The Bond Market's q^*

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Abstract

I propose an implementation of the q-theory of investment using bond prices instead of equity prices. Credit risk makes corporate bond prices sensitive to future asset values, and q can be inferred from bond prices. The bond market's q performs much better than the usual measure in standard investment equations. With aggregate data, the fit is three times better, cash flows are driven out and the implied adjustment costs are reduced by more than an order of magnitude. The new measure also improves firm level investment equations.

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In his 1969 article, James Tobin argued that "the rate of investment – the speed at which investors wish to increase the capital stock – should be related, if to anything, to q, the value of capital relative to its replacement cost.[1]" Tobin also recognized, however, that q must depend on "expectations, estimates of risk, attitudes towards risk, and a host of other factors," and he concluded that "it is not to be expected that the essential impact of [..] financial events will be easy to measure in the absence of direct observation of the relevant variables (q in the models)." The quest for an observable proxy for q was therefore recognized as a crucial objective from the very beginning.

Subsequent research succeeded in integrating Tobin's approach with the neoclassical investment theory of Jorgenson (1963). Lucas and Prescott (1971) proposed a dynamic model of investment with convex adjustment costs, and Abel (1979) showed that the rate of investment is optimal when the marginal cost of installment is equal to $q-1$. Finally, Hayashi (1982) showed that, under perfect competition and constant returns to scale, marginal q (the market value of an *additional unit* of capital divided by its replacement cost) is equal to average q (the market value of *existing* capital divided by its replacement cost). Since average q is observable, the theory became empirically relevant. Unfortunately, its implementation proved disappointing. The investment equation fits poorly, leaves large unexplained residuals correlated with cash flows, and implies implausible parameters for the adjustment cost function (see Summers (1981) for an early contribution, and Hassett and Hubbard (1997) and Caballero (1999) for recent literature reviews).

Several theories have been proposed to explain this failure. Firms could have market power, and might not operate under constant returns to scale. Adjustment costs might not be convex (Dixit and Pindyck (1994), Caballero and Engle (1999)). Firms might be credit constrained (Fazzari, Hubbard, and Petersen (1988), Bernanke and Gertler (1989)). Finally, there could be measurement errors and aggregation biases in the capital stock or the rate of investment.

None of these explanations is fully satisfactory, however. The evidence for constant returns and price taking seems quite strong (Hall (2003)). Adjustment costs are certainly not convex at the plant level, but it is not clear that it really matters in the aggregate (Thomas (2002), Hall (2004)), although this is still a controversial issue (Bachmann, Caballero, and

[1] Tobin (1969), page 21.

Engel (2006)). Gomes (2001) shows that Tobin's q can capture most of investment dynamics even when there are credit constraints. Heterogeneity and aggregation do not seem to create strong biases (Hall (2004)).

In fact, an intriguing message comes out of the more recent empirical research: the market value of equity seems to be the culprit for the empirical failure of the investment equation. Gilchrist and Himmelberg (1995), following Abel and Blanchard (1986), use VARs to forecast cash flows and to construct q, and they find that it performs better than the traditional measure based on equity prices. Cumins, Hasset, and Oliner (2006) use analysts' forecasts instead of VAR forecasts, and reach similar conclusions. Erickson and Whited (2000) and Erickson and Whited (2006) use GMM estimators to purge q from measurement errors. They find that only 40 percent of observed variations are due to fundamental changes, and, once again, that market values contain large 'measurement errors'.

Applied research has therefore reached an uncomfortable situation, where the benchmark investment equation appears to be successful only when market prices are not used to construct q. This is unfortunate, since Tobin's insight was precisely to link observed quantities and market prices. The contribution of this paper is to show that a market-based measure of q can be constructed from corporate bond prices, and that this measure performs much better than the traditional one.

Why would the bond market's q perform better than the usual measure? There are several possible explanations. First, the bond market might be less susceptible to bubbles than the equity market. In fact, there is empirical and theoretical support for the idea that mispricing is more likely to happen when returns are positively skewed. Barberis and Huang (2007) show that cumulative prospect theory predicts that a positively skewed security can be overpriced. Brunnermeier, Gollier, and Parker (2007) argue that preference for skewness arises endogenously because investors chooses to be optimistic about the states associated with the most skewed Arrow-Debreu securities. Empirically, Mitton and Vorkink (2007) document that under-diversification is largely explained by the fact that investors sacrifice mean-variance efficiency for higher skewness exposure. Purely rational explanations can also be proposed. These explanations typically involve different degrees of asymmetric information, market segmentation, and heterogeneity in adjustment costs and stochastic processes.[2] They can create conceptual difficulties regarding the correct definition of firm value or the proper goals of managers. It is much too early at this stage to take a stand on which explanations are correct, and which ones are not. My goal is more modest: it is simply to provide a new measure of q based on market prices.

Of course, even if bond prices are somehow more reliable than equity prices, it is far from obvious that it is actually possible to use bond prices to construct q. The contribution of this paper is precisely to show how one can do so, by combining the insights of Black and Scholes (1973) and Merton (1974) with the approach of Abel (1979) and Hayashi (1982). In the Black-Scholes-Merton model, debt and equity are seen as derivatives of the underlying assets. In the simplest case, the market value of corporate debt is a function of its face value, asset volatility and asset value. But one can also invert the function, so that, given asset volatility and the face value of debt, one can construct an estimate of asset value from observed bond prices. I extend this logic to the case where asset value is endogenously determined by capital expenditures decisions.

Like in Hayashi (1982), I assume constant returns to scale, perfect competition, and convex adjustment costs. There are no taxes and no bankruptcy costs, so the Modigliani-Miller Theorem holds, and real investment decisions are independent from capital structure decisions.[3] Firms issue long-term, coupon-paying bonds as in Leland (1998), and the default boundary is endogenously determined to maximize equity value, as in Leland and Toft (1996). There are two crucial differences between my model and the usual asset pricing models. First, physical assets change over time. Under constant returns to scale, however, I obtain tractable pricing formulas, where the usual variables are simply scaled by the book value of assets. Thus, book leverage plays the role of the face value of principal outstanding, and q plays the role of total asset value. The second difference is that cash flows are endogenous, because they depend on adjustment costs and investment decisions.

I model an economy with a continuum of firms hit by aggregate and idiosyncratic shocks.

[2] Consider for instance the random arrival of a new technology. News about the technology can have a large impact on equity values, but if it is not possible to invest in the new technology before it actually arrives, there would be no corresponding change in capital expenditures. In addition, firms might be reluctant to use equity to finance capital expenditures, because of adverse selection, in which case the bond market might provide a better measure of investment opportunities (Myers (1984))

[3] One could introduce taxes and bankruptcy costs if one wanted to derive an optimal capital structure, but this is not the focus of this paper. See Hackbarth, Miao, and Morellec (2006) for such an analysis, with a focus on macroeconomic risk.

Even though default is a discrete event at the firm level, the aggregate default rate is a continuous function of the state of the economy. To build the economic intuition, I consider first a simple example with one-period debt, constant risk free rates, and *iid* firm level shocks. I find that, to a first order (i.e., for small aggregate shocks), Tobin's q is a linear function of the spread of corporate bonds over government bonds. The sensitivity of q to bond spreads depends on the risk neutral default rate, just like the delta of an option in the Black-Scholes formula. In the general case, I choose the parameters of the model to match aggregate and firm level dynamics, estimated with post-war U.S. data. Given book leverage and idiosyncratic volatility, the model produces a non-linear mapping from bond yields to q.

I then use the theoretical mapping to construct a time series for q based on the yield of Baa and government bonds, taking into account trends in book leverage and idiosyncractic risk, as well as changes in real risk free rates. This bond market's q fits the investment equation quite well with post-war aggregate US data. The R^2 is around 60%, cash flows become insignificant, and the implied adjustment costs are more than an order of magnitude smaller than with the usual measure of q. The fit is as good in levels as in differences.[4]

At the firm level, I use credit ratings to construct a proxy for bond prices, and I use the model's mapping to construct q. The coefficient estimated in the cross section is fairly similar to the one estimated in the time series. Recent work by Gilchrist and Zakrajsek (2007) also provides evidence that firm specific interest rates forecast firm level investment. At the firm level, however, equity and cash flows remain strongly significant.

The remaining of the paper is organized as follows. In section 1, I present the setup of the model. In section 2, I study a simple case to build the economic intuition. In section 3, I present the numerical solution for the general case. In section 4, I present evidence based on aggregate data. In section 5, I present evidence based on firm level data. Section 6 concludes.

[4] Cochrane (1996) finds a significant correlation between stock returns and the growth rate of the aggregate capital stock, but Hassett and Hubbard (1997) argue that it is driven by the correlation with residential investment, not corporate investment. In any case, I find that the bond market's q outperforms the usual measure both in differences and in levels.

1 Model

1.1 Firm value and investment

Time is discrete and runs from $t = 0$ to ∞. The production technology has constant returns to scale and all markets are perfectly competitive. All factors of production, except physical capital, can be freely adjusted within each period. Physical capital is predetermined in period t and, to make this clear, I denote it by k_{t-1}. Once other inputs have been chosen optimally, the firm's profits are therefore equal to $p_t k_{t-1}$, where p_t is the exogenous profit rate in period t. Let the function $\Gamma(k_{t-1}, k_t)$ capture the total cost of adjusting the level of capital from k_{t-1} to k_t. For convenience, I include depreciation in the function Γ, and I assume that it is homogenous of degree one, as in Hayashi (1982).[5]

Let r_t be the one period real interest rate, and let $E^{\pi}[.]$ denote expectations under the risk neutral probability measure π.[6] The state of the firm at time t is characterized by the endogenous state variable k_t and a vector of exogenous state variables ω_t, which follows a Markov process under π. The profit rate and the risk free rate are functions of ω_t. The value of the firm solves the Bellman equation:

$$V(k_{t-1}, \omega_t) = \max_{k_t \geq 0} \left\{ p(\omega_t) k_{t-1} - \Gamma(k_{t-1}, k_t) + \frac{E^{\pi}[V(k_t, \omega_{t+1}) | \omega_t]}{1 + r(\omega_t)} \right\}. \quad (1)$$

Since the technology exhibits constant returns to scale, it is convenient to work with the scaled value function:

$$v_t \equiv \frac{V_t}{k_{t-1}}. \quad (2)$$

Similarly, define the growth of k as: $x_t \equiv \frac{k_t}{k_{t-1}}$. After dividing both sides of equation (1) by k_{t-1}, and using the shortcut notation ω' for ω_{t+1}, we obtain:

$$v(\omega) = \max_{x \geq 0} \left\{ p(\omega) - \gamma(x) + \frac{x}{1 + r(\omega)} E^{\pi}\left[v(\omega') | \omega \right] \right\}, \quad (3)$$

[5] For instance, the often used case of quadratic adjustment costs corresponds to: $\Gamma(k_t, k_{t+1}) = k_{t+1} - (1-d) k_t + 0.5\Gamma_0 (k_{t+1} - k_t)^2 / k_t$, where d is the depreciation rate, and Γ_0 is a constant that pins down the curvature of the adjustment cost function.

[6] Pricing a bond is like pricing a derivative. Using risk neutral probabilities instead of pricing kernels is therefore going to simplify the notations and the algebra. In any case, it is crucial to account for risk premia. For instance, Berndt, Douglas, Duffie, Ferguson, and Schranz (2005) show that objective probabilities of default are much smaller than risk-adjusted probabilities of default. Lettau and Ludvigson (2002) also emphasize the role of time varying risk premia.

where γ is the re-normalized version of Γ. The function γ is assumed to be convex and to satisfy $\lim_{x \to 0} \gamma(x) = \infty$ and $\lim_{x \to \infty} \gamma(x) = \infty$. The optimal investment rate $x(\omega)$ solves:

$$\frac{\partial \gamma}{\partial x}(x(\omega)) = q(\omega) \equiv \frac{E^{\pi}[v(\omega') | \omega]}{1 + r(\omega)}. \tag{4}$$

Equation (4) defines the q-theory of investment: it says that the marginal cost of investment is equal to the expected discounted marginal product of capital. The most important practical issue is the construction of the right-hand side of equation (4).

1.2 Measuring q

The value of the firm is the value of its debt plus the value of its equity. Let B_t be the market values of the bonds outstanding at the end of period t, and define b_t as the value scaled by end of period physical assets:

$$b_t \equiv \frac{B_t}{k_t}. \tag{5}$$

Similarly, let $e(\omega)$ be the ex-dividend value of equity, scaled by end of period assets. Then, q is simply:

$$q(\omega) = e(\omega) + b(\omega). \tag{6}$$

The most natural way to test the q-theory of investment is therefore to use equation (6) to construct the right hand side of equation (4). Unfortunately, it fits poorly in practice (Summers (1981), Hassett and Hubbard (1997), Caballero (1999)). Equation (6) has been estimated using aggregate and firm level data, in levels or in first differences, with or without debt on the right-hand-side. It leaves large unexplained residuals correlated with cash flows, and it implies implausible values for the adjustment cost function $\gamma(x)$.

As argued in the introduction, there are potential explanations for this empirical failure, but none is really satisfactory. Moreover, a common finding of the recent research is that "measurement errors" in equity seem to be responsible for the failure of q-theory (Gilchrist and Himmelberg (1995), Erickson and Whited (2000), Cumins, Hasset, and Oliner (2006), Erickson and Whited (2006)). I do not attempt in this paper to explain the meaning of these "measurement errors". I simply argue that, even if equity prices do not provide a good measure of q, it is still possible to construct another one using observed bond prices.

1.3 Corporate debt

I assume that there are no taxes and no deadweight losses from financial distress. The Modigliani and Miller theorem implies that leverage policy does not affect firm value or investment. Leverage does affect bond prices, however, and I must specify debt dynamics before I can use bond prices to estimate q. The model used here belongs to the class of structural model of debt, with endogenous default boundary. In this class of models, default is chosen endogenously to maximize equity value (see Leland (2004) for an illuminating discussion).

There are many different types of long term liabilities, and my goal here is not to study all of them, but rather to focus on a tractable model of long term debt. To do so, I use a version of the exponential model introduced in Leland (1994), and used by Leland (1998) and Hackbarth, Miao, and Morellec (2006) among others. In this model, the firm continuously issues and retires bonds. Specifically, a fraction ϕ of the remaining principal is called at par every period. The retired bonds are replaced by new ones. To understand the timing of cash flows, consider a bond with coupon c and principal normalized to 1, issued at the end of period t. The promised cash flows for this particular bond are:

$$\begin{array}{llllll} t+1 & t+2 & ... & \tau & ... \\ c+\phi & (1-\phi)(c+\phi) & ... & (1-\phi)^{\tau-t-1}(c+\phi) & ... \end{array}$$

Let $F_{\tau-1}$ be the sum of the face values of all the bonds outstanding at the beginning of period τ. I use the index $\tau-1$ to make clear that this variable, just like physical capital, is predetermined at the beginning of each period. The timing of events in each period is the following

1. The firm enters period τ with capital $k_{\tau-1}$ and total face value of outstanding bonds $F_{\tau-1}$.

2. The state variable ω_τ is realized. The value of the firm is then $V_\tau = v_\tau k_{\tau-1}$, defined in equations (1) and (3).

   (a) If equity value falls to zero, the firm defaults and the bond holders recover V_τ.

   (b) Otherwise, the bond holders receive cash flows $(c+\phi) F_{\tau-1}$.

3. At the end of period τ, the capital stock is k_τ, the face value of the bonds (including newly issued ones) is F_τ, and their market value is $B_\tau = b_\tau k_\tau$.[7]

In Leland (1994) and Leland (1998), book assets are constant, since there is no physical investment, and the firm simply chooses a constant face value F. In my setup, the corresponding assumption is that the firm chooses a constant book leverage ratio. I therefore make the following assumption:

Assumption: *Firms maintain a constant book leverage ratio:* $f \equiv F_t/k_t$

A bond issued at the end of period t has a remaining face value of $(1-\phi)^{\tau-t-1}$ at the beginning of period τ. In case of default during period τ, all bonds are treated similarly and the bond issued at time t receives $(1-\phi)^{\tau-t-1} V_\tau/F_{\tau-1}$. Since all outstanding bonds are treated similarly in case of default, we can characterize the price without specifying when this principal was issued. The following proposition characterizes the debt pricing function.

Proposition 1 *The scaled value of corporate debt solves the equation:*

$$b(\omega) = \frac{1}{1+r(\omega)} E^\pi \left[\min\left\{(c+\phi) f + (1-\phi) b\left(\omega'\right); v\left(\omega'\right)\right\} |\omega\right]. \tag{7}$$

Proof. See appendix. ■

The intuition behind equation (7) is relatively simple. Default happens when equity value falls to zero, i.e., when $v - (c+\phi) f - (1-\phi) b = 0$. There are no deadweight losses and bond holders simply recover the value of the company. When there is no default, bond holders receive the cash flows $(c+\phi) f$ and they own $(1-\phi)$ remaining bonds. A few special cases are worth pointing out. Short term debt corresponds to $\phi = 1$ and $c = 0$, and the pricing function is simply:

$$b^{short}(\omega) = \frac{1}{1+r(\omega)} E^\pi \left[\min\left(f;\ v\left(\omega'\right)\right) |\omega\right]. \tag{8}$$

The main difference between short- and long-term debt is the presence of the pricing function b on both sides of equation (7), while it appears only on the left hand side in equation (8). A perpetuity corresponds to $\phi = 0$, and, more generally, $1/\phi$ is the average maturity of the

[7] New issuances represent a principal of $F_\tau - (1-\phi) F_{\tau-1}$.

debt. The value of a default-free bond with the same coupon and maturity structure would be:

$$b^{free}(\omega) = \frac{(c+\phi) f + (1-\phi) E^{\pi}\left[b^{free}(\omega') \,|\omega\right]}{1+r(\omega)}. \tag{9}$$

With a constant risk free rate, b^{free} is simply equal to $(c+\phi) f/(\phi+r)$.

2 A simple example

This section presents a simple example in order to build the intuition for the more general case. The specific assumptions made in this section, and relaxed later, are: the risk free rate is constant; firms issue only short term debt; and idiosyncratic shocks are *iid*. Let us first decompose the state ω into its aggregate component s, and its idiosyncratic component η. The aggregate state follows a discrete Markov chain over the set $[1, 2.., S]$, and it pins down the aggregate profit rate $a(s)$, as well as the conditional risk neutral expectations. The profit rate of the firm depends on the aggregate state and on the idiosyncratic shock:

$$p(s,\eta) = a(s) + \eta. \tag{10}$$

The shocks η are independent over time, and distributed according to the density function $\zeta(.)$. Since idiosyncratic profitability shocks are *iid*, the value function is additive and can be written: $v(s,\eta) = v(s) + \eta$. I assume that s and η are such that $v(s,\eta)$ is always positive, so that firms never exit. Tobin's q is the same for all firms, and I normalize the mean of η to zero, therefore:[8]

$$q(s) = \frac{E^{\pi}\left[v(s')\,|s\right]}{1+r}. \tag{11}$$

Let $\bar{v} \equiv E^{\pi}[v(s)]$ be the unconditional risk neutral average asset value, and define $q \equiv \bar{v}/(1+r)$.

We can write the value of the aggregate portfolio of corporate bonds by integrating (8) over idiosyncratic shocks:

$$b(s) = \frac{1}{1+r} E^{\pi}\left[f + \int_{-\infty}^{f-v(s')} \left(v\left(s'\right) + \eta' - f\right) \zeta\left(\eta'\right) d\eta' |s\right]. \tag{12}$$

[8] All the firms choose the same investment rate. This will not be true in the more general model with persistent firm level shocks.

In equation (12), f is the promised payment, and the integral measures credit losses. Let δ be the default rate estimated at the risk neutral average value:

$$\delta \equiv \int_{-\infty}^{f-\bar{v}} \zeta\left(\eta'\right) d\eta'. \tag{13}$$

Let $\bar{b}$ be the corresponding price for the aggregate bond portfolio.[9] Using (13) and (11), we can write (12) as:

$$b\left(s\right)-\bar{b}=\delta\left(q\left(s\right)-q\right)+\frac{E^{\pi}\left[o\left(v'\right)\right]}{1+r}, \tag{14}$$

where $o\left(v'\right)$ is first order small, in the sense that $o\left(\bar{v}\right)=0$ and $\partial o/\partial v'=0$ when evaluated at $\bar{v}$.[10] When aggregate shocks are small, so that v stays relatively close to $\bar{v}$, $E^{\pi}\left[o\left(v'\right)\right]$ is negligible.

Equation (14) is the equivalent of the Black-Scholes-Merton formula, applied to Tobin's q. The value of the option (debt) depends on the value of the underlying (q), and the 'delta' of the option is the probability of default. If this probability is exactly zero, bond prices do not contain information about q. The fact that the sensitivity of b to q is given by δ is intuitive. Indeed, b respond to q precisely because a fraction δ of firms default on average each period. A one unit move in aggregate q therefore translates into a δ move in the price of a diversified portfolio of bonds.

To make equation (14) empirically relevant, we need to express it in terms of bond yields. All the prices we have discussed so far are in real terms but, in practice, we observe nominal yields. Let $r^{\$}$ be the nominal risk free rate, and let $y^{\$}$ be the nominal yield on corporate bonds. With short term debt, the market value is equal to the nominal face value divided by $1+y^{\$}$. Under the assumption we have made in this section, and neglecting the terms that are first order small, a simple manipulation of equation (14) leads to the following proposition.

Proposition 2 *To a first order approximation, Tobin's q is a linear function of the relative yields of corporate and government bonds:*

$$q_t \approx \frac{f}{\delta\left(1+r\right)}\frac{1+r_t^{\$}}{1+y_t^{\$}}+constant,$$

[9] $(1+r)\,\bar{b} \equiv f+\int_{-\infty}^{f-\bar{v}}\left(\bar{v}+\eta'-f\right)\zeta\left(\eta'\right)d\eta'$

[10] $o\left(v'\right) \equiv \int_{f-\bar{v}}^{f-v'}\left(v'+\eta'-f\right)\zeta\left(\eta'\right)d\eta'$

where r is the real risk free rate, f is average book leverage, and δ is the risk neutral default rate.

The proposition sheds light on existing empirical studies, such as Bernanke (1983), Stock and Watson (1989), and Lettau and Ludvigson (2002), showing that the spread of corporate bonds over government bonds predicts future output.[11] This finding is consistent with q-theory, since the proposition shows that corporate bond spreads are, to a first order, *proportional* to Tobin's q.

3 General case

I now consider the case of long term debt and realistic firm level shocks. The goal is to obtain a mapping from bond yields to Tobin's q that extends the simple case presented above. As in the previous section, let s denote the aggregate state and let η denote the idiosyncratic component of the profit rate, defined in equation (10). With persistent idiosyncratic shocks, Tobin's q and the investment rate depend on both s and η, and the value function is no longer additively separable. There is no closed form solution for bond prices, and I turn directly to numerical simulations with a model calibrated to U.S. data.

3.1 Calibration

The data used in the calibration are summarized in Table 1. The data are also described in more details in Section 4, when the model is used to construct a new series for q over the post-war period. All the parameters used in the calibration, and the empirical moments used to infer them, are presented in Table 2.

Some parameters are chosen a priori. The real risk free interest rate is assumed constant at $r = 3\%$ per year.[12] Book leverage is set to $f = 0.45$ and average debt maturity to 10 years ($\phi = 0.1$), based on Leland (2004) who uses these values as benchmark for Baa bonds. I use a quadratic adjustment cost function:

$$\gamma(x) = \gamma_1 x + 0.5\gamma_2 x^2. \tag{15}$$

[11] In the proposition, I use the ratio instead of the difference because this is more accurate when inflation is high. The approximation of small aggregate shocks made in this section refers to real shocks, but does not require average inflation to be small.

[12] This is only to simplify the exposition. I take into account changes in real interest rates in Section 4.

With this functional form, the investment equation is simply $x = (q - \gamma_1)/\gamma_2$. The parameter γ_1 is irrelevant and is normalized to one. There is much disagreement about the parameter γ_2 in the literature. Shapiro (1986) estimates a value around 2.2 years, and Hall (2004) finds even smaller adjustment costs.[13] On the other hand, Gilchrist and Himmelberg (1995) find values around 20 years, and estimates from macro data are often implausibly high (Summers (1981)). I pick a value of $\gamma_2 = 8$ years, which is in the middle of the set of existing estimates, and also corresponds to the slow adjustment case in Hall (2001). It turns out, however, that the mapping from bond yields to q is not very sensitive to this parameter.

Some parameters are directly observed in the data. Idiosyncratic profitability is assumed to follow an AR(1) process:

$$\eta_t = \lambda_\eta \eta_{t-1} + \varepsilon_t^\eta. \tag{16}$$

Let σ_η be the standard deviation of ε_t^η. Equation (16) is estimated with firm level data from Compustat. The profit rate is operating income divided by the net stock of property plants and equipment, and η as the idiosyncractic component of this profit rate. Firms in finance and real estate are excluded. The panel regression includes firm fixed effects to remove mechanical differences in average profitability across firms or industries.[14] The estimated parameters, $\lambda_\eta = 0.47$ and $\sigma_\eta = 14\%$, are consistent with many previous studies.[15] Similarly, I specify aggregate dynamics as:

$$a_t - \bar{a} = \lambda_a (a_{t-1} - \bar{a}) + \varepsilon_t^a. \tag{17}$$

Using annual NIPA data on corporate profits and the stock on non-residential capital over the post-war period, I estimate $\lambda_a = 0.7$.

The parameters $\bar{a}$ and σ_a cannot be calibrated with historical aggregate profit rates

[13] Shapiro (1986) estimates between 8 and 9 using quarterly data, which corresponds to 2 to 2.2 at annual frequencies.

[14] These are mostly due to accounting choices, fixed technological differences, whereas the model is explicitly about firm level shocks. Not including fixed effects would lead to overestimation of the persistence parameter λ.

[15] For instance, Gomes (2001) uses a volatility of 15% and a persistence of 0.62 for the technology shock. Hennessy, Levy, and Whited (2007) report a persistent of the profit rate of 0.51 and a volatility of 11.85%, which they match with a persistence of 0.684 and a volatility of 11.8% for the technology shocks. Note that in both of these papers, firms operate a technology with decreasing returns. Here, by contrast, the technology has constant returns to scale. This explains why some details of the calibration are different.

because they must capture risk adjusted values, not historical ones.[16] Instead, the model must be consistent with observed bond prices. Three parameters are thus not directly observed in the data: these are c (the coupon rate), $\bar{a}$ and σ_a. Their values are inferred by matching empirical and simulated moments. The processes (16) and (17) are approximated with discrete-state Markov chains using the method in Tauchen (1986). The investment rate $x(s,\eta)$ and the value of the firm value $v(s,\eta)$ are obtained by solving the dynamic programming problem in equation (3). Equation (7) is then used to compute the bond pricing function $b(s,\eta)$. The aggregate bond price $b(s)$ and the aggregate corporate yield $y(s)$ are obtained by integrating over the ergodic distribution of η.

The empirical moments are the mean and standard deviation of the price of Baa bonds relative to Treasuries, defined as $\left(\phi + r_t^{\$}\right) / \left(\phi + y_t^{\$}\right)$, where $y^{\$}$ is the yield on Baa corporate bonds and $r^{\$}$ is the yield on government bonds. The final requirement is that the average bond be issued at par, as in Leland (1998). The three parameters c, $\bar{a}$ and σ_a are chosen simultaneously to match the par-value requirement and the two empirical moments. The parameters inferred from the simulated moments are: $c = 4.35\%$, $\bar{a}/r = 0.93$ and $\sigma(\varepsilon_t^a) = 5.46\%$.

3.2 Simulation and interpretation

We can now use the model to understand the relationship between bond prices and Tobin's q. The model is simulated with the parameters described in the previous section. Figure 1 presents the main result. It shows the model-implied aggregate $q(s)$ as a function of the model-implied average relative bond price $(\phi + r)/(\phi + y(s))$. Tobin's q is an increasing and convex function of the relative price of corporate bonds. Figure 1 therefore extends Proposition 2 to the case of long term debt, persistent firm level shocks, and large aggregate shocks.

The mapping from bond yields to Tobin's q is conditional on the calibrated parameters, in particular on book leverage and idiosyncratic volatility. Figure 2 shows the comparative statics with respect to book leverage (f) and firm volatility (σ_η). The comparative statics

[16] Note that, in theory, the same applies to λ_a, because persistence under the risk neutral measure can be different from persistence under the physical measure. In practice, however, the difference for λ_a is much smaller than for $\bar{a}$ or σ_a. I therefore take the historical persistence to be a good approximation of the risk neutral persistence.

are intuitive. For a given value of q, an increase in leverage leads to more credit risk, and lower bond prices, so the mapping shift left when leverage increases. Similarly, for a given value of q, an increase in idiosyncratic volatility increases credit risk, and the mapping shifts left when volatility increases. In this case, the slope and the curvature of the mapping also change, and the intuition is given by Proposition 2: idiosyncratic volatility increases the delta of the bond with respect to q.

In the next section, mappings like the ones displayed on Figure 2 are used to construct a new measure of q from observed bond yields.

4 Aggregate Evidence

In this section, I construct a new measure of q using only data from the bond market. I then compare this measure to the usual measure of q, and I assess their respective performances in the aggregate investment equation.

4.1 Data and measurement issues

Bond yields

Moody's Baa index, denoted y_t^{Baa}, is the main measure of the yield on risky corporate debt. Moody's index is the equal weighted average of yields on Baa-rated bonds issued by large non financial corporations.[17] Following the literature, the 10-year treasury yield is used as the benchmark risk free rate. Both r_t^{10} and y_t^{Baa} are obtained from FRED.[18]

Expected inflation and real rate

The Livingston survey is used to construct expected inflation, and the yield on the 10-year

[17] To be included in the index, a bond must have a face value of at least 100 million, an *initial* maturity of at least 20 years, and most importantly, a liquid secondary market. Beyond these characteristics, Moody's has some discretion on the selection of the bonds. The number of bonds included in the index varies from 75 and 100 in any given year. The main advantages of Moody's measure are that it is available since 1919, and that it is broadly representative of the U.S. non financial sector, since Baa is close to the median among rated companies.

[18] Federal Reserve Economic Data: http://research.stlouisfed.org/fred2/ . The issue with using the 10-year treasury bond is that it incorporates a liquidity premium relative to corporate bonds. To adjust for this, it is customary to use the LIBOR/swap rate instead of the treasury rate as a measure of risk free rate (see Duffie and Singleton (2003) and Lando (2004)), but these rates are only available for relatively recent years. I add 30 basis points to the risk free rate in order to adjust for liquidity (see Almeida and Philippon (2007) for a discussion of this issue).

treasury to construct the ex-ante real interest rate.[19] The mappings are conditional on the current value of the real rate.

Idiosyncratic risk and leverage

The model described in section 3 constructs q from the relative price of corporate bonds, conditional on book leverage and idiosyncratic risk. Using Compustat, I find a slow increase in average book leverage from 0.4 to 0.55 over the post-war period.[20] The idiosyncratic volatility of publicly traded companies has also changed over time.[21] Campbell and Taksler (2003) show that changes in idiosyncratic risk have contributed to changes in yield spreads. Following the standard practice in the literature, I estimate idiosyncratic volatility with a 6-month backward moving average of the monthly cross-sectional standard deviation of individual stock returns.[22] The corresponding measure in the simulated data is used to infer the evolution of σ_η over time. This leads to a family of mappings, conditional on leverage and volatility, similar to the ones in Figure 2.

Creating q^{bond}

To summarize, q^{bond} is a function of four observed inputs: average book leverage, average idiosyncratic volatility, the ex-ante real rate, and the relative price of corporate bonds. Figure 3 displays the three main components: leverage, volatility and the relative price. In the short run, q^{bond} depends mostly on the relative price component. Year to year changes in $\left(\phi + r_t^{10}\right) / \left(\phi + y_t^{Baa}\right)$ account for 85% of the year to year changes in q^{bond}. In the long run, leverage and, especially, idiosyncratic risk are also important.

Classic measure of Tobin's q

The usual measure of Tobin's q is constructed from the flow of funds as in Hall (2001). The usual measure is the ratio of the value of ownership claims on the firm less the book value of inventories to the reproduction cost of plant and equipment. All the details on the construction of this measure can be found in Hall (2001).

[19] Piazzesi and Schneider (2006) analyze the consequences on asset prices of disagreement about inflation expectations. My model has nothing new to say about these issues.

[20] The sample includes non financial firms, with at least five years of non missing values for assets, stock price, operating income, debt, capital expenditures, and property plants and equipment.

[21] See Campbell, Lettau, Malkiel, and Xu (2001), Comin and Philippon (2005) for a discussion, and Davis, Haltiwanger, Jarmin, and Miranda (2006) for evidence on privately held companies.

[22] There are other ways to define idiosyncratic risk at the firm level (the standard deviation of the growth rate of sales) but they produce similar trends. See Comin and Philippon (2005) for a comparison of various measures of firm volatility. The measure used here is easy to construct and forward looking.

Investment and capital stock

I use the series on private non residential fixed investment, and the corresponding current stock of capital, from the Bureau of Economic Analysis. Table 1 displays the summary statistics.

4.2 Results

Figure 4 shows the two measures of q: q^{usual} constructed from the flow of funds as in Hall (2001), q^{bond} constructed using bond yields, leverage, idiosyncratic volatility, expected inflation, and the theoretical mappings described in the previous sections. The average value of q^{bond} is arbitrary: γ_1 is normalized to 1, so q is just above 1 on average.

It is meaningful, however, to compare the variations of the two measures. The standard deviation of q^{usual} is 0.848, while the standard deviation of q^{bond} is only 0.115, seven times less volatile. It is also interesting to notice that q^{bond} is approximately stationary, because the mappings take into account the evolution of idiosyncratic volatility and book leverage, as explained above.

Figure 5 shows q^{usual} and the investment rate in structure and equipment. Figure 6 shows q^{bond} and the same investment rate. The corresponding regressions are reported in the upper panel of Table 3. They are based on quarterly data. The investment rate in structure and equipment is regressed on the two measures of q, measured at the end of the previous quarter:

$$x_t = \phi^b q_{t-1}^{bond} + \phi^e q_{t-1}^{usual} + \varepsilon_t.$$

The standard errors control for auto-correlation in the error terms up to four quarters. q^{bond} alone accounts for more than half of aggregate variations in the investment rate. q^{usual} accounts for only 14% of aggregate variations. Moreover, once q^{bond} is included, the standard measure has no additional explanatory power. Looking at Figure 5, the fit of the investment equation is uniformly good, except during the 1991 recession, where the drop in the investment rate exceeds the one predicted by the bond market.[23]

q^{bond} is more correlated with the investment rate, hence the better fit of the estimated equation, but it is also much less volatile than the standard measure of q. As a result, the elasticity of investment to q is 15 times higher with this new measure, which is an

[23] This is consistent with the interpretation of the 1991 recession as a credit crunch.

encouraging result since the low elasticity of investment to q has long been a puzzle in the academic literature. The estimated coefficient still implies adjustment costs that are too high, around 16 years but, as Erickson and Whited (2000) point out, there are many theoretical and empirical reasons why the inverse of the estimated coefficient is likely to underestimate the true elasticity.

Figure 7 shows the 4 quarter difference in the investment rate, a measure used by Hassett and Hubbard (1997) among others, because of the high auto-correlation of the series in levels. The corresponding regressions are presented in the bottom panel of Table 3. The fit of the equation in difference is even better than the fit in levels, with an R^2 around 60%. In the third regression, the change in corporate cash flows over capital is added to the right hand side of the equation, but it is insignificant and does not improve the fit of the equation.

The conclusions from this empirical section are the following:

- With aggregate US data, the fit of the investment equation with the bond market's q is good, both in levels and in differences;
- The estimated elasticity of investment to q is more than 10 times higher than the one estimated with the usual measure of q;
- Corporate cash flows do not have significant explanatory power once the bond market's q is included in the regression.

5 Firm Level Evidence

Testing the theory at the firm level is more difficult because there are no panel data set on corporate bond prices comparable to Compustat and CRSP for equity prices. I create a panel of corporate bond yields by matching the rating of a company in a given year, available from Compustat, to the average yield in the same year and rating category. Thus, my measure of yields is rating-year specific, not firm-year specific.

I obtain data on corporate yields from Citigroup's yield book, which covers the period 1985-2004. For bonds rated A and BBB, these data are available separately for maturities 1-3, 3-7, 7-10, and 10+ years and I use the 10+ maturity. For bonds rated BB and below, these data are available only as an average across all maturities. The firm level data come from Compustat and are entirely standard in the literature. The data are described in Table 4.

I construct the bond market's q using the model without aggregating across firms. To test the theory in the cross section, I include a full set of year dummies and firm-fixed effects. The time dummies capture any change in the real interest rate and any common trend in leverage or risk. The firm fixed effect capture any permanent heterogeneity at the firm level.[24] These controls offer a cross-sectional test of the model that is orthogonal to the one performed earlier in the time series. I estimate the following equation:

$$x_{it} = \alpha_i + \xi_t + \phi^b q_{t-1}^{bond}(rating_{it}) + \phi^u q_{i,t-1}^{usual} + \varepsilon_{it}. \qquad (18)$$

I write explicitly $q_{i,t-1}^{bond}$ as a function of the rating to emphasize that all firms with the same rating in the same year have the same imputed yield. By contrast, $q_{i,t-1}^{usual}$ is really firm specific. To avoid outliers, I truncate the investment rate at 100%, and q at 10, as can be seen in Table 4.

Table 5 presents the regression results. For q^{bond}, the results are consistent with the ones obtained earlier. The coefficients are highly significant and the point estimates are similar to the ones obtained in the time series. For q^{usual}, the results are different. The regression coefficients are significant, and much larger than the ones obtained in the time series. In other words, relative stock market values are consistently linked to relative investment rates, even though the relation is weak in the aggregate. Finally, in contrast with the macro data, cash flow variables retain significant explanatory power at the firm level, even after the inclusion of q^{bond} and q^{usual}.

These results are consistent with the ones obtained by Gilchrist and Zakrajsek (2007) with a large panel data set of firm level bond prices. They regress the investment rate on a firm specific measure of the cost of capital, based firm level bond yields and industry specific prices for capital. They find a strong negative relationship between the investment rate and the corporate yields, and they also find that q^{usual} and cash flows remain significant.

[24] I include fixed effects rather than estimating firm level measures of risk because, while the trends in average or median volatility are consistent across a wide variety of measures (see Comin and Philippon (2005)), the cross sectional dispersion of the measures is quite large. In any case, the combination of fixed effects and time dummies actually accounts for most of the variations in idiosyncratic risk in the panel of rated firms.

One reason q^{bond} might perform better in the aggregate is that idiosyncratic shocks make the mapping from bond yields to aggregate q very smooth. At the firm level, however, the mapping is very nonlinear and small errors in the parameters translate into large errors in q^{bond}. Returns to scale might also be decreasing at the level of an individual firm, even though they are constant for the economy as a whole. This could explain why cash flows are significant in the micro data but not in the macro data. In addition, Vuolteenaho (2002) shows that much of the volatility at the firm level reflects cash flow news, whereas discount rate shocks are much more important in the aggregate. Finally, to the extent that mispricing explains some of the discrepancy between q^{usual} and q^{bond}, the results are consistent with the argument in Lamont and Stein (2006) that there is more mispricing at the aggregate level than at the firm level.

6 Conclusion

This paper has shown that it is possible to construct Tobin's q using bond prices, by bringing the insights of Black and Scholes (1973) and Merton (1974) to the investment models of Abel (1979) and Hayashi (1982). The bond market's q performs much better than the usual measure of q when used to fit the investment equation using post-war US data. The explanatory power is good (both in level and in differences), cash flows are no longer significant, and the inferred adjustment costs are more than ten times smaller.

Two interpretations of these results are possible. The first is that the equity market is subject to severe mispricing, while the bond market is not, or at least not as much. This interpretation is consistent with the arguments in Shiller (2000) and the recent theoretical work of Barberis and Huang (2007) and Brunnermeier, Gollier, and Parker (2007).

Another interpretation is that the stock market is mostly right, but that it measures something else than the value of the existing stock of physical of capital. This is the view pushed by Hall (2001) and McGrattan and Prescott (2007). According to this view, firms accumulate and decumulate large stocks of intangible capital. If the payoffs from intangible capital are highly skewed, then they could affect equity prices more than bond prices, and this could explain the results presented in this paper. The difficulty of this theory, of course, is that it rests on a stock of intangible capital that we cannot readily measure (see Atkeson and Kehoe (2005) for a plant level analysis).

Looking back at Figure 4, it is difficult to imagine a satisfactory answer that does not mix both theories. Moreover, these theories are not as contradictory as they appear, because the fact that intangible capital is hard to measure increases the scope for disagreement and mispricing. One can hope that future research will be able to reconcile them.

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A Proof of proposition 1

Let θ_τ be the marginal default rate during period τ. Let $\Theta_{t,\tau}$ be the cumulative default rate in periods $t+1$ up to $\tau-1$. In other words, if a bond has not defaulted at time t, the probability that it enters time $\tau > t$ is $1-\Theta_{t,\tau}$. Thus, by definition, $\Theta_{t,t+1} = 0$ and the default rates satisfy the recursive structure: $1-\Theta_{t,\tau} = (1-\theta_{t+1})(1-\Theta_{t+1,\tau})$. The value at the end of period t of *one unit of outstanding principal* is:

$$b_t^1 = E_t^\pi \left[\sum_{\tau=t+1}^{\infty} (1-\Theta_{t,\tau}) \frac{(1-\phi)^{\tau-t-1}}{(1+r_{t,\tau})^{\tau-t}} \left((1-\theta_\tau)(c+\phi) + \theta_\tau V_\tau / F_{\tau-1}\right) \right]. \tag{19}$$

Similarly, and just to be clear, the price of one unit of principal at the end of $t+1$ is:

$$b_{t+1}^1 = E_{t+1}^\pi \left[\sum_{\tau=t+2}^{\infty} (1-\Theta_{t+1,\tau}) \frac{(1-\phi)^{\tau-t-2}}{(1+r_{t+1,\tau})^{\tau-t-1}} \left((1-\theta_\tau)(c+\phi) + \theta_\tau V_\tau / F_{\tau-1}\right) \right]. \tag{20}$$

Using the recursive structure of Θ and the law of iterated expectations, we can substitute (20) into (19), and obtain:

$$b_t^1 = \frac{1}{1+r_t} E_t^\pi \left[(1-\theta_{t+1})(c+\phi) + \theta_{t+1} V_{t+1}/F_t\right] + \frac{1-\phi}{1+r_t} E_t^\pi \left[(1-\theta_{t+1})\, b_{t+1}^1\right] \tag{21}$$

Default happens when equity value reaches zero, that is, when:

$$V_t < F_{t-1}\left(\phi + c + (1-\phi)\, b_t^1\right)$$

Therefore, the pricing function satisfies:

$$b_t^1 = \frac{1}{1+r_t} E_t^\pi \left[\min\left\{\phi + c + (1-\phi)\, b_{t+1}^1;\ V_{t+1}/F_t\right\}\right] \tag{22}$$

Now recall that b_t^1 is the price of one unit of outstanding capital. Let us define b_t as the value of bonds outstanding at the end of time t, scaled by end of period physical assets:

$$b_t \equiv f b_t^1, \tag{23}$$

where book leverage was defined in the main text as $f \equiv \frac{F_t}{k_t}$, and assumed to be constant. Multiplying both sides of (22) by f, we obtain:

$$b_t = \frac{1}{1+r_t} E_t^\pi \left[\min\left\{(\phi+c)\, f + (1-\phi)\, b_{t+1};\ v_{t+1}\right\}\right]$$

In recursive form, and with constant book leverage, this leads to equation (7). Note that if book leverage is state contingent, the first term in the *min* function would simply be $(\phi + c)\, f_t + (1-\phi)\, b_{t+1} \frac{f_t}{f_{t+1}}$.

Table 1: Aggregate Summary Statistics

Quarterly Aggregate Data. 1953:2-2005:3.

	Obs.	Mean	St. Dev.	Min	Max
I / K	210	0.103	0.009	0.082	0.123
E(inflation)	210	0.038	0.025	-0.016	0.113
y^{Baa}	210	0.082	0.031	0.035	0.170
r^{10}	210	0.065	0.027	0.023	0.148
$(0.1+ r^{10})/(0.1+y^{Baa})$	210	0.908	0.033	0.796	0.974
Classic Tobin's q	210	2.005	0.848	0.821	4.989
Bond market's q	210	1.035	0.115	0.721	1.267

Investment and replacement cost of capital are from NIPA. Expected inflation is from Livingston survey. Yields on 10-year Treasuries and Baa bonds are from FRED. Classic Tobin's Q is computed from the flow of funds, following Hall (2001).

Table 2: Parameters of benchmark model

Parameters chosen exogenously			
Real risk free rate	r	3%	
Curvature of adjustment cost function	γ_2	8 years	
Average maturity	$1/\varphi$	10 years	
Book leverage	f	0.45	
Parameters directly observed in the data			
		data	model
Persistence of idiosyncratic profit rate	λ_η	0.47	0.47
Volatility of idiosyncratic innovations	σ_η	0.14	0.14
Persitence of aggregate profit rate	λ_a	0.7	0.7
Moments matched			
Relative bond price (mean)	(0.1+r)/(0.1+y)	0.908	0.907
Relative bond price (volatility)	(0.1+r)/(0.1+y)	0.033	0.33
Average bond issued at par value	E[b]/f	1	1
Implied parameters			
Average profit rate	a/r	0.93	
Volatility of aggregate innovations	σ_a	0.0546	
Coupon Rate	c	0.0435	

Table 3: Aggregate Regressions

	Equation in Levels: I/K(t)		
Bond Q (t-1)	**0.0622**		**0.0584**
s.e.	*0.0057*		*0.0060*
Tobin's Q (t-1)		**0.0042**	0.0016
s.e.		*0.0014*	*0.0009*
N	209	209	209
R^2	0.5684	0.1396	0.5861
	Estimation in Changes: I/K(t) - I/K(t-4)		
Δ(Bond Q) (t-5,t-1)	**0.0478**		**0.0424**
s.e.	*0.0045*		*0.0053*
Δ(Tobin's Q) (t-5,t-1)		**0.0071**	0.0021
s.e.		*0.0017*	*0.0012*
Δ(Profit Rate) (t-5,t-1)			0.0756
s.e.			*0.0495*
N	205	205	205
R^2	0.5986	0.1208	0.6163

Fixed private non-residential capital and investment series are from the BEA. Quarterly data, 1953:3 to 2005:3. Tobin's Q is constructed from the Flow of Funds, as in Hall (2001). Bond Q is constructed by applying the structural model to Corporate and Treasury yields, expected inflation, book leverage and idiosyncratic firm volatility. Newey-West standard errors with autocorrelation up to 4 quarters are below the coefficients, in italics. Bold coefficients are significant at the 1% level or higher.

Table 4: Firm Level Summary Statistics

Annual Firm Level Data. 1986-2004					
	Obs.	Mean	St. Dev.	Min	Max
Capital Expenditures / Prop. Plant & Equip.	14360	0.184	0.116	-0.031	1
Yield Spread (corporate-treasury)	14360	0.028	0.031	0.005	0.337
Market Value over Book Assets	14360	1.588	0.907	0.445	10
Operating Income over Book Assets.	14360	0.133	0.075	-0.799	0.918

Notes: Capital expenditures, income, market and book values are from Compustat. Corporate yields are from Citibank's Yieldbook, and 10-year treasury yields are from FRED. Sample includes Compustat firms with a bond rating, and at least $10 millions in Property, Plants and Equipment.

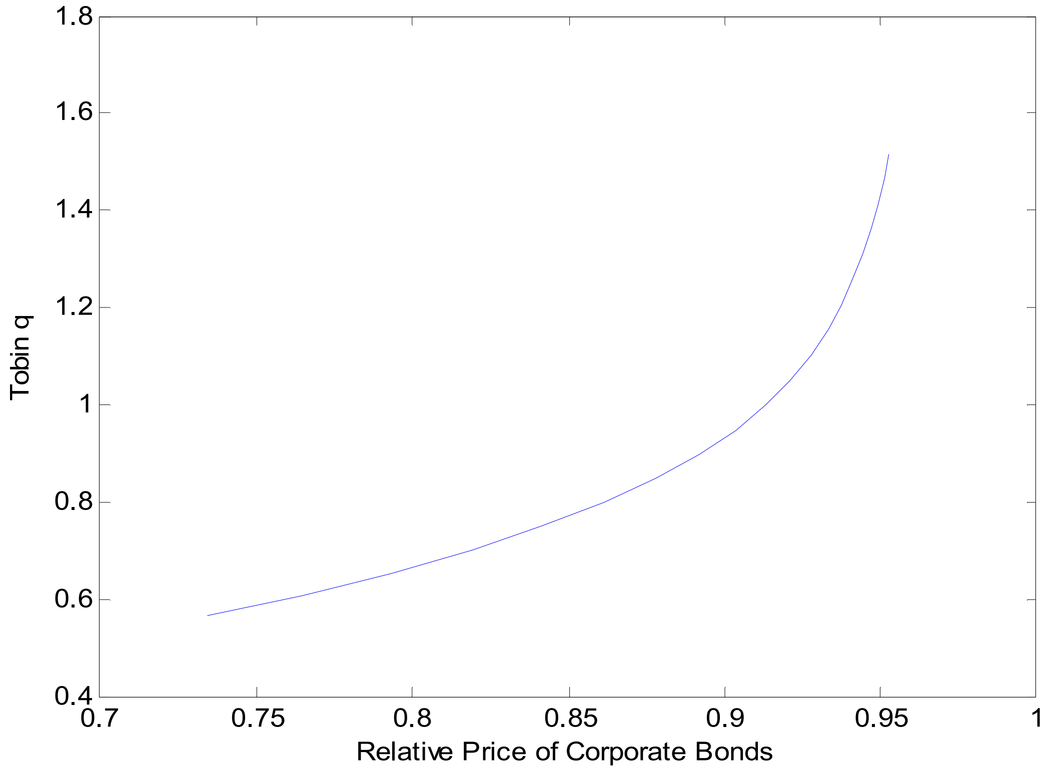
Table 5: Firm Level Regressions

Dependent variable is Capital Expenditure over lagged Property, Plants and Equipment. Panel regressions with fixed effects and robust standard errors.

Bond Q (lagged one year)	**0.0619**	**0.0535**
s.e	*0.005*	*0.0051*
Market Value over Book Assets (lagged one year)	**0.0298**	**0.0244**
s.e	*0.0018*	*0.002*
Operating Income over Book Assets (lagged one year)		**0.2003**
s.e		*0.0199*
Firm Fixed Effects	yes	yes
Year Dummies	yes	yes
N	14,360	14,360

Annual data 1986-2004. Sample includes Compustat firms with a bond rating, and at least $10 millions in Property, Plants and Equipment. Bond Q constructed from yield spreads matched by firm level rating. Standard errors corrected for firm level clustering. Bold coefficients are significant at the 1% level of higher. Sources: firm level data from Compustat, yields by rating from Citibank's yieldbook.

Figure 1: Tobin's q and the price of corporate bonds relative to treasuries.

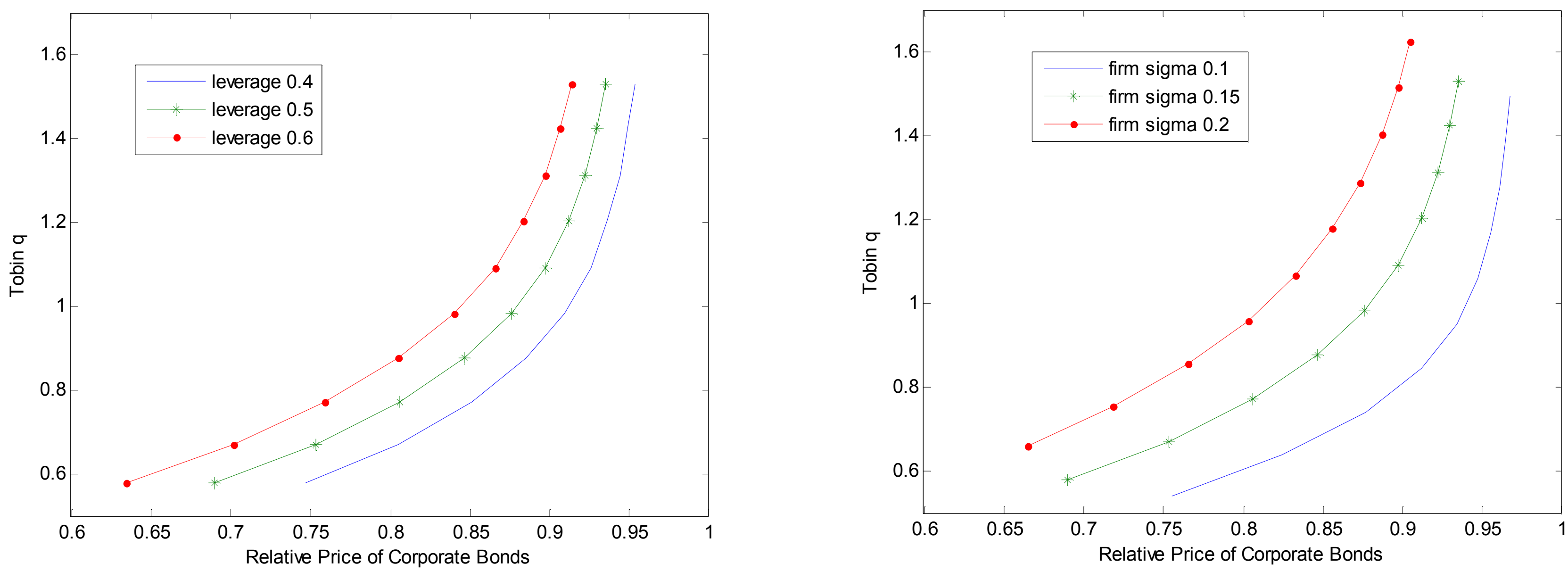


Note: Bonds are calibrated to an average maturity of 10 years. The relative price of corporate bonds is defined as (0.1+r)/(0.1+y), r is the risk free rate (assumed constant at 3%), and y is the average yield on corporate bonds. Parameters of the benchmark calibration are described in the text.

Figure 2: Impact of Leverage and Firm Volatility.

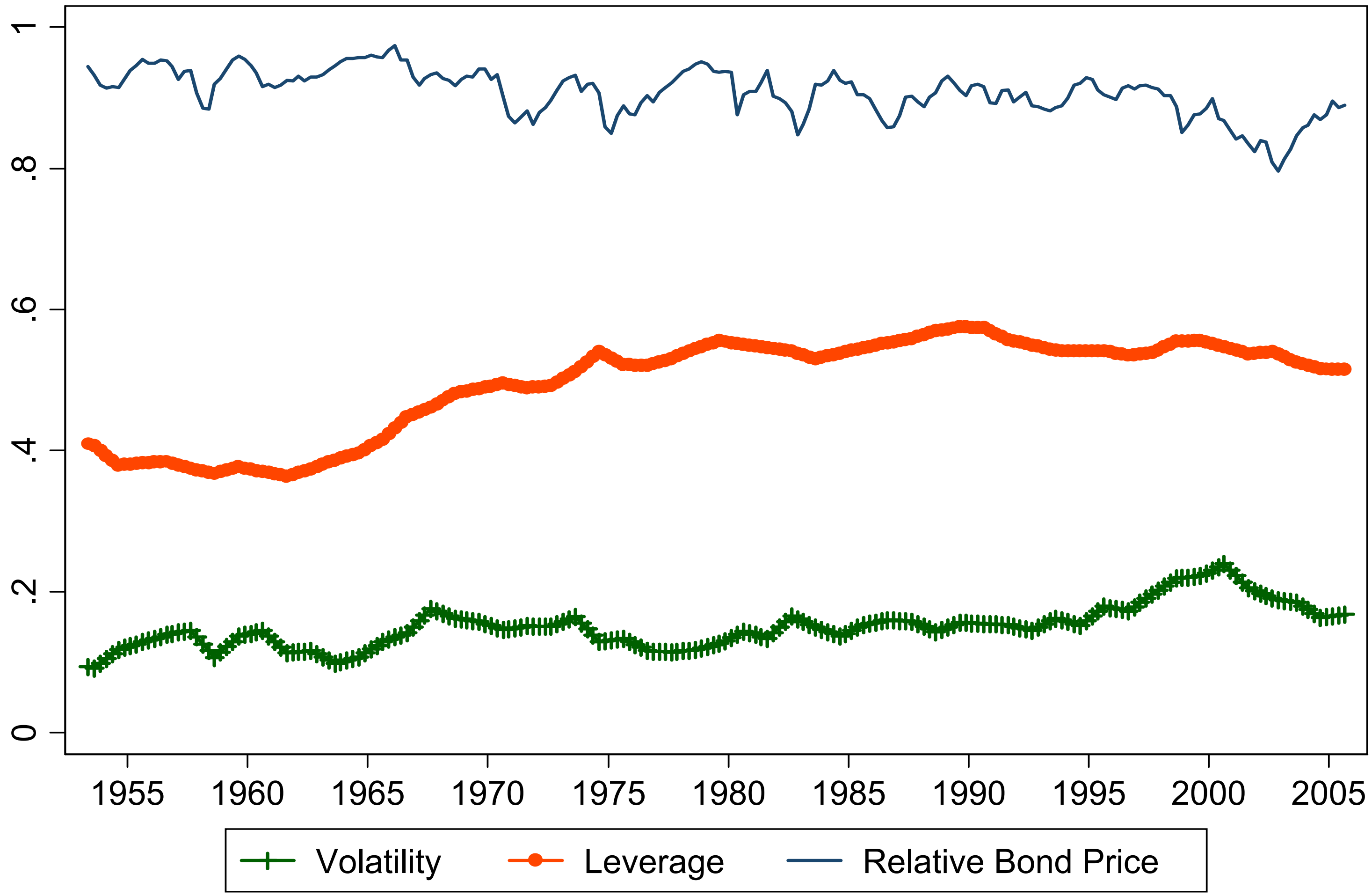
2a: Mapping for different values of book leverage

2b: Mapping for different volatilities of idiosyncratic shocks



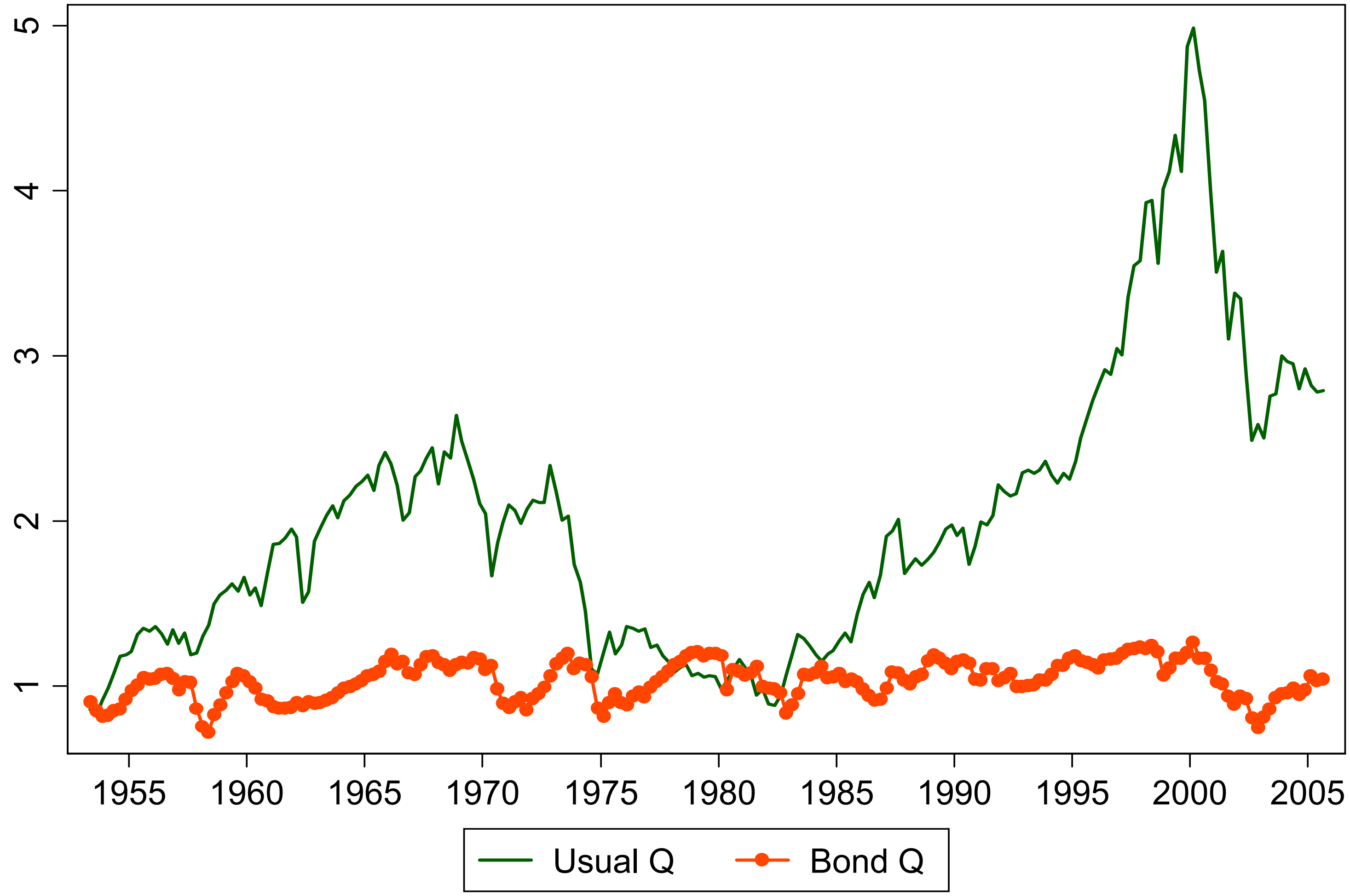
Note: Calibration like in Figure 1, except for book leverage in the left panel, and firm volatility in the right panel.

Figure 3: Three Components of Bond Market's Q



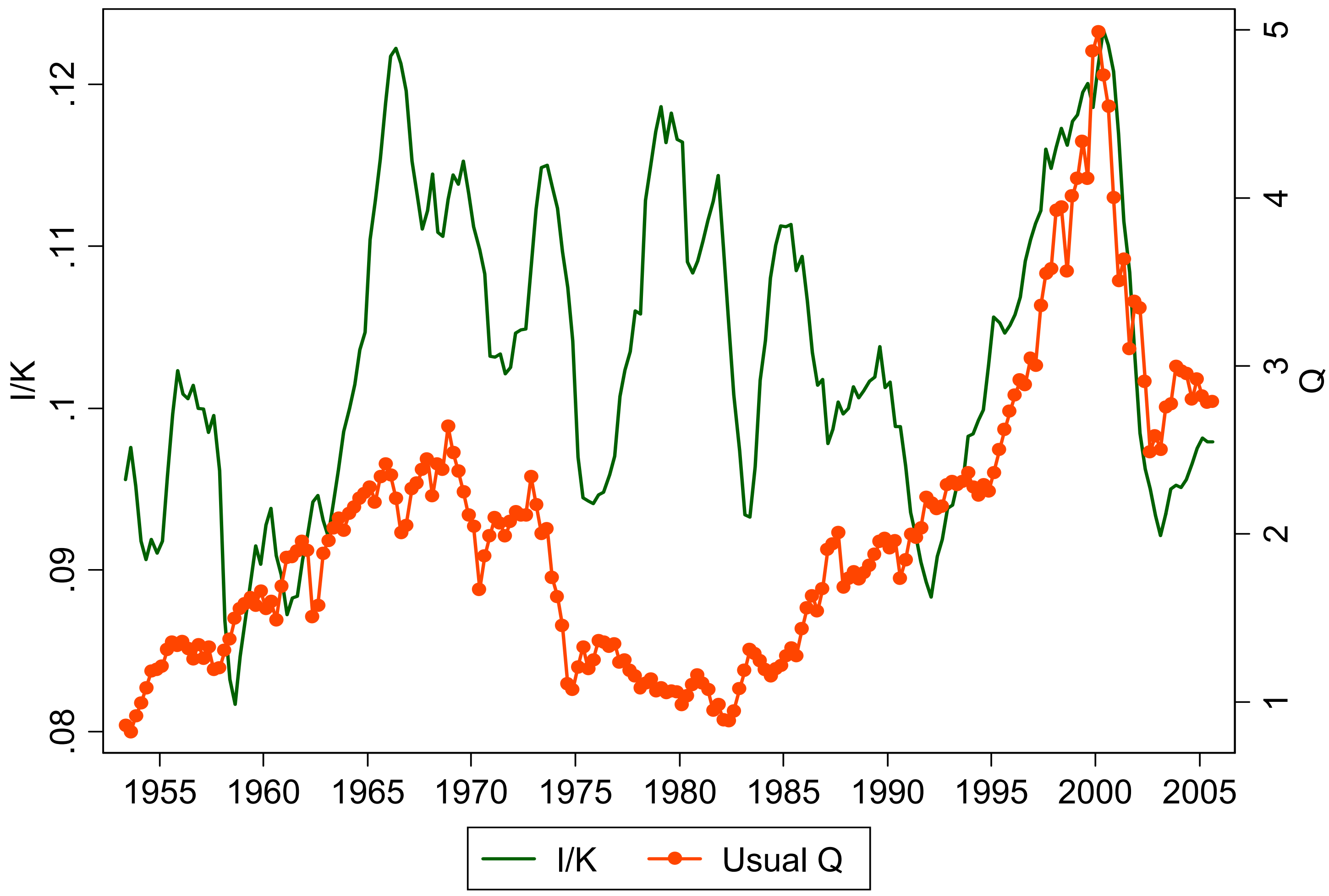
Notes: Leverage is average book leverage among non financial firms in Compustat. Idiosyncratic volatility is estimated using idiosyncratic stock returns and translated into parameter σ_η of the model. Relative Bond Price is the relative price of corporate and government bonds, defined as $(\varphi+r)/(\varphi+y)$, using Moody's Baa and 10-year Treasury yields (with $\varphi=0.1$).

Figure 4: Usual Measure of Q and Bond Market's Q



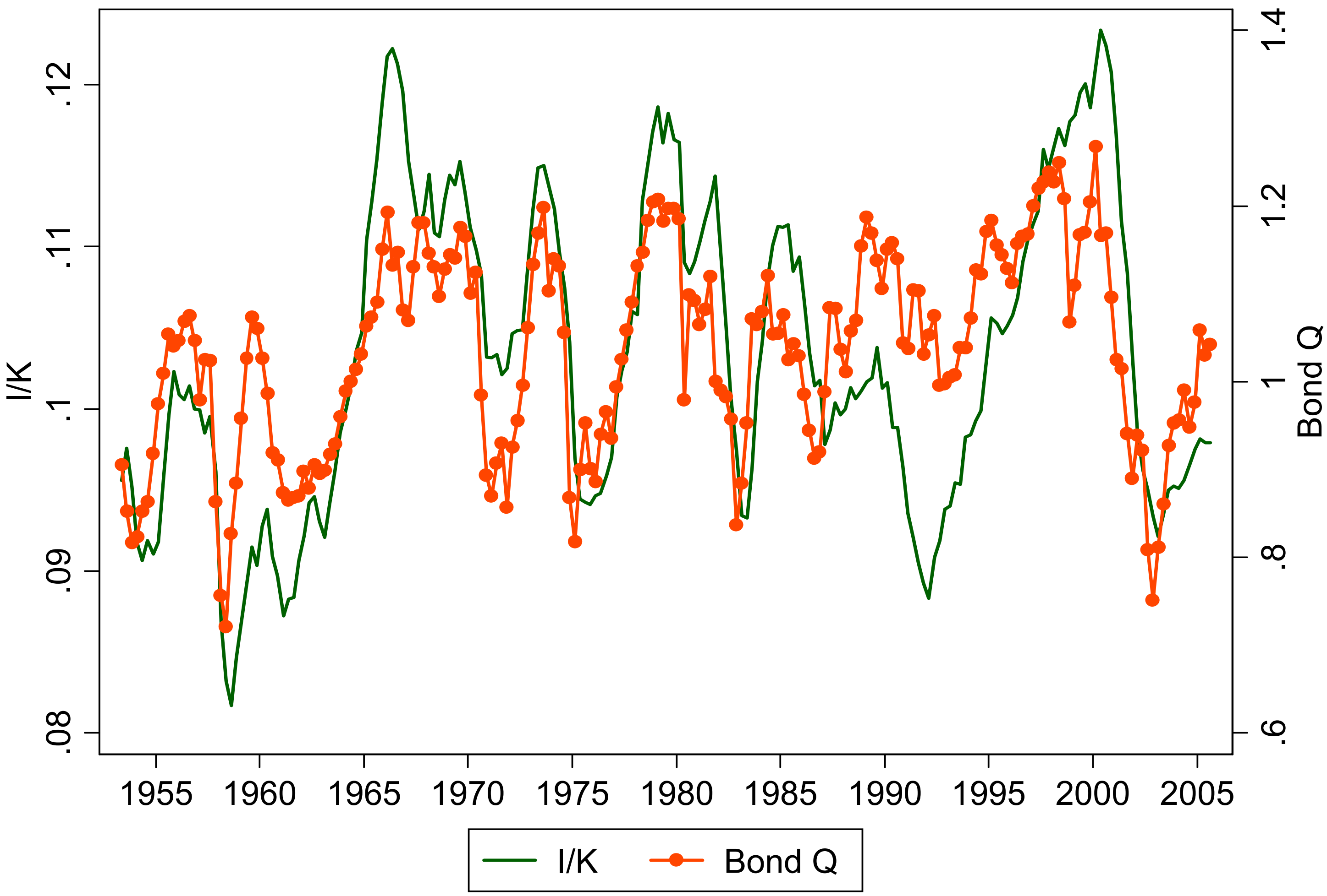
Notes: Tobin's Q is constructed from the Flow of Funds, as in Hall (2001). Bond Q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey and the yield on 10-year Treasury bonds.

Figure 5: Usual Measure of Q and Investment Rate (levels)



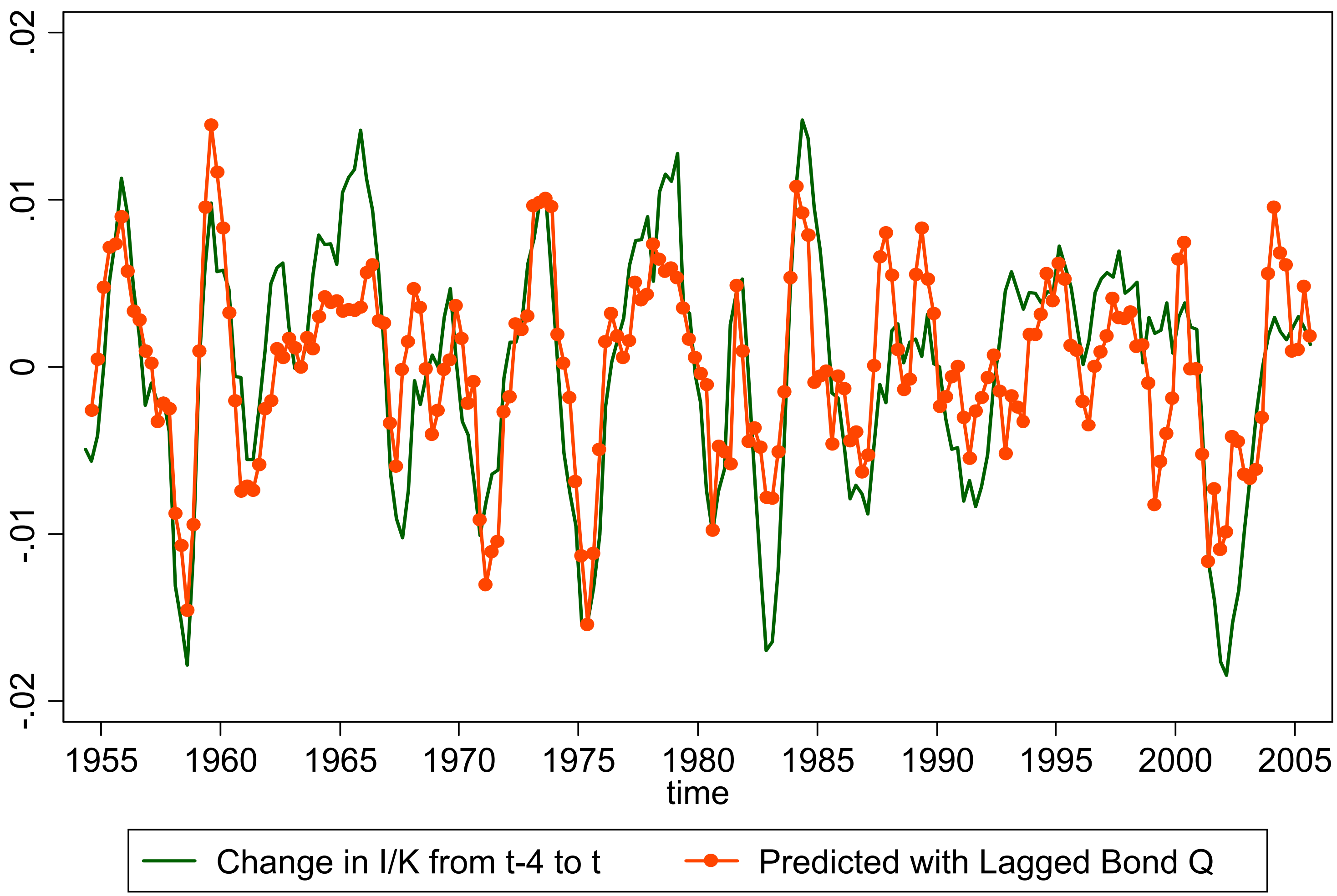
Notes: I/K is corporate fixed investment over the replacement cost of equipment and structure. Usual Q is constructed from the Flow of Funds, as in Hall (AER, 2001)

Figure 6: Bond Market's Q and Investment Rate (levels)



Notes: I/K is corporate fixed investment over the replacement cost of equipment and structure. Bond Q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey and the yield on 10-year Treasury bonds.

Figure 7: Investment Rate (4-quarter changes), actual and predicted with Bond Q.



Notes: I/K is corporate fixed investment over the replacement cost of equipment and structure. Bond Q is constructed from Moody's yield on Baa bonds, using the structural model calibrated to the observed evolutions of book leverage and firm volatility, expected inflation from the Livingston survey and the yield on 10-year Treasury bonds.