omega|x_1) = x_1$ ,  $\int z dF(z|0) = 0$ , and  $\int \mathbf{1}_{\{\omega \leq x_1 - z\}} dF(\omega|x_1) = F(x_1 - z|x_1) = F(-z|0)$ , we can rewrite this condition as

$$b - 2b \int F(-z|0)dF(z|0) = 0. \tag{8}$$

The incentive constraint for sender 2 (provided sender 1 reports truthfully) can be derived similarly. His deviation payoff is

$$V_2(x_2, \hat{x}_2) = \int \int \left[ - \left( \omega - b - \frac{\hat{x}_2 + \omega + z}{2} \right)^2 - 4b(\omega + z - \hat{x}_2)^+ \right] dF(\omega|x_2)dF(z|0).$$

Maximization in  $\hat{x}_2$  at  $\hat{x}_2 = x_2$  yields,

$$\int \int \left( -b + \frac{\omega - x_2 - z}{2} + 2b\mathbf{1}_{\{\omega \geq x_2 - z\}} \right) dF(\omega|x_2)dF(z|0) = 0,$$

which simplifies to

$$-b + 2b \int [1 - F(-z|0)] dF(z|0) = 0. \quad (9)$$

Since the integrands in equations (8) and (9) add up to 1 (for all  $z$ ), by the symmetry of the normal distribution and  $1 - F(-z|0) = F(z|0)$  we have

$$\int F(-z|0)dF(z|0) = \int [1 - F(-z|0)] dF(z|0) = \frac{1}{2}.$$

Hence both first-order conditions hold as claimed. ■

**Proof of Proposition 6.** The equilibrium (described in the text) is sustained by the following off-equilibrium beliefs and behavior. At any  $t \geq d(\hat{x}_1 - \hat{x}_2)$  neither sender has an incentive to report anything other than  $m^* = (\hat{x}_1 + \hat{x}_2)/2$ , given that the other reports  $m^*$ , because a deviation only leads to a longer delay.

Neither sender can force a resolution before the delay is over. This is so because neither sender can predict what the other sender reports during the period of babbling, hence he cannot match the other's announcement. Moreover, he cannot credibly report  $m^*$  to the receiver either because he is believed to be babbling.

By Lemma 1, sender  $i$  has no incentive to misreport his signal to the other at  $t = 0$ . Assume that if a sender fails to make his private report at  $t = 0$  then the other sender babbles forever; this eliminates any incentive for a sender to deviate at  $t = 0$ .

Finally, the receiver has no profitable deviation because the senders babble until  $t = d(\hat{x}_1 - \hat{x}_2)$ . By the diffuse prior assumption she infers nothing about the expected value of the state of nature (even as time passes and she learns lower bounds on  $\hat{x}_1 - \hat{x}_2$ ) until  $t = d(\hat{x}_1 - \hat{x}_2)$ , and she has an incentive to wait because she learns the posterior mean of  $\omega$  after finite expected delay. ■

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