

Capital Structure and Contract Enforcement

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Abstract

This paper studies how the degree of contract enforcement in a country influences firms' capital structures. We first document the capital structure for a new dataset of firms in two countries, Ecuador and the UK, that feature different degree of contract enforcement. We find that capital structure is different in these two countries in terms of mean leverage and the leverage-size relation. In Ecuador leverage ratios are lower and smaller firms have smaller leverage ratios than large firms. In the UK leverage ratios are higher and bigger firms have smaller leverage ratios. We build a model of heterogeneous firms in an environment with lack of enforcement in debt contracts that delivers the observed capital structure in the data. In the model, the degree of contract enforcement acts like a tax or subsidy on the amount of borrowing for all firms. Weak contract enforcement corresponds to a tax that limits loans for all firms but hurts small firms more because their firm value relative to the tax is smaller and thus debt financing is more constrained. Strong contract enforcement corresponds to a subsidy on all firms that enables them to issue more debt but also helps disproportionately small firms given that the subsidy relative to their value is large. We quantify our mechanisms by calibrating our model to the firm datasets in the two countries and find that different degrees of enforcement can provide a unified rationale for the differential capital structure observed in the data.

1. Introduction

How does the capital structure of firms differ across countries? We document the relation between firm size and leverage for two new comprehensive datasets of firms in two countries: Ecuador and the UK. We find that leverage, defined as the ratio of liabilities relative to assets, is on average higher in firms in the UK relative to firms in Ecuador. We also find that the relation between firm size and leverage ratios differ across the two countries. In the UK small firms tend to have larger leverage ratios than large firms. In contrast, small firms in Ecuador have smaller leverage ratios relative to larger firms inside the country. We also document the relation between growth and size. We find that on average firms' sales in Ecuador grow faster than sales from firms in the UK. However both countries feature the common size-growth relation: small firms grow faster than large firms conditional on survival.¹ We choose these countries due to availability of data and because we want to contrast the firms' capital structure and dynamics in economies with different degree of contract enforcement.

This paper builds a model of heterogeneous firms to study the link between enforcement in financial contracts and firms' capital structure. In particular we study how lack of enforcement in debt contracts and incomplete markets can provide a rational for the facts regarding leverage, growth and size across countries. In the model, the degree of contract enforcement acts like a tax or subsidy on the amount of borrowing for all firms. Weak contract enforcement corresponds to a tax that limits loans for all firms but hurts small firms more because their firm value relative to the tax is smaller and thus debt financing is more constrained. Strong contract enforcement corresponds to a subsidy on all firms that enables them to issue more debt but also helps disproportionately small firms given that the subsidy relative to their value is large. We quantify both mechanisms by calibrating our model to Ecuador and the UK and find that our model provides a unified rational for the relation between leverage, growth and size that is dependent on the degree of contract enforcement.

¹ See for example Rossi-Hansberg and Wright (2005).

The framework is a dynamic model of heterogeneous firms similar to Albuquerque and Hopenhayn (2004) but with incomplete markets. Firms in the model borrow from foreign investors to finance the working capital and set up costs needed for production. Firms' productivity consists of two components: a permanent component and an i.i.d. component. We assume that firms sign contracts with investors to finance working capital before their i.i.d. shock is known and that these contracts cannot be contingent on the shock realization. After observing their shock firms can choose to repay the capital borrowed and the debt due and remain in operation for the next period, or default and get a default value. Incentives to repay debt depend crucially on the value of keeping the firm in operation at every period relative to the value of default.

Firms with permanent productivity differences have a distinct degree of enforcement because of the larger value of more productive projects relative to a constant default value. The key insight from our model is that when enforcement is very weak this constant default value acts like a tax on firms borrowing. The tax is disproportionately more expensive for small firms and thus borrowing is limited mostly for these firms. Large firms have essentially a larger endogenous degree of enforcement in these economies because the surplus from the relation with lenders is larger for firms with high productivity, allowing greater borrowing. However very strong enforcement acts like a subsidy on firms that is disproportionately more beneficial for small firms that have a low value. Thus the borrowing capacity is relatively larger for small firms as they are the ones that benefit the most from the additional available loans. We show that without uncertainty and permanent productivity differences, our model delivers a monotonic relation between size and leverage that is increasing when enforcement is weak, and is decreasing when enforcement is strong.

In our model with uncertainty we find that the firm's value depends inversely on the amount of debt that it owes to creditors. We show that firms with low values and high debt are inefficiently small because they under-invest relative to an unconstrained first best level. Enforcement problems are severe for low value firms, limiting the amount of risk-free funds. If firms instead borrow risky and default in some states, the high interest rates charged on loans also distort downward the optimal investment and the size of firms. Thus low value firms uniformly underinvest and produce at inefficient scales. Incomplete contracts introduce rich dy-

namics in the debt firms owe and the value of firms. In our model firms with a history of bad shocks accumulate debt, while reducing its value because their low output is not enough to cover interest payments on outstanding debt. Incompleteness of contracts and enforcement problems are key for generating the result that firms decrease their value and are likely to exit after a sequence of bad shocks. The relation between debt and investment in our model also rationalized the fact that small firms on average grow faster than large firms. If an inefficiently small firm receives a sequence of good shocks, it can reduce debt and increase investment further, which translates into higher growth and a better scale. Thus our model generates ‘positive persistence’ for low and high productivity realizations in firms’ output because debt can adjust inducing a less or more efficient scale in firms which is magnified over time.

We calibrate the full model to match certain features of the firm size distribution in Ecuador. We then increase the parameter controlling the degree of contract enforcement and compare the results to the UK statistics. We find that the enforcement friction and incomplete markets can deliver the features of the data in terms of mean leverage and the capital structure observed in both countries. The model can quantitatively account for the mean leverage ratios in Ecuador and for the size-leverage relation ranging from about 0.5 for small firms to 0.7 for large firms. The model with stronger degree of enforcement can also account for a larger mean leverage and the decreasing relation between leverage and size in the UK with ratios ranging from 0.9 for small firms to 0.7 for large firms. Both mechanisms, precautionary savings and differential endogenous degree of enforcement for firms with different sizes that varies with the country wise degree enforcement level, allow the model to deliver these results. The model also qualitatively matches the data in terms of the growth-size relation: small firms grow faster than large firms. However the growth statistics the model generates are smaller than in the data. The reason why growth is not high enough is because firms precisely engage in precautionary savings exactly to avoid the costs of underinvestment and volatile growth.

The paper is related to the literature that studies the implications of financial frictions for the dynamics of firms and firm size. Cooley and Quadrini (2001) study how financial frictions can rationalize the relation of exit and growth with size. Our model shares many of the features of their paper, however we are concentrating on how enforcement frictions can help explain the relation between leverage and size. Moreover we focus on the distinct implications

for debt financing in the presence of permanent and temporary productivity differences across firms. Albuquerque and Hopenhayn (2004) focus on the effects of enforcement problems and solve for the optimal state contingent contract. Our environment is different from them in that we consider an incomplete set of assets. Incomplete markets allows for firms in the model with a history of bad shock to decrease the effective degree of enforcement through time by increasing their debt holdings and for precautionary savings play a role.

Clementi and Hopenhayn (2005) and Quadrini (2004) also study financial imperfections and firm dynamics. These papers study financial constraints that arise due to informational asymmetries between the lender and the entrepreneur and show that moral hazard considerations can also rationalize borrowing constraints which make investment sensitive to cash flows. They show that information asymmetries can also provide a rational for the relation between growth and size.

2. Empirical Facts

We have compiled a new dataset of firms in two countries with different degrees of contract enforcement: Ecuador and UK.² Here we document the regularities regarding the capital structure of firms in these countries and the dynamics of firms with regards to growth and exit. For Ecuador, we have balance sheets for a panel dataset of over 25,000 firms for the years 1996 to 1999 from the tax records in the local regulatory agency: Superintendencia de Compañías del Ecuador.³ The dataset includes the universe of firms registered as legal entities in Ecuador from all sectors except Agriculture and Real Estate. For the UK, we have balance sheet data for a panel of over 2 Million firms for the years 2000 to 2005 from Amadeus Database. The dataset covers all sectors in the economy. To analyze the datasets, we clean the sample by restricting it to include those firms that report positive and non-missing assets each year. We also take out all firms in the financial sector following Rajan and Zingales (1995) and throw away outlier firms with leverage ratios above 10. This procedure cuts the sample in Ecuador to about 20,000 firms per year, and in the UK to about 600,000 firms per

² From the Business Environmental Risk Intelligence, the contract enforcement index for Ecuador is 0.65, for the UK 1.25, and for the US 1.28.

³ We have data for Ecuadorean firms up to 2005, however Ecuador dollarized in 2000 and thus we have limited the sample to include balance sheets denominated only in the same currency.

year. For the sales growth statistics we also restrict the sample to firms who report positive and non-missing revenues.

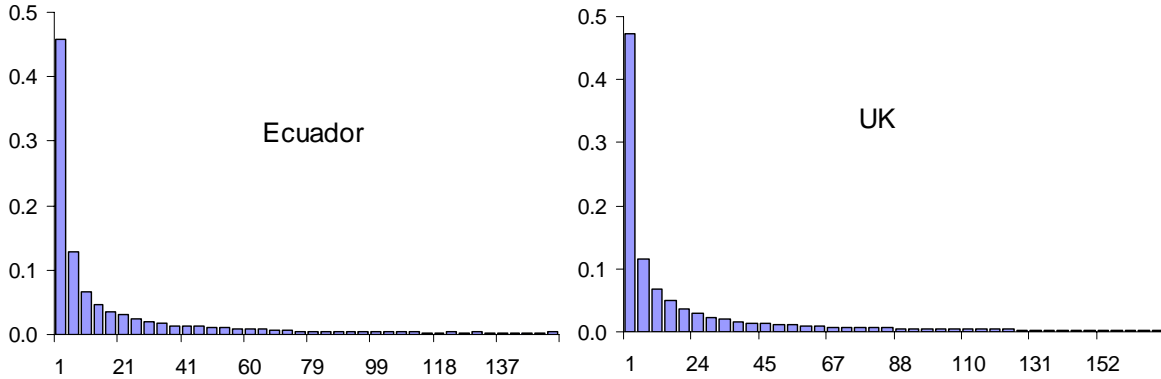


Figure 1. Asset Distributions

Our measure of firm size is the book value of total assets for the firm. Unfortunately we cannot use the more common size measure, employment, because the dataset for Ecuador provides no information regarding employment.⁴ Figure 1 plots the firm size distribution for the two countries with the clean sample. The histogram for Ecuadorian firms is for 1996 and the histogram for British firms is for 2000.⁵ The histograms consist of all firms in the sample of each country with the exception of the largest 10% of firms. The overall median level of assets in 1996 is 63,762 US dollars in Ecuador. The overall median asset level in 2000 is 184,952 US dollars in UK. The mean asset level in Ecuador is 1.06 Million US dollars, and in UK it is 18.21 millions US dollars. In the figure we normalize assets in both countries such that the mean asset in the first bin equals 1. The firm size distributions for both countries present similar patterns: most firms are small, and the distribution is highly skewed to the right. In Ecuador the 90 to 10 percentile of assets in 1996 is 423 and in the UK in 2000 it is 487. It is worth noting that although our samples have large number of small firms, they might still be underrepresented. In Ecuador a large informal sector exists which is not covered in our

⁴ We have also done all the analysis based on sales as the measure for size. Statistics reported in the paper are very similar for both countries with this alternative size measure.

⁵ Firm size distributions are very stable across time in both countries.

dataset and for the UK the Amadeus dataset coverage does not include the universe of registered firms which biases it against the small firms.

We divide firms in each country into quintiles according to assets to organize the data. We are interested in the relation of size with firms' capital structure. Leverage is defined as the broad measure of total liabilities over total assets of the firm. We use this broad definition because it is a more consistent measure across countries and because it provides the largest sample of firms. We are also interested in the relation between size and growth. Growth is defined as real sales growth per firm between one year and the next. Sales are deflated by the CPI of each country and year.

Table 1 reports descriptive statistics for the firms in Ecuador in 1996 and the United Kingdom in 2000. The mean leverage ratio for Ecuadorian firms of 0.57 is smaller to that for British firms of 0.76. The arithmetic average yearly growth in sales for firms in Ecuador of 64% is larger than that in the UK of 39%. Figure 2 illustrates graphically the relation of size with leverage with patterns completely different in both countries. In the UK the leverage size relation is downward sloping: small firms have relatively higher leverage ratios than large sized firms.⁶ In particular the mean leverage ratio of the firms in the first quintile is 1.01 and the mean leverage ratio in firms in the fifth quintile is 0.6. In Ecuador the leverage-size relation is monotonically increasing, ranging from 0.41 for the smallest firms to 0.68 for the largest firms.

It is interesting to see that the leverage ratios for the largest firms are similar in both countries. However small firms appear remarkably different in both countries, with small firms in the UK having larger debt to asset ratios than in Ecuador. The difference in the size-leverage relation across these two countries is robust across years, sectors, number of bins, and alternative size measures such as sales. The appendix shows these robustness checks regarding the differential leverage-size relation in the data between Ecuador and UK. Our model will address precisely how different degree of contract enforcement across both countries can affect most significantly the debt limits of small firms.

⁶ Rajan and Zingales (1995) document a monotonic increasing relation between leverage and size for the UK in a sample of the largest 608 firms. We find a similar pattern when considering only the largest 1000 firms from our dataset.

Table 1. Firm Size Statistics

	Leverage Liabilities/Assets		Sales Growth	
	Ecuador	UK	Ecuador	UK
Overall	0.57	0.76	0.64	0.39
Quintile 1	0.41	1.01	1.30	0.67
Quintile 2	0.55	0.82	0.58	0.31
Quintile 3	0.60	0.71	0.37	0.29
Quintile 4	0.64	0.64	0.50	0.39
Quintile 5	0.68	0.60	0.38	0.28

Data for Ecuador is for 1996, and for UK it is for 2000. Leverage = Total Liability/Total Asset. Growth is net real growth in sales deflated by CPI.

In terms of growth and size both countries present the patterns that have been documenting extensively for other countries such as the US: small firms grow faster than large firms conditional on surviving. The relation between growth and size in the UK featured only the smallest firms have a significantly higher growth rate than the rest of firms. Rossi-Hansberg and Wright (2006) report similar patterns for the US as US employment growth rates between the middle and large firms do not vary substantially, whereas employment growth for small firms is significantly higher. In Ecuador, mean growth rates for all firms are on average higher than in the UK and the difference in growth rates between firms in the fifth and the second quintile remains substantial.

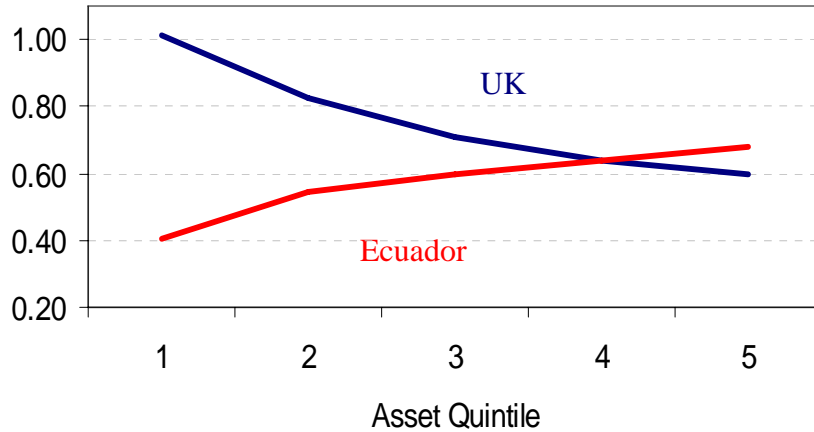


Figure 2. Size and Leverage in the Data

3. Model Economy

The economy consists of a small country populated by a continuum of infinitively lived entrepreneurs who have access to risky projects and finance those projects by borrowing from foreign lenders. Financial contracts are not enforceable and entrepreneurs can default on the debt they owe to creditors. Contracts in this economy are also incomplete and thus default can occur along the equilibrium.

3.1 Firms

Entrepreneurs in the economy have access to a mass of N project opportunities to produce consumption goods. We define each project opportunity as a firm. Every period a fraction ξ of the projects are available to new entrepreneurs. Entrepreneurs are risk neutral, infinitively lived and decide on entry, exit and production and financing plans to maximize the lifetime value of dividends from the project. Entrepreneurs discount time at rate $\beta < 1$ and face an exogenous probability of dying $\delta < 1$ each period. New entrepreneurs have no initial wealth and

their outside option is equal to 0. Thus, it is always worth it for an entrepreneur to enter and operate the firm if he draws a project that gives him positive expected present value of dividends. Each entrepreneur owns at most one firm.

Firms are funded by foreign lenders who offer contracts to cover two distinctive needs: initial set up costs and working capital requirements. Each new firm needs to pay an initial set-up cost I to start its operation. Since new firms have no initial wealth, they borrow from foreign long-lenders this initial set up cost. The set up cost is the initial loan for every firm $B = I$. This initial loan is associated with a starting long-debt position for the firm $B_R = I_R$ that is due the next period. Every subsequent period the firm can choose a new loan and long-debt position (B', B'_R) from the set of long term contracts offered. By choosing a different long-term contract corresponding to a smaller long debt position, $B'_R < B_R$, the firm effectively repays off part of its initial set up cost.

Every period each firm that is in operation produces output y with a stochastic production technology that transforms capital goods into consumption goods with a decreasing returns technology. For a given level of capital input K invested at the beginning of the period, the firm produces output y at the end of the period such that

$$y = z\varepsilon_z K^\alpha, \quad (1)$$

where $0 < \alpha < 1$. The productivity shock has two firm specific components. One is the permanent productivity level z , which is drawn from distribution $F(z)$. The other is the temporary productivity level ε_z , which is i.i.d. and drawn from distribution $G_z(\varepsilon_z)$. Capital depreciates completely every period.

The timing of decisions within the period is as follows. At the beginning of the period, each entrepreneur who owns a project with long-debt B_R , decides how much capital K to invest in the project depending on his permanent productivity level and his expectation of temporary productivity shocks. Firms borrow the working capital desired for production from foreign short-lenders. At the end of the period the temporary productivity shock ε_z is realized and the firm decides whether to repay or default on all debt due. If the firm repays the working capital and the long debt, then it chooses a long-debt contract from the ones offered and

continues operations for the next period. If the firm defaults on all the debt due, both short and long term, then it exits and the entrepreneur re-draws a new project with probability θ with productivity drawn from distribution $F(z)$. Every firm that exits frees up a project opportunity for the following period.

3.2 Contracts

The set of contracts between the firm and lenders are of two types: short-term contracts and long-term contracts. Short-term contracts are for capital requirements. These are traded in the beginning of the period before the temporary productivity shock is known and consist of two numbers (K, K_R) . K denotes the payment in the beginning of the period from the lender to the firm which is used as capital input for producing output. K_R is the payment the firm promises the lender at the end of the period conditional on not defaulting. The set of short-term contracts are a function of the firms' long-debt position in the beginning of the period B_R and its permanent productivity level z .

Long-term contracts are traded at the end of the period after the productivity shock is known and consist also of two numbers (B', B'_R) . B' is the immediate payment from the lender to the firm that is used for interest payments of the original set up costs and to increase period dividends. B'_R is payment the firm promises the lender next period conditional on not defaulting. B'_R is the new long debt position of the firm next period. The set of long-term contracts are independent of the firm's long debt in the beginning of the period B_R , but depends on the firm's permanent productivity level z . Both contracts are exogenously incomplete in that they are not a function of the output realization when the payments are due.

3.3 Incentives, Dividends and Production

The dividend each entrepreneur receives at the end of the period depends on whether he decides to repay his debts and remain in operation, or default and exit. After observing the

temporary productivity shock ε_Z at the end of the period, if repayment is chosen, then the entrepreneur of each firm receives as dividend

$$d = z\varepsilon_Z K^\alpha - K_R - B_R + B' . \quad (2)$$

The dividend conditional on repayment equals the output produced with capital K in the period minus payment of the short-term working capital debt K_R and long debt due B_R plus new long debt issuances B' . Also we define as ‘profit’ the period return from the project excluding long debt operations: $z\varepsilon_Z K^\alpha - K_R$.

If the entrepreneur chooses to default on his debt then the firm exits and the entrepreneur loses the project. The default welfare of the entrepreneur is given by V^d , which represents any costs and benefits the entrepreneur might get after closing the firm. The parameter V^d controls the degree of enforceability of financial contracts in the country. If V^d is very low, default is very unattractive for entrepreneurs and we interpret this as the case with higher enforceability of financial contracts, and vice versa.

Entrepreneurs consider benefits and costs when they decide on whether to default or not. The default decision for a particular firm that has a permanent productivity z , long debt B_R and short contract (K, K_R) after observing its temporary productivity shock ε_Z is given by

$$W^o(B_R, K, K_R, z, \varepsilon_Z) = \max \left\{ W^c(B_R, K, K_R, z, \varepsilon_Z), V^d \right\}, \quad (3)$$

where $W^o(B_R, K, K_R, z, \varepsilon_Z)$ denotes the present value of this firm and $W^c(B_R, K, K_R, z, \varepsilon_Z)$ is the present value conditional on repaying debt today.

If the entrepreneur decides to repay his debt, then he chooses a long-term contract (B', B'_R) to maximize the value of staying in the contract

$$W^c(B_R, K, K_R, z, \varepsilon_Z) = \max_{B', B'_R} \left(u + \beta(1 - \delta)W(B', z) \right), \quad (4)$$

subject to

$$d = z\varepsilon_Z K^\alpha - K_R - B_R + B'$$

$$u = \begin{cases} d & d \geq 0 \\ \gamma d & d < 0 \end{cases},$$

where $\gamma \geq 1$. The entrepreneur's current utility is increasing in dividend and the marginal utility for negative dividends is higher than those of positive dividends under the assumption that $\gamma \geq 1$. This assumption basically puts some concavity into the entrepreneur's preference. It can be interpreted in two ways. First, negative dividends are new equity issuances that come with costs. Second, stock holders are likely to think of negative dividends as bad news and might intervene more in firm's management, which the entrepreneur dislikes.

$W(B'_r, z)$ is the present value of a firm with debt B'_r and a permanent productivity z in the beginning of the following period before the temporary productivity shocks are realized. By paying the short-term debt and choosing the new long-term contract the entrepreneur can remain in operation for one more period. The entrepreneur understands that the decision to repay debts is a period-by-period decision and that tomorrow he will again have the option to default. The default policy of each firm can be summarized by repayment sets and default sets. For a given initial level of debt B_R and short contract is (K, K_R) , denote the default set $D(B_R, K, K_R, z)$ as the ε_Z set for which the value under the contract is less than the value under default

$$D(B_R, K, K_R, z) = \{ \varepsilon_Z : W^c(B_R, K, K_R, z, \varepsilon_Z) < V^d \}. \quad (5)$$

The repayment set is defined as the complement of the default set:

$$A(B_R, K, K_R, z) = [D(B_R, K, K_R, z)]^c$$

In addition to debt and default choices, the entrepreneur makes investment decisions every period. In the beginning of the period before the productivity shock is observed the entrepreneur decides on capital and production plans. Given an initial level of debt B_R the entrepreneur chooses the short-term contract (K, K_R) to finance investment such that it maximizes his value

$$W(B_R, z) = \max_{K, K_R} \int W^o(B_R, K, K_R, z, \varepsilon_Z) dG_Z(\varepsilon_Z). \quad (6)$$

The decision of how much capital to invest depends on the permanent productivity level, the expectations of temporary productivity and on the set of contracts available. For example, if temporary productivity is expected to be high, the optimal size of the project is larger and the entrepreneur has an incentive to invest a large level of capital. In fact in a world without enforcement frictions, the entrepreneur will choose the size of the project proportionally to temporary productivity expectations. However with enforcement frictions the optimal size of the project can be distorted by the set of short-term and long-term contracts available. In particular if the firm already starts the period with a large level of long term debt then default would be more likely and thus the set of short term and long term contracts will very limited. The entrepreneur might then not be able to run his project with an optimal size and the project will be inefficiently small.

3.4 Lenders

Lenders in the model are assumed to be competitive and discount time at the rate of the international risk free interest rate r . They behave passively and are willing to finance the firm's initial set up costs and working capital needs as long as they are compensated for the expected loss in case of default. In particular a short-term lender offers contracts (K, K_R) to a firm with long debt B_R such that

$$K = \frac{K_R}{(1+r)} \left(1 - \int_{D(B_R, K, K_R, z)} dG_Z(\varepsilon_Z) \right). \quad (7)$$

With every short term contract (K, K_R) the lender breaks even in expected value.

A long-term lender offers contracts (B', B'_R) such that with every contract the lender receives in expectation the risk free interest rate

$$B' = \frac{B'_R(1-\delta)}{1+r} \left(1 - \int_{D(B'_R, K'(B'_R, z), K'_R(B'_R, z), z)} dG_Z(\varepsilon_Z) \right) \quad (8)$$

Every long-term contract offered (B', B'_R) forecasts the firm's choice of the short-term contract next period $(K'(B'_R, z), K'_R(B'_R, z))$. This is because the firm's decision to default or repay is carried out after the capital stock is in place and the productivity is realized.

When a new firm starts its operation, its initial long-term contract (I, I_R) is such that the firm receives as first payment the initial set-up costs needed I with

$$I = \frac{I_R(1-\delta)}{1+r} \left(1 - \int_{D(I_R, K(I_R, z), K_R(I_R, z), z)} dG_Z(\varepsilon_Z) \right). \quad (9)$$

If initial set up costs are so high then there might not exist a long-term contract that can finance the entry of the firm because probabilities of default are too high.

3.5 Equilibrium

We now define the equilibrium:

Definition.

The recursive equilibrium for this economy is defined as: (i) the policy functions for short-term contracts $(K(B_R, z), K_R(B_R, z))$, long-term contracts $(B'(B_R, K, K_R, z, \varepsilon_z), B'_R(B_R, K, K_R, z, \varepsilon_z))$, dividends $d(B_R, K, K_R, z, \varepsilon_z)$, repayment sets $A(B_R, K, K_R, z)$ and default sets $D(B_R, K, K_R, z)$ for each firm, (ii) a menu of short-term contracts $(K(B_R, z), K_R(B_R, z))$ and long-term contracts $(B'(z), B'_R(z))$ offered to each firm given its productivity, (iii) a distribution of firms over permanent productivity level, temporary productivity shocks and long debt holdings $\Upsilon(B_R, z, \varepsilon_z)$ and (iv) a mass of new entrants $\xi(\Upsilon(B_R, z, \varepsilon_z))$, such that:

1. *Taking as given the menu of short-term contracts $(K(B_R, z), K_R(B_R, z))$, and long-term contracts $(B'(z), B'_R(z))$ offered, the policy functions $(K(B_R, z), K_R(B_R, z))$,*

$(B'(B_R, K, K_R, z, \varepsilon_z), B'_R(B_R, K, K_R, z, \varepsilon_z)), d(B_R, K, K_R, z, \varepsilon_z)$, repayment sets

$A(B_R, K, K_R, z)$ and default sets $D(B_R, K, K_R, z)$ satisfy the firm's optimization problem.

2. Short-term contracts and long-term contracts available to each firm reflect the firm's default probabilities such that lenders break even in expected value.
3. The distribution of firms $\Upsilon(B_R, z, \varepsilon_z)$ is consistent with individual decisions and shocks.
4. The mass of new entrants $\xi(\Upsilon(B_R, z, \varepsilon_z))$ is equal to the measure of all the firms that default and die in the limiting distribution $\Upsilon(B_R, z, \varepsilon_z)$.

3.6 First Best Benchmark

We will compare our model to the benchmark where contracts are perfectly enforceable but incomplete. We denote this benchmark model as the first best. The capital stock of each firm in the first best is such that the expected marginal product of capital equals the interest rate. Firms have different sizes because of different permanent productivity levels. The firm pays each period the perpetuity value of the set-up cost and receives as dividend the residual. In the first best the capital stock is determined by

$$zE(\varepsilon_z)\alpha K_{fb}(z)^{\alpha-1} = (1+r). \quad (10)$$

Though we have different sizes of firms, all the firm are producing at the efficient scales given their permanent productivity level. Note that under the first best, the short profit given by

$$\pi_{fb}(z) = \left(\frac{zE(\varepsilon_z)\alpha}{1+r} \right)^{\frac{\alpha}{1-\alpha}} - (1+r) \left(\frac{zE(\varepsilon_z)\alpha}{1+r} \right)^{\frac{1}{1-\alpha}}$$

and that profit over the investment is independent of productivity and given

$$\text{by: } \frac{\pi_{fb}(z)}{K_z} = (1+r) \left(\frac{1-\alpha}{\alpha} \right).$$

Given the model setup, we will next illustrate how the enforcement friction interacts with firms' productivity shocks under incomplete contracts. Especially we will focus on the stylized facts documented in section 2. In the next two sections we illustrate some numerical examples with only permanent productivity differences or only temporary shocks alone to study the model mechanisms for firm dynamics. In section 6 we will study the quantitative implications of the model by calibrating the parameters to the firms in Ecuador.

4. Degree of Enforcement and Leverage

A feature we want to study with our model is the relation between firm size and borrowing capacity across economies with different degree of enforcement. To this end this section provides an example that illustrates how our model generates leverage levels that are increasing in firm size when economies have weak enforcement while leverage decreasing in firm size in economies with strong enforcement.

Consider the case of no-uncertainty with firms differing in size because of permanent differences in productivity z . For this case firms' investment level is the first best level (equation 10) every period, yet high z firms invest larger K . The project value for a firm with productivity z and level of long debt B in a stationary equilibrium is given by:

$$W^c(B_R, z) = \frac{1}{1 - \beta(1 - \delta)} \left(\pi_{fb}(z) - \frac{r}{(1 + r)} B_R \right) \quad (11)$$

where $\pi_{fb}(z)$ is the short profit at the first best investment level.

Long debt contracts B_R for which the firm has incentives to repay are limited by the outside option V^d . In particular the maximum amount of funds that a firm with productivity z can borrow is given by the level of debt $\bar{B}(z)$ that makes the contract value equal to the outside option

$$\pi_{fb}(z) - \frac{r}{(1 + r)} \bar{B}(z) = V^d (1 - \beta(1 - \delta)).$$

Note that more productive firms are able to borrow larger loans independently on the degree of enforcement because short profits $\pi_{fb}(z)$ are larger for more productive firms.

Now leverage in the data is given by total liabilities relative to total assets for the firm, which corresponds in our model to the ratio $(K_R + B_R) / (K + I)$. Assume for now that $I = 0$ for all firms. The leverage ratio in this case is given by $(1+r) \left(1 + \frac{B_R}{K_R} \right)$. Thus the relation between size and leverage is driven by the ratio of $B_R(z) / K_R(z)$ across firms with different productivity. Now from the above equations the maximum ratio is:

$$\frac{\bar{B}(z)}{K(z)} = \left((1+r) \frac{(1-\alpha)}{\alpha} - \frac{V^d(1-\beta(1-\delta))}{K(z)} \right) \frac{(1+r)}{r}. \quad (12)$$

Given that $K(z)$ is increasing in z , we get the result that leverage capacity increases with size in economies with weak enforcement, positive V^d , and leverage decreases with size in economies with strong enforcement, negative V^d . Moreover the leverage-size relation is monotonically decreasing with size as the degree of enforcement becomes stronger.

This result implies that in countries with weak enforcement permanently more productive firms have higher degree of enforcement and can sustain more debt relative to their larger assets without having incentives to default. The reason is that the value of a more productive firm is bigger relative to a constant outside option that is high and positive. Weak enforcement in essence acts like tax on the amount of borrowing for all firms. This constant tax is disproportionately more costly for small low-productivity firms because their value is smaller. In countries with strong enforcement, small firms can sustain more debt relative to their assets while having incentives to repay their loans. Strong enforcement acts like a subsidy on the amount of borrowing for all firms, which is more disproportionately more beneficial for small firms.

5. Firm Dynamics with Incomplete Markets

Uncertainty and incomplete markets introduce dynamics in the investment and output of firms. This section studies the implications from lack of enforcement in contracts and incomplete assets in terms of firm size and dynamics to highlight the role of uncertainty in our model. We find that firms with high levels of debt are inefficiently small, and this inefficiency is exacerbated over time if firms receive a sequence of bad productivity realizations.

We first characterize default and investment decisions for firms for an example where shocks are *i.i.d.* and given by $\varepsilon = \{\varepsilon_L, \varepsilon_H\}$, $z = 1$ for all the firms and dividends are restricted to be non-negative, i.e. $\gamma = \infty$. The probability of the low shock is π_L and the probability for the high shock is $\pi_H = 1 - \pi_L$. We study how the firms' decisions vary as a function of the level of long debt it owes. With only temporary uncertainty that is *i.i.d.* over time, the set of long debt contracts offered to firms is constant over time and across firms. In particular the available set is independent of the level of level of long debt B_R , productivity realization ε and short-term contract (K, K_R) .

Lemma 1.

1. Default sets are increasing in long debt. If $\varepsilon \in D(B_R^1, K, K_R)$, then $\varepsilon \in D(B_R^2, K, K_R) \forall B_R^2 > B_R^1$.
2. Long debt contracts are bounded. There exists an $\bar{B} > 0$ such that $B' \leq \bar{B}$ for all (B', B'_R) contracts.

Firms default when they hold high long debt. This is because both current dividend $d(B_R, K, K_R, \varepsilon)$ and the contract value $W^c(B_R, K, K_R, \varepsilon)$ are decreasing in long debt while the default value is independent of long debt.

The set of long debt contracts are bounded by the maximum set up costs that a project can have while remaining profitable. In an environment without perfect enforcement, the entrepreneur will only participate in the project if the expected value delivers a positive value. The project at most will generate the expected lifetime discounted profit under the optimal investment K_{fb} , $M = \left[E(\varepsilon)K_{fb}^\alpha - K_{Rfb} \right] / r$. Thus projects with larger set up costs will for sure not be

financed because they can not deliver expected positive value to both the lender and the entrepreneur. Hence, the set of long contracts every period offered to firms that remain in operations are bounded such that $\bar{B} \leq M$.

Moreover, the value of each project in the economy with imperfect enforcement will be lower than in an economy with perfect enforcement due to the added constraints. We show below that in our model capital depends on the level of long debt and firms will be inefficiently small. This implies that the maximum set up cost for which projects will be financed are strictly smaller in our economy relative to the first best $\bar{B} < M$. In addition, if with this maximum loan of \bar{B} default sets are non-empty the corresponding long contract (\bar{B}, \bar{B}_R) will imply that $\bar{B}_R \geq \bar{B}(1+r)$.

Definition. *Debt Overhang.* A firm with long debt B_R faces ‘debt overhang’ if when it invests the first best capital stock, it cannot satisfy the non-negative dividend conditions for all shock realizations.

Proposition 1. Debt overhang exists. There exists a B^* such that for all $B_R > B^*$, $\varepsilon_L K_{fb}^\alpha - K_{Rfb} - B_R + B' < 0 \forall (B', B'_R)$.

Define $B^* = \bar{B} + \varepsilon_L K_{fb}^\alpha - K_{Rfb}$. For any $B_R > B^*$, the non-negative dividend condition for ε_L cannot be satisfied. The first best capital stock is chosen based on average productivity. However the non-negative dividend condition must be satisfied for every shock realization. Thus if ex-post the low shock is realized for some firm the first best capital stock is too big because the short-term loan is on an inefficiently big capital stock. Given that enforcement problems limit the resources long debt contracts provide, a firm with large long debt that invests the first best capital and receives the low productivity shock will have negative dividends. Note that in our model firms can have debt overhang for $B_R < \bar{B}$ if $\bar{B} + \varepsilon_L K_{fb}^\alpha - K_{Rfb} < \bar{B}_R$. Given that $\bar{B}_R \geq \bar{B}(1+r)$ the necessary condition is

that $\varepsilon_L K_{fb}^\alpha - K_{Rfb} < r\bar{B}$. We can then choose a particular distribution of shocks to guarantee this condition for any $r\bar{B} \geq 0$.

Proposition 2. Default sets are the lower set.

If $\varepsilon_2 \in D(B_R, K, K_R)$, then $\varepsilon_1 \in D(B_R, K, K_R)$ for any $\varepsilon_1 < \varepsilon_2$.

Proof. See Appendix.

Firms default when productivity is low. If default is involuntary then it is clear that default happens for low shocks because current dividends $d(B_R, K, K_R, \varepsilon)$ are decreasing in ε . If default is voluntary, then firms default for low productivity because the contract value $W^c(B_R, K, K_R, \varepsilon)$ is increasing in productivity at a faster rate than the default value, which is constant across the temporary productivity shock. The reason is that the set of long debt contracts the firm can choose under the low shock, such that the non-negative dividend condition is satisfied, are also feasible under the high shock.

Proposition 3. A firm with debt overhangs under-invests relative to the first best.

Proof. See Appendix.

A firm that faces ‘debt overhang’ has two choices: (1) the firm chooses not to default for both shocks, but adjusts the investment such that the non-negative dividend condition is satisfied for ε_L ; (2) the firm chooses to default for some shock and adjusts the investment decision according to default choices. The proposition shows both cases lead to underinvestment relative to the first best.

A firm that faces debt overhang and decides to repay debt, cannot have the first best capital because this would lead to having negative dividends. Thus the firm decreases its investment such that profits under ε_L are high enough to deliver a non-negative dividend. This necessarily leads to underinvestment relative to the first best because smaller sizes of firms increase ex-post profits for ε_L .

The firm with debt overhang could also decide to default for some shock realizations. If the firm chooses to default, this will happen only for the case of low productivity ε_L . This is because default sets are the lower set due to proposition 2, and because they are less than the whole set when projects produce positive output.

Capital choices are based on maximizing expected profits and relaxing the non-negative dividend condition ex-post state by state in the repayment states. Regarding the maximization of profits firms choose capital such that the marginal product equals the marginal cost. When firms choose to default under some shocks, the expected marginal product of capital is lower than if firms do not default because default states involve direct costs as the firm loses output in the low shock. However the expected marginal cost of capital is the same regardless of the default decisions because short-term lenders break even. These two features push the optimal capital stock to be smaller than the first best when default sets are non-empty. Regarding relaxing the non-negative dividend condition, this is only relevant for the choice of capital if it is binding under ε_H .⁷ However, under ε_H the non-negative dividend condition is relaxed by also decreasing the capital stock. The reason is that given that the short-term contract carries a default premium and the capital that maximizes $\varepsilon_H K^\alpha - (1+r)/\pi_H K$ is smaller than K_{fb} . Thus if the firm wants to use capital to relax the non-negative dividend condition in the high shock, this also implies that it must under-invest relative to the first best. It is key that the short-term contract adjusts according to default choices. If the short-term contract would not be adjusted to default choices a defaulter firm could find it optimal to over-invest relative to the first best.

The result that enforcement problems induce firms to be inefficiently small is also present in models of enforcement frictions with a complete set of assets (Quintin (2000), Cooley, Marimon and Quadrini (2004), and Albuquerque and Hopenhayn (2004)). Thus our added feature of incomplete markets is unnecessary for qualitatively getting this result. However, our key difference is that incomplete markets amplify the inefficiency in scale for some firms because the value of the firm can decrease over time due to higher debt holdings. Enforcement prob-

⁷ The non-negative dividend condition under z_L is irrelevant if default is chosen here.

lems are more severe when the value of the firm is low, thus as firms increase debt holdings the enforcement friction limits further their ability to produce efficiently. For a given initial start up cost, lack of enforcement plus incomplete markets generates a larger portion of inefficiently small firms in the invariant distribution.

To illustrate the relation between long debt B and investment K we plot the decision rule and dynamics in terms of capital and output after a stream of positive or negative shocks. The detailed calibration of these parameters is reported in Table 5 in the in the next section.

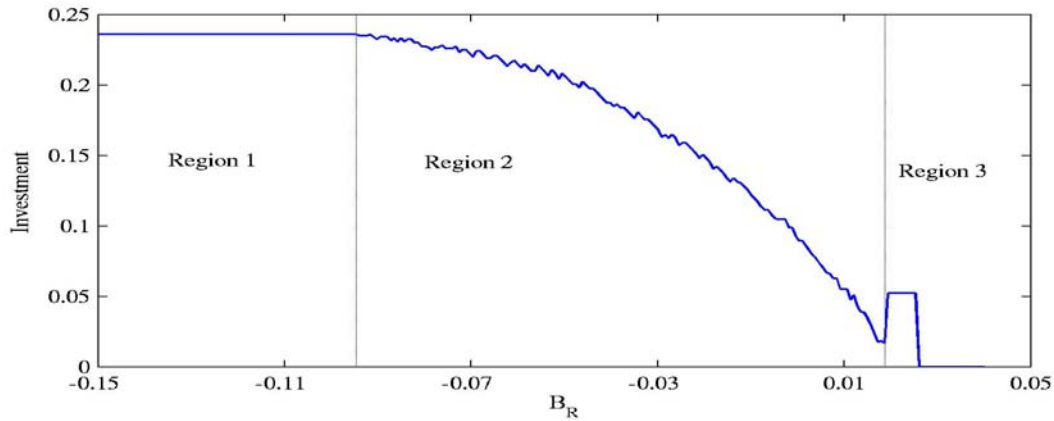


Figure 3 plots the investment decision a firm as a function of its long debt. We can classify 4 distinct regions across long debt according to investment decisions. In the first region long debt is small enough such investment is equal to the first best. The second region corresponds to the case where the firm faces a binding non-negative dividend condition when the low shock is realized. Here the firm under-invests as prescribed in Proposition 3 to avoid default by increasing profits in the low state such that the non-negative dividend condition is satisfied. The reason that the firm is willing to be small is that the continuation value of keeping the project $W(B')$ is very large once the initial setup costs have been already paid.

When long debt is big enough, the firm finds it optimal to default if the low shock is realized; this is region 3. In anticipation of such an event the firm under-invests relative to the first best as in Proposition 3, but increases investment relative to region 2. The reason for the increase in investment is that given that in the low shock the short loan will not be repaid, the

firm finds it optimal to adjust its size for a more appropriate scale in the high shock. However given the more expensive short contracts, this increase in investment is never big enough such that capital is higher than in the first best. For the highest debt levels in region 3, the firm can further under-invest because the non-negative dividend condition binds for the high shock, and decreasing investment relaxes this constraint.

Finally in region 4, long debt is so high, that for any positive output the firm will find it optimal to default. Thus in equilibrium short lenders are unwilling to provide the firm with any contracts that would give positive output. An alternative way to think about region 4 is that projects with the initial set up costs corresponding to region 4 will not be financed in equilibrium. Note though that under perfect enforcement some of these projects would be financed because the value of the firm under perfect enforcement is uniformly higher than under imperfect enforcement for all initial set up costs. This is because the value of the firm is lower due to underinvestment when debt is high.

We now present the time series plots. The dynamics of a firm's investment and output after a series of shocks is driven by the dynamics of long debt, as short-term investment contracts do not carry any persistent effects in our model.⁸ Figure 4 presents the time series dynamics for two firms: the upper panel plots output and long-term bond dynamics for a firm that experiences a sequence of only low productivity shocks; the lower panel plots the same statistics for a firm that experiences a sequence of only high productivity shocks. The initial debt position was chosen such that firms start in region 2.

Let's first consider the case of a firm that faces low productivity shocks. This firm produces less and less output while it increases its long debt holdings over time. The firm chooses to remain in operation because of better future prospects, but it produces very little contemporaneously. After a long enough sequence of low shocks, 6 periods in this simulation, long debt grows so much, such that the firm finds it optimal to go bankrupt and exit. Anticipating a default event in period 6, the capital invested increases because the firm does not repay back the short loan in the low shock and thus dividends are strictly positive.

⁸ This is because we have assumed that depreciation of capital equals 1. If depreciation was to be lower, capital dynamics would interact with long debt dynamics.

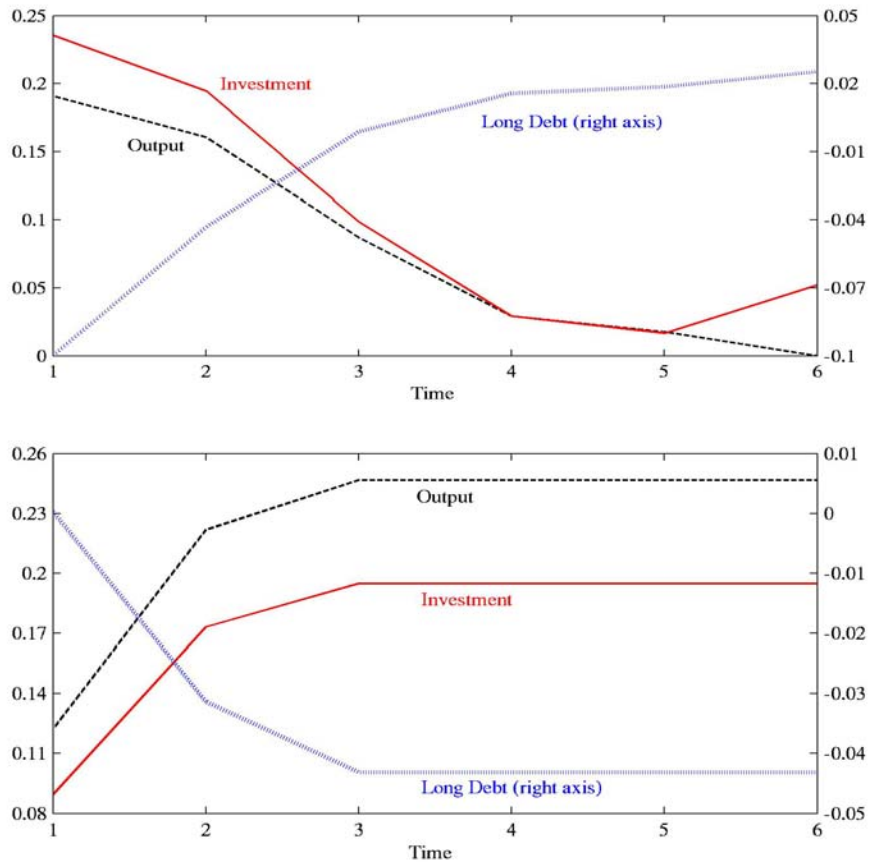


Figure 4. Dynamics of firm's output, capital and long debt

Incomplete markets are essential for enforcement frictions to amplify bad productivity shocks over time. Our interpretation is that enforcement problems are exacerbated for firms that have a history of low productivity. A sequence of low shocks lowers the value of the firm because uncontingent long debt has to grow over time to cover interest payments of an increasingly higher stock of debt. This feature differentiates our model from existing models that look at the implications of enforcement problems under a complete set of assets. In those models the value of the firm cannot go down ever because state contingent long debt allows the firm to efficiently pay off the set up costs only when high shocks are realized. By introducing incomplete markets our model is able to maintain the underinvestment feature of those models, while producing the additional feature of amplification for low shocks

Let's now consider the case of a firm who faces a sequence of good productivity shocks as in the lower panel of Figure 4. A firm who initially has a high initial long debt, starts operat-

ing at an inefficient scale. However if the firm receives a high productivity shock then it is able to repay enough of its long debt, such that in the next period it becomes unconstrained. This shows that our model also generates amplification of good shocks because after a sequence of high shocks the value of the firm increases enough such that it becomes unconstrained. Our model shares the ‘positive persistence’ feature of models with enforcement frictions and a complete set of assets have.

Our model matches the data in that small firms pay less dividends and their investment is sensitive to cash flow. Furthermore, our model has several empirical testable implications in terms of the dynamics of firms. First it predicts that firms with high debt will have more volatile output conditional on a given volatility of shocks. Second, our model predicts that the history of productivity matters for a long time in terms of firms’ output. In particular even under temporary shocks as in this example, firms can become smaller and smaller over time because they face very more restrictive financial contracts after a history of low productivity.

How does different degree of contract enforcement affect dynamics of output and investment? Worse enforcement makes borrowing constraints tighter and so for given initial level of debt, underinvestment is faster and more severe in the economy with worse enforcement. Thus firms in the worse enforcement economies grow and shrink more after a sequence of good or bad shocks. This feature allows our model to address the higher growth rates of small firms in Ecuador relative to those in the UK.

However in the model firms have an extra asset, the level of long debt, to precisely insure against the effects of borrowing constraints and underinvestment. Thus the tighter constraints and more severe underinvestment in economies with worse contract enforcement gives firms greater incentives to engage in precautionary savings. Thus firms in economies with worse enforcement would want to hold relatively lower levels of debt because of the difficulties of obtaining loans. This mechanism allows our model to match the feature of the data that the mean leverage of firms is smaller in Ecuador than in the UK.

5. Quantitative Implications of the Model

We now calibrate the full model with both transitory and permanent. The goal is to assess quantitatively our mechanism in reproducing the facts regarding leverage and size for the

dataset of firms in the UK and Ecuador. We first describe the calibration of the model, and then present the quantitative results.

We calibrate the permanent and transitory productivity process to match two features of the firm level data in Ecuador. First are the sizes of the decile firms according to asset. Second is the mobility matrix across the quintile bins of assets in the empirical part. We assume that the standard deviation of transitory shocks is independent of the permanent shock and that the permanent shocks have the same mass. We choose 9 permanent shock levels $\{z_i\}_{i=1}^9$ and the variance of the transitory shock $\sigma_z = \sigma$ such that the model produces the asset deciles and the mobility matrix closest to those in the data.

We set the interest rate r at 4 percent per annum. The decreasing return parameter α is chosen to be 0.90 following the empirical estimates by Basu and Fernald (1997), which study the manufacturing firms and find this parameter close to one. We calibrate the death rate δ to the exit rate in the largest asset bin of 5% and the discount factor β to the mean leverage ratio in Ecuador. We set the preference parameter γ to be 5 and the startup cost I to zero and will conduct the sensitive analysis. Finally, the enforcement parameter V^d for each model economy is chosen to match the leverage ratio of the first bin in the UK and in Ecuador respectively. See Table 5 for the summary of parameters.

	Parameter	Values	Target
Permanent productivity	(z_1, z_2, \dots, z_9)	(1.075, 1.15, 1.22, 1.26, 1.35, 1.43, 1.53, 1.60, 1.84)	Asset level of the decile firms
Temporary productivity	σ_z	0.15	Mobility matrix across asset bins
Discount factor	β	0.96	Mean leverage in Ecuador
Death rate	δ	0.05	Exit rate of the 5 th bin in Ecuador
Technology parameter	α	0.90	Basu and Fernald (1997)
Interest rate	r	0.04	US real interest rate annual
Preference	γ	5.0	Sensitivity analysis

Startup cost	I	0.0	Sensitivity analysis
Enforcement parameter	V_d	Ecuador: 1.2	Leverage of the first asset bin
		UK: -1.2	

We simulate the model over 500 periods for 3000 firms for both Ecuador and the UK. In each economy, at every point in time there is a cross section distribution of firms that we use to compute the model statistics. Given that we don't have aggregate uncertainty, the model delivers a stationary distribution of firms over debt and assets. We divide at every point in time, the cross section of firms into 5 quintiles based on size K such that an equal number of firms is in each bin. We then compute for every quintile and for the whole distribution the average leverage and sales growth. Table 6 reports averages of these statistics across the last 100 periods for both the UK and Ecuador.

Table 6. Model Results

Ecuador	Assets	Leverage	Growth
Overall	27.43	0.74	1.03
Quintile 1	1.00	0.64	1.04
Quintile 2	2.76	0.72	1.03
Quintile 3	7.47	0.76	1.03
Quintile 4	22.10	0.78	1.03
Quintile 5	103.84	0.78	1.03
UK	Assets	Leverage	Growth
Overall	26.66	0.82	1.03
Quintile 1	1.00	0.92	1.03
Quintile 2	2.68	0.83	1.03
Quintile 3	7.27	0.81	1.03
Quintile 4	21.48	0.79	1.03

Quintile 5

100.88

0.77

1.03

The model produces a higher mean leverage ratio for the UK than that for Ecuador, 0.82 versus 0.74. The reason is as follows. The financing constraints that firms face in the good enforcement economy are looser than those in a bad enforcement economy. Given the looser financing constraints, firms in the UK have less incentive to engage in precautionary savings, which drives up the mean leverage in the UK relative to Ecuador. The model also produces an upward size-leverage relation for Ecuador and a downward size-leverage relation for the UK, which are both consistent with the data. Also see Figure 8 for the contrasting pattern of the size-leverage relations. These results are coming from the intuition developed in section 4. In the UK a strong enforcement provides additional loans for all firms. And the smallest firms benefit mostly from the additional loans available relative to their small values. In Ecuador, the weak enforcement limits the aggregate level of loans for all firms. However the firms that are most affected by the lack of loans are the small firms because it is more costly relative to their small value. Moreover the model matches quantitatively the patterns in the data, for example the leverage ratio for the smallest quintile in Ecuador of 0.64 which is much smaller than the one in the UK of 0.92. It is interesting to see that the large firms in both countries have similar leverage ratios in our model and in the data. This is because large firms are the least affected in both countries by the particular lack or abundance of loans. Their high productivity projects give them enough revenues and profits to be able to self insure over and above any enforcement considerations.

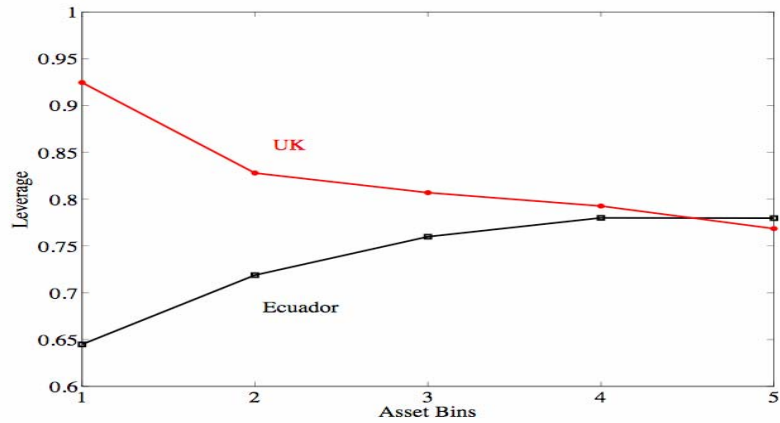


Figure 8. Size and Leverage in the Model

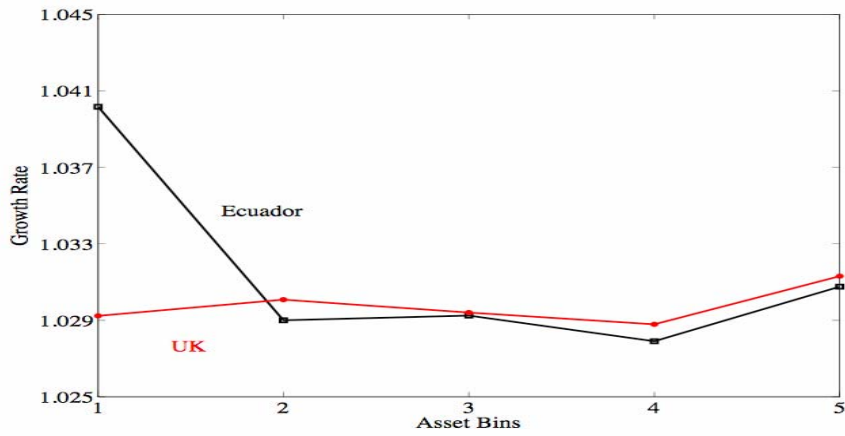


Figure 9. Size and Growth in the Model

We also report the statistics for the size-growth relation (see also figure 9). In the Ecuadorian model economy small firms grow faster than large firms, but the differences are much smaller than the data. In the UK, we produce a flat size-growth relation. To generate the size-growth relation we need to have underinvestment and inefficient scale to be high probability events in the distribution of firms. As discussed in Section 5 with uncertainty, firms in the first bin are small because they have received a sequence of bad shocks. Thus they are very sensitive to good shocks because they can reduce their debt holdings and increase their capital

stock towards a better size. However, in our model these are low probability events because the level of precautionary savings is calibrated to match a relatively low leverage ratio observed. Firms in our model still display the dynamics of output and investment after a sequence of good and bad shocks, but the severe underinvestment does not happen frequently enough in the limiting distribution of the model to generate the high growth rates observed.

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Appendix 1

Proposition 2. Default sets are the lower set.

If $z_2 \in D(B, K, P)$, then $z_1 \in D(B, K, P)$ for any $z_1 < z_2$.

Proof: We need to show that for any $z_1 < z_2$, $W^c(B, K, P, z_1) < \eta z_1 K^\alpha$, i.e.,

$(1 - \eta)z_1 K^\alpha - P - B + L_1^* + \beta W^o(B_1^*) < 0$, where (L_1, B_1^*) is the optimal long debt contract.

From $z_2 \in D(B, K, P)$, we have $(1 - \eta)z_2 K^\alpha - P - B + L_2^* + \beta W^o(B_2^*) < 0$, where (L_2, B_2^*) is the optimal long debt contract. Define the feasible set of long debt contract (L, B') under

$W^c(B, K, P, z)$ as $\Gamma(K, P, B, z) = \{(L, B') \mid zK^\alpha - P - B + L \geq 0\}$.

Clearly $\Gamma(K, P, B, z_1) \subset \Gamma(K, P, B, z_2)$. Thus, we have

$$0 > (1 - \eta)z_2 K^\alpha - P - B + L_2^* + \beta W^o(B_2^*) \geq (1 - \eta)z_2 K^\alpha - P - B + L_1^* + \beta W^o(B_1^*)$$

$$\geq (1 - \eta)z_1 K^\alpha - P - B + L_1^* + \beta W^o(B_1^*). \text{ Q.E.D.}$$

Proposition 3. A firm with debt overhangs under-invests relative to the first best.

Proof: A firm with debt overhangs has two choices: either repays under both shocks or defaults under the low shock, but repays under the high shock. We show that the firm will under-invest relative to the first best under either choice.

Choice 1: To repay debt under both shocks, the firm has to adjust investment away from the first best to guarantee the non-negative dividend (NND) condition under z_L . That is, the firm has to increase the profit under z_L , only by lowering investment from K_{fb} .

Choice 2: We first show that K_{fb} is dominated by some investment level smaller than the first best, and then show that K_{fb} dominates any investment higher than the first best.

(1) Underinvestment: the first best investment K_{fb} is dominated by some K_a , smaller

than K_{fb} , i.e., $K_a = K_{fb} - \varepsilon < K_{fb}$. We need to show that the expected welfare under K_a , denoted by $A(B, K_a, P_a)$ is higher than that under K_{fb} , denoted by $A(B, K_{fb}, P_{fb})$, where the expected welfare is defined as $A(B, K, P) = E(\max\{W^c(B, z, K, P), \eta z K^\alpha\})$.

Assume that the short contracts are designed according to defaulting only for the low shock. We can find an $\varepsilon_0 > 0$ such that choosing $K_a > K_{fb} - \varepsilon_0$ increases short profits under both shocks and the expected profit relative to choosing K_{fb} , where ε_0 is defined

as $\varepsilon_0 = K_{fb} - \left(\alpha(\pi_H z_H + \eta \pi_L z_L) / (1+r)\right)^{1/(1-\alpha)} > 0$. Consider two cases when the firm chooses K_a : defaults under z_L and repays under both shocks.

(a) When choosing K_a , the firm defaults under z_L .

In this case, we know $A(B, K_a, P_a) = \pi_H W^c(B, z_H, K_a, P_a) + \eta \pi_L z_L K_a^\alpha$

and $A(B, K_{fb}, P_{fb}) = \pi_H W^c(B, z_H, K_{fb}, P_{fb}) + \eta \pi_L z_L K_{fb}^\alpha$. Assume that $(L^*(K_{fb}), B'^*(K_{fb}))$ is the optimal long contract of $W^c(B, z_H, K_{fb}, P_{fb})$. Since the profit under K_a is higher than

that under K_{fb} when z_H occurs, $L^*(K_{fb})$ is feasible under $W^c(B, z_H, K_a, P_a)$. Thus, we have

$$W^c(B, z_H, K_a, P_a) \geq z_H K_a^\alpha - (1+r)K_a / \pi_H - B + L^*(K_{fb}) + \beta \delta W^o(B'^*(K_{fb})).$$

To show $A(B, K_a, P_a) > A(B, K_{fb}, P_{fb})$, it is suffice to show

$$(\pi_H z_H + \pi_L \eta z_L) K_a^\alpha - (1+r)K_a > K_{fb}^\alpha - (\pi_H z_H + \pi_L \eta z_L)(1+r)K_{fb}, \quad (1)$$

which is guaranteed by our choice of K_a .

(b) When choosing K_a , the firm repays for both shocks.

$$\text{In this case, we have } A(B, K_a, P_a) = \pi_H W^c(B, z_H, K_a, P_a) + \pi_L W^c(B, z_L, K_a, P_a).$$

We know that $A(B, K_a, P_a) \geq \pi_H W^c(B, z_H, K_a, P_a) + \pi_L \eta z_L K_a^\alpha$ because the firm chooses to repay under the low shock. As in case (a), we have

$$\begin{aligned} W^c(B, z_H, K_a, P_a) &\geq z_H K_a^\alpha - (1+r)K_a - B + L^*(K_{fb}) + \beta \delta W^o(B'^*(K_{fb})) \\ &\geq z_H K_a^\alpha - (1+r)K_a / \pi_H - B + L^*(K_{fb}) + \beta \delta W^o(B'^*(K_{fb})). \end{aligned}$$

And $A(B, K_a, P_a) > A(B, K_{fb}, P_{fb})$ is true for our choice of K_a .

(2) No overinvestment: any investment greater than the first best K_{fb} is dominated by K_{fb} .

The profits under both shocks and also the expected profit will decrease when the firm over-invests. Follows the similar argument above, we could show that any investment greater than the first best K_{fb} is dominated by K_{fb} . Q.E.D.

Appendix 2: Computational Algorithm

When solving the model numerically, we follow the following computation algorithm:

(1) Given the world interest rate R , we start the initial guess of long and short contracts as $P = (1+r)K$ and $B' = (1+r)L$.

(2) Given the long and short contracts, value functions and decision rules are solved through the value function iterations:

$\{W(B, z_{-1}), W^o(B, K, P, z), W^c(B, K, P, z), V^d(K, z),$
 $K(B, z_{-1}), P(B, z_{-1}), L(B, K, P, z), B'(B, K, P, z), d(B, K, P, z), A(B, K, P), D(B, K, P)\}$.

- Start with an initial guess of the value function $W_j(B, z_{-1})$.
- Compute value functions $V^d(K, z)$ and $W^c(B, K, P, z)$ with optimal long contract $(L(B, K, P, z), B'(B, K, P, z))$. Set $W^c(B, K, P, z) = -\infty$ if the set of the feasible allocation is empty.
- Compare $V^d(K, z)$ with $W^c(B, K, P, z)$ and get the default decision $d(B, K, P, z)$. Thus, we get the repayment set $A(B, K, P)$, the default set $D(B, K, P)$ and the value function $W^o(B, K, P, z)$.
- Update the value function $W_{j+1}(B, z_{-1})$ by choosing the optimal short contract $(K(B, z_{-1}), P(B, z_{-1}))$.
- Iterate the above procedures until W converges.

(3) Given the decision rules and value functions, we can updated the long and short contracts according to

$$K = \frac{P}{(1+r)} \left(1 - \int_{D(B, K, P)} dG(z, z_{-1}) \right) \text{ and } L = \frac{B'}{1+r} \left(1 - \int_{D(B', K'(B', z), P'(B', z))} dG(z', z) \right).$$

- (4) Iterate (1)-(3) until the short and long contracts converges.

Appendix 3

1. Leverage Robustness Check: different years

	Table 2. Ecuador Leverage			Table 2. UK Leverage		
	1997	1998	1999	2001	2002	2003
Overall	0.60	0.61	0.64	0.84	0.85	0.85
Quintile 1	0.51	0.52	0.56	1.10	1.10	1.10

Quintile 2	0.58	0.59	0.61	0.89	0.89	0.89
Quintile 3	0.62	0.63	0.64	0.77	0.78	0.80
Quintile 4	0.64	0.66	0.68	0.73	0.72	0.72
Quintile 5	0.66	0.67	0.72	0.73	0.73	0.74

2. Leverage Robustness Check: sectors

	Table 2. Ecuador (1996)			Table 2. UK (200)		
	Manufacturing	Services	Trade	Manufacturing	Services	Trade
Overall	0.59	0.50	0.59	0.79	0.88	0.89
Quintile 1	0.59	0.38	0.43	1.40	1.26	1.10
Quintile 2	0.60	0.46	0.66	0.90	0.89	0.89
Quintile 3	0.59	0.50	0.59	0.74	0.78	0.80
Quintile 4	0.59	0.51	0.63	0.70	0.72	0.72
Quintile 5	0.59	0.64	0.63	0.71	0.74	0.74

3. Leverage Robustness Check: number of bins

	Ecuador (1996)	UK (2001)
Overall	0.57	0.76
Quantile 1	0.34	1.06
Quantile 2	0.47	0.95
Quantile 3	0.55	0.86
Quantile 4	0.54	0.79

Quantile 5	0.59	0.73
Quantile 6	0.60	0.69
Quantile 7	0.62	0.65
Quantile 8	0.66	0.63
Quantile 9	0.68	0.60
Quantile 10	0.67	0.59

4. Leverage Robustness Check: sale bins

	Ecuador (1996)	UK (2001)
Overall	0.60	0.83
Quintile 1	0.46	0.86
Quintile 2	0.59	0.89
Quintile 3	0.64	0.85
Quintile 4	0.67	0.81
Quintile 5	0.66	0.73