Monetary policy with Heterogeneous Agents and Borrowing constraints*

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Abstract

This paper revisits the macroeconomic effects of monetary policies in the neoclassical growth model, when incomplete markets and idiosyncratic income shocks are introduced. In the benchmark complete markets model without borrowing constraints, a permanent increase in money growth rate has no long-run effect on capital accumulation and output, but only on money demand. This result no longer holds once we allow for incomplete markets and borrowing constraints. First, we show theoretically that inflation has a long-run real effect by affecting the demand for real balances and capital differently between constrained and unconstrained agents. This heterogeneity leads to a new precautionary savings motive. Second we quantify the importance of this new channel in an incomplete markets model which closely matches the US wealth distribution. Inflation turns out to have a non-trivial positive impact on capital accumulation and output compared to the complete markets set-up.

JEL: E2, E5

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1 Introduction

The long-run relation between inflation, capital accumulation and growth, is one of the most celebrated issue in modern macroeconomics. This tradition dates back at least to the classic monetary neutrality result of Sidrauski (1967), who showed that money has no long-run effect on capital accumulation and output in the neoclassical growth model. Recent studies have challenged this neutrality result by taking into account frictions such as distortionary capital taxes (Phelps, 1973 and Chari et al., 1996 among others) or search frictions (Shi, 1999 for instance), and by looking at redistributive issues of the seigniorage rents across households (Grandmont and Younès, 1973; and Kehoe et al., 1992) or across generations (Weiss 1980 ; Weil, 1991). But these studies have maintained the convenient assumption of the absence of borrowing constraints, and thus have abstracted from the possibility that saving decisions in capital and money, following changes in monetary policy, could depend on the extent of financial market imperfections.

The aim of this paper is to contribute towards filling this gap. The focus on incomplete markets and borrowing constraint is motivated by two main considerations. The first reason is that this framework offers a straightforward mechanism under which the long-run neutrality of money is challenged. If households can save both in money and capital to partially self-insure against idiosyncratic shocks, they substitute away money for capital when inflation rises and the real return on money falls.[1] But in presence of financial market imperfections, borrowing-constrained households are not able to undertake such portfolio adjustment and thus adjust their money holdings differently compared to unconstrained households. Inflation thus triggers endogenous heterogeneity in money holdings in presence of borrowing constraints, providing incentives for unconstrained households with positive income shocks to increase their precautionary savings. Inflation could thus affect the aggregate stock of capital and output in the long-run. The second reason is that this friction is empirically relevant. The tightness of borrowing constraints is a well-established empirical fact (Jappelli 1990, Budria Rodriguez et *al.*, 2002), and it is thus important to understand how they could interact with monetary policy.

To investigate this effect, we model capital market imperfections in a production economy in which ex-ante identical infinitely-lived agents face idiosyncratic income shocks. They can accumulate interest-bearing financial assets in the form of capital to partially insure against income risks, but they face borrowing constraints. In this framework we embed money in the utility function (MIU). Money is valued both for its liquidity service and as a store of value

[1] The effect of inflation on the portfolio allocation was first suggested by Tobin (1965). But the borrowing constraints channel through which portfolio composition affects the real economy is absent from Tobin's analysis.

which provides additional insurance against labor-market risks. Obviously, assuming money in the utility function is a reduced form to provide motives for money demand. This modelling choice is less microfounded than the one proposed in the search literature (Lagos and Wright, 2005 among many others). But it has the key advantage to introduce only simple departures, namely incomplete market and borrowing constraints, from the textbook MIU model in which money is neutral absent frictions.

The first contribution of this paper is theoretical. To the best of our knowledge, we provide a new channel for the non-neutrality of money on capital accumulation and output only due to borrowing constraints. In an economy with deterministic income shocks à la Woodford (1990), we show that inflation has a long-run effect as long as borrowing constraints are binding. Inflation affects the demand for real balances differently for constrained and unconstrained households. This leads to higher precautionary savings and consequent increase in capital accumulation and output. This real effect occurs even in the absence of the other potential channels which have been proposed in the existing literature, such as tax distortions or leisure-labor supply distortions. Importantly, this non-neutrality result shows up even when we shut down the redistributive effect of the seigniorage rent which could provide insurance against idiosyncratic risks and thus have real effect, as in Kehoe et al. (1992) or Molico and Zhang (2006). Our non-neutrality result only stems from the heterogeneity in household portfolio adjustment due to borrowing constraints.

The second contribution of this paper is to provide a quantitative evaluation of the role of incomplete markets on the real effect of inflation in the standard neoclassical monetary growth model with money in the utility function, augmented with uninsurable idiosyncratic risks à la Aiyagari (1994). The analysis is carried on in an economy in which the wealth distribution and the fraction of borrowing-constrained households closely resemble that of the United States. We first gauge the specific quantitative role played by borrowing constraints and incomplete markets by eliminating all other frictions. Next, we quantify the potential interactions between incomplete markets and borrowing constraints on the one hand and the other principal traditional distortions put forward in the existing literature. Specifically, we disentangle the quantitative real effects of inflation transiting through i) the non-neutral redistribution of the seigniorage rent across households, ii) the distorting effect on the capital tax, and eventually iii) the distorting effect on labor supply. We evaluate the contribution of incomplete markets to these real effects of inflation by comparing the outcomes with those obtained in the corresponding complete market economies.

The deviation from the complete market paradigm turns out to be significant not only qualitatively, but also quantitatively. Following a ten point rise in inflation from 0% to 10%, borrowing constraints per se account for a rise in quarterly capital stock by 1.05 percent. This result is obtained when we eliminate all the traditional channels through which inflation might be expected to have a long-run real effect. Namely, this framework abstracts from potential redistributive effects of the seigniorage rent, distorting tax on capital, and adjustment of labor supply (by assuming exogenous hours). Note that in this set-up, inflation has no long-run real effect at all on capital and output with complete markets.

We then show that borrowing constraints amplify the traditional other frictions stressed in the literature. First, with respect to distortionary taxes on capital, it has long been recognized that the seigniorage rent could alleviate capital taxes and induce greater capital accumulation. Yet, this so-called Phelps effect (Phelps, 1973; Chari *et al.* 1996) is quantitatively much larger in an incomplete market set-up, since the presence of borrowing constraints gives rise to precautionary savings motives. A ten point rise in the inflation rate triggers an increase in aggregate capital by 1.93 percent in the incomplete market set-up, against 0.39 percent in the complete market economy. Regarding the distorting effect of inflation on labor supply, we show that this channel matters quantitatively much more with incomplete markets: inflation triggers greater precautionary saving in the presence of borrowing constraints, which leads in turn to a rise in labor productivity and the incentives to work. The capital increases by 3.28 percent, the impact being nearly four times as high under incomplete markets compared to complete markets economies.

We shall stress that a property of our framework is to lead to a positive effect of inflation on capital accumulation. Naturally, other frictions such as search frictions (Aruoba et al., 2006) or cash-in advance frictions could lead to a negative effect of inflation by hampering transactions, but looking at all the other frictions is beyond the scope of our paper. Yet the fact to identify contradictory channels for inflation (see Shi (1999) for similar positive effect of inflation on capital in the monetary search literature) might explain why the empirical evidence on the long-run effect of inflation on capital accumulation are mixed, at best. Barro (1995) showed in a cross-country analysis that inflation is negatively correlated with investment in the long-run, but only when high inflation episodes were included in the sample. More recent empirical evidence shows that for low-inflation countries, a small increase in money growth has actually a long-run positive effect on the capital stock (Kahn et al., 2006, Loayza, et al., 2000) and output (Bullard and Keating, 1995).

To the best of our knowledge, this paper is the first one to study the real macroeconomic effects of monetary policy stemming from incomplete markets and borrowing constraints. The most closely related paper is that of Erosa and Ventura (2002) on the distributional effect of inflation in a neo-classical model with incomplete markets. But their analysis was focused on the welfare effect of long-run inflation. And the real effect of inflation hinged on a specific transactions technology. The authors thus did not look at the specific contribution of incomplete markets and borrowing constraints to understand the real effect of money on capital accumulation and output. The novelty of our paper is to study the specific implications of such financial imperfections on macroeconomic activity, letting aside the normative welfare analysis done by Erosa and Ventura (2002). In the tradition of Bewley models (1980, 1983), various papers have also revisited the welfare effect of inflation under incomplete markets in context of endowment economies (Kehoe et al., 1992, Imrohoroglu, 1992, Akyol, 2004). But these analysis did not stress the specific contribution of incomplete markets and borrowing constraint to the real effect of inflation on capital accumulation and output.

Our paper is organized as follows. Section 2 first provides a simple model with deterministic individual shocks to show analytically the non-neutrality of money transiting only through borrowing constraints. Section 3 lays out the full model with stochastic uninsurable individual shocks. Section 4 quantifies the real effect of inflation and reports sensitivity analysis.

2 A Simple Model

2.1 The model

In this section, we provide a theoretical model to show that inflation is no longer neutral in a production economy with binding borrowing constraints. To obtain closed-form solutions, we set out a simplified version of the fully-fledged model used in the next quantitative section. The model draws upon a standard heterogeneous agent production economy à la Aiyagari (1994) in which agents face individual income fluctuations and borrowing constraints. But we make the key assumption that households alternate *deterministically* between the different labor market states. This liquidity-constrained model has been used, for instance, by Woodford (1990) to study the effect of public debt and by Kehoe and Levine (2001) to characterize the equilibrium interest rate. We extend this framework to monetary policy issues by taking account of the value of money in the utility function[2]. We show analytically that Sidrauski's neutrality result no longer holds when borrowing constraints are binding in this framework. Inflation affects the long-run interest rate, even when seigniorage revenue is redistributed in the most neutral way, and regardless of any other potential frictions.

[2] In the literature, both the money-in-the-utility (MIU) function hypothesis and cash-in-advance (CIA) assumption are used. We use the first hypothesis because on the theoretical side, MIU appears more general and flexible than CIA. In particular, the result of the simple model holds with CIA. On the quantitative side, we will calibrate the model on the quarterly basis and it seems too extreme to assume that households must choose their money holdings one quarter in advance. This last effect is all the more undesirable as individual income will be fluctuating.

Preferences and technology

Households are infinitely-lived and have identical preferences[3]. Each household can be in two states, H or L. In state H (resp. L), households have a high labor endowment e^H (resp. e^L). For the sake of simplicity we assume that $e^H = 1$ and $e^L = 0$. Households alternate deterministically between state H and L at each period. At the initial date, there is a unit mass of the two household types. Type 1 households are in state H at date 1, type 2 households are in state L at date 1. Consequently, type 1 (resp. 2) households are in state H (resp. L) every odd period and in state L (resp. H) every even period. Type i ($i = 1, 2$) households seek to maximize an infinite-horizon utility function over consumption c^i and real money balances m^i which provide liquidity services. The period utility function u of these households is assumed to have the simple form

$$u\left(c_t^i, m_t^i\right) = \phi \ln c_t^i + (1-\phi) \ln m_t^i$$

where $1 > \phi > 0$ weights the marginal utility of consumption and money. For the sake of simplicity we use a log-linear utility function in this section. The result holds for general utility functions as shown in a technical appendix of the paper available upon request.

At each period $t \geq 1$, a type i household can use her revenue for three different purposes. She can first buy an amount c_t^i of final goods. We denote by P_t the price of the final good in period t, and Π_{t+1} is the gross inflation rate between period t and period $t+1$, that is $\Pi_{t+1} = P_{t+1}/P_t$. She also saves an amount a_{t+1}^i of financial assets yielding a return of $(1 + r_{t+1})\, a_{t+1}^i$ in period $t+1$, where $1+r_{t+1}$ is the gross real interest rate between period t and period $t+1$. A borrowing constraint is introduced in its simplest form, in that we assume that no household is able to borrow: $a_{t+1}^i \geq 0$. Finally, type i household buys a nominal quantity of money M_t^i, which corresponds to a level of real balances $m_t^i = M_t^i/P_t$. This yields revenue m_t^i/Π_{t+1} in period $t+1$. In addition to labor income and to the return on her assets, each household receives by helicopter drop a monetary transfer from the State, denoted μ_t^i in nominal terms.

The problem of the type i household, $i = 1, 2$, is given by

$$\max_{\{c_t^i, m_t^i, a_{t+1}^i\}_{t=1..\infty}} \sum_{t=1}^{\infty} \beta^t u\left(c_t^i, m_t^i\right) \text{ with } 0 < \beta < 1 \tag{1}$$

$$\text{s.t.} \quad c_t^i + m_t^i + a_{t+1}^i = (1 + r_t)\, a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i + \frac{\mu_t^i}{P_t} \qquad \text{with} \qquad a_t^i, c_t^i, m_t^i \geq 0 \tag{2}$$

[3] These assumptions are key in cancelling out the potential real effects of inflation stemming from the OLG structure or from exogenous heterogeneity in preferences. Significantly, we do not use Kiotaky and Moore's (1997) assumption of different discount factors which ensure that credit constraints are binding in equilibrium in this kind of model. However, we will establish sufficient conditions under which credit constraints are binding in our simple framework with identical preferences.

where β stands for the discount factor, a_1^i and $M_0^i = P_0 m_0^i$ are given, and a_t^i and m_t^i are subject to the standard transversality conditions.

The production function of the representative firm has a simple Cobb-Douglas form $K^\alpha L^{1-\alpha}$ where L stands for total labor supply and K is the amount of total capital which fully depreciates in production. Profit maximization is given by $\max_{K_t, L_t} F(K_t, L_t) - (1 + r_t) K_t - w_t L_t$, and yields the standard first-order conditions

$$1 + r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}, \;\; w_t = (1 - \alpha) K_t^\alpha L_t^{-\alpha} \tag{3}$$

In period $t \geq 1$, financial market equilibrium is given by $K_{t+1} = a_{t+1}^1 + a_{t+1}^2$. Labor market equilibrium is $L_t = e_t^1 + e_t^2 = 1$. Goods market equilibrium implies $F(K_t, L_t) = K_{t+1} + c_t^1 + c_t^2$.

Monetary policy with neutral redistribution

Let $\bar{M}_t$ denotes the nominal quantity of money in circulation and $\Omega_t = \bar{M}_t / P_t$ the real quantity of money in circulation at the end of period t. Money market equilibrium implies $m_t^1 + m_t^2 = \Omega_t$ in real terms and $M_t^1 + M_t^2 = \bar{M}_t$ in nominal terms.

Let π denotes the growth rate of money. Monetary authorities provide a new nominal quantity of money in period t, which is proportional to the nominal quantity of money in circulation at the end of period $t - 1$. As a result, $\mu_t^1 + \mu_t^2 = \pi \bar{M}_{t-1}$, where the initial nominal quantity of money, $\bar{M}_0 = M_0^1 + M_0^2$, is given. The law of motion of the nominal quantity of money is thus

$$\bar{M}_t = (1 + \pi) \bar{M}_{t-1} \tag{4}$$

In order to focus on the specific role of borrowing constraints on the non-neutrality of inflation, it is assumed that monetary authorities follow the "most" neutral rule, which is to distribute by lump-sum transfer the exact amount of resources paid by private agents due to the inflation tax. Obviously this assumption is unrealistic and its only aim is to stress the specific role of borrowing constraints regardless of any redistributive effects. As a consequence, new money is distributed proportionally to the level of beginning-of-period money balances. In period t, type i agents have a beginning-of-period quantity of money M_{t-1}^i. Hence, we assume that $\mu_t^i = \pi M_{t-1}^i$, and the real transfer is

$$\frac{\mu_t^i}{P_t} = \frac{\pi}{\Pi_t} m_{t-1}^i \tag{5}$$

2.2 Stationary Equilibrium

Given the initial conditions a_1^1, a_1^2, M_0^1, and M_1^2, and given π, an equilibrium in this economy is a sequence $\{c_t^1, c_t^2, m_t^1, m_t^2, a_{t+1}^1, a_{t+1}^2, P_t, r_t, w_t\}_{t=1\ldots\infty}$ which satisfies the households' problem

(1), the first-order condition of the firms' problem (3), and the different market equilibria. More precisely, we focus on symmetric stationary equilibria[4], where all real variables are constant, and where all agents in each state H and L, denoted H and L households, have the same consumption and savings levels. The variables describing agents in state H will be denoted m^H, c^H, a^H, and those for in state L will be described by m^L, c^L, a^L. As a consequence, since the real quantity of money in circulation $\Omega = \bar{M}_t/P_t$ is constant in a stationary equilibrium, equation (4) implies that the price of the final good grows at rate π, and hence $\Pi = 1 + \pi$.

Note that under our assumption of a neutral redistributive monetary policy, we can use the budget constraint (2), and the amount μ_t^i/P_t given by (5), to obtain the budget constraints of H and L households at the stationary equilibrium

$$\text{H households} \quad : \quad c^H + m^H + a^H = (1+r)\, a^L + m^L + w \tag{6}$$

$$\text{L households} \quad : \quad c^L + m^L + a^L = (1+r)\, a^H + m^H \tag{7}$$

The inflation rate does not appear in these equations since the creation of new money does not imply any transfer between the two types of households. The redistributive effects of the seigniorage rent analyzed for instance by Kehoe et al. (1992) are cancelled out.

Using standard dynamic programming arguments, the households' problem can be solved easily. This is done in Appendix A.

For H households, we have the following optimality conditions

$$u_c'\left(c^H, m^H\right) = \beta\,(1+r)\, u_c'\left(c^L, m^L\right) \tag{8}$$

$$u_c'\left(c^H, m^H\right) - u_m'\left(c^H, m^H\right) = \frac{\beta}{\Pi} u_c'\left(c^L, m^L\right) \tag{9}$$

Equation (8) is the Euler equation for H households, who can smooth their utility thanks to positive savings. H households are high-income and are never borrowing constrained. The second equation is the arbitrage equation, which determines the demand for real money balances. H households equate the marginal cost of holding money in the current period, (i.e. the left-hand side of equation 9), to the marginal gain of transferring one unit of money to the following period when they are in state L, (i.e. the right-hand side of equation 9). The marginal utility of money shows up here as a decrease in the opportunity cost of holding money. And the gain from money holdings takes into account the real return $1/\Pi$ of cash.

The solution of the program of L households depends on whether borrowing constraints are

[4]In liquidity-constraint models, the path of the economy converges toward a steady state, or even begins at a steady state if a period 1 transfer is made to households consistently with steady state values (Kehoe and Levine, 2001)

binding or not. If borrowing constraints are binding, the solution is $a^L = 0$ and

$$u_c'\left(c^L, m^L\right) > \beta\left(1+r\right)u_c'\left(c^H, m^H\right) \quad (10)$$

$$u_c'\left(c^L, m^L\right) - u_m'\left(c^L, m^L\right) = \frac{\beta}{\Pi}u_c'\left(c^H, m^H\right) \quad (11)$$

The first inequality shows that L households would be better off if they could transfer some income from the next period to the current period. The second equation involves the same trade-off as that for H households discussed above. Finally, if borrowing constraints are not binding for L households, inequality (10) becomes an equality and $a^L > 0$.

Using expression (8) together with condition (10), we find that borrowing constraints are binding if and only if $1+r < 1/\beta$. If borrowing constraints are not binding, equation (8) and the relationship (10) taken with equality imply $1+r = 1/\beta$. The following proposition[5] summarizes this standard result.

Proposition 1 *Borrowing constraints are binding for L households if and only if* $1+r < 1/\beta$. *If borrowing constraints are not binding then* $1+r = 1/\beta$.

When borrowing constraints are binding, the gross real interest rate $1+r$ is lower than the inverse of the discount factor. As a result, there is always capital over-accumulation due to the precautionary saving motive, which is is a standard result in this type of liquidity-constrained model (see Woodford, 1990; Kehoe and Levine, 2001, amongst others). The next section establishes sufficient conditions for borrowing constraints to be binding in this simple framework.

2.3 Monetary Policy with binding borrowing constraints

Perfect financial markets

As a starting point, we present the conditions required to produce Sidrauski's neutrality result in this simple framework. If markets were complete and borrowing constraints were not binding, the Euler equation would hold with equality whatever the state of the labor market. In this case, money demand would be identical across households of types H and L. Using a log utility specification and taking the Euler equation with equality, we can rewrite money demand as follows

[5] Note that $1+r$ cannot be lower than $1/\Pi$, otherwise financial markets cannot clear. As such, an equilibrium with binding credit constraints can exist only if $1/\Pi < 1/\beta$. Moreover, we assume that the surplus left for consumption is positive at the Friedman rule, which implies $\alpha < 1/\Pi$.

$$\frac{m^H}{c^H} = \frac{m^L}{c^L} = \frac{1-\phi}{\phi} \frac{1}{1 - \frac{1}{\Pi} \frac{1}{1+r}} \tag{12}$$

In this case, whatever the current state and the history of the labor market, the ratio of money over consumption is determined only by the preference parameters and the opportunity cost of holding money. To see this, assume that r and π are small, so that $1 - 1/(1+\pi)(1+r) \simeq r - (-\pi)$, which is the difference between the real net return on financial titles and the real net return on money or, in other words, the nominal interest rate.

In this case, inflation has no real effect on savings since households adjust their money demand in exactly the same proportion following a rise in inflation. Inflation does not then bring about any intra-period heterogeneity between household H and L; it therefore has no effect on saving patterns for inter-period smoothing motives, or on the equilibrium interest rate. This is the traditional Sidrauski result regarding the long-run neutrality of money.

Binding borrowing constraints

This long-run neutrality result no longer holds in this simple framework when borrowing constraints are binding.

Since H households are never borrowing-constrained and profit from their good employment state to accumulate a buffer financial stock, their Euler equation always holds with equality. The money demand of H households is therefor still only determined by the opportunity cost of holding money

$$\frac{m^H}{c^H} = \frac{1-\phi}{\phi} \frac{1}{1 - \frac{1}{\Pi} \frac{1}{1+r}} \tag{13}$$

By contrast, the money demand of L households might be affected, depending on whether borrowing constraints are binding since the Euler equation no longer holds with equality. When borrowing constraints are binding, that is when $1+r < \frac{1}{\beta}$, we have the following money demand equation from (8) and (11):

$$\frac{m^L}{c^L} = \frac{1-\phi}{\phi} \frac{1}{1 - \frac{\beta^2}{\Pi}(1+r)} \tag{14}$$

The equilibrium ratio for L households is not simply determined by the opportunity cost of holding money, but by the difference between consumption in the current period and the return on money holdings two periods hence. The ratio $\beta^2(1+r)/\Pi$ is the discounted value of one unit of money held in state L, transferred to state H, and then saved via financial market on to the next period, where the household is in state L again. As this ratio rises, L households increase the ratio of their money holdings over their consumption. L households then increase the relative demand for money as the real interest increases, contrary to H households. The real interest rate appears here as the remuneration of future savings and not as the opportunity cost of holding money. The following proposition summarizes this key property of the model. The proof can be found in the Appendix.

Proposition 2 *If $\alpha < 1/(2+\beta)$, there exists an unique equilibrium with binding borrowing constraints. In such an equilibrium, the real interest rate falls as inflation rises.*[6]

When borrowing constraints are binding, a rise in inflation triggers a heterogeneous response in money demand across households. L households decrease their money holdings m^L proportionately less than do H households, because money is their only available store of value. As a result, H households have more resources since their budget constraint is $c^H + a^H = w + m^L - m^H$. An increase in inflation thus provides an incentive for H households to save more in order to smooth consumption between periods. Thus in this simple framework with binding borrowing constraints, inflation unambiguously favors capital accumulation and output, in line with the traditional result of Tobin (1965).

This simple model has shown that imperfections on financial markets give rise to heterogeneity in money demand, which is at the core of the non-neutrality of inflation. The next section provides a quantitative evaluation of this new channel.

3 The General Model

We now describe a fully-fledged model including more general assumptions about idiosyncratic risks, endogenous labor supply and distorting taxes in order to investigate quantitatively the role of inflation. The economy considered here is based on the traditional heterogeneous agent framework *à la* Aiyagari (1994). However, we embed money in the utility function in this framework. This section presents the most general model. Different specifications of this model will be used in the simulation exercise to disentangle the various channels through which inflation affects the real economy.

3.1 Agents

3.1.1 Households

The economy consists of a unit mass of *ex ante* identical and infinitely-lived households. They maximize expected discounted utility from consumption c, from leisure and real balances $m = \frac{M}{P}$. Labor endowment per period is normalized to 1, working time is l and thus leisure is $1 - l$. For

[6] Under this condition, we have $1+r<1/\beta$. Note that this condition holds for fairly standard parameter values such as $\alpha = 1/3$, $\beta < 1$ and $\Pi > 1$.

the sake of generality, we follow the literature which directly introduces money m in the utility function of private agents to capture its liquidity services. For the benchmark version of the model, we assume that the utility function has a general CES specification, following Chari *et al.* (2000). The utility of agent i is given by:

$$u\left(c_i, m_i, l_i\right) = \frac{1}{1-\sigma}\left[\left(\omega c_i^{\frac{\eta-1}{\eta}} + (1-\omega)\, m_i^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} (1-l_i)^{\psi}\right]^{1-\sigma} \tag{15}$$

where ω is the share parameter, η is the interest elasticity of the demand for real balances, ψ is the weight of leisure and σ is risk aversion.

Individuals are subject to idiosyncratic shocks on their labor productivity e_t. We assume that e_t follows a three-state Markov process over time with $e_t \in E = \left\{e^h, e^m, e^l\right\}$, where e^h stands for high productivity, e^m for medium productivity, and e^l for low productivity. The productivity process follows a 3×3 transition matrix[7] Q. The probability distribution across productivity is represented by a vector $n_t = \{n_t^h, n_t^m, n_t^l\}$: $n_t \geq 0$ and $n_t^h + n_t^m + n_t^l = 1$. Under technical conditions, that we assume to be fulfilled, the transition matrix has a unique vector $n^* = \{n^h, n^m, n^l\}$ such that $n^* = n^* Q$. Hence, n_t converges toward n^* in the long run. n^* is distribution of the population in each state. For instance, n^h is the proportion of the population with high labor productivity. In the general model, there is endogenous labor supply for each productivity level.

Markets are incomplete and no borrowing is allowed. In line with Aiyagari (1994), households can self-insure against employment risks by accumulating a riskless asset a which yields a return r. But they can also accumulate real money assets $m = M/P$, which introduces a new channel compared to the previous heterogeneous agent literature. With the price level of the final good at period t being denoted P_t, the gross inflation rate between period $t-1$ and period t is $\Pi_t = \frac{P_t}{P_{t-1}}$. If a household holds a real amount m_{t-1} of money at the end of period $t-1$, the real value of her money balances at period t is $\frac{m_{t-1}}{\Pi_t}$. As long as $\Pi_t > \frac{1}{1+r_t}$, money is a strictly-dominated asset, but which will nonetheless be in demand for its liquidity services. Households are not allowed to borrow and cannot issue any money.

The budget constraint of household i at period t is given by:

$$c_t^i + m_t^i + a_{t+1}^i = (1+r_t)\, a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i l_t^i \qquad t = 0, 1, .. \tag{16}$$

[7] This assumption is based on Domeij and Heathcote (2004), who found that at least three employment states are needed to fit crucial empirical features of the employment process and wealth distribution. See the section devoted to the specification of the model parameters.

where $(1+r_0)\,a_0^i$ and m_{-1}^i are given The sequence of constraints on the choice variables is

$$a_{t+1}^i \geq 0,\ 1 \geq l_t^i \geq 0, c_t^i \geq 0,\ m_t^i \geq 0 \quad t = 0, 1... \tag{17}$$

The value r_t is the after-tax return on financial assets, e_t^i is the productivity level of the worker in period t, and w_t is after-tax labor income per efficient unit.

For the sake of realism, we assume that there is a linear tax on private income. The tax rate on capital at period t is denoted χ_t^a and the tax rate on labor is denoted χ_t^w. Letting $\tilde{r}_t$ and $\tilde{w}_t$ denote capital cost and labor cost per efficient unit, the returns for households then satisfy the following relationships

$$\begin{aligned} r_t &= \tilde{r}_t(1-\chi_t^a) \\ w_t &= \tilde{w}_t(1-\chi_t^w) \end{aligned}$$

Let q_t^i denote total wealth in period t

$$q_t^i = (1+r_t)\,a_t^i + \frac{m_{t-1}^i}{\Pi_t}$$

With this definition, the program of agent i can be written in the following recursive form

$$v\left(q_t^i, e_t^i\right) = \max_{\{c_t^i, m_t^i, l_t, a_{t+1}^i,\}} u\left(c_t^i, m_t^i, l_t^i\right) + \beta E\left[v\left(q_{t+1}^i, e_{t+1}^i\right)\right]$$

$$\text{s.t} \quad c_t^i + m_t^i + a_{t+1}^i = q_t^i + w_t e_t l_t^i$$

with the sequence of constraints on the choice variables in (17) and the transition probabilities for labor productivity given by the matrix Q.

Since the effect of inflation on individual behavior depends heavily on whether borrowing constraints are binding, we distinguish two cases.

- *Non-Binding borrowing constraints*

In this case, the first-order conditions of agent i are as follows

$$u_c'\left(c_t^i, m_t^i, l_t^i\right) = \beta\,(1+r_{t+1})\,E\left[v_1'\left(q_{t+1}^i, e_{t+1}^i\right)\right] \tag{18}$$

$$u_c'\left(c_t^i, m_t^i, l_t^i\right) - u_m'\left(c_t^i, m_t^i, l_t^i\right) = \frac{\beta}{\Pi_{t+1}} E\left[v'\left(q_{t+1}^i, e_{t+1}^i\right)\right] \tag{19}$$

$$u_l'\left(c_t^i, m_t^i, l_t^i\right) = -w_t e_t u_c'\left(c_t^i, m_t^i, l_t^i\right) \tag{20}$$

Equation (20) only holds if the solution satisfies $l_t^i \in [0;1]$. Otherwise, l_t^i takes on a corner value, and the solution is given by (18) and (19).

Let γ_{t+1} denote the real cost of money holdings

$$\gamma_{t+1} \equiv 1 - \frac{1}{\Pi_{t+1}} \frac{1}{(1 + r_{t+1})}$$

This indicator measures the opportunity cost of holding money. When the after-tax nominal interest rate r^n_{t+1}, defined by $1 + r^n_{t+1} = \Pi_{t+1} (1 + r_{t+1})$, is small enough, then $\gamma_{t+1} \simeq r^n_{t+1}$. With this notation and the expression of the utility function given in (15) above, the first-order conditions (18) and (19) yield

$$m^i_t = \left(\frac{1 - \omega}{\omega} \frac{1}{\gamma_{t+1}} \right)^\eta c^i_t$$

This equation shows that the money demand of unconstrained households is only affected by the substitution effect, which depends on the opportunity cost of holding money.

- *Binding borrowing constraints*

When the household problem yields a negative value for financial savings, borrowing constraints are binding, $a_{t+1} = 0$, and the first-order condition yields the inequality

$$u'_c \left(c^i_t, m^i_t, l^i_t \right) > \beta (1 + r_{t+1}) E \left[v'_1 \left(q^i_{t+1}, e^i_{t+1} \right) \right]$$

The first-order conditions of the constrained problem are given by

$$\begin{aligned} u'_c \left(c^i_t, m^i_t \right) - u'_m \left(c^i_t, m^i_t \right) &= \frac{1}{\Pi_{t+1}} \beta E \left[v' \left(\frac{m^i_t}{\Pi_{t+1}}, e^i_{t+1} \right) \right] & (21) \\ u'_l \left(c^i_t, m^i_t, l^i_t \right) &= -w_t e_t u'_c \left(c^i_t, m^i_t, l^i_t \right) & (22) \end{aligned}$$

There is no simple expression for money demand in the case of binding constraints. The static trade-off between money demand and consumption demand appears on the left-hand side of (21). Were money not to be a store of value, this expression would be equal to 0. However, as money allows individuals to transfer income to the next period, this introduces an additional motive for holding money.

The right-hand side of equation (21) makes clear that inflation has two opposing effects on the demand for money by borrowing-constrained households. On the one hand, inflation induces a substitution effect which serves to decrease money demand as inflation rises (represented by the term $1/\Pi_{t+1}$); on the other hand, as inflation enters the value function via a revenue effect, there might be an increase in money demand as inflation increases.

The core reason for this result is that money is the only store of value which can be adjusted if households are borrowing-constrained. If the function v is very concave, and for realistic parameter values, this second effect may dominate, and the demand for money can increase with inflation. We will show in the quantitative analysis that this result holds for the poorest agents. As a consequence, this case proves that the change in money demand resulting from inflation, the so-called *Tobin effect*, can be decomposed into a revenue effect and a substitution effect for borrowing-constrained households.

Finally, working hours are determined by equation (22). If the value of l_t from (22) is negative, then $l_t = 0$ and the fist order condition (22) holds with inequality.

The solution of the households' program provides a sequence of functions which yield at each date the policy rules for consumption, financial savings, money balances and leisure as a function of the level of labor productivity and wealth:

$$\left.\begin{array}{r} c_t(.,.) : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ a_{t+1}(.,.) : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ m_t(.,.) : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ l_t(.,.) : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \end{array}\right\} t = 0, 1, ...$$

3.1.2 Firms

We assume that all markets are competitive and that the only good consumed is produced by a representative firm with aggregate Cobb-Douglas technology. Let K_t and L_t stand for aggregate capital and aggregate effective labor used in production respectively. It is assumed that capital depreciates at a constant rate δ and is installed one period ahead of production. Since there is no aggregate uncertainty, aggregate employment and, more generally, aggregate variables are constant at the stationary equilibrium

Output is given by

$$Y_t = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha} \quad 0 < \alpha < 1$$

Effective labor supply is equal to $L_t = L_t^h e^h + L_t^m e^m + L_t^l e^l$, where L_t^h, L_t^m and L_t^l are the aggregate demand for each type of labor. Prices are set competitively:

$$\tilde{w}_t = (1-\alpha)(K_t/L_t)^{\alpha} \tag{23}$$

$$\tilde{r}_t + \delta = \alpha(K_t/L_t)^{\alpha-1} \tag{24}$$

As high, medium and low productivity workers are perfect substitutes with different productivity, we necessarily have

$$\tilde{w}_t^h = e^h \tilde{w}_t, \qquad \tilde{w}_t^m = e^m \tilde{w}_t, \qquad \tilde{w}_t^l = e^l \tilde{w}_t \tag{25}$$

The aggregate demand for capital is given by

$$K_t = L_t(\alpha/(\tilde{r}_t + \delta))^{\frac{1}{1-\alpha}}$$

3.1.3 Government

The government levies taxes to finance a public good, which costs G units of final goods in each period. Taxes are proportional to the revenue of capital and labor, with coefficients χ_t^a and χ_t^w in period t. In addition, the government receives the revenue of the new money created at period t, which is denoted τ_t^{tot} in real terms. It is assumed that the government does not issue any debt. The government budget constraint is given by

$$G = \chi_t^a \tilde{r}_t K_t + \chi_t^w \left(L_t^h e_t^h + L_t^l e_t^l + L_t^m e_t^m \right) \tilde{w}_t + \tau_t^{tot} \tag{26}$$

3.1.4 Monetary Policy

Monetary policy is assumed to follow a simple rule. In each period, the monetary authorities create an amount of new money which is proportional with factor π to the nominal quantity of money in circulation, $P_t\Omega_t = P_{t-1}\Omega_{t-1} + \pi P_{t-1}\Omega_{t-1}$. As is standard in the monetary literature, we assume that the State receives all the revenue from the inflation tax[8], which is a more realistic assumption than the helicopter drops of money. As a result the real quantity of money in circulation at period t is

$$\Omega_t = \frac{\Omega_{t-1}}{\Pi_t} + \pi \frac{\Omega_{t-1}}{\Pi_t} \tag{27}$$

The real value of the inflation tax in period t is

$$\tau_t^{tot} = \pi \frac{\Omega_{t-1}}{\Pi_t} \tag{28}$$

Note that if the real quantity of money in circulation is constant (which is the case in equilibrium), equation (27) implies that $\Pi = 1 + \pi$, and hence $\tau^{tot} = \frac{\pi}{1+\pi}\Omega$, which is the standard expression for the inflation tax.

3.2 Equilibrium

Market Equilibria

Let $\lambda_t : E \times \mathbb{R}^+ \longrightarrow [0,1]$ denote the joint distribution of agents over productivity and wealth. Aggregate consumption C_t, aggregate real money holdings M_t, aggregate effective labor

[8]In practice, the profits of Central Banks are redistributed to the State and are not used for specific purposes.

L_t^s and aggregate financial savings A_{t+1} are respectively given by

$$\begin{aligned} C_t &= \int\int c_t\left(e^k, q\right) d\lambda_t\left(e^k, q\right) \\ M_t &= \int\int m_t\left(e^k, q\right) d\lambda_t\left(e^k, q\right) \\ L_t^s &= e^h \int l_t\left(e^h, q\right) \lambda_t\left(e^h, q\right) dq + e^l \int l_t\left(e^l, q\right) \lambda_t\left(e^l, q\right) dq + e^m \int l_t\left(e^m, q\right) \lambda_t\left(e^m, q\right) dq \\ A_{t+1} &= \int\int a_{t+1}\left(e^k, q\right) d\lambda_t\left(e^k, q\right) \end{aligned}$$

Equilibrium in the final good market implies

$$C_t + K_{t+1} + G_t = Y_t + (1-\delta) K_t \tag{29}$$

Equilibrium in the labor market is

$$L_t = L_t^s$$

Equilibrium in the financial market implies

$$K_{t+1} = A_{t+1} \tag{30}$$

Last, money-market equilibrium is defined by

$$M_t = \Omega_t \tag{31}$$

where Ω_t is the real quantity of money in circulation at period t.

Competitive equilibrium

A stationary competitive equilibrium for this economy consists of constant decision rules $c(e,q)$, $m(e,q), l(e,q)$ and $a(e,q)$ for consumption, real balances, leisure and capital holdings respectively, the steady state joint distribution over wealth and productivity $\lambda(e,q)$, a constant real return on financial assets r, a constant real wage w, the real return on real balances $1/\Pi$, and tax transfers χ^a , χ^w , consistent with the exogenous supply of money π and government public spending G such that

1. The long-run distribution of productivity is given by a constant vector n^*.
2. The functions $a(.,.), c(.,.), m(.,.), l(.,.)$ solve the households' problem
3. The joint distribution λ over productivity and wealth is time invariant.
4. Factor prices are competitively determined by equations (23)-(25).
5. Markets clear: equations (29)-(31).
6. The quantity of money in circulation follows the law of motion (27).

7. The tax rates χ^a and χ^w are constant and are defined to balance the budget of the State (26), where the seigniorage rent from the inflation tax τ^{tot} is given by (28).

Note that equilibrium on the money market and stationary of the joint distribution imply that the real quantity of money in circulation is constant.

3.3 Parameterization

We parameterize the model by using data from the US economy. The first critical point of the parametrization is the choice of the model period to generate a reasonable inflation tax base. Since real balances consist of liquid assets, we choose a model period equals to one quarter rather than one year, consistently with the previous quantitative literature on the inflation tax (see Erosa and Ventura, 2002, or Cooley and Hansen, 1989, among others). The key targets of this parametrization are the wealth distribution, including the share of borrowing-constrained individuals, the individual process of income fluctuations, the interest-elasticity of money demand and the key ratio of M1 over output and capital over output. In what follows we focus on the benchmark incomplete market economy with endogenous prices, proportional taxes, and endogenous labor supply for an inflation rate π of 3 percent, which corresponds to the average inflation rate since the early 1980s.

Technology and preferences

Table 1 shows the parameters for preferences and technology. Parameter values for the utility function (15) are chosen as follows. Estimates of interest elasticity of money since the 1980s yield an average value of η about 0.5 (e.g. Chari et al. 2000, Holman 1998, Hoffman, Rasche and Tieslau 1995). Moreover, for the period 1982-2006, the ratio of M1/GNP averaged about 13.2 percent at the annual level for a average inflation rate of 3 percent. We thus pin down $\omega = 0.988$ to match the corresponding ratio M1/GNP at a quarterly frequency. The weight on leisure ψ is set to reproduce a steady state fraction of labor of 33 percent of total time endowment. The risk aversion is set at a standard value of $\sigma = 1$ as in Chari et al. (2000) baseline case. The parameters relating to the production technology and the capital's share also take on their standard values: α is set equal to 0.36 and the capital depreciation rate is 0.0025. The value of the discounting factor is then set equal to $\beta = 0.99$ so as to reproduce a capital/output ratio of 12 at the quarterly level (Cooley, 1995). Eventually, we set $G = 0.28$ to reproduce a share of G over GDP of 20 percent and an average tax rate on labor and capital $\chi = 0.30$ close to that observed (Domeij and Heathcote, 2004).

Employment Process

Table 1: Benchmark calibration

Parameters	β	α	δ	ω	η	ψ	σ
Values	0.99	0.36	0.025	.988	0.5	2	1

An important aspect of the parameterization is to find a stylized process for wages which is both empirically relevant and able to replicate the US wealth distribution such as the Gini coefficient and the share of people who are borrowing-constrained. We follow the traditional quantitative macroeconomic literature by assuming a first-order autoregressive process for wages or earnings. Various authors have estimated these parameters by using PSID data, and found a coefficient of autocorrelation close to 0.9 and a standard deviation of innovation in the range .12 and .25 (see Card, 1991, Hubbard et al., Heathcote et al., 2005). However the stochastic process of relevance in our framework pertains to wages since hours are endogenous. We thus draw on Floden and Lindé (2001) who estimated, at the annual level, a model with a labor supply choice and thus focused on a process for wage rather than earnings. We use their findings by imposing that the quarterly Markov process can reproduce a coefficient of autocorelation equal to 0.91 and a standard deviation in the innovation term equal to 0.22 at the annual level. [9]

The second issue is to find a process able to match the observed Gini coefficient of wealth and the extent of households who are borrowing constrained. To that respect we follow the current literature (see among others Domjei and Heathcote, 2004) which shows that a Markov chain with three states and nil probabilities to transit between extreme states could make a good job in matching the Gini index. We thus assume a set of employment states represented by $E = \left\{e^h, e^m, e^l\right\}$ where e^h stands for high productivity, e^m for medium productivity, and e^l for low productivity. And we assume that $p_{h,l} = p_{l,h} = 0$. This leaves us with four restrictions to identify the Markov process. Two restrictions are given by the previous autocorrelation process and the standard error of the innovation in the wage process. The two other parameters are chosen so that to reproduce a Gini index for wealth equals to 0.76 and a share of borrowing constraint households equals to 6 percent. The Gini coefficient in wealth is fairly close to the recent findings of Budria Rodriguez et al. (2002); and the associated Gini coefficient in consumption is 0.30, consistent with Krueger and Perri (2005). The empirical measure of the share of borrowing constrained households heavily depends on the choice of the indicator chosen. By using information on the number of borrowing requests which were rejected in the Survey of Consumer Finance (SCF), Jappelli showed that up to 19 percent of families are liquidity

[9]Naturally, these previous estimates are generally based on annual data on the PSID. But we have chosen a model period of one quarter to be consistent with the more liquid nature of money and avoid any overestimation of the inflation tax base. We thus parameterize the endowment process on a quarterly base so that it could reproduce annual data estimations.

constrained. But by using updated SCF data, Budria Rodriguez et al. (2002) reported that only 2.5 percent of household have zero wealth, which might correspond to our theoretical borrowing limit in the model. Obviously this figure does not mean that these households are liquidity-constrained. In particular, Budria Rodriguez et al. (2002) also report that 6 percent of households have delayed their debt repayments for two months or more, which could be used as another proxy for liquidity constraints. To this extent, our measure of 6 percent of liquidity-constrained individuals in our model can be considered as an intermediate value, which prevents us from over-estimating the effect of borrowing constraints on the non-neutrality of inflation. [10] This calibration procedure delivers parameters values that satisfy all four criteria. The implied probability of transitions are $p_{l,l} = p_{h,h} = 0.9750$ and $p_{m,m} = 0.9925$, and the ratio for productivity values are $e_1/e_2 = 4.64$ and $e_2/e_3 = 5.65$.

4 Results

4.1 Individual policy rules

We start by discussing the impact of inflation on individual policy rules in the benchmark economy with endogenous hours and taxes.

Figure 1 illustrates the main policy rules in the benchmark economy with an inflation rate of 3 percent. Consumption, real balances and financial assets are an increasing function of labor productivity and current total wealth q, made up of financial assets and cash. But due to the presence of borrowing constraints, the value functions and the implied policy rules for consumption and money demand are concave at the low values of wealth and productivity. Moreover the policy rule for financial assets held by medium- and low-productivity workers displays kinks at low levels of wealth, indicating that these two types of workers are net-dissavers. By contrast high productivity workers are net-savers in order to smooth consumption across less favorable productivity states.

Figure 2 shows the impact of a one point permanent increase in inflation, from $\pi = 2\%$ to $\pi = 3\%$, on next-period asset holdings and money balances as a function of total beginning of

[10] It is worth noting at this point that our model with capital and real balances yields quite naturally a positive number of people who are liquidity-constrained with the employment process at stake. This result is due to the introduction of real balances in the traditional Aiyagari model. For instance Domeij and Heathcote (2004) and Heathcote (2005) found that no-one is borrowing-constrained for the same kind of employment process. By contrast, introducing money in the utility function naturally entails that wealth-poor people need to carry over real balances into the next period in order to be able to consume. They thus draw down their financial assets to zero to be able to keep a positive amount of real balances when they are affected by negative labor productivity shocks. Note that the previous literature generally uses stochastic discounting factors to fit this dimension (Krusell and Smith, 1998, Carroll, 2000). We do not follow this strategy since the goal of this paper is to look at the specific role of credit constraints and incomplete markets in the non-neutrality of money regardless of any additional heterogeneity, in particular with respect to preferences.

period wealth. The focus is on policy rules around the kink where the main non-linearity lies. We focus on the high productivity state and the low productivity state, as households in the medium state have similar policy rules to low-productivity households. For the high value of productivity, an increase in inflation provides more incentives to save via financial assets at the expense of real money balances whose value has been slashed by inflation. This behavior stands in sharp contrast with that of households in lower productivity states. These households are borrowing-constrained on asset holdings at the low level of total wealth. In this case they have no alternative but to carry over higher level of money balances following a rise in inflation in order to sustain their level of consumption. Money is used as a store of value, and the revenue effect dominates the substitution effect when wealth is low, as explained in the discussion of equation (21). Their level of real-money balances decreases only at the higher level of total wealth for which borrowing constraints on financial assets are no longer binding and households can thus use their capital as a buffer stock. This contrasting effect suggests that the impact of inflation on the real economy and welfare crucially depends on borrowing constraints. Moreover, these policy rules show that wealth-poor households hold a higher fraction of their wealth in real balances compared to wealth-rich households. This endogenous outcome is consistent with the data (see Erosa and Ventura, 2002).

4.2 Aggregate results

This section quantifies the impact of monetary policy on the real economy and welfare. We look at a policy experiment in which the inflation rate varies by one point between $\pi = 2$ percent to $\pi = 3$ percent. Note that the model has been calibrated on a stationary level of 3 percent of inflation. But for the clarity of the exposition, all the results are presented in terms of a rise of inflation from 2 percent to 3 percent. The quantitative theoretical analysis proceeds as follows: we quantify the aggregate impact of inflation depending on different assumptions made regarding the redistribution of the seigniorage rent, the tax structure and the adjustment of labor supply, to be able to disentangle the various effects of inflation in this economy.

First we consider a version of the model in which hours are exogenous and money creation is made by helicopter drops. The new money is redistributed proportionally to the begining-of-period real balances of households, who consider these transfers as lump-sum. We thus abstract from any redistributive and distortionary issues discussed in the previous literature. Consistent with our theoretical results in section 2, this set-up allows us to quantify the non-neutrality of monetary policy which *only* transits through borrowing constraints. This framework is thus

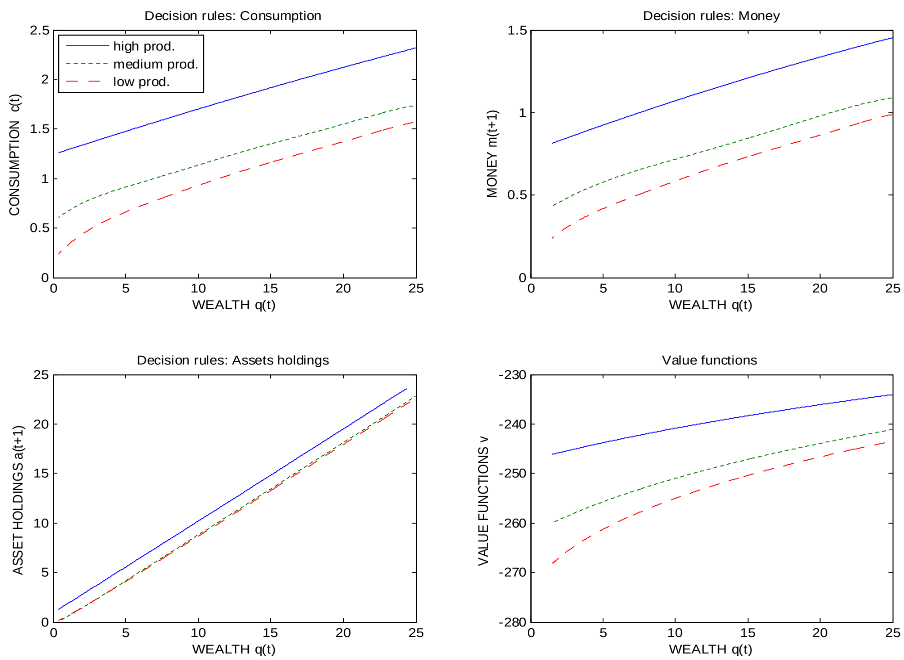

Figure 1: Individual policy rules

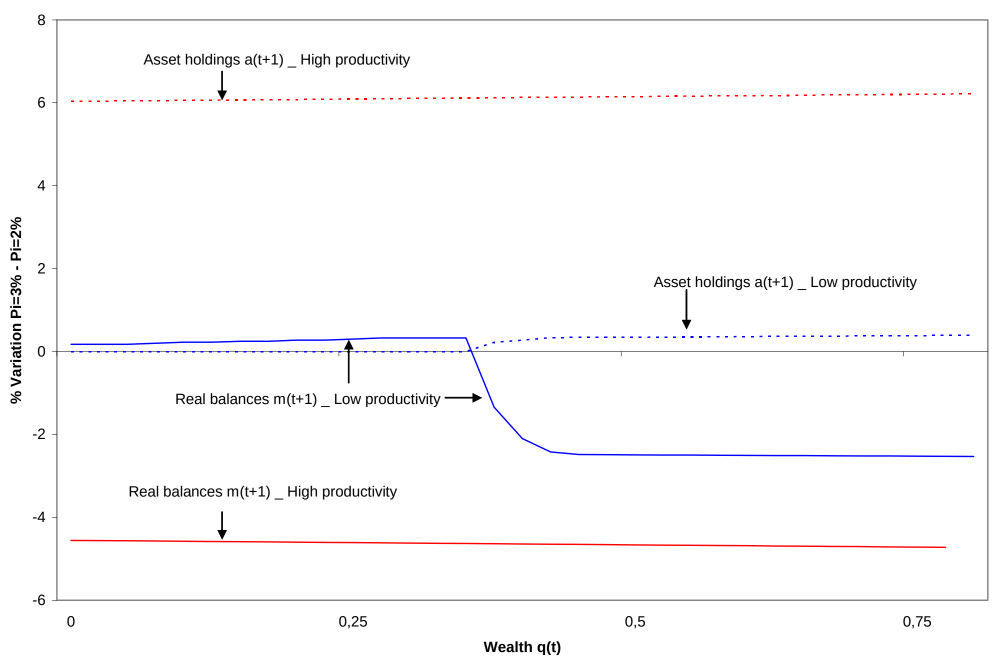

Figure 2: Effect of inflation on individual policy rules

mainly illustrative since the neutrality of money would apply under these assumptions were markets to be complete.

Second, we take into account the traditional redistributive and distortionary effects of inflation which will interact with borrowing constraints. Labor supply is still assumed to be exogenous but there are now proportional taxes on labor and capital income. In this case, borrowing constraints give rise to two new inflation effects. The redistributive effect is due to the seigniorage rent being redistributed unevenly across wealth-poor and wealth-rich agents. The distortionary tax effect is due to the seigniorage rent allowing a reduction in capital taxes and thus increasing incentives to save. This traditional Phelps effect is amplified by the presence of borrowing constraints via precautionary savings motives. We assess the contribution of borrowing constraints to the magnitude of these effects by comparing complete markets and incomplete markets with borrowing constraints.

Third, we extend the model by introducing endogenous labor supply. Due to borrowing constraints, inflation gives rise to heterogeneous labor supply responses depending on the endogenous heterogeneity in wealth. We measure this new effect by comparing the incomplete and complete market set-ups with endogenous hours of work and distortionary taxation.

For each economy, we change the parameters relating to the household productivity process so that each economy matches the same targeted feature of the US wealth distribution. Moreover, we adjust the discounting factor β and the amount of public spending G so that to start from exactly the same initial capital-output ratio, money-output ratio and public spending-output ratio in each economy. The calibration described above refers to the benchmark model with endogenous labor supply.

4.2.1 Lump-Sum Transfers

Environment

In the first stage of the analysis, we define a special case of our model in which the real effect of monetary policy only transits through borrowing constraints, regardless of other potential distortions. To do so we consider the following environment. First, we assume that each household supplies inelastically $l = \bar{l}$ hours of labor. We set $\bar{l} = 0.33$, which corresponds to the steady state value of labor with endogenous labor supply. Second, we assume that there are no taxes on labor and capital, and all (net) transfers are lump-sum. Third, government spending is equal to 0, and the government distributes new money proportionally to the beginning of period level of real balances held by each household. This environment corresponds to the simple model presented in Section 2 but with a more general labor income process.

The budget constraint (16) of household i can be written in this case as

$$c_t^i + m_t^i + a_{t+1}^i = (1 + \tilde{r}_t)\, a_t^i + \tilde{w}_t e_t^i \bar{l} + \frac{m_{t-1}^i}{\Pi_t} + \tau_t^i$$

where τ_t^i stands for the lump-sum transfer of the seigniorage tax, and $\tilde{r}_t$ and $\tilde{w}_t$ are the levels of the interest rate and wages paid by the firm (without tax), defined by equations (23) and (24) respectively.

The seigniorage tax is redistributed *ex-post* to each agent as a lump-sum transfer proportional to the beginning of period money holdings

$$\tau_t^i = \frac{\pi}{\Pi_t} m_{t-1}^i$$

As a consequence, the *ex-post* individual budget constraint is

$$c_t^i + a_{t+1}^i + m_t^i = (1 + \tilde{r}_t)\, a_t^i + \tilde{w}_t e_t^i \bar{l} + m_{t-1}^i$$

Here inflation no longer appears in the individual budget constraint. But since the seigniorage tax is redistributed *ex-post*, the inflation rate is still taken into account by households as the anticipated inflation rate affects the arbitrage conditions to hold money.

Aggregate results

We now consider the aggregate impact of a one-point variation in the inflation rate from $\pi = 2$ percent to 3 percent in this environment. The aggregate results for the economy with neutral lump-sum taxes and exogenous hours are reported in Line 1 of Table 2. We focus on the main aggregate variables: output Y, capital K, real balance M/P , aggregate consumption C and prices w and r. In this table, we give the percentage change in each variable compared to its value with inflation of 2%.

With complete markets and non-distortionary taxes, inflation has no real effect on the stationary values of the real aggregate variables. Each household adjusts its demand for real money and financial asset holdings in the same proportions, leading to a neutral effect of inflation on aggregate consumption, capital and output. The effect of inflation only transits through nominal variables, with the aggregate stock of money decreasing by 12.80 percent

By contrast, when markets are incomplete and households face borrowing constraints, those who are borrowing constrained cannot adjust their money and capital holdings in the same way as unconstrained agents, as illustrated in figure 2. Constrained agents have no other choice than increasing their demand for money to restore their level of real balances and to be able to consume tomorrow. At the other extreme, unconstrained agents increase their level of financial assets, whose returns relative to cash increase with inflation. Consequently, aggregate money demand turns out to decrease less under incomplete markets compared to complete market set-up due to the behavior of borrowing constrained households. Aggregate capital rises by 0.127 percent, leading to an increase in output and consumption of 0.046 percent and 0.009 percent respectively.

4.2.2 Redistributive effects of seigniorage rent

Environment

We now consider the sensitivity of the role played by borrowing constraints and incomplete markets in the non-neutrality of money when we take into account the redistribution of the inflation tax. We thus introduce proportional taxes in line with the benchmark incomplete markets model described in section 3, and compare the results to those from a complete market economy. We still avoid the labor supply channel by assuming that the number of hours is fixed at its stationary level $\bar{l} = 0.33$. In this case, the individual budget constraint and the government budget constraint are respectively

$$c_t^i + m_t^i + a_{t+1}^i = (1 + \tilde{r}_t(1 - \chi_t^a))a_t^i + (1 - \chi_t^w)\tilde{w}_t e_t^i \bar{l} + \frac{m_{t-1}^i}{\Pi_t}$$

and

$$G = \chi_t^a \tilde{r}_t K_t + \chi_t^w \left(n^h e_t^h + n^m e_t^l + n^m e_t^m \right) \bar{l} \tilde{w}_t + \tau_t^{tot}$$

with $\tau_t^{tot} = \pi \frac{\Omega_t}{\Pi_t}$.

Non-distorting redistribution of the inflation tax

We first focus on the redistributive effect of the seigniorage rent. In particular we assume that the seigniorage rent is redistributed proportionally to labor income. To isolate this redistributive effect, we need to cancel out the Phelps effect which works via a reduction in capital tax. We then assume that the distorting proportional tax on capital χ^a is not affected by inflation. This tax is held constant between the two economies and takes on its value for an inflation rate of 2 percent. Yet a variation in the inflation rate from $\pi = 2$ percent to $\pi = 3$ percent triggers an increase in the seigniorage tax τ_t^{tot}, allowing a reduction in the proportional tax on labor χ^w. Thus everything works as if the government was engineering a transfer of the seigniorage rent proportionally to labor income. As we assume in this section that labor supply is exogenous, these transfers are not distortionary.

Line 2 of Table 2 shows that the tax on labor sharply decreases by -0.505 percent due to higher seigniorage rents. However, since the redistribution of the seigniorage rent is proportionally more favorable to high productivity workers, these latter have a greater incentive to save in order to smooth their consumption. The increase in aggregate capital and output (by 0.189 percent and 0.068 percent respectively) is greater compared to the previous environment with neutral redistribution of the seigniorage rent. As a consequence, wages are higher and the labor income of borrowing constrained household is greater in this environment. These households are thus less willing to carry on real balances to smooth consumption, which explains the sharper decline in aggregate real balances compared to the previous environment. As regards the complete market economy, monetary policy remains neutral since labor supply is still assumed to be exogenous and taxes on labor are thus non-distortionary.

4.2.3 Capital taxation distortion

We now discuss the interplay between borrowing constraints and distortionary taxes on capital. We retain the same environment as above with exogenous hours. However, we do take into account the adjustment of capital tax following a rise in inflation. Due to the seigniorage rent, a rise in inflation allows a reduction in the capital tax rate required to balance the government budget constraint. This phenomenon, traditionally known as the Phelps effect, interacts in our framework with the borrowing constraints which amplify incentives to save via the precautionary savings motive. We quantify the contribution of borrowing constraints to this traditional Phelps effect by comparing the incomplete with the complete market set-up.

Line 3 of Table 2 first indicates that the tax on capital decreases by 0.47 percent with incomplete markets. This provides a greater incentive to save. Significantly, the precautionary saving motive due to the existence of borrowing constraints amplifies the rise in aggregate capital and the Phelps effect is much higher under incomplete markets. Due to the Phelps effect, aggregate capital increases by around 0.05 percent (from 0.18 to 0.24 percent, by comparing Line 4 and Line 5) in the incomplete markets set-up whereas it increases by 0.04 percent (from 0 to 0.04 percent) in the complete markets economy. And the Phelps effect leads to a increase in output by 0.02 percent in the incomplete markets economy against 0.014 percent under the complete markets economy.

4.2.4 Endogenous labor supply

We end-up this analysis by taking into account the interplay between borrowing constraints and labor supply. Line 4 of Table 2 compares the benchmark incomplete market economy described in section 3 with a complete market set-up. Note that taxes on labor and capital income are now both distortionary.

The primary channel through which inflation affects labor is by altering the productivity of labor (measured by wages) and thus the marginal rate of substitution between leisure and con-

Table 2: Aggregate impact of inflation

Economies	Percentage change following a rise in inflation $\pi = 2\% \rightarrow 3\%$								
	Y	K	M/P	C	L	χ^w	χ^a	$\tilde{r}$	$\tilde{w}$
	Neutral lump-sum tax - Exogeneous Hours (1)								
Incomplete markets	0.046	0.127	-11.300	0.009	0	0	0	-0.566	0.004
Complete markets	0	0	-12.802	0	0	0	0	0	0
	Non-distorting redistribution - Exogenous hours (2)								
Incomplete markets	0.068	0.189	-12.11	0.019	0	-0.505	0	-0.923	0.072
Complete markets	0	0	-12.81	0	0	-0.437	0	0	0
	Distorting.tax - Exogeneous hours (3)								
Incomplete markets	0.088	0.240	-12.13	0.024	0	-0.472	-0.472	-1.061	0.082
Complete markets	0.014	0.041	-12.81	0.004	0	-0.454	-0.454	-0.188	.0148
	Distorting tax - Endogoneous.hours (4)								
Complete markets	0.182	0.380	-12.73	0.121	0.062	-0.540	-.540	-1.454	0.112
Incomplete markets	0.056	0.085	-12.79	0.063	0.041	-0.467	-0.467	-0.207	0.221

sumption. As aggregate capital increases, the productivity of labor rises and wages increase by 0.11 percent following the rise in inflation rate. This entails a substitution effect in labor supply, which rises by 0.06 percent. Conversely, the rise in labor supply increases capital productivity and provides greater incentives to save. This effect leads to a steady increase in aggregate capital and output by 0.38 percent and 0.18 percent respectively. As a result, the role of incomplete market on the real effect of inflation is sizeable: By combining the previous different channels through which inflation has a real effect, the overall impact is about three times higher in an economy with incomplete markets compared to an economy with complete markets.

4.3 Sensitivity analysis

This section discusses the role of key parameters driving the real effect of inflation on capital accumulation. We carry on this investigation under the benchmark model with distorting taxes and endogenous hours, and adjust the calibration to have comparable steady states with that of the previous section.

We start by looking at the value of the elasticity of substitution between goods and money, which drives the elasticity of money demand towards the interest rate. In particular, we look at the case $\eta = 1$ (the Cobb-Douglas case), corresponding to the value estimated on long-run data (Holman, 1998). To get comparable results, we changed the value of ω from $\omega = 0.988$ to $\omega = 0.982$ to start from the same steady state value for M/P. The first line in Table 3 show that substitution effect away from money is now very large. The real stock of money decreases by 22.25% (compared to 12.73 % in the benchmark case $\eta = 0.5$), while the resources devoted to consumption and financial savings steadily increase. The capital stock increases by 0.64% instead of 0.38%, and consumption almost doubled from 0.12% in the benchmark case to 0.22% with $\eta = 1$. Table 3 also reports the results for the complete markets economy. It turns out that the higher η is, the more important the differences between the complete and the incomplete market economies are.

Second, we investigate the sensitivity of our results to the coefficient of risk aversion which drives the precautionary saving motives. We investigate the effect of an increase of σ from 1 to 2. To get initial steady-states K/Y similar to the previous benchmark economy, we had to decrease the subjective discount factor from $\beta = 0.99$ to $\beta = 0.9858$. The results are reported in the second line in Table 3. The substitution away from money is roughly the same as before (-12.47 instead of -12.73), but as households are more risk adverse, high-income workers self-insure more using financial assets. Hence, the capital stock increases by 0.55% instead of 0.38% with $\sigma = 1$. It is worthwhile to stress that under complete markets, the increase in the capital stock would have been equal to 0.08% (instead of 0.056% with $\sigma = 1$). Thus the differences between complete markets and incomplete markets economies increase when risk aversion increases.

We end-up this analysis by looking at the real effect of inflation for larger inflation differentials. In particular, we look at the traditional experiment in the literature of an increase from 0% to 10% in the inflation rate. This analysis is run for the benchmark calibration to compare our results with Table 2.

The results are reported in Table 4. The contribution of incomplete markets to the real effects of inflation by comparison with complete markets becomes larger. A ten point increase in the inflation rate leads to a rise by 1.052 percent in aggregate capital in the incomplete market environment without any other distortion, while the effect remains nil in the complete markets framework. When considering the benchmark economy with distorting taxes and endogenous hours, a 10 point rise in the inflation rate would increase aggregate capital and output by 0.73 percent and 0.48 percent respectively in the complete markets environment. The effect is around four times larger under incomplete markets, the aggregate capital and output increasing by 3.28 percent and 1.52 percent respectively.

5 Conclusion

This paper has proposed a new theoretical and quantitatively significant channel for the non-neutrality of money which directly transits through borrowing constraints. Higher inflation induces heterogeneity in money demand to the extent that borrowing-constrained households cannot substitute away their real balances for financial assets in the same way as unconstrained

Table 3: Effect of Alternative Assumptions

Economies	Percentage change following a rise in inflation $\pi = 2\% \rightarrow 3\%$								
	Y	K	M/P	C	L	χ^w	χ^a	$\tilde{r}$	$\tilde{w}$
$\eta = 1$ (1)									
Incomplete markets	0.30	0.64	-22.25	0.22	0.12	-0.41	-0.41	-2.02	0.18
Complete markets	0.08	0.09	-24.24	0.10	0.07	-0.30	-0.30	-0.12	0.13
$\sigma = 2$ (1)									
Incomplete markets	0.26	0.55	-12.47	0.19	0.12	-0.56	-0.56	-2.08	0.16
Complete markets	0.05	0.08	-12.95	0.06	0.04	-0.49	-0.49	-0.20	0.22

Table 4: Aggregate impact of inflation: Ten points rise

Economies	Percentage change following a rise in inflation $\pi = 0\% \rightarrow 10\%$								
	Y	K	M/P	C	L	χ^w	χ^a	$\tilde{r}$	$\tilde{w}$
	Neutral lump-sum tax - Exogenous hours (1)								
Incomplete markets	0.37	1.05	-61.99	0.07	0	0	0	-4.60	0.36
Complete markets	0	0	-63.652	0	0	0	0	0	0
	Non-distorting redistribution - Exogeneous hours (2)								
Incomplete markets	0.59	1.66	-66.88	0.18	0	-4.27	0	-7.14	0.59
Complete markets	0	0	-67.615	0	0	-4.13	0	0	0
	Distorting tax - Exogeneous hours (3)								
Incomplete markets	0.69	1.93	-66.88	0.20	0	-4.00	-4.00	-9.52	0.68
Complete markets	0.10	0.39	-53.95	0.03	0	-3.57	-3.57	-1.53	0.23
	Distorting tax - Endogoneous.hours (4)								
Incomplete markets	1.52	3.28	-66.98	1.05	0.54	-4.59	-4.59	-11.76	0.963
Complete markets	0.48	0.73	-67.59	0.54	0.35	-4.17	-4.17	-1.76	0.391

households do. This endogenous heterogeneity across households due to borrowing constraints gives rise to a real effect of inflation on aggregate capital for precautionary savings motives. We have shown that this specific channel has a sizeable quantitative impact in incomplete market economies with an empirically-relevant wealth distribution. Not only do incomplete markets and borrowing constraints have a real quantitative impact on their own, they also amplify significantly the other potential distorting channels of inflation compared to the case in complete market economies.

This paper has focussed on the long-run properties of money with borrowing constraints. A promising route for future research would be to analyze the short-run effects of monetary shocks in this type of incomplete market economy with borrowing constraints. This framework could provide a new relevant channel to account for the persistence and non-neutrality of monetary shocks. Borrowing constraints and household heterogeneity would also offer a useful framework for the analysis of the short-run redistributive effects of monetary policies.

A Solution to the Households' Problem

Using the Bellman equations, the households' problem can be written in recursive form. Stationary solutions satisfy, of course, the usual transversality conditions. As a consequence, we can focus on the first-order condition of the households' problem. This is given by the program

$$
\begin{aligned}
V\left(q_t^i, e_t^i\right) &= \max_{\{c_t^i, m_t^i, a_{t+1}^i\}} u\left(c_t^i, m_t^i\right) + \beta V\left(q_{t+1}^i, e_{t+1}^i\right) \\
c_t^i + m_t^i + a_{t+1}^i &= q_t^i + w_t e_t^i + \frac{\mu_t^i}{P_t} \qquad (32)\\
q_{t+1}^i &= R_{t+1} a_{t+1}^i + \frac{m_t^i}{\Pi_{t+1}} \qquad (33)\\
c_t^i, m_t^i, a_{t+1}^i &\geq 0 \qquad (34)
\end{aligned}
$$

where q_1^1, q_1^2 are given, $R_{t+1} = 1 + r_{t+1}$, and income shocks are deterministic: $e_{t+1}^i = 0$ if $e_t^i = 1$, and $e_{t+1}^i = 1$ if $e_t^i = 0$. Using (32) and (33) to substitute for c_t^i and q_{t+1}^i, we can maximize only over a_{t+1}^i and m_t^i. Using the first-order conditions, together with the envelope theorem (which yields in all cases $V'\left(q_t^i, e_{t+1}^i\right) = u_c'\left(c_t^i, m_t^i\right)$), we have

$$
\begin{aligned}
u_c'\left(c_t^i, m_t^i\right) &= \beta R_{t+1} u_c'\left(c_{t+1}^i, m_{t+1}^i\right) \qquad (35)\\
u_c'\left(c_t^i, m_t^i\right) - u_m'\left(c_t^i, m_t^i\right) &= \frac{\beta}{\Pi_{t+1}} u_c'\left(c_{t+1}^i, m_{t+1}^i\right) \qquad (36)
\end{aligned}
$$

If the equations above yield a quantity $a_{t+1}^i < 0$, then the borrowing constraint is binding and the solution is given by $a_{t+1}^i = 0$ and $u_c'\left(c_t^i, m_t^i\right) > \beta R_{t+1} u_c'\left(c_{t+1}^i, m_{t+1}^i\right)$, together with (36). In a stationary equilibrium, all H agents become L agents the next period and *vice versa*. Since H agents are in the good state, they always take the opportunity to save for precautionary motives and their borrowing constraints are never binding (see next section). We can rewrite the previous equations using the state of the households instead of their type. With the logarithm utility function, this yields the expressions given in section 2.

B Proof of Proposition 2 on binding borrowing constraints and the non-neutrality of money

In this proof, we assume as a first step that borrowing constraints are binding for L households to derive the equilibrium interest rate. In a second step, we check that borrowing constraints are actually binding for L agents but not for H agents. By using proposition 1, it will suffice to check that the equilibrium interest rate satisfies $1 + r < \frac{1}{\beta}$

First, by using the first-order condition (8), we obtain $\frac{c^L}{c^H} = \beta(1 + r)$. Equilibrium on the goods market implies that $c^H + c^L = K^\alpha - K$, and the first-order conditions of the firm imply that $1+r=\alpha K^{\alpha-1}$ and $w=(1-\alpha)K^{\alpha}$. Substituting for c^H, w and K we obtain

$$c^L = \beta \frac{1+r-\alpha}{\beta(1+r)+1}\left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}$$

The budget constraint of L agents, given by (7), yields

$$\frac{m^L}{c^L} - \frac{m^H}{c^H}\frac{c^H}{c^L} = \frac{a^H(1+r)-c^L}{c^L}$$

Using the value of the ratio $\frac{c^L}{c^H}=\beta(1+r)$ and the expressions (13) and (14), one finds

$$f(r) = g(r,\Pi) \tag{37}$$

with

$$f(r) \equiv \frac{\phi}{1-\phi}\left(\alpha\frac{\beta(1+r)+1}{1+r-\alpha}-\beta\right) \text{ and } g(r,\Pi) \equiv \frac{\beta}{1-\frac{\beta^2}{\Pi}(1+r)} - \frac{1}{1+r-\frac{1}{\Pi}}$$

Equation (37) determines the equilibrium interest rate as a function of the parameters of the model and Π. We now have to prove the existence and uniqueness of the equilibrium.

Existence of a solution with binding borrowing constraints

Recall that we assume that $\alpha < 1/\Pi < 1/\beta$. We then look for the existence of a solution r^* such that $1+r^* \in (1/\Pi; 1/\beta)$. If such a solution exists, borrowing constraints are binding and both money and financial titles are held in equilibrium.

Note that $f(r)$ is continuous in r, for $1+r \in (\frac{1}{\Pi};\frac{1}{\beta})$ and f takes finite values at the boundaries $\frac{1}{\Pi}$ and $\frac{1}{\beta}$. For a given value of Π, $g(r,\Pi)$ is continuous in r for $1+r \in (\frac{1}{\Pi};\frac{1}{\beta})$. However, $g(r,\Pi) \longrightarrow -\infty$ when $1+r \in (\frac{1}{\Pi};\frac{1}{\beta})$ and $1+r \longrightarrow 1/\Pi$. And $g\left(\frac{1}{\beta}-1,\Pi\right)=0$. As a result, a sufficient condition for an equilibrium to exist is $f\left(\frac{1}{\beta}-1\right)<0$. This condition is equivalent to $\alpha < 1/(2+\beta)$. Hence, if $\alpha < 1/(2+\beta)$, there exists an equilibrium interest rate r^* such that $1+r^* \in (1/\Pi; 1/\beta)$. From proposition 1, borrowing constraints are binding in such an equilibrium. QED

Uniqueness and variations

Note that $f(r)$ is decreasing in r when $1+r \in (\frac{1}{\Pi};\frac{1}{\beta})$ as $\alpha < 1/\Pi$ (a simple derivative of f). We can show that $g(r,\Pi)$ is increasing in r. As a result, the solution is unique, for continuity reasons. Finally, we can show that $g(r,\Pi)$ is increasing in Π. Define a function h such that

$$h(y) = \frac{y^3(1+r)^3}{\left(1+r-\frac{y^2}{\Pi}(1+r)^2\right)^2} \tag{38}$$

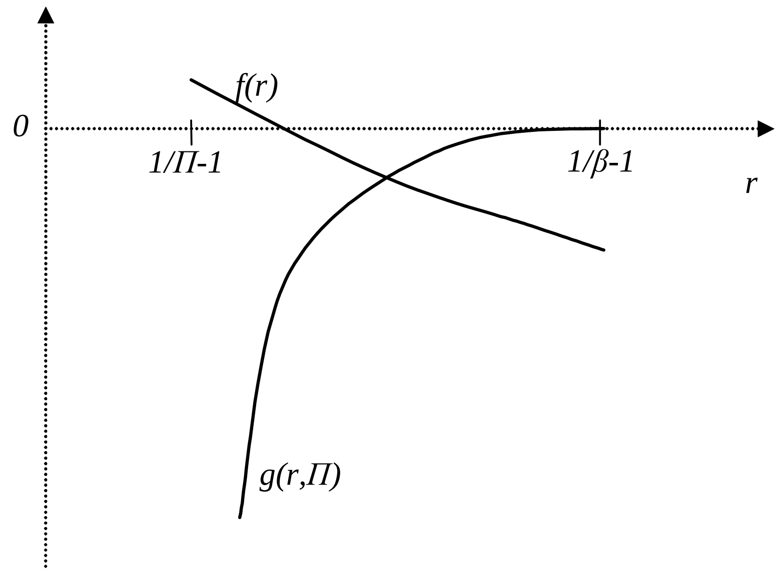

Figure 3: Existence of an Equilibrium with $1{+}r{<}1/\beta$

The function h is positive and increasing in y. Now, the derivative $g'_{\Pi}(r,\Pi)$ can be written as $g'(\Pi) = \frac{1}{\Pi^2}\left(h\left(\frac{1}{1+r}\right) - h(\beta)\right)$. At the equilibrium $1/(1+r^*) > \beta$, and hence we have $g'_{\Pi}(r^*,\Pi) > 0$.

Consequently, by the implicit function theorem $f(r^*) = g(r^*,\Pi)$ defines implicitly r^* as a decreasing function of Π. Figure 3 illustrates the existence and the uniqueness of the equilibrium with binding borrowing constraints. QED

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